

# Soft Computing Based Epidemical Crisis Prediction

Dan E. Tamir, Naphtali D. Rische, Mark Last and Abraham Kandel

**Abstract** Epidemical crisis prediction is one of the most challenging examples of decision making with uncertain information. As in many other types of crises, epidemic outbreaks may pose various degrees of surprise as well as various degrees of “derivatives” of the surprise (i.e., the speed and acceleration of the surprise). Often, crises such as epidemic outbreaks are accompanied by a secondary set of crises, which might pose a more challenging prediction problem. One of the unique features of epidemic crises is the amount of fuzzy data related to the outbreak that spreads through numerous communication channels, including media and social networks. Hence, the key for improving epidemic crises prediction capabilities is in employing sound techniques for data collection, information processing, and decision making under uncertainty and exploiting the modalities and media of the spread of the fuzzy information related to the outbreak. Fuzzy logic-based techniques are some of the most promising approaches for crisis management. Furthermore, complex fuzzy graphs can be used to formalize the techniques and methods used for the data mining. Another advantage of the fuzzy-based approach is that it enables keeping account of events with perceived low possibility of occurrence via low fuzzy membership/truth-

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D.E. Tamir (✉)

Department of Computer Science, Texas State University, San Marcos, TX, USA  
e-mail: dan.tamir@txstate.edu

N.D. Rische · A. Kandel

School of Computing and Information Sciences, Florida International University,  
Miami, FL, USA  
e-mail: rishen@cs.fiu.edu

A. Kandel

e-mail: abraham.kandel.fiu@gmail.com

M. Last

Department of Information Systems Engineering, Ben-Gurion University of the Negev,  
Beer-Sheva, Israel  
e-mail: mlast@bgu.ac.il

values and updating these values as information is accumulated or changed. In this chapter we introduce several soft computing based methods and tools for epidemic crises prediction. In addition to classical fuzzy techniques, the use of complex fuzzy graphs as well as incremental fuzzy clustering in the context of complex and high order fuzzy logic system is presented.

## 1 Introduction

In the context of this Chapter, the term *epidemic* (or *epidemic*) refers to a disease which spreads widely and attacks many persons at the same time. An epidemic crisis is an epidemic that spreads very fast and affects numerous people in different countries and continents. In this sense, the term *epidemic crisis* and *pandemic* are almost synonyms.

Epidemic crisis prediction (ECP) is a special case of disaster prediction and management (DPM). DPM is one of the most challenging examples of decision making under uncertain information. One of the main issues related to ECP/DPM is the amount of a priori information available, i.e., the amount of surprise affiliated with the epidemic crisis as well as the risk and devastation that follow the crisis outbreak. A closer look at these parameters shows that the velocity and acceleration (and higher derivatives) of these parameters are highly important. Another important aspect includes secondary adverse effects (i.e., secondary epidemic crises) that are triggered by the initial disaster.

One of the main concerns about epidemic crises is the amount of surprise that accompanies the outbreak. Emergencies may produce a wide range of surprise levels. The terror attack of 9/11 is an example of a disaster with very high level of surprise. On the other hand the landfall of a hurricane in Florida in the middle of a hurricane season is not as surprising.

Even in the extreme cases where the nature of the disaster is known, preparedness plans are in place, and analysis, evaluation, and simulations of the disaster management procedures have been performed, the amount and magnitude of “surprises” that accompany the real disaster pose an enormous demand. In the more severe cases, where the entire disaster is an unpredicted event, the disaster management and response system might fast run into a chaotic state. Hence, the key for improving disaster preparedness and mitigation capabilities is in employing sound techniques for data collection, information processing, and decision making under uncertainty.

Analysis of epidemic crises presents three types of challenges: the first is the ability to predict the occurrence of epidemic crises, the second is the need to produce a preparedness plan, and the third is the actual real time response activities related to providing remedies for a currently occurring disaster.

As a special case of DPM, ECP is a highly challenging example of decision making under uncertain information. Epidemic outbreaks might pose various degrees of surprise as well as various degrees of the “derivatives” of the surprise. One of the

unique features of epidemical crises is the amount of fuzzy data related to an outbreak that spreads through numerous communication channels and social networks.

The key for improving epidemical crises prediction and management capabilities is in employing sound techniques for data collection, information processing, and decision making under uncertainty and in exploiting the modalities and media of the spread of the fuzzy information related to the outbreak. Hence, fuzzy logic-based techniques are some of the most promising approaches for crisis management. Furthermore, complex fuzzy graphs can be used to formalize the techniques and methods used for the data mining. Another advantage of the fuzzy-based approach is that it enables keeping account of events with perceived low possibility of occurrence via low fuzzy membership/truth-values and updating these values as information is accumulated or changed.

In this chapter we introduce several soft computing based methods and tools for epidemical crises prediction. In addition to classical fuzzy techniques, the use of complex fuzzy graphs as well as incremental fuzzy clustering in the context of complex and high order fuzzy logic systems is presented.

The rest of this chapter elaborates on some of the important aspects of ECP and DPM, concentrating on the related uncertainty which can be addressed via fuzzy logic-based tools as well as the geospatial-temporal analytics/Big Data aspects of the problem. Section 2 provides the background, Sect. 3 provides an overview of several fuzzy logic-based tools for ECP, and Sect. 4 concludes and proposes future research.

## 2 Background

The topic of predicting epidemical crises falls under the more general subject of disaster prediction management and mitigation (DPM). In this section, we discuss DPM (Sect. 2.1) and elaborate on some of the distinguishing factors of ECP (Sect. 2.2). In addition, we describe the geospatial-temporal analytics of the correlation of environmental factors and incidence of disease and report on a set of tools and concept demonstrations that show the solvability of Big Data problems involving geospatial data correlated with publically available medical data.

Epidemical crises occur with different degrees of unpredictability and severity, which are manifested in two main facets. First, the actual occurrence of the pandemics might be difficult (potentially impossible) to predict. Second, regardless of the level predictability of the crisis, it is very likely that it will be accompanied by secondary effects. Hence, epidemical crises are a major source of “surprise” and uncertainty and their mitigation and management require sound automatic and intelligent handling of uncertainty.

Often, the stakeholders of ECP programs are classifying the unpredictability of epidemical crises as two types of unknowns: unknown unknowns and known unknowns. The first type of unknowns (unknown unknowns) is often referred to by the metaphor of a *black swan*, coined by Taleb [30, 31], while the second type

of unknowns (known unknowns) is referred to as a *gray swan*. Arguably, however, there is no such animal as a black swan and every swan is a gray swan depending on the amount of surprise it carries and the potential devastation associated with it. In almost all the cases of epidemical crises recorded so far there was some *a priori* information concerning the disaster yet this information was filtered, ignored, or just did not pass a threshold of being classified as significant.

## 2.1 Severity and Predictability of Epidemical Crises

In his excellent books [30, 31], Nassim Nicholas Taleb describes the following features of blackswan events:

1. The blackswan is an outlier; lying outside of the space of regular expectations.
2. It has very low predictability and it carries an extremely adverse impact.
3. The unknown component of the event is far more relevant than the known component.
4. Finally, we can explain the event *post factum* and through those explanations make it predictable in retrospect.

In this sense, the black swan represents a class of problems that can be referred to as the “unknown unknowns.” However, a thorough investigation of many of the events that are widely considered as black swans, e.g., the September 11, 2001 attack in NYC, shows that there had been available information concerning the evolving event; yet, this information did not affect the decision making and response prior to the attack. Hence, the term “not connecting the dots” is often used to describe these phenomena. This brings to the forefront the problem of predicting the occurrence of epidemical crises. More important is the issue of identifying (and not ignoring) anomalies.

