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Technical Paper

Disruption management in a constrained multi-product imperfect production system

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ABSTRACT

Over several decades, production and inventory systems have been widely studied in different aspects, but only a few studies have considered the production disruption problem. In production systems, the production may be disrupted by priorly unknown disturbance and the entire manufacturing plan can be distorted. This research introduces a production-disruption model for a multi-product single-stage production-inventory system. First, a mathematical model for the multi-item production-inventory system is developed to maximize the total profit for a single-disruption recovery-time window. The main objective of the proposed model is to obtain the optimal manufacturing batch size for multi-item in the recovery time window so that the total profit is maximized. To maintain the matter of multi-product, budget and space constraints are used. A genetic algorithm and pattern search techniques are employed to solve this model and all randomly generated test results are compared. Some numerical examples and sensitivity analysis are given to explain the effectiveness and advantages of the proposed model. This proposed model offers a recovery plan for managers and decision-makers to make accurate and effective decisions in real time during the production disruption problems.

1. Introduction

A supply chain comprises of facilities and entities that are involved in transforming raw materials into completed products and later delivered to end customers through the supplier. Mostly, conventional supply chains are designed for problem-free and smoothly-operating environments. However, in reality, some unexpected events may occur during the production process such as labor strikes, natural disasters, machine breakdowns, raw materials shortage, and transportation problems. These events are almost unavoidable and cause disruptions at different stages of a supply chain system. In real-life supply chain environments, production disruption is one of very familiar disruptions/interruptions. In literature, production disruption is defined as any form of disturbance during a production process, including power cut, tool failure, machine breakdown, material shortage, or any type of man-made or accidental interruption [39,40]. In 1995, a disastrous earthquake hit Kobe and destroyed all available transportation links to the site. This earthquake affected the Toyota production setup negatively and a large amount of revenue was lost due to this unavoidable event [50]. Recovering from production interruptions due to these kinds of events without having a proper response results in an additional cost for the organization, which is considerably high [31]. The most recent example of this type of production disruptions is the COVID-19 epidemic and it caused manufacturers like Hyundai and Fiat Chrysler Automobiles NV to halt their productions [16].

Batch production systems are a well-known and very popular technique in advanced manufacturing setups [5,46]. In these manufacturing setups, both single and multiple products depending on the type of products are produced and delivered in batches. Batch production helps to reduce cost and increase profitability, the production lot size is determined to minimize the cost of the manufacturing system. There are numerous industries that produce single or multiple products using the batch production technique [27,48,53]. However, in real-life cases, there are lots of risk factors involved and these should be taken into consideration when a system is analyzed [37]. The system may face production interruptions due to the above mentioned inevitable events or any other type of production system failure. Because of the imperfect production, the process reliability (which is usually less than 100%) is also an important factor in real-life production systems. It is very difficult to obtain a system which produces all perfect products.

Production disruption may affect the organization financially and the reputation of the organization in the market may suffer due to the shortage of goods and customer demand that is not fulfilled during this period. To avoid financial and reputation losses, the organization

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should have a recovery plan for these types of inevitable events [38]. The organization can use such a plan in a time of need and quickly alleviate the disruption effect. Another aspect of production systems is the impact they have on the environment and human lives. This is why production disruption has recently gathered the interest of researchers. Moreover, reliability of the system is highly related to production disruption, and if the system is more reliable, the chances of moving from *in-control* to *out-of-control* are rare, resulting in a less imperfect production [19]. Usually, rejected imperfect products are either reworked to obtain perfect quality or disposed. Disposing of rejected products is more dangerous for the environment and should be avoided.

1.1. Literature review

One of the most widely studied research topics is the analysis of production-inventory systems. In early studies, researchers developed models under ideal conditions with simple assumptions. Later, a single-item imperfect production inventory control model with reworking was considered by Sana [43]. Pal et al. [30] incorporated price dependent production rate in an inventory model under imperfect conditions. Sarkar et al. [45] studied a single-stage production-inventory model for rework at the same production setup with random defective rates and backorders. Cárdenas-Barrón and Sana [8] investigated a two-echelon supply chain model for multiple items under promotion dependent demand and delayed payments. Moreover, Cárdenas-Barrón et al. [6] proposed a multi-period supplier selection problem for multiple items in an inventory model. An improved multi-stage imperfect production model was analyzed by Kim and Sarkar [20] under controllable lead time. Further, Cheikhrouhou et al. [9] developed an inventory inspection model for order size and sample size with the return of imperfect products. Recently, Malik and Sarkar [26] studied an inventory control model for a coordination supply chain management to reduce the leadtime by considering two different modes of transportation. Malik and Kim [24] introduced a flexible production model with production rate and economic lot size optimization considering the relation between production rates and carbon emissions.

