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Distinct influences of affective and cognitive factors on children's non-verbal and verbal mathematical abilities

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Abstract

Individual differences in children's math performance have been associated with math anxiety, attention problems, working memory (WM), and reading skills, but the mechanisms by which these factors jointly contribute to children's math achievement are unknown. Here, we use structural equation modeling to characterize the relation between these factors and their influence on non-verbal Numerical Operations (NO) and verbal Math Reasoning (MR) in 330 children (M = 8.34 years). Our findings indicate that WM plays a central role in both non-verbal NO and verbal MR, whereas math anxiety and reading comprehension have unique and more pronounced influences on MR, compared to NO. Our study elucidates how affective and cognitive factors distinctly influence non-verbal and verbal mathematical problem solving.

Keywords

Early math learning; Math anxiety; Working memory; Attention; Reading; Math problem solving

1. Introduction

An extensive body of research has investigated the roles of cognitive factors such as working memory, attention, and processing speed, in mathematics achievement. In contrast, studies examining the role of affective factors, such as math anxiety, are relatively few, and the mechanisms by which affective and cognitive factors collectively influence different components of mathematical reasoning are poorly understood. In particular, it has been suggested that the impact of affective factors such as math anxiety on early math learning

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and achievement are also related to numerous cognitive factors such as attention and working memory (WM; Fuchs, Geary, Fuchs, Compton, & Hamlett, 2015). However, previous studies have predominantly examined the effect of each variable in iso-lation, and mainly in relation to fluency with numerical operations that place little demand on verbally-based math problem solving. Little is known about how these affective and cognitive factors differentially contribute to individual differences in competence in basic non-verbal and verbal math problem solving skills, the latter of which place a greater load on working memory and attention (Bailey, Watts, Littlefield, & Geary, 2014). This question has added significance in light of the introduction of the Common Core curriculum, which places a greater emphasis on applying mathematical knowledge in real world applications. The cognitive and affective demands for these kinds of applications are likely to be different from those invoked in basic numerical fluency and procedural skills.

The overarching goal of the current study was to investigate the distinct, but interrelated, roles of affective and cognitive factors on two components of mathematical problem solving: basic non-verbal computational skills and verbal problem solving. Our specific aims were to investigate the differential roles of (1) math anxiety and WM, (2) attention and WM, and (3) reading achievement and WM in relation to computational skills and word-based problem solving using theoretically-informed structural equation modeling. Here, we first review relevant literature then highlight the open questions in each case. We then describe our use of statistical models to uncover the interrelations among these factors and their impacts on participants' performance on two standardized measures of mathematical achievement: Wechsler Individual Achievement Test, 2nd edition (WIAT-II; Wechsler, 2002) Numerical Operations (NO) for basic non-verbal computational skills and Math Reasoning (MR) for more complex, verbally-based problem solving.

1.1. Aim 1: Differential role of math anxiety and working memory in math achievement

1.1.1. Math anxiety—Math anxiety is defined as a negative emotional reaction to situations involving numerical problem solving (Ashcraft, 2002; Richardson & Suinn, 1972). Previous studies have shown that having stressful emotional and anxious responses to math-related situations and problem solving demands were significantly correlated with disruptions in online math performance and math achievement (Ashcraft, 2002; Maloney, Risko, Ansari, & Fugelsang, 2010). Most importantly, these performance decrements could not be attributed to general trait anxiety (Wu, Barth, Amin, Malcarne, & Menon, 2012). Recent studies using standardized and age-appropriate math achievement measures have shown that math anxiety is negatively correlated with math achievement, even at the earliest stages of math learning (Ramirez, Gunderson, Levine, & Beilock, 2013; Wu et al., 2012)

1.1.2. Working memory—As described by Baddeley and Hitch (1974) and Baddeley (1992, 2003), WM is a cognitive system that facilitates the acquisition of new knowledge and general problem solving by maintaining and storing information from recent past experience. In both children and adults, mathematics skills are highly dependent on WM because of the multiple procedures involved with calculations, such as carrying and storing numerical values and intermediate computational results, and remembering task rules (Geary, Hoard, Byrd-Craven, & Catherine DeSoto, 2004; see Raghubar, Barnes, & Hecht, 2010 for a review). In young children, lower math achievement scores were associated with lower WM capacity as compared to children with average math achievement (Friso-Van Den Bos, Van Der Ven, Kroesbergen, & Van Luit, 2013; Mabbott & Bisanz, 2008).

With respect to math anxiety and WM, studies in young adults have shown that math anxiety interferes with the WM processes that support mathematical computations, thereby resulting in a decrement to performance (Ashcraft & Kirk, 2001; Eysenck & Calvo, 1992). This interaction was clarified in a study by Beilock and Carr (2005) that found a differential impact of pressure on math performance as a function of WM capacity. Unfortunately, the preponderance of research examining math anxiety, WM, and math achievement has relied on samples consisting of young adults, and the limited available research in young children is mixed. Regarding the relation between math anxiety and WM, the only available study from Ramirez et al. (2013) showed that math anxiety had a particularly salient effect on math performance in 1st grade children with high WM capacity.

Taken together, these results suggest a differential influence of math anxiety as a function of math problem complexity (Ashcraft & Krause, 2007) as well as a potential role of WM in mediating the relation between math anxiety and early math achievement. Thus, the first aim of the current study is to investigate the distinct role of math anxiety on early math achievement in the context of other cognitive factors and, to characterize how the mediational effect of WM relates differentially to basic computational skills versus word-based problem solving. We hypothesized that the effect of math anxiety on math performance would be mediated by WM and that the strength of the effects would be different for MR compared to NO.

1.2. Aim 2: Differential role of attention and working memory in math achievement

1.2.1. Attention problems—The most relevant evidence for the role of attention problems in math performance comes from studies on children with behavioral disorders that involve attentional weaknesses such as Attention Deficit/Hyperactivity Disorder (ADHD; Capano, Minden, Chen, Schacher, & Ickowicz, 2008; Wu, Willcutt, Escovar, & Menon, 2014). Attentional problems in children diagnosed with ADHD have been shown to interfere with basic computational processes, such as fact retrieval (Zentall, 1990) as well as with more complex math processes (Raghubar et al., 2009). Several explanations for why attention can impair mathematical performance have been suggested in literature, including (1) a lack of continual focus on and rehearsal of repetitive stimuli, as is necessary when internalizing addition or multiplication facts, and (2) an impaired ability to switch between the various processes required to solve math problems. These preliminary results, although based on studies in clinical populations, suggest that attentional difficulties may also differentially influence performance on basic computational and word-based problems in typically-developing children.

1.2.2. Attention problems and working memory—As previously discussed, studies to date have suggested that both attentional and WM problems can have a detrimental effect on math performance. However, their differential roles relative to one another in basic computational as opposed to word-based problem solving skills are still relatively unclear.