This suggests that the term *black swan* is a bit too extreme and one should consider using the term *gray swan* where the gray level relates to the level of surprise. A gray swan represents an unlikely event that can be anticipated and carries an extremely adverse impact. In this respect, gray swans represent a two dimensional spectrum of information. The first dimension represents the predictability of the event where black swans are highly unpredictable and white swans are the norm. The second dimension represents the amount of adverse outcome embedded in the event; with black swans representing the most adverse outcomes. Consequently, the black swan is a special case of a gray swan.

To further elaborate, one type of unpredictability relates to a set of events that can be considered as known unknowns. For example, a hurricane occurring in Florida during the hurricane season should not surprise the responsible authorities. Moreover, often, there is a span of a few days between the identification of the hurricane and the actual landfall. Regardless, even a predictable hurricane landfall carries numerous secondary disastrous events that are hard to predict.

We refer to these secondary events as second generation gray swans. Second generation swans are generated and/or detected while the disaster is occurring. The

collapse of the Twin Towers in 9/11 is an example of a second-generation gray swan. A gray swan might (and according to the Murphy laws is likely to) spawn additional gray swans. Generally, second generation swans evolve late and fast. Hence, they introduce a more challenging detection problem and require specialized identification tools, such as dynamic clustering. Detecting relatively slow evolving [first generation] gray swans before the disaster occurs and relatively fast evolving second generation gray swans requires an adequate set of uncertainty management tools.

The following is a partial list of well-known gray swans, each of which has a different degree of surprise as well as different degree of severity.

1. Philippines typhoon disaster
2. Bangladeshi factory collapse
3. Iceland volcano eruption
4. Fukushima—tsunami followed by nuclear radiation and risk of meltdown of nuclear facilities in the area
5. HIV, HSV2, Swine, SARS, and West Nile Virus infection outbreaks
6. 9/11 NYC attack followed by the collapse of the Twin Towers
7. Financial Markets' falls (1987, 2008), Madoff's fraud
8. The 1998 fall and bailout of Long-Term Capital Management L.P. hedge fund
9. Yom Kippur War
10. December 7, 1941—Pearl Harbor
11. Assassinations of Lincoln, Kennedy, Sadat, and Rabin

Fuzzy logic is one of the suggested tools that can help create a better understanding of ECP tools, including, but not limited to, intelligent robotics, learning and reasoning, language analysis and understanding, and data mining. Hence, research in fuzzy logic and uncertainty management is critical for producing a successful ECP programs.

Recently, the academic community and government agencies have effected spurring growth in the field of data mining in Big Data systems. These advances are beginning to find their way into ECP programs and are redefining the way we address potential disaster and mitigate the effects of epidemical crises. Nevertheless, academia, industry, and governments need to engage as a unified entity to advance new technologies as well as apply established technologies in preparation and response to the specific emerging problems of epidemical crises.

## ***2.2 Epidemical Crises Information Spread***

Predicting the epidemical outbreak is an important component of the management and mitigation of a pandemic. It can enable early setup of a mitigation plan. Nevertheless, the fact that an epidemical crisis is somewhat predictable does not completely reduce the amount of surprise that accompany the actual occurrence of the crisis.

Hence, any mitigation plan, that is, a set of procedures compiled in order to address the adverse effects of epidemical crises, should be flexible enough to handle additional surprises related to secondary adverse effects of the epidemical crisis. Finally, the real time ramification program is the actual set of procedures enacted and executed as the epidemical crisis occurs. There are two preconditions for successful remedy of epidemical crisis affects. First, the authorities/leadership have to attempt following the original mitigation plan as close as possible while instilling a sense of trust and calmness in the people that experience the crisis and the mitigation provider teams. Second, and as a part of their leadership traits, the authorities must possess the ability to adapt their remedy procedures to the dynamics of the epidemical crisis, potentially providing effective improvisations aimed at handling secondary disastrous events that evolve from the main disaster. A simple example for such events is looting and violence that might accompany a major disaster. For this end, fast automatic assessment of the dynamics is paramount.

One of the distinguishing features of pandemics' outbreaks is the modalities of sharing information in communication and social networks. Often, due to the panic that accompanies outbreaks there is an explosion of data which is characterized by large amount of information and high "velocity" and higher derivatives of velocity of information spread. Using the current terminology to describe the phenomenon: pandemic news are likely to become viral. Notably, many countries would try to completely ban, control, or limit, the news spread. But experience show that these attempts are not likely to be fruitful: citizens of these countries can still find numerous ways to spread the information through social networks. Hence, data mining in electronic versions of newspapers and related media as well as in social networks is an important cyber warfare ammunition. In addition, it is well known (but not published) that many national security agencies are digitizing "hard forms" of publicly available information of other countries such as newspapers. These might be related to official, semi-official, or private media-outlets. This media, however, is easier to control by a country that is trying to conceal an epidemical outbreak. One interesting pattern in this situation is the appearance of a few news reports at the beginning of the outbreak, followed by acceleration in reports, followed by a complete seizure of these reports due to a discovery of the news by the country's leaders, followed by a complete ban on this news.

Interestingly, there is another cyber source of information which relates to restrictions that a country might put on travelers entering the country. For example, during the SARS pandemic several countries has required that people entering the country would go through a fast automatic screening of body temperatures.

### ***2.3 Geospatial-Temporal Analytics of Correlation of Environmental Factors and Incidence of Disease***

We have developed tools and concept demonstrations that show the solvability of Big Data problems involving geospatial data correlated with publically available medical data. We bring the Big Data approach to geospatial epidemiology, a field of

study focused on describing and analyzing geographic variations of disease spread, considering demographic, environmental, behavioral, socioeconomic, genetic, and infectious risk factors [8]. Our work in this area assists the development of the related field of personalized medicine by correlating clinical, genetic, environmental, demographic, and other background geospatial data.

Our TerraFly GeoCloud [18] system combines several diverse technologies and components in order to analyze and visualize geospatial data. In this system, a user can upload a spatial dataset and display it using the TerraFly Map API [26]. Datasets can be subsequently analyzed using various functions, such as Kriging, a geo-statistical estimator for unobserved locations, and Spatial Clustering, which involves the grouping of closely related spatial objects. Various analysis functions related to spatial epidemiology have been integrated into TerraFly GeoCloud. Analysis functions can be used by selecting the appropriate dataset and function in the interface menu, along with the variables to be analyzed. TerraFly GeoCloud then processes the data and returns a result that can be visualized on the TerraFly Map or on a chart. Results displayed on the map include a legend, which identifies certain range values by color. Certain visualizations are interactive, allowing additional information to be displayed.

Our Spatial Epidemiology System provides four kinds of API algorithms for data analysis and results visualization, based on the TerraFly GeoCloud System: (1) disease mapping (mortality/morbidity map, SMR map); (2) disease cluster determination (spatial cluster, HotSpot analysis tool, cluster and outlier analysis); (3) geographic distribution measurement (mean central, median central, standard distance, distributional trends); and (4) regression (linear regression, spatial auto-regression). The system is interfaced with our Health Informatics projects [4, 13, 25, 27–29, 40, 43].

We work on tools and methodologies that will assist in operational and analytical Health Informatics. The TerraFly Geospatial Analytics System (<http://terrafly.com>) demonstrates correlation of location to environment-related disorders, enabling clinicians to more readily identify macro-environmental exposure events that may alter an individual's health. It also enables applications in targeted vaccine and disease management, including disease surveillance, vaccine evaluation and follow-up, intelligent management of emerging diseases, cross-analysis of locations of patients and health providers with demographic and economic factors, personalized medicine, and other geospatial and data-intensive applications.