Multi-product production inventory models have been considered by several researchers for previous two decades. A multi-product optimal production quantity model was developed by Pasandideh et al. [32] with imperfect production, permissible rework, and a space constraint. Taleizadeh et al. [51] analyzed a multi-item production model with limited production capacity and service level constraint. Further, Taleizadeh et al. [52] updated the imperfect production model with a random defective rate under limited production capacity. They minimized total cost on optimal values of back ordered quantity, cycle length, and production quantity. Pirayesh and Poormoaied [41] solved a multi-item imperfect production model under limited production capacity by using the genetic algorithm and swarm optimization. Pasandideh et al. [33] developed a multi-item lot size inventory model considering budget and space constraints. Recently, Malik and Sarkar [25] analyzed a multi-product model with uncertain demand, service level and storage space constraints to optimize the order quantity and process quality.

Recently, researchers focused on developing different types of disruption recovery models for supply chain and production-inventory systems. They investigated supply chain systems for several types of disruptions such as supply disruption [21,29], demand disruption [14,1,17], transportation disruption [57,13,34], machine breakdowns [22,2,59] and production disruption [39]. One can find a recent review on rescheduling of production systems in Uhlmann and Frazzon [55].

Studies for optimal recovery strategies in production disruptions are scarce and limited. Xia et al. [58] introduced a disruption management general approach for the two-stage production-inventory system. They incorporated a penalty cost of deviations from the original plan to the new plan and a recovery time window for production disruption. The work of Xia et al. [58] was extended for real-time recovery strategy in a single-stage perfect production-inventory system, by Hishamuddin et al. [12]. The model considered constrained non-linear optimization problem for a known disruption period. Two different solution methodologies, heuristic and evolutionary algorithms were used and compared the results. Paul et al. [37] suggested a real-time production disruption management plan for a two-stage batch production system under the reliability considerations. Initially, they developed a single (independent) production disruption recovery model and then extended it to multiple (dependent) disruptions scenarios.

Further, Paul et al. [39] extended model of Hishamuddin et al. [12] to imperfect production system with random production disruptions and a uniform random distribution was followed by disruption occurrences. They introduced a mathematical model that deals with both dependent and independent (single and multiple) disruptions on a real-time basis. Moreover, a three-stage production-inventory model with single and multiple disruptions was proposed by Paul et al. [38]. They considered perfect production and presented a heuristic solution and a standard search algorithm to solve the model. Paul et al. [40] proposed a three-stage supply chain model for mitigation of production disruptions. They developed three different models for different scenarios, an ideal plan with infinite planning horizon, a predictive disruption mitigation plan, and a reactive disruption mitigation plan. The model was solved by a heuristic approach and optimization with LINGO software and compared the results of both.

Another important factor that should be considered when developing a production-inventory model is the process reliability, which has a significant impact on system cost and profit. Lulu and Black [23] studied the multi-component manufacturing and assembly environment and analyzed the impact of process unreliability on the performance of the manufacturing production system. They proved that process unreliability results in lower system utilization. Cheng [11] defined the production process reliability as the percentage of non-defective products produced in a system. He considered process reliability for a single-period inventory system and formulated it as a geometric problem. A production model with production process reliability was developed by Bag et al. [3]. They considered the set-up cost, production period, and process reliability as the decision variables for that model. Sarkar [44] analyzed an economic manufacturing quantity model for an imperfect production process under the effects of inflation while considering reliability as the decision variable. Paul et al. [36] analyzed a production-inventory system using process reliability and uncertain demand.

Furthermore, Paul et al. [35] introduced a two-stage batch production-inventory system for real-time disruption management under consideration of the process reliability. They formulated the model for both kinds of disruptions, single and multiple (dependent and independent), and revised the solution with changed parameters. See Table 1 for the contributions from different authors, only the most relevant papers are included. The research contributions regarding the production type, disruption type, reliability, and constraints are given.

In previous studies, no one has considered the production-disruption problem for multi-item production-inventory systems. In this study, the production disruption model for a single-stage single-product production-inventory system introduced by Hishamuddin et al. [12] is considered. They considered a perfect production-inventory system facing production disruptions during the production cycle. In their model, they assumed all the produced products are perfect. However, in real life situations production systems also produce some defective products due to system limitations. Production disruptions due to inevitable events such as natural disasters or machine breakdowns cannot be predicted in advance. However, when the production system experiences disruption, the original plan is revised and updated quickly to come to alleviate the effects of the disruption. Till now in literature, no study is available regarding multi-product imperfect production systems under disruption.