Lucangeli and Cabrele (2006) found that children with attention difficulties who have trouble inhibiting irrelevant information are also more likely to have lower WM scores and severe difficulties in arithmetic word problem solving due to their tendency to remember tangential instead of target information. In a small exploratory study, Kercood and Grskovic (2009) found that providing a method of organizing information helped children with ADHD increase their recall of relevant information and improve their performance on tasks of mathematical computations. These findings provide initial evidence that WM may mediate the relation between attention and math achievement in children. Therefore, our second aim was to specifically probe the extent to which WM mediates the influence of attention problems on math achievement, and whether it has a differential impact as a function of type of mathematical achievement. Because MR likely places more demands on attentional and cognitive systems than NO, we hypothesized that the effect of attention would not be completely mediated by WM in MR but would be in NO.

Although math anxiety was previously found to positively correlate with attention problems (Wu et al., 2014), the distinct roles of these two factors on early math achievement are still unclear (Schatz & Rostain, 2006; Wine, 1971). No study to date has explored how math anxiety specifically relates to attentional problems in the context of different components of math achievement in a non-clinical sample. Here, we address this question with an exploratory analysis examining whether math anxiety mediates the relation between attentional difficulties and math achievement by testing a direct path from attentional problems to math achievement.

1.3. Aim 3: Reading achievement, working memory and their differential contribution to different components of math achievement

Previous research has found a consistent correlation between dimensional measures of math achievement and reading performance on standardized tests (Dunn & Markwardt, 1970; Wechsler, 2002), in school settings (Vilenius-Tuohimaa, Aunola, & Nurmi, 2008), and in a variety of empirical contexts (see Menon, 2015 for a review). Despite this, the importance of reading skills in mathematical proficiency has been largely ignored in numerical cognition, as most studies approach reading achievement as a confound and control for its contributions to math by constructing samples out of participants matched on reading achievement. Other studies have examined math performance only in the context of reading disabilities (De Smedt & Boets, 2010; Evans, Flowers, Napoliello, Olulade, & Eden, 2014). While it is important to understand the domain-specific aspects of numerical cognition (e.g. symbolic and non-symbolic quantity representation and manipulation), it is also important to understand how reading skills relate to mathematical ability, particularly due to the high comorbidity and genetic overlap between math and reading disabilities. It is estimated that 60% of the math disability population also meets criteria for a reading disability (Archibald, Oram Cardy, Joanisse, & Ansari, 2013; Willcutt et al., 2013). Other related research has also reported a differential relation between reading achievement and more reading-dependent math problems versus simpler computational problems (Fuchs & Fuchs, 2002; Fuchs et al., 2015).

Several mechanisms have been proposed to account for how reading achievement is related to math achievement. First, poor phonological skills and awareness that underlie reading proficiency could also impede basic symbolic representations and mapping of numerical quantity (Zebian & Ansari, 2012). Second, difficulties in memorizing and retrieving basic arithmetic facts (via a verbally-mediated retrieval process) from long-term memory could also contribute to poorer math performance (Geary et al., 2004). Finally, as previously discussed, shared weaknesses in attentional abilities may impact both math and reading performance (Houdé, Rossi, Lubin, & Joliot, 2010). While some studies have explored these hypotheses to varying degrees (Fuchs et al., 2015; Swanson & Fung, 2016), the mechanism through which RC affects MR and NO, and whether its effects are mediated by other cognitive and affective factors remains unclear.

Thus, the third aim of our study was to test these hypotheses by examining the strength of the paths between reading comprehension and MR relative to NO, with the prediction of a stronger relation between reading abilities and MR than NO. Additionally, we test the shared attentional weaknesses hypothesis (Houdé et al., 2010) by testing for the existence of indirect paths from attention to MR and NO via reading.

1.3.1. Current study—While previous studies have implicated various affective and cognitive factors underlying math achievement, little is known about their interrelations and how they jointly affect different types of mathematical achievement in children. Here, we develop and test theoretically-informed structural equation models to rigorously investigate three specific theoretical questions. First, math anxiety has been shown to have detrimental effects on math performance by taxing WM resources. However, the mechanism by which math anxiety impacts children's performance on different types of mathematical problem solving tasks remains unknown. We address these gaps by determining whether the effect of math anxiety on math performance is fully mediated by WM (see Fig. 1d, and Methods section for description of the Full Mediation model), or whether there are additional direct influences of math anxiety on math performance above and beyond WM (Fig. 1c, Partial Mediation model). Secondly, although WM and attentional problems have been shown to jointly contribute to math performance, their specific relations have not been established. In particular, it is unclear whether attention acts through WM or has independent influence on different components of math achievement. Lastly, despite past literature suggesting associations between reading and math performance, the extent to which the reading achievement acts on non-verbal and verbal math achievement remains unknown. Our study is novel in that it examines how these key affective and cognitive factors influence non-verbal and verbal math abilities in children through their complex interrelations.

2. Methods

2.1. Participants

Participants were recruited from a wide range of schools in the Greater San Francisco Bay Area using parent mailing lists, flyers at schools and libraries, and postings in newspapers. The final participants included 330 children, 158 of whom were boys and 172 of whom girls, with a mean age of 8.34 years (*SD* = 0.70 years). 129 of the participants were 2nd graders, 184 were 3rd graders, and 4 were 4th graders. All protocols were approved by the Institutional Review Board at Stanford University, and participants were treated in accordance with the American Psychological Association Code of Conduct. Informed consent was obtained by guardians and parents for participation in the study.

Guardians or parents were asked to complete a demographic questionnaire and the Child Behavioral Checklist (Achenbach & Rescorla, 2001) to assess for behavioral and emotional problems. Children with significant reading and attentional difficulties were excluded. Participants who had full-scale IQs greater than 80 were selected for this study. 11 children who were initially screened were excluded due to having received a clinical diagnosis from their psychiatrist or pediatrician. All tests were administered in one neuropsychological assessment session that lasted about 2 h.

2.2. Measures and procedures

2.2.1. Outcome variables – mathematical achievement—The WIAT-II (Wechsler, 2002) was used to assess mathematical abilities. The Numerical Operations (NO) subtest is a paper-and-pencil test that measures the ability to identify and write numbers, rote counting, number production, and solve written calculation problems and simple equations. The Mathematical Reasoning (MR) subtest is a verbal problem solving test that measures the ability to count, identify geometric shapes, and solve single- and multistep word problems. Problems were read to children and also presented to them in written form, and they were asked to complete the problem within specific time limits.

2.3. Independent variables

2.3.1. Reading achievement—Reading achievement was assessed by the Reading Comprehension subtest of the WIAT-II (Wechsler, 2002). Children are asked to read short passages of increasing difficulty and answer multiple-choice questions about content.