### 3 Tools for Predictions and Evaluations of Fuzzy Events

Fuzzy logic-based techniques are some of the most promising approaches for ECP. The advantage of the fuzzy-based approach is that it enables keeping account on events with perceived low possibility of occurrence via low fuzzy membership/truth-values and updating these values as information is accumulated or changed. Numerous fuzzy logic-based algorithms can be deployed in the data collection, accumulation,

and retention stage, in the information processing phase, and in the decision making process. In this section we describe several possible fuzzy tools to try and predict epidemical crises and cope with evolving epidemical crises via sound ECP programs. We consider the following fuzzy logic-based tools:

1. Fuzzy switching mechanisms
2. Fuzzy expectation and variance
3. Fuzzy relational databases (FRDB), fuzzy data-mining and fuzzy social network architectures (FSNA)
4. Neuro Fuzzy-based Logic, and Systems
5. Complex and multidimensional fuzzy Sets, Logic, and Systems
6. Complex fuzzy graphs
7. Dynamic and incremental fuzzy clustering

### 3.1 Making Decisions with no Data

As an example for this idea we use the fuzzy treatment of the transient behavior of a switching system and its static hazards [12]. Perhaps the major reason for the ineffectiveness of classical techniques in dealing with a static hazard and obtaining a logical explanation of the existence of a static hazard lies in their failure to come to grips with the issue of fuzziness. This is due to the fact that the hazardous variable implies imprecision in the binary system, which does not stem from randomness but from the lack of a sharp transition between members in the class of input states. Intuitively, fuzziness is a type of imprecision that stems from a grouping of elements into classes that do not have sharply defined boundaries—that is, where there is no sharp transition from membership to non-membership. Thus, the transition of a state has a fuzzy behavior during the transition time.

Any fuzzy-valued switching function can be expressed in disjunctive and conjunctive normal forms, in a similar way to two-valued switching functions. A fuzzy-valued switching function over  $n$  variables can be represented by a mapping  $f: [0, 1]^n \rightarrow [0, 1]$ . We define a V-fuzzy function as a fuzzy function  $f(x)$  such that  $f(\xi)$  is a binary function for every binary  $n$ -dimensional vector  $\xi$ . It is clear that a V-fuzzy function  $f$  induces a binary function  $F$  such that  $F: [0, 1]^n \rightarrow [0, 1]$  determined by  $F(\xi) = f(\xi)$  for every binary  $n$ -dimensional vector  $\xi$ .

If a V-fuzzy function  $f$  describes the complete behavior of a binary combinational system, its steady-state behavior is represented by  $F$ , the binary function induced by  $f$ . Let  $f(x)$  be an  $n$ -dimensional V-fuzzy function, and let  $\xi$  and  $\rho$  be adjacent binary  $n$ -dimensional vectors. The vector  $T_{\xi_j}^\rho$  is a static hazard of  $f$  iff  $f(\xi) = f(\rho) \neq f(T_{\xi_j}^\rho)$ .

If  $f(\xi) = f(\rho) = 1$  then  $T_{\xi_j}^\rho$  is a 1-hazard. If  $f(\xi) = f(\rho) = 0$  then  $T_{\xi_j}^\rho$  is a 0-hazard. If  $f$  is V-fuzzy and  $T_{\xi_j}^\rho$  is a static hazard, then  $f(T_{\xi_j}^\rho)$  has a perfect fuzzy value, that is,  $f(T_{\xi_j}^\rho) \in (0, 1)$ . Consider the static hazard as a malfunction



represented by an actual or potential deviation from the intended behavior of the system. We can detect all static hazards of the V-fuzzy function  $f(x)$  by considering the following extension of Shannon normal form. Let  $f(\bar{x})$ ,  $\bar{x} = (x_1, x_2, \dots, x_n)$ , be a fuzzy function and denote the vector

$$(x_1, x_2, x_{j-1}, x_{j+1}, \dots, x_n) \text{ by } x^j.$$

By successive applications of the rules of Fuzzy Algebra, the function  $f(x)$  may be expanded about  $x_j$  as follows:

$$f(x) = x_j f_1(x^j) + \bar{x}_j f_2(x^j) + x_j \bar{x}_j f_3(x^j) + f_4(x^j),$$

where  $f_1, f_2, f_3$ , and  $f_4$  are fuzzy functions. It is clear that the same expansion holds when the fuzzy functions are replaced by B-fuzzy functions of the same dimension. Let  $\xi$  and  $\rho$  be two adjacent n-dimensional binary vectors that differ only in their  $j^{\text{th}}$  component. Treating  $\xi_j$  as a perfect fuzzy variable during transition time implies that  $T_{\xi_j}^\rho$  is a 1-hazard of  $f$  iff  $f(\xi) = f(\rho) = 1$  and  $f(T_{\xi_j}^\rho) \in [0, 1)$ . We show that the above conditions for the vector  $T_{\xi_j}^\rho$  to be 1-hazard, yielding the following result:

**Theorem 1** ([12]): *The vector  $T_{\xi_j}^\rho$  is a 1-hazard of the B-fuzzy function  $f(x)$  given above iff the binary vector  $\xi_j$  is a solution of the following set of Boolean equations:*

$$f_1(\bar{x}^j) = 1, \quad f_2(x^j) = 1, \quad f_4(x^j) = 0.$$

*Proof*

State 1:  $\xi_j = 1$  and  $\bar{\xi}_j = 0$  imply  $f_1(\xi^j) + f_4(\xi^j) = 1$ .

State 2:  $\xi_j = 0$  and  $\bar{\xi}_j = 1$  imply  $f_2(\xi^j) + f_4(\xi^j) = 1$ .

Transition state:  $\xi_j \in (0, 1)$  [which implies  $\bar{\xi}_j \in (0, 1)$ ], and thus:

$$0 \leq \max\{\min[\xi_j, f_1(\xi^j)], \min[\bar{\xi}_j, f_2(\xi^j)], \min[\xi_j, \bar{\xi}_j, f_3(\xi^j)], f_4(\xi^j)\} < 1.$$

It is clear from the transition state that  $f_4(\xi_j)$  cannot be equal to one, and thus:

$$f_4(\xi^j) = 0, \quad f_1(\xi^j) = f_2(\xi^j) = 1.$$

Several items must be pointed out. The system is not a fuzzy system. It is a Boolean system. The modeling of the system as a fuzzy system is due to the lack of knowledge regarding the behavior of  $x_j$  during the transition. It is providing us with a tool to make decisions (regarding the Boolean values of  $f_1, f_2$  and  $f_4$ ) with no data whatsoever regarding  $x^j$ . Thus, we were able to make non-fuzzy decisions in a deterministic environment with no data.

### 3.2 Fuzzy Expectation and Variance

Ordinarily, imprecision and indeterminacy are considered to be statistical, random characteristics and are taken into account by the methods of the Probability Theory. In real situations, a frequent source of imprecision is not only the presence of random variables, but the impossibility, in principle, of operating with exact data as a result of the complexity of the system, or the imprecision of the constraints and objectives. At the same time, classes of objects that do not have clear boundaries appear in the problems; the imprecision of such classes is expressed in the possibility that an element not only belongs or does not belong to a certain class, but that intermediate grades of membership are also possible. The membership grade is subjective; although it is natural to assign a lower membership grade to an event that have a lower probability of occurrence. The fact that the assignment of a membership function of a fuzzy set is “non-statistical” does not mean that we cannot use probability distribution functions in assigning membership functions. As a matter of fact, a careful examination of the variables of fuzzy sets reveals that they may be classified into two types: statistical and non-statistical.