This research considered a single-stage multi-product imperfect

Table 1
Summary of different authors' contributions.

Author(s)	Production type	Disruption	Reliability	Constraint
Banerjee [4]	Single-item, perfect production	–	–	–
Widyadana and Wee [56]	Single-stage, single-item, perfect production	–	–	–
Sarkar [44]	Single-stage, single-item, imperfect production	–	Product reliability	Production capacity
Paul et al. [36]	Single-stage, single-item, perfect production	Single and multiple production disruptions	System reliability	Production capacity
Cárdenas-Barrón and Sana [7]	Single-stage, single-item, perfect production	–	–	–
Paul et al. [38]	Three-stage, single-item, perfect production	Production disruption	System reliability	Production capacity
Kim and Sarkar [20]	Multi-stage, multi-item, imperfect production	–	–	Budget constraint
Saha et al. [42]	Single-stage, single-item, perfect production	–	–	–
This model	Single-stage, multi-item, imperfect production	Production disruption	System reliability	Production capacity, budget, and space constraint

" – " This key word is not available in paper.

production-inventory system that faces random production disruptions at different stages of the production cycle. In this paper, the authors first developed a multi-constrained non-linear mathematical model to deal with the production disruption in an imperfect production system where multiple products are produced. The developed mathematical model is solved by two standard search algorithms, genetic algorithm (GA) and pattern search (PS). Authors have generated number of test problems for the production disruption by using a uniform distribution. Numerical examples are presented to demonstrate the effectiveness of the proposed research and the obtained results from both search techniques, GA and PS, are compared. The production system is faced with eight types of costs: setup cost, production cost, holding cost, inspection cost, imperfect products rejection cost, interest and depreciation cost, backorder cost, and lost sales cost. The research contributions regarding the production type, disruption type, reliability, and constraints are given in Table 1. Based on the research gap from the literature review and authors' contribution table, the overall contribution of this paper is twofold

1 First, a multi-item single-stage production system is studied under the consideration of unavoidable production disruption. To the best of authors' knowledge, in existing literature, only a very few studies exist with production disruption scenario while none of them has considered multi-item production system. In practical environments, multi-item production systems are getting more famous among manufacturers. Therefore, it is need of time for managers and manufacturers to have multi-item production recovery models with disruptions during production cycles.
2 Second, most of the existing models do not consider budget and space limitations for economic and production decisions while these two factors are the most important for multi-item production-inventory systems. This model considers limitations over both, budget and space, to make the model more practical and real-life problem-based.

The remainder of the paper is formulated as follows. In Section 2, the problem definition, notation, and assumptions are given. A mathematical model is given in Section 3. Section 4, presents the solution approach, and numerical examples are given in Section 5. Section 6 consists of a sensitivity analysis. Finally, conclusions are given in Section 7.

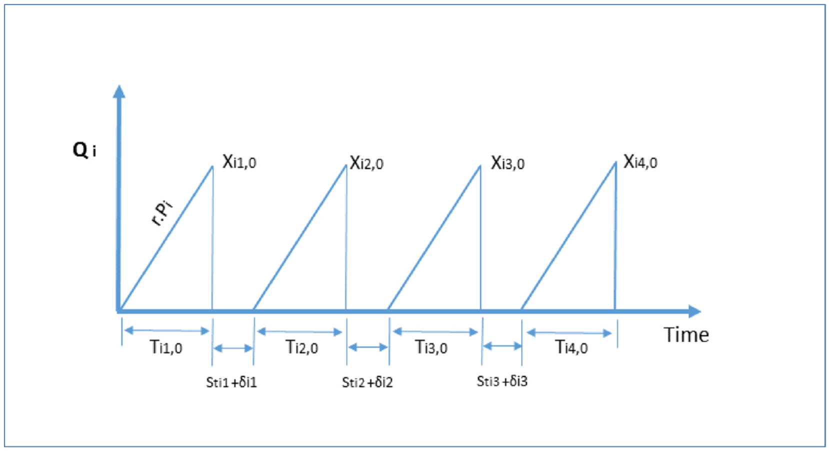
2. Problem definition, notation, and assumptions

In this section, first the problem definition is given, which indicates the major motivation for this research. After that, notation and assumptions are given for the mathematical model.