2.3.2. Working memory—Four subtests of the Working Memory Test Battery – for Children (WMTB-C; Pickering & Gathercole, 2001) were used: Counting Recall, Backward Digit Recall, Digit Recall, and Block Recall. Two central executive subsets were administered: Counting Recall and Backward Digit Recall. Counting Recall requires the child to count a set of 4, 5, 6, or 7 dots on a card, and then to recall the number of counted dots at the end of a series of cards. With Backward Digit Recall, the experimenter states a string of number words and the child repeats them in reverse order. Digit Recall was used to assess the phonological loop. The task requires the child to repeat a string of numbers in the same order spoken by the experimenter. Block Recall was used to assess the visuospatial sketchpad. The stimuli consist of a board with nine raised blocks in what appears as a random arrangement. The blocks have numbers on one side that can only be seen by the experimenter. The experimenter taps a block (or series of blocks), and the child's task is to duplicate the tapping in the same order as the experimenter.

2.3.3. Socioemotional behavioral assessment—The Child Behavior Checklist for Ages 6–18 (CBCL/6–18; Achenbach & Rescorla, 2001) is a well-validated standardized measure that rates social and behavioral problems in children between the ages of 6 and 18.

Parents or guardians were asked to rate 113 items describing whether the child was currently exhibiting or had exhibited specific behavioral and emotional problems or traits within the last 6 months. Trait anxiety was measured by the DSM-IV Anxiety Problems subscale, while the attention difficulties were measured by the Syndrome-Oriented Attention Problems subscale.

2.3.4. Math anxiety—The Scale for Early Math Anxiety (SEMA; Wu et al., 2012) was used to assess anxiety related to the early experience of learning math. The SEMA assesses specifically for anxiety related to numbers and calculations, as well as situational and social anxiety related to completing math. Descriptive data of all measures are summarized in Table 1.

2.3.5. Full scale IQ—Full Scale IQ was estimated using the Wechsler Abbreviated Scales of Intelligence (Wechsler, 1999), an abbreviated measure that is composed of Verbal Comprehension and Perceptual Reasoning indices. We included the full-scale IQ in our models in light of research conducted by Bailey et al. (2014) which suggested including developmentally stable factors such as IQ when explaining individual differences in math achievement.

2.4. Analyses and statistical approach

We tested the fit of our observed data to five theoretically driven SEM models (Fig. 1). The models illustrate the hypothesized relations between the independent variables of math anxiety, attention problems, WM, and IQ with the outcome variables of Numerical Operations (NO) and Math Reasoning (MR). Factors such as traitanxiety and age that have been shown in previous literature to be associated with math achievement were included in the SEM models (Meyer, Salimpoor, Wu, Geary, & Menon, 2010; Wu et al., 2012) but are not shown in the graph for simplicity. WM was modeled as a latent construct that was comprised of the four subscales of WMBT-C test battery (see Fig. 1; Digit Recall, Counting Recall, Block Recall, Backwards Digit Recall).

SEM models 1 through 4 are intended to address our main questions about the relation between math anxiety, attention problems and WM on NO and MR (Aim 1 and 2). In order to address the third aim regarding the role of reading skills in math achievement, we chose the best fitting of the first 4 models for NO and MR separately, and then included reading achievement as an independent variable for Model 5 (Fig. 2b). It is important to note that, for all five models, NO and MR were tested separately as math achievement outcome variables in order to determine whether different math skill complexity differentially relates to these independent variables (IV). In greater detail, the models are as follows:

(1) Direct Effect Model (Model 1): This model is based on a priori literature, in which all affective and cognitive predictors are assumed to only have independent and direct effects on math achievement. We included this model as a corollary of previous research that examined the relation between these independent variables and math achievement without consideration of any interrelations. As has been done in previous research (Hembree, 1990; Wu et al., 2012), traitanxiety was included as a control variable in order to demonstrate that the influence of math anxiety is domain-specific and still present even when trait anxiety is accounted for. As shown in Fig. 1a, Model 1 for NO has only direct paths from IQ, age, trait anxiety, attention, math anxiety, and WM (latent variable) to NO. For our sequential SEM analyses, this model can be considered as the basic model to test the direct effects of affective and cognitive factors on NO and MR.

(2) Correlational Model (Model 2): The Correlational Model is an extension of the Direct Effect Model (Model 1) with the assumption that all IVs were correlated in a bidirectional manner (Fig. 1b). This model was included because it provided full estimates of all of the interrelationships among the IVs.

(3) Partial Mediation Model (Model 3): This model was based on a priori research and theory in order to explain how the effects of math anxiety and attention on math performance are mediated by WM. In this model, math anxiety and attention were assumed to have indirect paths on math skill measures via WM. We also tested whether the relation between math achievement and attention problems was mediated by math anxiety by adding a direct path from attention to math anxiety. Residual correlations were assumed for IQ with math anxiety, attention and WM as observed in our correlation matrices and Correlational Model (Model 2; Fig. 1c). This type of model was named the ‘‘Partial Mediation” model because the predictors of math anxiety and attention have direct paths as well as indirect paths to math skill measures via WM.

(4) Full Mediation Model (Model 4): This model was an extension of the Partial Mediation Model in that non-significant direct effects from the independent variables to math achievement and other non-significant mediation paths were excluded (Fig. 1d). By comparing the fit of the Partial Mediation (Models 3) and Full Mediation models, we can explore the underlying mechanisms of math anxiety and attention on math performance by examining WM as a potential mediator. The Full Mediation model would be supported if no significant loss of model fit was observed when the direct paths were excluded. Note that we have used the terms full/complete and partial mediation to be consistent with the current literature (Preacher, 2015). However, this distinction should be interpreted with caution as these effects are dependent on sample size (Hayes, 2013).

(5) Reading Effect Model (Model 5): The fifth model was designed to specifically address our third aim of clarifying the effect of reading comprehension on math achievement. Using the best fitting model from Models 1 through 4 for NO and MR, we added reading comprehension as an independent variable. We hypothesized that reading comprehension has a direct effect on math achievement, and is related to IQ, attention and WM, but not math anxiety, given that the latter is content-specific (Wu et al., 2012). We directly tested how reading skills in children may differentially contribute to math skills as a function of type of math achievement. More specifically, we tested whether reading achievement has direct effect on math achievement and whether IQ, WM and attention had indirect paths to the math achievement measures via reading.

2.4.1. Model selection and model fit—

We used the Lavaan package (version 0.5–17) in R (version 3.2.2) for calculating estimates and model fit parameters. For both NO and MR, we used multiple model fitting parameters to select the best-fitting model from Models 1 through 4, including Chi-square test, CFI, TLI and RMSEA, AIC, and BIC measures. Alternate models were built with modifications based on our theoretical hypotheses. Once the best-fitting model from Models 1–4 was determined, we examined whether the model fit could be further improved by dropping any statistically non-significant relation between variables. Lastly, as described earlier, Model 5 (the Reading Effect Model) was built based on the best fitting model in order to examine the association between reading achievement and math achievement in children.

3. Results

3.1. General descriptive and correlational results

General descriptive statistics and correlational data for independent and dependent variables of interest are presented in Tables 1 and 2.