**Definition 1** ([12]): Let  $B$  be a Borel field ( $\sigma$ -algebra) of subsets of the real line  $\Omega$ . A set function  $\mu(\cdot)$  defined on  $B$  is called a fuzzy measure if it has the following properties:

1.  $\mu(\Phi) = 0$  ( $\Phi$  is the empty set);
2.  $\mu(\Omega) = 1$ ;
3. If  $\alpha, \beta \in B$  and  $\alpha \subset \beta$  then  $\mu(\alpha) \leq \mu(\beta)$ ;
4. If  $\{\alpha_j \mid 1 \leq j < \infty\}$  is a monotonic sequence, then

$$\lim_{j \rightarrow \infty} [\mu(\alpha_j)] = \mu[\lim_{j \rightarrow \infty} (\alpha_j)].$$

Clearly,  $\Phi, \Omega \in B$ ; also, if  $\{\alpha_j \mid 1 \leq j < \infty, \alpha_j \in B\}$  is a monotonic sequence then  $\lim_{j \rightarrow \infty} (\alpha_j) \in B$ . In the above definition, (1) and (2) mean that the fuzzy measure is bounded and nonnegative, (3) means monotonicity (in a similar way to finite additive measures used in probability), and (4) means continuity. It should be noted that if  $\Omega$  is a finite set, then the continuity requirement can be deleted.  $(\Omega, B, \mu)$  is called a fuzzy measure space;  $\mu(\cdot)$  is the fuzzy measure of  $(\Omega, B)$ . The fuzzy measure  $\mu$  is defined on subsets of the real line. Clearly,  $\mu[\chi_A \geq T]$  is a non-increasing, real-valued function of  $T$  when  $\chi_A$  is the membership function of set  $A$ . Throughout our discussion, we use  $\xi^T$  to represent  $\{x \mid \chi_A(x) \geq T\}$  and  $\mu(\xi^T)$  to represent  $\mu[\chi_A \geq T]$ , assuming that the  $A$  set is well specified. Let  $\chi^A: \Omega \rightarrow [0, 1]$  and  $\xi^T = \{x \mid \chi_A(x) \geq T\}$ . The function  $\chi_A$  is called a  $B$ -measurable function if  $\xi^T \in B, \forall T \in [0, 1]$ . Definition 2 introduces the fuzzy expected value (*FEV*) of  $\chi_A$  when  $\chi_A \in [0, 1]$ . Extension of this definition when  $\chi_A \in [a, b], a < b < \infty$ , is presented in [12].

**Definition 2** ([12]): Let  $\chi_A$  be a  $B$ -measurable function such that  $\chi_A \in [0, 1]$ . The fuzzy expected value (*FEV*) of  $\chi_A$  over a set  $A$ , with respect to the measure  $\mu(\cdot)$ , is defined as  $\left( T \in [0, 1] \right)^{sup} \{ \min[T, \mu(\xi^T)] \}$ , where  $\xi^T = \{x \mid \chi_A(x) \geq T\}$ . Now,  $\mu\{x \mid \chi_A(x) \geq T\} = f_A(T)$  is a function of the threshold  $T$ . The actual calculation of  $FEV(\chi_A)$  then consists of finding the intersection of the curves  $T = f_A(T)$ . The intersection of the two curves will be at a value  $T = H$ , so that  $FEV(\chi_A) = H \in [0, 1]$ . It should be noted that when dealing with the  $FEV(\eta)$  where  $\eta \in [0, 1]$ , we should not use a fuzzy measure in the evaluation but rather a function of the fuzzy measure,  $\eta'$ , which transforms  $\eta$  under the same transformation that  $\chi$  and  $T$  undergo to  $\eta$  and  $T'$ , respectively. In general the *FEV* has the promise and the potential to be used as a very powerful tool in developing ECP technologies.

### 3.3 Fuzzy Relational Databases and Fuzzy Social Network Architecture

The FRDB model which is based on research in the fields of relational databases and theories of fuzzy sets and possibility is designed to allow representation and manipulation of imprecise information. Furthermore, the system provides means for “individualization” of data to reflect the user’s perception of the data [42]. As such, the FRDB model is suitable for use in fuzzy expert system and other fields of imprecise information-processing that model human approximate reasoning such as FSNA [15, 19].

The objective of the FRDB model is to provide the capability to handle imprecise information. The FRDB should be able to retrieve information corresponding to natural language statements as well as relations in FSNA. Although most of these situations cannot be solved within the framework of classical database management systems, they are illustrative of the types of problems that human beings are capable of solving through the use of approximate reasoning. The FRDB model and the FSNA model retrieve the desired information by applying the rules of fuzzy linguistics to the fuzzy terms in the query.

The FRDB as well as the FSNA development [15, 19, 42] were influenced by the need for easy-to-use systems with sound theoretical foundations as provided by the relational database model and theories of fuzzy sets and possibility. They address the following issues:

1. representation of imprecise information,
2. derivation of possibility/certainty measures of acceptance,
3. linguistic approximations of fuzzy terms in query languages,
4. development of fuzzy relational operators (IS, AS...AS, GREATER, ...),
5. processing of queries with fuzzy connectors and truth quantifiers,
6. null-value handling using the concept of the possibilities expected value,
7. modification of the fuzzy term definitions to suit the individual user.

The FRDB and the FSNA are collections of fuzzy time-varying relations which may be characterized by tables, graphs, or functions, and manipulated by recognition (retrieval) algorithms or translation rules.

As an example, let us take a look at one of these relations, the similarity relation. Let  $D_i$  be a scalar domain,  $y \in D_i$ . Then  $s(x, y) \in [0, 1]$  is a similarity relation with the following properties: Reflexivity:  $s(x, x) = 1$ ; Symmetry:  $s(x, y) = s(y, x)$ ;  $\Theta$ -transitivity: where  $\Theta$  is most commonly specified as max-min transitivity. If,  $y, z \in U$ , then  $s(x, z) \geq \max(y \in D_i) \min(s(x, y), s(y, z))$ . Another example is the proximity relation defined below. Let  $D_i$  be a numerical domain and,  $y, z \in D_i$ . Here  $p(x, y) \in [0, 1]$  is a proximity relation that is reflexive, and symmetric with transitivity of the form  $p(x, z) \geq \max(y \in D_i) p(x, y) * p(y, z)$ .

The generally used form of the proximity relations is  $p(x, y) = e^{-\beta|x-y|}$ , where  $\beta > 0$ . This form assigns equal degrees of proximity to equally distant points. For this reason, it is referred to as the absolute proximity in the FRDB and FSNA models. Similarity and proximity are used in evaluation of queries of the general form: "Find  $X$  such that  $X.A \ominus d$ " Where  $X.A$  is an attribute of  $X$ ,  $d \in D$  is a value of attribute  $A$  defined on the domain  $D$ , and  $\ominus$  is a fuzzy relational operator. Clearly, both FRDS and FSNA may have numerous applications in epidemical outbreak prediction.

In many ECP/DPM programs the amount of information is determined by the amount of the uncertainty—or, more exactly, it is determined by the amount by which the uncertainty has been reduced; that is, we can measure information as the decrease of uncertainty. The concept of information itself has been implicit in many ECP models. That is, both as a substantive concept important in its own right and as a consonant concept that is ancillary to the entire structure of ECP.

### 3.4 Neuro-Fuzzy Systems

The term Neuro-Fuzzy systems refers to combinations of artificial neural networks and Fuzzy logic. Neuro-Fuzzy systems enable modeling human reasoning via fuzzy inference systems along with the modeling of human learning via the learning and connectionist structure of neural networks. Neuro-Fuzzy systems can serve as highly efficient mechanisms for inference and learning under uncertainty. Furthermore, incremental learning techniques can enable observing outliers and the Fuzzy inference can allow these outliers to coexist (with low degrees of membership) with "main-stream" data. As more information about the outliers becomes available, the information, and the derivatives of the rate of information flow, can be used to identify potential epidemical crises that are hidden in the outliers. The classical model of Neuro-Fuzzy systems can be extended to include multidimensional Fuzzy logic and inference systems in numerical domains and in domains characterized by linguistic variables.