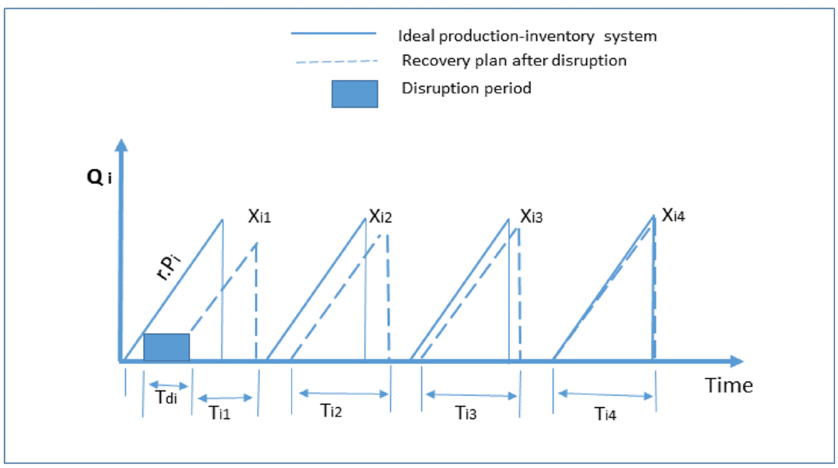
2.1. Problem definition

In manufacturing systems, it is no surprise that disruption problem can emerge at any time within the manufacturing uptime. In this particular section, the disruption problem in a multi-item production system is described. Almost every manufacturing system may face disruption during the production run-time and recover from disruption; it is a difficult and formidable task for the manufacturer. Usually, it is very difficult to devise a recovery plan after the occurrence of the disruption. Therefore, manufacturers should have a recovery plan prior to disruption. When disruption happens, it takes time to get to a point, where the manufacturer can start production again. Thus, the system loses time due to this recovery process. Occasionally the system may even face another disruption. However, if the system faces another disruption after recovering from the previous one, researchers will consider it as a single-disruption case. This is because the second disruption will not be affected by the previous disruption and recovery plan, and the system recovers from the second disruption on the basis of the original recovery plan. Paul et al. [39] developed a single item production disruption model, but this model does not consider any constraint for the budget or space. Most of the studies have been for the single-item production system because of the complexity of the multi-item production system. This paper presents the first attempt at a multi-item production system recovery plan for a disrupted system. This model considers a multi-item production system for the manufacturer, with limits on the available budget and storage space for the inventory.

Fig. 1(a) shows an ideal plan for the single-stage batch production-inventory system where multiple products are produced. These products are produced in batches. After the completion of each batch, there is a production down time which is the summation of idle time and setup. In an ideal production-inventory system, the production quantity for each cycle i is $X_{in,0}$ $(i = 1, 2, .., M)$, and summation of these quantities is equal to Q_n. The recovery plan is a updated schedule that includes the revised production quantities in each cycle, and ensuring the maximization of the total profit in the recovery time window. The number of future cycles allocated to return to the original production schedule from the disrupted cycle, is known as the recovery time



a



b

Fig. 1. a: Ideal production plan; **b**: Recovery plan after disruption.

window [39], and is decided by the management of the organization. Fig. 1(b) presents a disrupted production-inventory system and a recovery plan after the occurrence of production disruption. The production system becomes inoperable for a certain period (T_{dn}) due to a disruption and after then it operates normally. The recovery plan starts just after the ending point of the disruption period and continues during the recovery time window. It takes M number of production cycles during the recovery time window to returns the production process to its original or normal schedule.

2.2. Notation

Decision variablesX_{in}production quantity of product n for cycle i of the recovery time window, $i = 1, 2, .. ., M$ParametersS_{tn}setup time of machine n for each cycle (year)δ_{in}idle time of machine n for cycle i (year)$D_n$demand of product n per year (units/year)$H_n$holding cost for product n per unit per year ($/unit/year)$r$reliability of the production process, which is known from historical data of the production system$Q_n$economic lot size for product n per ideal production cycle with process reliability r (units/cycle)$A_n$setup cost of machine n per setup ($/setup)$P_n$production rate for product n (units/year) in a 100% reliable system$M$number of cycles in the recovery time window to get to the original production plan (given from the management)T_{dn}disruption period (years)$q_n$pre-disruption production quantity of product n (units)T_{0n}production time (years) for $q_n(=\frac{q_n}{rP_n})X_{in0}$production quantity of product n for normal cycle i (units/cycle)u_{in}production down time for cycle i (years) of machine n (setup time + idle time $= St_n + \delta_{in} = \frac{X_{in0}}{D_n} - \frac{X_{in0}}{rP_n}$)$T_{in0}$production up time (years) of machine n for cycle i of an ideal cycle $(=\frac{X_{in0}}{rP_n})T_{in}$production up time (years) of machine n for cycle i in the recovery time window $(=\frac{X_{in}}{rP_n})B_n$unit backorder cost of product n per unit ($/unit)$L_n$unit lost-sales cost of product n ($/unit)$C_{P_n}$per unit production cost of product n ($/unit)$C_{R_n}$rejection cost of product n per unit ($/unit)$C_{I_n}$inspection cost for product n ($/unit)$m_1$mark-up of selling price ($m_1 C_{P_n}$) of each acceptable unit of product n, this must be greater than 1$s_n$space occupied per unit of product n (meter sq./ unit)Wtotal budget ($)$S$total space(meter sq./ unit)Nmaximum number of machines and products, a positive integer