The two math achievement measures (NO and MR) were highly correlated with one another ($r = 0.64$, $p < 0.001$) and also significantly positively correlated with all four WM subscales. Both of the math achievement measures, NO and MR, were significantly and negatively correlated with math anxiety and attention problems, but positively with FSIQ. Interestingly, although both MR and NO were significantly correlated with reading comprehension, MR was more strongly correlated with reading than ($r = 0.46$) than NO ($r = 0.29$), Fisher $Z = 2.54$, $p = 0.011$. Most notably, the CBCL anxiety problems subscale was not significantly correlated with math anxiety or either of the math achievement subscales.

3.2. SEM analysis of numerical operations

The analyses indicated that the Direct Effect model (Model 1) for NO ($\chi^2(25) = 115.99$, $p < 0.001$; see Table 3 for other model fit parameters) did not fit the data well. Since age and trait anxiety were not significantly correlated with the other IVs, they were dropped from subsequent models. Although attention did not have a significant direct path onto NO, it was included in subsequent models because it had significant correlations with the other IVs and because of a priori hypotheses suggesting that attention may have an indirect effect on math achievement via other IVs, such as WM. The Correlational model (Model 2) for NO showed acceptable fit to the data ($\chi^2(14) = 19.77$, $p = 0.14$, $CFI = 0.99$, $TLI = 0.97$), and the direct effect of math anxiety, attention, and IQ on NO were not statistically significant ($p > 0.10$). The fit statistics for the Partial Mediation model (Model 3) for NO indicated that it fit the data well ($\chi^2(14) = 19.77$, $p = 0.14$, $CFI = 0.99$, $TLI = 0.97$). However, as observed in the Correlational model (Model 2), the direct paths from math anxiety, attention and IQ were non-significant ($p > 0.10$). In contrast, the effects of math anxiety, and attention on WM were significant ($p < 0.01$).

The Full Mediation model (Model 4) for NO showed high fidelity to the data ($\chi^2(17) = 22.54$, $p = 0.17$, $CFI = 0.99$, $TLI = 0.98$) and had the lowest AIC and BIC values (see Table 3). This model accounted for 44.1% variance in NO (see Fig. 2a). Although a v^2 deviation test did not reveal a significant difference between the Partial Mediation (Model 3) and Full Mediation models for NO ($\chi^2(3) = 2.782$, $p > 0.05$), the AIC and BIC values both suggest that the Full Mediation model was the best fitting model for NO (for all estimates, see Table 4), In this model, only WM had significant and direct paths to NO. The effects of math anxiety and attention on NO were fully mediated by WM ($p < 0.001$). In comparing the Partial and Full Mediation models, we observed that the effects of both math anxiety and attention problems were fully mediated by WM, and therefore had no additional effect on NO other than their influence via WM. A bootstrapping approach revealed that the 95% Confidence Intervals (C.I.) for both indirect paths did not contain zero, respectively, for math anxiety → WM → NO, 95% C.I. = [0.388, 0.108], and for attention → WM → NO, 95% C.I. = [0.661, 0.238].

In Model 5, we added reading achievement as an IV into the Full Mediation model (Model 4) for NO. The results indicated there was no direct effect of reading achievement on NO ($p = 0.92$), and the model showed a decrement of fit to the data ($\chi^2(21) = 34.02$, $p < 0.05$). Therefore, these results suggested that reading achievement did not significantly contribute to basic computational skill development, despite the correlation between reading and NO.

3.3. SEM analysis of mathematical reasoning

As in the case of NO, the Direct Effect model (Model 1) for MR did not fit the data well, and age and trait anxiety were dropped from the subsequent analyses (see Table 3). The Correlational model (Model 2) fit the data well. The direct effects of math anxiety, WM, and IQ on MR were all statistically significant. The Partial Mediation model (Model 3) for MR fit the data well ($\chi^2(14) = 13.63$, $p = 0.49$, $CFI = 1.00$, $TLI = 1.00$) and showed lower AIC and BIC values compared to direct effect models. Therefore, for MR, the Partial Mediation model was the best fitting model; math anxiety, WM, and IQ all had direct effects on MR. Moreover, the mediation effects of WM from math anxiety and attention were also highly significant ($p < 0.001$). In addition, because the direct effect of attention was non-significant, we created a modified partial mediation model by removing the direct paths from attention to MR. The modified partial mediation model showed slight improvements on AIC and BIC measures (for all the estimates, see Table 5). Bootstrap analysis showed that the 95% C.I. for math anxiety → WM → MR was [0.278, 0.049], and for attention → WM → MR was [0.486, 0.130]. This model accounted for 60.4% variance in MR.

We also explored the fit of the Full Mediation model (Model 4) by examining the effect of removing the direct paths of math anxiety and attention onto MR that were tested in the Partial Mediation model (Model 3). Eliminating the direct paths resulted in a decrement in model fit, suggesting that the Partial Mediation model still provided the best fit for MR. Moreover, the effect of math anxiety on MR was divided into both direct and indirect effects via WM, whereas the effect of attentional problems on MR was still fully mediated by WM similar to NO. However, given that the Full Mediation model resulted in a poorer fit to the data, and that the deviance test between the Full Mediation and Partial Mediation models was significant ($\chi^2(2) = 28.718$, $p < 0.001$), the Partial Mediation model provided a significantly better fit for MR.

Model 5, based on the modified Partial Mediation model (Model 3) with reading achievement as an IV, indicated a significant and direct effect of reading achievement on MR ($p = 0.05$), and good model fit ($v^2(19) = 25.36$, $p = 0.15$, CFI = 0.99, TLI = 0.98). A combined model with the Full Mediation for NO and a Partial Mediation for MR was created (since these were the two best fitting models for NO and MR, respectively) to test the differential effect of reading skills on NO and MR. In this model, we observed that, there was no significant effect of reading on NO, $\beta = 0.005$, $SE = 0.082$, $z = 0.058$, $p > 0.10$, whereas a significant effect of reading on MR, $\beta = 0.125$, $SE = 0.061$, $z = 2.062$, $p < 0.05$. We compared the difference between the two path coefficients based on Clogg, Petkova, and Haritou (1995), and found a significant difference in the influence of reading on MR vs NO ($z = 6.41$, $p < 0.001$).

In this model, attention, WM and IQ showed significant direct effects on reading achievement (see Fig. 2b and Table 6). Additionally, anxiety showed an additional effect on MR that was not mediated by the other factors, and in particular, reading achievement. However, attention had an additional effect on MR but was still mediated by other factors such as reading achievement.

4. Discussion

Our study advances knowledge of how multiple affective and cognitive factors – math anxiety, working memory, attention, and reading achievement –contribute to the development of early math skills in children. Much of the previous research has only examined the relation between these factors and math achievement in isolation, and many of these studies have examined these relations in adults or in clinical populations (Ashcraft & Kirk, 2001; Hopko, Ashcraft, Gute, Ruggiero, & Lewis, 1998; Swanson & Beebe-Frankenberger, 2004). Our analysis of theoretically-motivated SEM models revealed three main findings. First, math anxiety had a consistent significant indirect effect on non-verbal (Numerical Operations; NO) and verbal math (Math Reasoning; MR) achievement via WM, but the presence of a direct effect depended on verbal load. In the more complex MR task which involves verbal problem solving, math anxiety had both a direct and indirect influence via WM on math achievement. In contrast, the relation between math anxiety and NO was completely mediated by WM. Second, regardless of verbal task complexity, the effect of attention on math skills was indirect in both NO and MR, and was mediated by WM or reading achievement. Finally, reading achievement directly influenced MR, but not NO, which is more calculation-oriented. Our findings are the first to dissociate the joint influence of key affective and cognitive factors related to different components of children's math achievement.