Assuming that people form opinions that are fuzzy and that the information exchange between people influences the opinion formation, the opinion formation process is naturally modeled by structures such as fuzzy coupled map networks

and fuzzy and neuro-fuzzy networks [35–39]. In these networks of information aggregation and personal and collective opinion formation, interesting dynamic processes that eventually produce self-organization in structured opinion groups is developed [35, 39].

### 3.5 Complex Fuzzy Systems

Several aspects of the ECP program can utilize the concept of complex fuzzy logic [7, 12, 14, 23, 24, 32–34, 41].

Complex fuzzy logic can be used to represent the two-dimensional information embedded in the description of an epidemical crisis; namely, the severity and uncertainty. In addition, inference based on complex fuzzy logic can be used to exploit the fact that variables related to the uncertainty that is a part of epidemical crises are multi-dimensional and cannot be readily defined via single dimensional clauses connected by single dimensional connectives. Finally, the multi-dimensional fuzzy space defined as a generalization of complex fuzzy logic can serve as a media for clustering of epidemical crisis information in a linguistic variable-based feature space.

Tamir et al. introduced a new interpretation of complex fuzzy membership grade and derived the concept of pure complex fuzzy classes [32]. This section introduces the concept of a pure complex fuzzy grade of membership, the interpretation of this concept as the denotation of a fuzzy class, and the basic operations on fuzzy classes.

To distinguish between classes, sets, and elements of a set we use the following notation: a class is denoted by an upper case Greek letter, a set is denoted by an upper case Latin letter, and a member of a set is denoted by a lower case Latin letter.

The Cartesian representation of the pure complex grade of membership is given in the following way:

$$\mu(V, z) = \mu_r(V) + j\mu_i(z),$$

where  $\mu_r(V)$  and  $\mu_i(z)$ , the real and imaginary components of the pure complex fuzzy grade of membership, are real value fuzzy grades of membership. That is,  $\mu_r(V)$  and  $\mu_i(z)$  can get any value in the interval  $[0, 1]$ . The polar representation of the pure complex grade of membership is given by:

$$\mu(V, x) = r(V)e^{j\sigma\phi(z)},$$

where  $r(V)$  and  $\phi(z)$ , the amplitude and phase components of the pure complex fuzzy grade of membership, are real value fuzzy grades of membership. That is, they can get any value in the interval  $[0, 1]$ . The scaling factor  $\sigma$  is in the interval  $[0, 2\pi]$ . It is used to control the behavior of the phase within the unit circle according to the specific application. Typical values of  $\sigma$  are  $\{1, \frac{\pi}{2}, \pi, 2\pi\}$ . Without loss of generality, for the rest of the discussion in this section we assume that  $\sigma = 2\pi$ .

The difference between pure complex fuzzy grades of membership and the complex fuzzy grade of membership proposed by Ramot et al. [23, 24] is that both components of the membership grade are fuzzy functions that convey information about a fuzzy set. This entails different interpretation of the concept as well as a different set of operations and a different set of results obtained when these operations are applied to pure complex grades of membership. This is detailed in the following sections.

### 3.5.1 Complex Fuzzy Class

A fuzzy class is a finite or infinite collection of objects and fuzzy sets that can be defined in an unambiguous way and comply with the axioms of fuzzy sets given by Tamir and Kandel [33] and the axioms of fuzzy classes given by [3, 6]. While a general fuzzy class can contain individual objects as well as fuzzy sets, a *pure fuzzy class of order one* can contain only fuzzy sets. In other words, individual objects cannot be members of a pure fuzzy class of order one. A pure fuzzy class of order  $M$  is a collection of pure fuzzy classes of order  $M - 1$ . We define a *Complex Fuzzy Class*  $\Gamma$  to be a pure fuzzy class of order one, i.e., a fuzzy set of fuzzy sets. That is,  $\Gamma = \{V_i\}_{i=1}^{\infty}$ ; or  $\Gamma = \{V_i\}_{i=1}^N$  where  $V_i$  is a fuzzy set and  $N$  is a finite integer. Note that despite the fact that we use the notation  $\Gamma = \{V_i\}_{i=1}^{\infty}$  we do not imply that the set of sets  $\{V_i\}$  is enumerable. The set of sets  $\{V_i\}$  can be finite, countably infinite, or uncountably infinite. The use of the notation  $\{V_i\}_{i=1}^{\infty}$  is just for convenience.

The class  $\Gamma$  is defined over a universe of discourse  $U$ . It is characterized by a pure complex membership function  $\mu_{\Gamma}(V, z)$  that assigns a complex-valued grade of membership in  $\Gamma$  to any element  $z \in U$ . The values that  $\mu_{\Gamma}(V, z)$  may receive lie within the unit square or the unit circle in the complex plane, and are in one of the following forms:

$$\begin{aligned}\mu_{\Gamma}(V, z) &= \mu_r(V) + j\mu_i(z), \\ \mu_{\Gamma}(z, V) &= \mu_r(z) + j\mu_i(V),\end{aligned}$$

where  $\mu_r(\alpha)$  and  $\mu_i(\alpha)$ , are real functions with a range of  $[0, 1]$ .

Alternatively:

$$\begin{aligned}\mu_{\Gamma}(V, z) &= r(V)e^{j\theta\phi(z)}, \\ \mu_{\Gamma}(z, V) &= r(z)e^{j\theta\phi(v)},\end{aligned}$$

where  $r(\alpha)$  and  $\phi(\alpha)$ , are real functions with a range of  $[0, 1]$  and  $\theta \in (0, 2\pi]$ .

In order to provide a concrete example we define the following pure fuzzy class. Let the universe of discourse be the set of all the pandemics that hit the U.S. (in any time in the past) along with a set of attributes related to the pandemic, such as spread mechanism, speed of spread, symptoms, etc. Let  $M_i$  denote the set of pandemics that hit the U.S. in the last  $i$  years. Furthermore, consider a function ( $f_1$ ) that associates

a number between 0 and 1 with each set of pandemics. For example, this function might reflect the severity of the pandemics in terms risk to affected people. In addition, consider a second function ( $f_2$ ) that associates a number between 0 and 1 with each specific epidemic. For example, this function might denote the incubation time of the relevant micro-organisms. The functions ( $f_1, f_2$ ) can be used to define a pure fuzzy class of order One. A compound of the two functions in the form of a complex number can represent the degree of membership in the pure fuzzy class of “high risk pandemics” in the set of pandemics that have occurred in the last 10 years.

Formally, let  $U$  be a universe of discourse and let  $2^U$  be the powerset of  $U$ . Let  $f_1$  be a function from  $2^U$  to  $[0, 1]$  and let  $f_2$  be a function that maps elements of  $U$  to the interval  $[0, 1]$ . For  $V \in 2^U$  and  $z \in U$  define  $\mu_\Gamma(V, z)$  to be:

$$\mu_\Gamma(V, z) = \mu_r(V) + j\mu_i(z) = f_1(V) + jf_2(z)$$

Then,  $\mu_\Gamma(V, z)$  defines a pure fuzzy class of order one, where for every  $V \in 2^U$ , and for every  $z \in U$ ,  $\mu_\Gamma(V, z)$ , is the degree of membership of  $z$  in  $V$  and the degree of membership of  $V$  in  $\Gamma$ . Hence, a complex fuzzy class  $\Gamma$  can be represented as the set of ordered triples:  $\Gamma = \{V, z, \mu_\Gamma(V, z) \mid V \in 2^U, z \in U\}$

Depending on the form of  $\mu_\Gamma(\alpha)$  (Cartesian or polar),  $\mu_r(\alpha)$ ,  $\mu_i(\alpha)$ ,  $r(\alpha)$ , and  $\phi(\alpha)$  denote the degree of membership of  $z$  in  $V$  and/or the degree of membership of  $V$  in  $\Gamma$ . Without loss of generality, however, we assume that  $\mu_r(\alpha)$  and  $r(\alpha)$  denote the degree of membership of  $V$  in  $\Gamma$  for the Cartesian and the polar representations respectively. In addition, we assume that  $\mu_i(\alpha)$  and  $\phi(\alpha)$  denote the degree of membership of  $z$  in  $V$  for the Cartesian and the polar representations respectively. Throughout this chapter, the term *complex fuzzy class* refers to a pure fuzzy class with pure complex-valued membership function, while the term *fuzzy class* refers to a traditional fuzzy class such as the one defined by [3].