4.1. Differential role of math anxiety and working memory in math achievement

Our study extends previous analyses indicating that math achievement is independently correlated with both WM and math anxiety (Ashcraft & Kirk, 2001) by specifically examining the mechanisms through which math anxiety and WM collectively impact math achievement. First, our findings replicated previous results in that higher levels of math anxiety were indeed associated with poor working memory performance (Ashcraft & Kirk, 2001; Ashcraft & Krause, 2007; Hopko et al., 1998). Second, our analyses indicated that WM mediates the relation between math anxiety and math achievement; the best fitting models for both MR and NO suggested that math anxiety had an indirect effect on math achievement through WM. Moreover, math anxiety had a direct negative effect on MR but not NO. Although previous studies have also found an increased effect of math anxiety on more complex math problems in adult participants (Hopko et al., 1998), our results indicate this to also be true in children and provides a further distinction between the factors that influence non-verbal and verbal math achievement.

Our findings also provide a possible explanation for why math anxiety has a more pronounced effect on more complex verbal math problems. In more non-verbal computational tasks such as NO, the effect of math anxiety is less pervasive and acts only through WM. In tasks such as MR that require more reading, however, math anxiety has an additional effect that is independent of WM. It is important to note that our results indicated that the effect of math anxiety was not mediated by reading achievement. This finding also suggests that the additional effect of math anxiety on MR cannot be attributed to reading, a key component of verbal math problem solving. One possible explanation for these findings is that multiple forms of math anxiety might operate through different cognitive mechanisms. This is supported by prior research pointing to at least two factors in math anxiety wherein one factor is related to the anxiety of performing computations and the other is related to the social and performance anxiety (Richardson & Suinn, 1972). The effects of math anxiety may accumulate over time during presentation of verbal math problems and thereby trigger multiple mechanisms of math anxiety. Further research is needed to identify the specific mechanisms by which different types of anxiety impact complex math problem solving.

Our findings are consistent with the study conducted by Ramirez et al. (2013) which examined the relation between categorical WM, math anxiety and performance on Woodcock-Johnston-III Applied Problems, which has similar computational demands as MR. Briefly, they found that, in 1st and 2nd grade children classified as having high WM, high levels of math anxiety had uniquely disruptive effects on math performance. In contrast, children who had low levels of WM did not exhibit a similar decrement in performance even when they had high levels math anxiety. In the current study, we used a mediational model, as opposed to Ramirez and colleagues’ moderation analyses. Consequently, we could not directly address the interaction of WM capacity and math anxiety. In contrast, Ramirez and colleagues could not specifically test for multiple direct and indirect paths, as we do in the current study. Nevertheless, both results indicate that high levels of math anxiety have a significant and negative impact on WM in early math achievement. Moreover, both studies highlight the need to account for the impact of WM in future studies that examine math anxiety and different types of non-verbal and verbal math achievement.

4.2. Differential role of attention and working memory in math achievement

We examined the interrelation between attention problems and WM on math performance and found that higher levels of attention problems were related to lower levels of WM ability, and for both NO and MR, attention problems were negatively associated with math performance via WM. Most importantly, in contrast to math anxiety, the effect of attention problems was fully mediated by WM regardless of the non-verbal or verbal nature of the math task. Consequently, attention problems did not have any direct paths to either NO or MR. These results suggest that attention difficulties do not directly impact math performance, but instead negatively impact other cognitive resources, such as WM, that supports both basic and more complex computations.

Previous studies in adults have demonstrated that WM capacity can function as a gating mechanism to filter out distractions and prioritize relevant information (de Fockert, Rees, Frith, & Lavie, 2001). Therefore, our results suggest that WM may play a similar role in the context of attention problems and math learning in children given that the effect of attention problems on math problems was completely mediated by WM. Thus, during mathematical computations, children who have greater difficulty concentrating on relevant tasks and ignoring irrelevant information may also be overloading their limited WM resources. This in turn results in poorer performances on math tasks, regardless of the verbal task demands imposed.

4.3. The impact of attention is mediated by both math anxiety and WM

Our findings clarify the impact of attention on math achievement, one issue that has not been adequately addressed in previous empirical studies. While it has been suggested that math anxiety may attenuate attentional difficulties, others have theorized that anxiety may have a pejorative effect on attentional difficulties (Wine, 1971). We found that attention problems had a direct impact on math anxiety which was in turn was negatively correlated with WM, suggesting additional pathways through which attention may impact math achievement in the context of math anxiety. Our results indicate that math anxiety partially mediated the relation between attention problems and both MR and NO achievement. In both non-verbal NO and verbal MR, math anxiety mediated this relation via WM, whereas in MR, there was an additional direct mediational relation. Taken together, these results support the hypothesis that math anxiety may attenuate the effect of attentional weaknesses, and more directly in problems with greater mathematical complexity. One interpretation of our results is that higher levels of anxiety may increase vigilance (Eysenck & Calvo, 1992), and reduce the negative influence of attentional problems. That is, children who are highly anxious about their math performance may also be more likely to put forth effort to pay attention and focus on the math tasks at hand. Despite this protective effect, however, overall, high levels of attention problems are associated with weaker performance likely due to a negative impact on WM.

4.4. Reading achievement and its differential contribution to math achievement

Reading achievement was significantly correlated with MR, which includes verbally framed word problems, but to a lesser degree with NO, which is composed entirely of calculation problems. Our study extends previous findings (Fuchs et al., 2006, 2015) by showing that, in the context of WM and attentional difficulties, reading achievement did not significantly contribute to basic non-verbal NO skills. In contrast, attention problems and WM showed significant direct effects on reading, which then mediated their effects on MR.

Behavioral research has put forth several hypotheses on the mechanisms through which reading and math interact. One plausible explanation is that weaknesses in core language skills, such as phonological awareness, impede basic symbolic representations and mapping of numerical quantity (Zebian & Ansari, 2012). Another theory is that difficulties in memorizing and retrieving basic arithmetic facts from long-term memory are due to verbally mediated processes (De Smedt & Boets, 2010; Geary et al., 2004). Under these models, reading achievement should have significantly predicted both NO and MR abilities. However, our results indicated the opposite and suggest that, at least in our nonclinical sample, alternative explanations are needed. Indeed, our model of reading and MR performance is more consistent with research suggesting that math and reading performance are related via WM and shared attentional abilities (Houdé et al., 2010).