### Degree of Membership of Order $N$

The traditional fuzzy grade of membership is a scalar that defines a fuzzy set. It can be considered as degree of membership of order 1. The pure complex degree of membership defined in this chapter is a complex number that defines a pure fuzzy class. That is, a fuzzy set of fuzzy sets. This degree of membership can be considered as degree of membership of order 2 and the class defined can be considered as a pure fuzzy class of order 1. Additionally, one can consider the definition of a fuzzy set (a class of order 0) as a mapping into a one-dimensional space and the definition of a pure fuzzy class (a class of order 1) as a mapping into a two-dimensional space. Hence, it is possible to consider a degree of membership of order  $N$  as well as a mapping into an  $N$ -dimensional space. The following is a recursive definition of a fuzzy class of order. Note that part 2 of the definition is not really necessary, it is given in order to connect the terms pure complex fuzzy grade of membership and the term grade of membership of order 2.

**Definition 3** ([32]):

- (1) A fuzzy class of order 0 is a fuzzy set; it is characterized by a degree of membership of order 1 and a mapping into a one dimensional space.
- (2) A fuzzy class of order 1 is a set of fuzzy sets. It is characterized by a pure complex degree of membership. Alternatively, it can be characterized by a degree of membership of order Two and a mapping into a two-dimensional space.
- (3) A fuzzy class of order  $N$  is a fuzzy set of fuzzy classes of order  $N - 1$ ; it is characterized by a degree of membership of order  $N + 1$  and a mapping into an  $(N + 1)$ -dimensional space.

Generalized Complex Fuzzy Logic

A general form of a complex fuzzy proposition is: “ $x \dots A \dots B \dots$ ” where  $A$  and  $B$  are values assigned to linguistic variables and ‘ $\dots$ ’ denotes natural language constants. A complex fuzzy proposition  $P$  can get any pair of truth values from the Cartesian interval  $[0, 1] \times [0, 1]$  or the unit circle. Formally a fuzzy interpretation of a complex fuzzy proposition  $P$  is an assignment of fuzzy truth value of the form  $p_r + jp_i$ , or of the form  $r(p)e^{j\theta(p)}$ , to  $P$ . In this case, assuming a proposition of the form “ $x \dots A \dots B \dots$ ,” then  $p_r(r(p))$  is assigned to the term  $A$  and  $p_i(\theta(p))$  is assigned to the term  $B$ .

For example, under one interpretation, the complex fuzzy truth value associated with the complex proposition:

$x$  is a young person that lives close to the north pole of jupiter

can be  $0.1 + j0.5$ . Alternatively, in another context, the same proposition can be interpreted as having the complex truth value  $0.3e^{j0.2}$ . As in the case of traditional propositional fuzzy logic, we use the tight relation between complex fuzzy classes / complex fuzzy membership to determine the interpretation of connectives. For example, let  $C$  denote the complex fuzzy set of “young people that live close to the north pole of jupiter,” and let  $f_c = c_r + jc_i$ , be a specific fuzzy membership function of  $C$ , then  $f_c$  can be used as the basis for interpretations of  $P$ . Next we define several connectives along with their interpretation.

Table 1 includes a specific definition of connectives along with their interpretation. In this table,  $P$ ,  $Q$  and  $S$  denote complex fuzzy propositions and  $f_S$  denotes the complex fuzzy interpretation of  $S$ . We use the fuzzy Łukasiewicz logical system as

**Table 1** Basic propositional fuzzy logic connectives

Operation	Interpretation
Negation	$f(\neg P) = 1 + j1 - f(P)$
Disjunction	$f(P \oplus Q) = \max(p_R, q_R) + j \times \max(p_I, q_I)$
Conjunction	$f(P \otimes Q) = \min(p_R, q_R) + j \times \min(p_I, q_I)$
Implication	$f(P \longrightarrow Q) = \min(1, 1 - p_R + q_R) + j \times \min(1, 1 - p_I + q_I)$



the basis for the definitions [5, 9]. Hence, the max t-norm is used for conjunction and the min t-conorm is used for disjunction. Nevertheless, other logical systems, such as Gödel fuzzy systems, can be used [5, 20].

The axioms used for fuzzy logic are used for complex fuzzy logic, and Modus ponens is the rule of inference.

### Complex Fuzzy Propositions and Connectives Examples

Consider the following propositions ( $P$ ,  $Q$ , and  $S$  respectively):

$P$ : “x is a young person that lives close to the north pole of Jupiter”.

$Q$ : “x has elevated body temperature with a severe headache”.

$S$ : “x is closely monitored due to high risk of acquiring the pandemic”.

Let  $A$  be the term “*young person*,” and let  $B$  denote the term “*close to the north pole of Jupiter*.” Furthermore, let  $C$  be the term “*elevated body temperature*,” (alternatively, the term “*high fever*” can be used) and let  $D$  denote the term “*severe headache*” Hence,  $P$  is of the form: “x is a  $A$  that  $B$ ,” and  $Q$  is of the form “x is  $C$  with  $D$ .” In this case, the terms “*young person*,” “*close to the north pole of Jupiter*,” “*high fever*,” and “*severe headache*” are values assigned to linguistic variables. Furthermore, a term such as “*headache*,” can get fuzzy truth values (between 0 and 1) or fuzzy linguistic values such as “*minor*,” “*mild*,” and “*severe*,” (the terms “*that*,” and “*with*,” are linguistic constants). Assume that the complex fuzzy interpretation (i.e., degree of confidence or complex fuzzy truth value) of  $P$  is  $p_r + jp_i$ , while the complex fuzzy interpretation of  $Q$  is  $q_r + jq_i$ . Thus, the truth value of “x is a *young person*,” is  $p_r$ , and the truth value assigned to “*x lives close to the north pole of Jupiter*,” is  $p_i$ . The truth value of “x has high fever,” is  $q_r$ , and the truth value of “x has a severe headach,” is  $q_i$ . Suppose that the term “*old*” stands for “*not young*,” the term “*far*,” stands for “*not close*,” the term “*low*,” stands for “*not high*,” and the term “*no headache*” denotes the negation of “*severe headache*.” In a similar way,  $S$  is of the form: “*x is E due to F*,” where the complex fuzzy interpretation of  $S$  is  $s_r + js_i$ . Note that this is not the only way to define these linguistic terms and it is used to exemplify the expressive power and the inference power of the logic. Then, the complex fuzzy interpretation of the following composite propositions is:

$$(1) f('P) = (1 - p_r) + j(1 - p_i)$$

That is,  $'P$  denotes the proposition

“x is an old person that lives close to the north pole of Jupiter”.