It should be noted that we are not suggesting that phonological skills and verbal-mediated memory retrieval are entirely unrelated to math achievement. In the current study, we used the measure of reading comprehension to assess reading achievement, but did not include a measure of phonological skills that serve as the basis of reading proficiency in the current model. However, in order to explore whether there might exist a differential relation, we ran our models with word reading (i.e., WIAT-II Word Reading subtest) instead of reading comprehension. The analyses yielded the same results with respect to the best fitting models and significance. These results suggested that the influence of reading-related skills on NO and MR could also be observed at a more basic cognitive level.

It is also important to consider that reading may have a differential impact on math achievement across the developmental spectrum. For example, phonological skills may be more salient during the initial stage of the acquisition of arithmetic skills (e.g., learning the counting sequence, using fingers to count for answers), whereas comprehension skills may be more important in later complex word problem solving, such as in the age range we examined.

Finally, with respect to attentional difficulties, previous research has established a relation between attentional difficulties and weaknesses in reading (Willcutt et al., 2001). In the present study, we conducted an exploratory analysis examining the relation between reading achievement and attention problems in the context of performing non-verbal and verbal math problems. The direction of the association was in agreement with previous research, with a negative correlation between attention problems and reading proficiency and a positive correlation between reading achievement and math achievement. However, this relation differed across verbal and non-verbal problems. In addition to the influences via WM and math anxiety, attention problems influenced verbal MR abilities via reading. This was not the case in NO, however, where the influence of attention problems was mediated by WM and math anxiety only. Our findings indicate that the impact of attentional weaknesses on more complicated verbal and reading-dependent math problems may be mitigated by reading proficiency, whereas the same is not true for more calculation-specific math problems.

4.5. Limitations and future directions

Using structural equation modeling and multiple affective and cognitive measures that had not previously been examined together to our knowledge, the current study represents an advance over past research and provides new insights into how math anxiety, attention, working memory, and reading proficiency jointly relate to non-verbal and verbal math skills. Importantly, we examined these relations in a nonclinical sample of 2nd and 3rd grade children who are in the earliest stage of formal math learning. Such studies are a crucial step in helping to identify possible areas of intervention, as well as possibly clarifying how comorbidities, such as the one between math and reading disorders, may relate to one another. Crucially, our study elucidates how word-based reasoning problems place greater demands on cognitive and affective resources relative to more basic non-verbal computational problems. These inherent differences should be acknowl-edged not only in remediation, but in diagnosis and conceptualization as well (Fuchs, Fuchs, & Compton, 2013; Gilbert et al., 2013).

We conclude with limitations that need to be addressed in future research. First, although there were significant correlations between IQ and the other variables of WM, attention, and math anxiety, we only incorporated the latter variables as covariates in order to test more parsimonious models. While the associations between IQ and the other variables were not entirely surprising due to prior research suggesting the underlying impact of a general ‘‘g” intelligence factor on many different executive functions and reading and math abilities, this is one area that merits further exploration. Additionally, due to the cross-sectional nature of our study, we were unable to determine the direction of causality in our results. One future direction would be to address this gap by conducting similar analyses with longitudinal data.

4.6. Summary

We examined the association between affective and cognitive factors on the early development of math skills. The results indicated that the effect of math anxiety on math performance is fully mediated by WM for non-verbal numerical operations, but only partially mediated by WM for verbal math reasoning. Attentional difficulties had consistent indirect effects on both types of math abilities via WM. Reading ability contributed to individual differences in performance in MR, but not NO. These findings suggest that (1) WM is a domain-general ability pivotal to children’s math performance on both non-verbal and verbal problems, (2) that WM mediates the effects of math anxiety and attention specifically on verbal problems, and (3) that the influences of reading and attention become more pronounced for mathematical tasks that require additional cognitive resources. Our study elucidates how affective and cognitive factors distinctly influence non-verbal and verbal math problem solving.

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Appendix A.: Supplementary material

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.cognition.2017.05.016.

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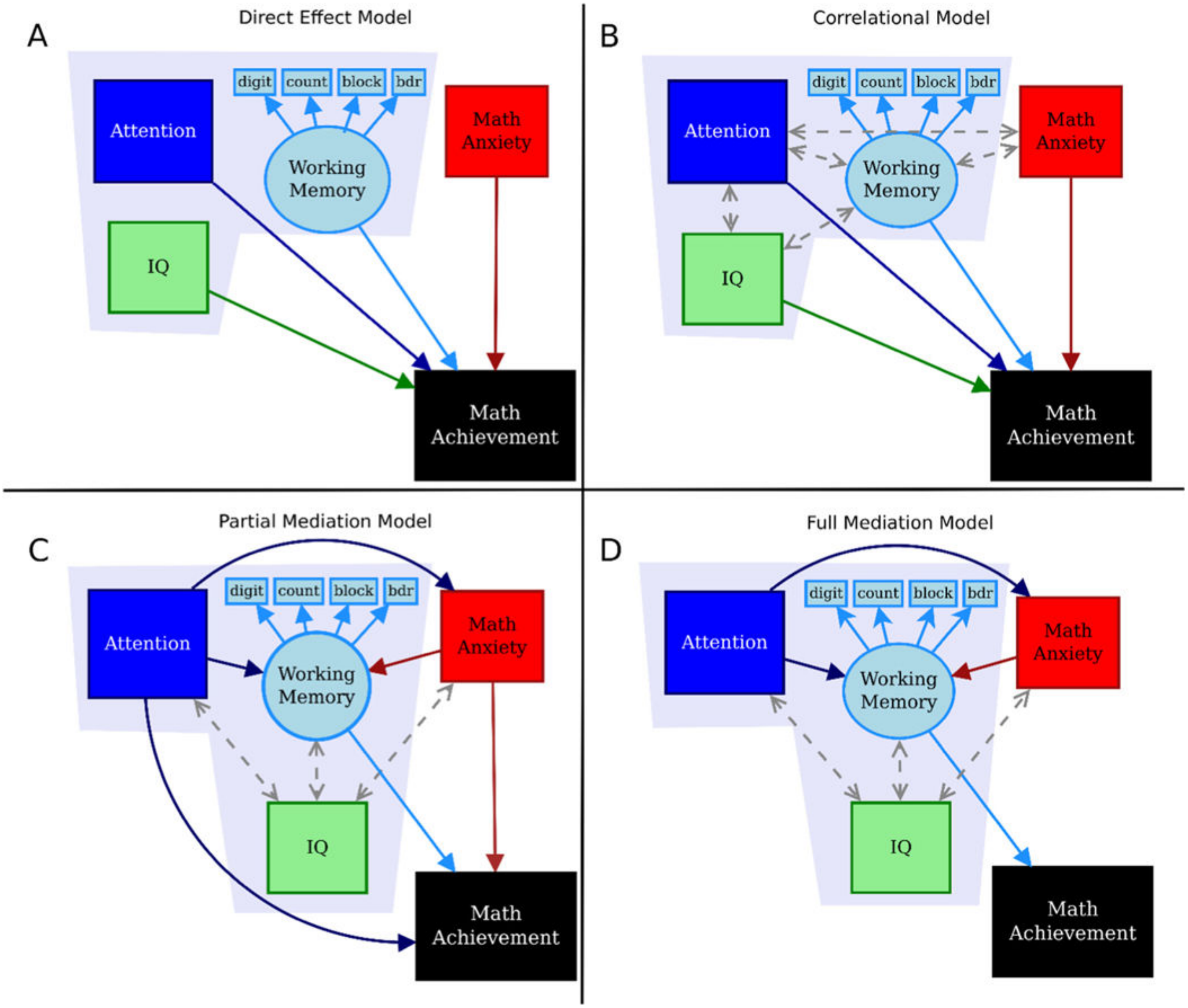

Fig. 1.

Illustration of four theoretically-informed structural equation models examining affective and cognitive factors that influence non-verbal (Numerical Operations) and verbal (Math Reasoning) math achievement. (A) Direct effect Model: assumes only direct effects from affective and cognitive factors to math achievement; (B) Correlational Model: assumes additional correlational structure between affective and cognitive factors; (C) Partial Mediation Model and (D) Full Mediation Model: focus on testing whether the effects of attention and math anxiety are mediated by working memory. In the partial mediation model, both direct and indirect effects of attention and math anxiety were included in the model, whereas in the full mediation model, no direct effect of attention and math anxiety on math achievement was assumed.

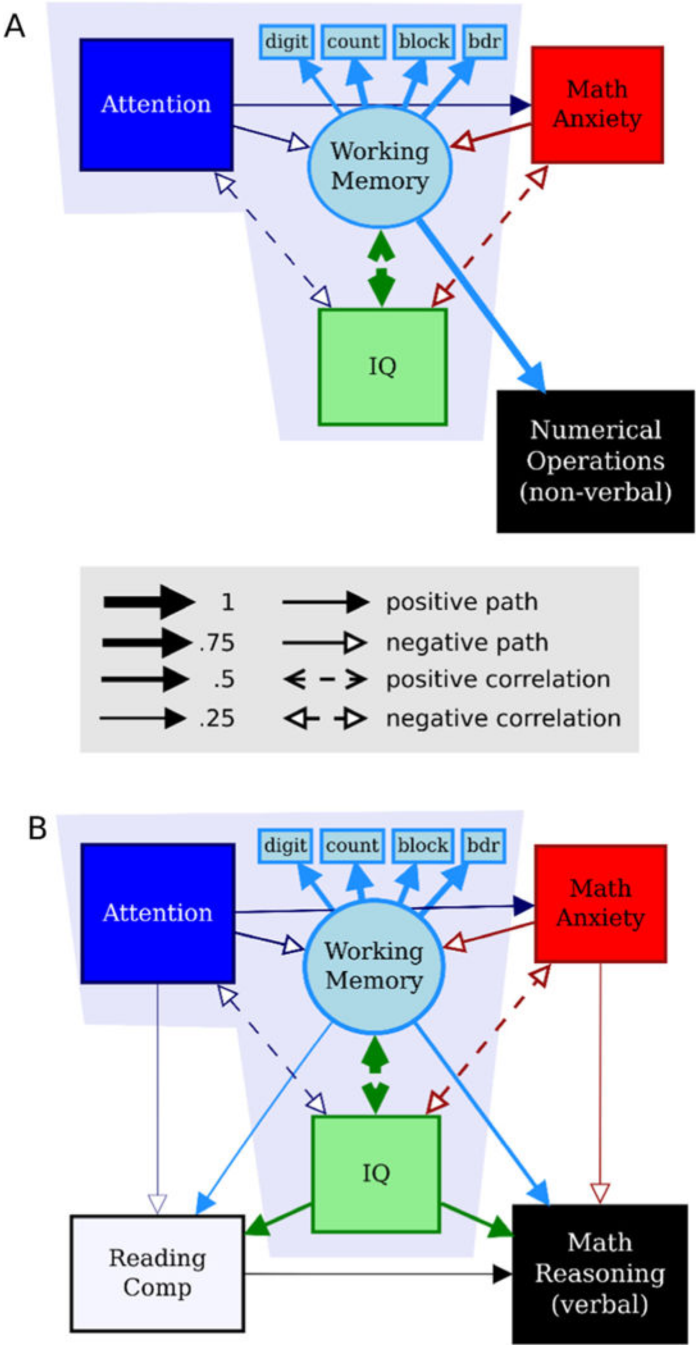

Fig. 2.

Best fitting SEM models for non-verbal and verbal math abilities. (A) Full mediation model for (non-verbal) Numerical Operations. (B) Partial mediation model for (verbal) Math Reasoning. The single-ended arrows depict direct effects between variables whereas double-ended arrows depict correlations between variables. The hollow-headed arrows depict negative estimates and the thickness of the arrows correspond to magnitude of each estimate.

Table 1

Descriptive statistics for entire sample.

Variable	**Overall sample**	
	M	***SD***
Age	8.34	0.70
Math Reasoning	111.40	16.65
Numerical Operations	107.76	17.08
Reading Comprehension	108.00	12.97
Math Anxiety	15.60	11.84
Digit Recall	102.50	17.09
Block Recall	96.43	15.41
Counting Recall	88.64	17.78
Backwards Digit Recall	96.63	16.39
CBCL Anxious/Depressed	53.76	5.59
CBCL Attention Problems	55.72	6.55
FSIQ	113.00	15.63

Table 2

Pearson correlation between dependent and independent variables of interest.

Measure	1	2	3	4	5	6	7	8	9	10	11
1. WIAT-II Math Reasoning	\										
2. WIAT-II Numerical Operations	0.64^{***}	\									
3. Reading Comprehension	0.46^{***}	0.29^{***}	\								
4. Math Anxiety	−0.38	-0.23^{***}	-0.18^{**}	\							
5. Digit Recall	0.32	0.15^{*}	0.3^{***}	-0.14^{*}	\						
6. Block Recall	0.36^{***}	0.36	0.15^{*}	-0.21^{***}	0.12^{*}	\					
7. Counting Recall	0.42^{***}	0.44	0.24^{***}	-0.18^{**}	0.27^{**}	0.33^{***}	\				
8. Backward Digit Recall	0.25^{***}	0.3^{***}	0.25^{***}	$-0.11^{\#}$	0.25^{***}	0.31^{***}	0.34^{***}	\			
9. CBCL Anxious/Depressed	−0.08	−0.1	−0.05	$0.11^{\#}$	−0.04	−0.05	−0.08	−0.05	\		
10. CBCL Attention Problems	-0.27^{**}	-0.2^{***}	0.2^{***}	0.18^{*}	-0.18^{**}	-0.13^{*}	-0.17^{**}	-0.17^{**}	0.36^{***}	\	
11. FSIQ	0.64^{***}	0.41^{***}	0.5^{***}	-0.29^{***}	0.28^{***}	0.30^{***}	0.35^{***}	0.24^{***}	−0.04	-0.18^{**}	\
12. Age	−0.04	−0.01	$0.12^{\#}$	−0.01	$-0.11^{\#}$	−0.07	−0.02	0	0.03	0	0.05

$^{***}p < 0.001$.