The confidence level in  $'P$  is  $(1 - p_r) + j(1 - p_i)$ ; where the fuzzy truth value of the term “x is an old person,” is  $(1 - p_r)$  and the fuzzy truth value of the term “...lives far ...,” is  $(1 - p_i)$

$$(2) f(P \oplus 'Q) = \max(p_r, 1 - q_r) + j \times \max(p_i, 1 - q_i).$$

That is,  $(P \oplus 'Q)$  denotes the proposition

“x is a young person that lives close to the north pole of Jupiter”. OR

“x has low fever and no headach”. The truth values of individual terms, as well as the truth value of  $P \oplus 'Q$  are calculated according to Table 1.

(3)  $f(P \oplus Q) = \min(1 - p_r, q_r) + j \times \min(1 - p_i, q_i)$ .

That is,  $(P \oplus Q)$  denotes the proposition

“x is an old person that lives far from the north pole of Jupiter”. AND

“x has high fever and severe headach”. The truth values of individual terms, as well as the truth value of  $P \oplus Q$  are calculated according to Table 1.

(4) Let the term  $R$  stand for  $(P \oplus Q)$ , (the complex fuzzy interpretation of  $R$  is  $r_r + jr_i$ .) then,  $R \longrightarrow S = \min(1, 1 - r_r + s_r) + j \times \min(1, 1 - r_i + s_i)$

Thus,  $(R \longrightarrow S)$  denotes the proposition

IF “x is a young person that lives close to the north pole of Jupiter”. AND

“x has high fever and severe headach”.

THEN

“x is closely monitored due to high risk of acquiring the pandemic disease”. The truth values of individual terms, as well as the truth value of are calculated according to Table 1.

**Complex Fuzzy Inference Example**

Assume that the degree of confidence in the proposition  $R$  defined above is  $r_r + jr_i$ , and assume that the degree of confidence in the fuzzy implication  $T = R \longrightarrow S$  is  $t_r + jt_i$ . Then, using Modus ponens

$$\begin{array}{c} R \\ R \longrightarrow S \\ \hline S \end{array}$$

one can infer  $S$  with a degree of confidence  $\min(r_r, t_r) + j \times \min(r_i, t_i)$ .

In other words if one is using:

“x is a young person that lives close to the north pole of Jupiter”

AND “x has high fever and severe headach”.

IF

“x is a young person that lives close to the north pole of Jupiter”

AND “x has high fever and severe headach”.

THEN

“x is closely monitored due to high risk of acquiring the pandemic”.

“x is closely monitored due to high risk of acquiring the pandemic”.

Hence, using Modus ponens one can infer:

“x is closely monitored due to high risk of acquiring the pandemic disease”. with a degree of confidence of  $\min(r_r, t_r) + j \times \min(r_i, t_i)$  .

**3.6 Fuzzy Graph Theory**

Graph theory and in specific fuzzy graph theory can be used for deriving algorithms for early identification of pandemic outbreaks. In this section we provide the basic definitions and list some of the relevant algorithms. A literature search performed

has revealed inconsistencies in the definitions. For this reason, we review some of the definitions of non-fuzzy graphs and provide a detailed and precise definition of the basic terms related to fuzzy graphs.

### 3.6.1 Non-fuzzy Graphs

A directed graph  $G$  is a tuple of the form  $G = (V, E, \varphi)$ , where  $V$  is a set referred to as the set of vertices.  $E$  is a set of edges, and  $\varphi$  is a function  $\varphi: E \rightarrow V \times V$ , such that for every  $e \in E$ ,  $\varphi(e) = (u, v)$ , where  $u \in V$  and  $v \in V$ . We assume that  $V \cap E = \emptyset$  and in general, we use the form  $e = (a, b)$  to denote a specific edge that is “said” to connect the vertices  $a$  and  $b$ . For an undirected graph, both  $e_1 = (u, v)$  and  $e_2 = (v, u)$  are in the domain/range of  $\varphi$ . A relation  $E \subseteq V \times V$  can be used as an implicit definition of an undirected graph with a set of vertices  $V$  and a set of edges  $E$ . A weighted graph  $G$  is a quintuple  $G = (V, E, \varphi, w_1, w_2)$ , where  $V, E$ , and  $\varphi$  are defined before and  $w_1: V \rightarrow R; w_2: E \rightarrow R$ ; are functions that map vertices and or edges to the set of real numbers  $R$  (it is possible but less common to assign complex weights to vertices and edges).

We list some of the important terms and algorithms related to non-fuzzy graphs. These terms and algorithms can be found in numerous textbooks [2]. The fundamental terms related to graphs are the *order of the graph*, the *order of a vertex*, and the *connectedness of the graph*. Other fundamental terms related to graphs are *complete graphs*, *planner graphs* and *simple graphs*, *sub-graphs*, *spanning sub-graphs*, *cliques*, *paths*, *cycles*, *tours*, *connectivity*, *Euler tours*, *Euler cycles*, *Hamiltonian tours*, *Hamiltonian cycles*, *forests*, and *trees*. The fundamental algorithms applied to weighted graphs are: (1) finding the shortest path between vertices, (2) finding the minimum spanning tree, (3) identifying maximal cliques, (4) finding the minimal Euler tour/cycle and (5) finding the minimal Hamiltonian tour/cycle. In this context, *short*, *max* and *min* might relate to the number of vertices/edges or the sum of weights of the relevant vertices/edges.

### 3.6.2 Fuzzy Graphs

A fuzzy directed graph  $G$  is a quadruple of the form  $\hat{G} = (\hat{V}, \sigma, \hat{E}, \varphi)$ , where  $\hat{V}$  is a set referred to as the set of vertices and  $\hat{E} \subseteq \hat{V} \times \hat{V}$  is a set of edges,  $\sigma: \hat{V} \rightarrow [0, 1]$  is a mapping (function) from  $\hat{V}$  to  $[0, 1]$  (i.e.,  $\sigma$  is the assignment of degrees of membership to members of  $\hat{V}$ ), and  $\varphi: \hat{E} \rightarrow [0, 1]$  is a function that maps elements of the form  $e \in \hat{E} = (u, v)$ , to  $[0, 1]$  (i.e.,  $\varphi$  is the assignment of degrees of membership to members of  $\hat{E}$ ), where  $u \in \hat{V}$  and  $v \in \hat{V}$ . We assume that  $\hat{V} \cap \hat{E} = \emptyset$  and in general, we use the form  $e = (a, b)$  to denote a specific edge that is “said” to connect the vertices  $a$  and  $b$ . For an undirected graph, both  $e_1 = (u, v)$  and  $e_2 = (v, u)$  are in the domain of  $\varphi$ . A weighted fuzzy graph  $G$  is an sextuple  $\hat{G} = (\hat{V}, \sigma, w_1 \hat{E}, \varphi, w_2)$ , where  $\hat{V}, E, \sigma$ , and  $\varphi$  are as defined previously,  $w_1: \hat{V} \rightarrow R$ ; and  $w_2: \hat{E} \rightarrow R$  are functions that map vertices and or edges to the

set of real numbers  $R$  (it is possible but less common to assign complex weights to vertices and edges).

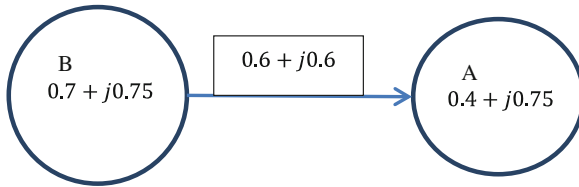
Following these definitions, a fuzzy graph is a special case of a non-fuzzy weighted graph. Hence, many of the algorithms applied to weighted graphs might be of interest when fuzzy graph and their semantics are considered. Moreover, without loss of generality, we can assume that weights represented by real numbers are normalized to the range of  $[0, 1]$ . In this case the vertices and/or the edges can be represented via a complex number representing the degree of membership of the vertex/edge in the graph. Clearly, a non-weighted fuzzy graph is a special case of a “regular” fuzzy graph. Fuzzy graphs were used in neuro-fuzzy models of information propagation and aggregation, including opinion formation [37, 39]. In the next section we provide important definitions related to complex fuzzy graphs.

### 3.6.3 Complex Fuzzy Graphs

A complex fuzzy directed graph  $\tilde{G}$  is a quadruple of the form  $\tilde{G} = (\tilde{V}, \sigma, \tilde{E}, \varphi)$ , where  $\tilde{V}$  is a complex fuzzy set referred to as the set of vertices and  $\tilde{E} \subseteq \tilde{V} \times \tilde{V}$  is a complex fuzzy set of edges,  $\sigma: \tilde{V} \rightarrow [0, 1] \times [0, 1]$  is a mapping from  $\tilde{V}$  to  $[0, 1] \times [0, 1]$  (i.e.,  $\sigma$  is the assignment of a complex degrees of membership to members of  $\tilde{V}$ ), and  $\varphi: \tilde{E} \rightarrow [0, 1] \times [0, 1]$  is a function that maps elements of the form  $e \in \tilde{E} = (u, v)$ , to  $[0, 1] \times [0, 1]$  (i.e.,  $\varphi$  is the assignment of complex degrees of membership to members of  $\tilde{E}$ ), where  $u \in \tilde{V}$  and  $v \in \tilde{V}$ . We assume that  $\tilde{V} \cap \tilde{E} = \emptyset$  and in general, we use the form  $e = (a, b)$  to denote a specific edge that is “said” to connect the vertices  $a$  and  $b$ . For an undirected graph, both  $e_1 = (u, v)$  and  $e_2 = (v, u)$  are in the domain of  $\varphi$ . Note that the use of complex fuzzy logic is a very strong “tool” that enables dealing with the edges/vertices as carrying complex fuzzy membership values. Hence, it enables exploiting the features of complex fuzzy set theory, complex fuzzy set theory, and complex fuzzy inference. In general, one can use the two components of the complex number assigned to vertices/edges as denoting complex fuzzy information. Alternatively one can use one of the two components as a real fuzzy value and the second component as a weight. To illustrate we provide the following example.

#### Complex Fuzzy Graph Example

Consider a pandemic that adversely affects “young” people that live in the “north” part of Jupiter. The main initial symptoms of the pandemic disease are: (1) “high” fever, (2) “severe” headaches. Many of the affected people are starting to post status and queries to a social network. While they do not clearly disclose infection, their status/queries might be indicative of a pandemic outbreak. Furthermore, assume that Bob, who is 27 years old and lives “close” to the north pole of Jupiter, sends a “Twitter<sup>®</sup>” type of message to Alice, who is 57 years old and lives in the same area. The message reads “Staying home today.” Alice responds with “What’s wrong?” Bob response is “I have a headache and a bit of fever”. Figure 1 depicts some of this information in a complex fuzzy graph.



**Fig. 1** A complex fuzzy graph representing pandemic related communication

In this graph the vertex ‘B’ represents Bob along with its degree of membership in a complex fuzzy class. The vertex ‘A’ represents the same for Alice and the directed edge from Bob to Alice represents the part of the communication between Bob and Alice where Bob reveals his membership in yet another complex fuzzy class. This type of graphs can be used to represent information, actions, and inference.

### 3.6.4 Complex Fuzzy Graphs’ Features and Algorithms

The order of a complex fuzzy graph  $\tilde{G}$  is the cardinality of  $\tilde{V}$  (denoted as  $|\tilde{V}|$ ). The order of a vertex in  $\tilde{G}$  is the number of edges incident to the vertex. Other terms related to complex fuzzy graphs are *complete graphs*, *planner graphs* and *simple graphs*, *sub graphs*, *spanning sub-graphs*, *cliques*, *paths*, *cycles*, *tours*, *Euler tours*, *Euler cycles*, *Hamiltonian tours*, *Hamiltonian cycles*, *forests*, and *trees*. The fundamental algorithms applied to complex fuzzy graphs are: (1) finding the shortest path between vertices, (2) finding the minimum spanning tree, (3) identifying maximal cliques, (4) finding the minimal Euler tour/cycle, and (5) finding the minimal Hamiltonian tour/cycle. In this context, *short*, *max* and *min*, might relate to the number of vertices/edges or the sum of weights of the relevant vertices/edges. These terms and algorithms are derived from their definitions in the context of non-fuzzy graphs [2] and fuzzy graphs [22]. We are currently working on the extension of these algorithms to complex fuzzy graphs. One interesting and relevant example is finding the maximal complex fuzzy clique in a complex fuzzy graph. In this example, we expand the work reported in [22] and use a neuro-fuzzy system as a “tool” for addressing the problem.

## 3.7 Dynamic and Incremental Fuzzy Clustering

Clustering is a widely used mechanism for pattern recognition and classification. Fuzzy clustering (e.g., the Fuzzy C-means) enables patterns to become members of more than one cluster. Additionally, it enables maintaining clusters that represent outliers through low degree of membership. These clusters would be discarded in clustering of hard (vs. Fuzzy) data. Incremental and dynamic clustering

(e.g., the incremental Fuzzy ISODATA) enable the clusters' structures to change as information is accumulated. Again, this is a strong mechanism for enabling identification of unlikely events without premature discarding of these events. The clustering can be performed in a traditional feature space composed of numerical measurements of feature values. Alternatively, the clustering can be performed in a multidimensional Fuzzy logic space where the features represent values of linguistic variables. The combination of powerful classification capability, adaptive and dynamic mechanisms, as well as the capability to consider uncertain data, maintain data with low likelihood of occurrence, and use a combination of numerical and linguistic values makes this tool, one of the most promising tools for predicting epidemical crisis outbreaks. We currently conduct research on dynamic and incremental fuzzy clustering and it is evident that the methodology can serve as a highly efficient tool for identifying outliers. We plan to report on this research in the near future.

## 4 Conclusion

In this chapter, we have outlined features of epidemical crises outbreaks. We have shown that an important challenge related to an epidemical crisis is the identification of slow-evolving uncertain events that points to the potential of occurrence of the crisis before it occurs and of fast-evolving data concerning the secondary effect of epidemical crises after the occurrence of primary crisis. We have outlined a set of fuzzy logic-based tools that can be used to address these and other challenges related to ECP.

Recent epidemical crises are showing that there is still a lack of technology-based tools, particularly specific decision-support tools, for addressing epidemical crises, mitigating their adverse impact, and managing crisis response programs. Additional activities that will assist in ECP programs include [17]:

1. Accelerated delivery of technical capabilities for ECP
2. Preparation for an uncertain future
3. Development of world-class science, technology, engineering and mathematics (STEM) capabilities

On top of these important tasks, one should never forget that in the development of ECP programs we do not have the luxury of neglecting human intelligence [16]. In any fuzzy event related to a gray swan, investigation after the fact reveals enough clear data points which had been read correctly but had not been treated properly.

In the future, we intend to investigate the ECP utility of several additional fuzzy logic-based tools including:

1. Value-at-Risk (VaR) under fuzzy uncertainty
2. Non-cooperative fuzzy games
3. Fuzzy logic-driven web crawlers and web-bots
4. Fuzzy Expert Systems and Fuzzy Dynamic Forecasting

Finally, we plan to expand our research on complex fuzzy graphs and their applications, complex fuzzy logic-based neuro-fuzzy systems, and research on incremental and dynamic fuzzy clustering. These research threads are expected to provide significant advancement to our capability to identify and neutralize (as much as possible) primary and secondary adverse effects of epidemical crises.

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