$^{**}p < 0.01$.

$^{*}p < 0.05$.

$^{\#}p < 0.10$.

Table 3

SEM model parameters for Numerical Operations and Math Reasoning.

Math Achievement (Dependent Variable)	***χ2(df)***	***CFI***	***TLI***	***AIC***	***BIC (adjusted)***	***RMSEA***
Numerical operations						
Model 1 (Direct Effect Model)	120.40 (25)***	0.73	0.62	23445.14	23470.23	0.11
Model 2 (Correlation Model)	19.77 (14)	0.99	0.97	20731.43	20750.24	0.04
Model 3 (Partial Mediation Model)	19.77 (14)	0.99	0.97	20731.43	20750.24	0.03
Model 4 (Full Mediation Model)	22.49 (17)	0.99	0.98	20728.33	20745.26	0.03
Model 5 (Full Mediation Model + Reading)	34.11 (21)*	0.98	0.96	23231.88	23252.57	0.04
Math reasoning						
Model 1 (Direct Effect Model)	115.99 (25)***	0.81	0.74	23281.59	23306.67	0.11
Model 2 (Correlation Model)	13.63 (14)	1.00	1.00	20566.54	20585.36	0.00
Model 3 (Partial Mediation Model)	13.63 (14)	1.00	1.00	20566.54	20585.36	0.00
Model 3 (Partial Mediation Model Improved)	15.68 (15)	1.00	1.00	20566.15	20584.33	0.01
Model 4 (Full Mediation Model)	44.10 (17)***	0.95	0.92	20591.01	20607.95	0.07
Model 5 (Partial Mediation Model + Reading)	25.37 (19)	0.99	0.98	23061.48	23083.42	0.03

***$p < 0.001$.

**$p < 0.01$.

*$p < 0.05$.

Table 4

SEM estimates of the Full Mediation model for Numerical Operations.

	β	*Std. β*	*Std. Error*	*Z-value*
Latent variable				
Working memory –> Digit recall	1.00	0.38		
Working memory –> Count recall	1.77	0.63	0.31	5.52^{***}
Working memory –> Block recall	1.30	0.55	0.25	5.17^{***}
Working memory –> Digit backward recall	1.37	0.55	0.26	5.38^{***}
Direct paths				
Working memory –> Numerical operations	1.73	0.66	0.32	5.43^{***}
Math anxiety –> Working memory	–0.18	−0.33	0.05	-4.00^{***}
Attention –> Working memory	–0.22	−0.22	0.07	-3.58^{**}
Attention –> math anxiety	0.37	0.20	0.10	3.58^{***}
Indirect paths				
Math anxiety –> WM –> NO	−0.31	−0.22	0.07	-4.80^{***}
Attention –> WM –> NO	−0.38	−0.15	0.11	-3.39^{**}
Covariance				
Working memory ~~ IQ	47.77	47.77	9.92	4.78^{***}
Math anxiety ~~ IQ	−56.44	−0.31	10.50	-5.38^{***}
Attention ~~ IQ	−19.23	−0.19	5.92	-3.24^{**}

Note. WM = working memory; NO = Numerical operations. "–>" = direct path; "~~" = correlation. The same in the following tables.

$^{***}p < 0.001$.

$^{**}p < 0.01$.

$^{*}p < 0.05$.

Table 5

SEM estimates of the improved Partial Mediation model for verbal Math Reasoning.

	β	*Std.* β	*Std. Error*	*Z-value*
Latent variables				
Working memory –> Digit recall	1.00	0.44		
Working memory –> Count recall	1.46	0.63	0.24	6.00***
Working memory –> Block recall	1.07	0.53	0.20	5.48***
Working memory –> Digit backward recall	1.20	0.56	0.21	5.85***
Direct paths				
Math anxiety –> Math reasoning	−0.25	−0.17	0.06	−4.09***
Working memory –> Math reasoning	0.83	0.38	0.19	4.47***
IQ –> Math reasoning	0.42	0.40	0.06	6.87***
Math anxiety –> Working memory	−0.19	−0.29	0.05	−3.73***
Attention –> Working memory	−0.28	−0.24	0.09	−3.26**
Attention –> Math anxiety	0.37	0.20	0.10	3.56***
Indirect paths				
Math anxiety –> WM –> MR	−0.16	−0.11	0.05	−3.28**
Attention –> WM –> MR	−0.23	−0.09	0.08	−3.01**
Covariance				
Working memory ~~ IQ	53.63	53.63	10.69	5.02***
Math anxiety ~~ IQ	−56.51	−0.31	10.49	5.39***
Attention ~~ IQ	−19.30	−0.19	5.92	−3.26**

Note. WM = working memory; MR = Math reasoning.

*** $p < 0.001$.

** $p < 0.01$.

* $p < 0.05$.

Table 6

SEM estimates of the Partial Mediation model for verbal Math Reasoning.

	β	*Std.* β	*Std. Error*	*Z-value*
Latent variables				
Working memory –> Digit recall	1.00	0.46		
Working memory –> Count recall	1.42	0.62	0.24	6.06^{***}
Working memory –> Block recall	1.03	0.52	0.19	5.48^{***}
Working memory –> Digit backward recall	1.18	0.56	0.20	5.96^{***}
Direct paths				
Math anxiety –> Math reasoning	−0.24	−0.17	0.06	-4.10^{***}
Working memory –> Math reasoning	0.76	0.36	0.18	4.23^{***}
IQ –> Math reasoning	0.38	0.36	0.06	6.39^{***}
Reading –> Math reasoning	0.12	0.09	0.06	1.92^{*}
Math anxiety –> Working memory	−0.20	−0.30	0.05	-3.81^{***}
Attention –> Working memory	−0.29	−0.24	0.09	-3.24^{**}
Attention –> Math anxiety	0.37	0.20	0.10	3.61^{***}
Attention –> Reading	−0.17	−0.08	0.10	$-1.66^{\#}$
Working memory –> Reading	0.37	0.22	0.15	2.52^{*}
IQ –> Reading	0.31	0.38	0.06	5.48^{***}
Indirect paths				
Math anxiety –> WM –> MR	−0.15	−0.11	0.05	-3.20^{**}
Attention –> WM –> MR	−0.22	−0.09	0.08	-2.88^{**}
Covariance				
Working memory ~~ b	54.90	54.90	10.80	5.08^{***}
Math anxiety ~~IQ	−56.45	−0.31	10.48	-5.39^{***}
Attention ~~IQ	−19.40	−0.19	5.92	-3.28^{**}

Note. WM = working memory; MR = Math Reasoning.

$^{***}p < 0.001$.

[**] $p < 0.01$.

[*] $p < 0.05$.

[#] $p < 0.10$.