2.3. Assumptions

This paper is based on the following assumptions:

(1) This model is developed for a multi-product single-machine imperfect production system. In this system, multiple products are produced on the same machine; for example, different types of gears or bearings manufacturing.
(2) To fulfill the customer's demand, the production rate is always

higher than the demand for each product. Therefore, shortages are not allowed in the model.

(3) During the production cycle, the system may go under the disruption due to unexpected factors (i.e. labors strikes, material shortages, or power cut off) and production stops.

(4) The recovery cycle starts immediately after a production disruption occurs. The recovery cycle is the time in which management update the current production plan. The management would like to get to the original production plan within the reasonable number of production cycles after a production disruption happens. In this time window, the managers utilize the ideal time to produce products to over come the delay in production process caused by the production disruption during the disruption period [12].

(5) For the total cost of interest and depreciation per production cycle $F(A, r)$, Cheng's model (1989) is considered. In this model, $F(A, r)$ is inversely related to the setup cost (A) and is directly related to the process reliability (r), according to the following general power function:

$$F(A,\, r) = \mathrm{a}\mathrm{A}^{-b} r^{c},$$

where a, b and c are positive constants chosen to provide the best fit to the estimated cost function.

(6) This model is for an imperfect production system. However, all the produced products are well inspected before dispatch. In this system, all defective products are separated and rejected [39]. Hence, no rework is done in the system.

(7) The production system has a restriction on the available space for inventory storage. The total available storage space is S and the system should not violate this limit (see [28]).

(8) There is an investment limit for the manufacturer for the production cost. The total available budget for production cost is B and the production cost should not exceed this budget limit.

3. Mathematical model formulation

In this Section, the disruption problem for a multi-item production-inventory system is described and formulated. In real life, for multi-item production systems, different types of disruptions may occur, and to manage the system, it is necessary to devise a recovery plan. If a system faces another disruption after the recovery time window of the first disruption, it can still be managed as a single-disruption problem because the second disruption is not affected by the previous disruption and recovery plan. This model considers the multiple products single-disruption problem.

Using the lot-for-lot (LFL) policy of Banerjee [4] and the single-item batch production of Sarker and Khan [49], the proposed model obtains the economic lot size (Q_n).

The annual setup cost is calculated as the product of the number of setups per year $\frac{D_n}{Q_n}$ and the cost per setup A_n

$$\frac{D_n A_n}{Q_n}.$$

The average inventory per cycle is $\frac{Q_n}{2}$ and the per unit holding cost is H_n. Thus, the annual holding cost is

$$\frac{H_n Q_n D_n}{2\mathrm{rP}_n}.$$

The total cost for the inventory system (by adding the above-given equations) is given by

$$\frac{D_n A_n}{Q_n} + \frac{H_n Q_n D_n}{2\mathrm{rP}_n}.$$

and to minimize the total cost

$$\frac{d}{\mathrm{dQ}_n}\left(\frac{D_n A_n}{Q_n} + \frac{H_n Q_n D_n}{2\mathrm{rP}_n}\right) = 0$$

The economic lot size is obtained by solving the above equation:

$$Q_n = \sqrt{\frac{2A_n \mathrm{rP}_n}{H_n}}. \tag{1}$$

The mathematical expressions for the setup cost, production cost, holding cost, inspection cost, rejection cost, and interest and depreciation cost are needed to calculate for total cost calculation. These costs are considered in this model for the disruption recovery time window.

All the costs involved in the total cost of the production system TC per item are derived as follows:

Setup cost (SC)

The first step for starting production is devising the production/manufacturing setup. All the fixed costs, which are experienced every time a single item is produced, are consolidated as the setup cost, and all these fixed costs are mostly associated with the physical activities required within the setup to produce the quantity or lot. These physical activities include setting up the machine, work center, or assembly line. The total setup cost is determined as the cost per setup multiplied by the number of setups in the recovery time window:

$$\mathrm{SC} = \sum_{n=1}^{Z} A_n M. \tag{2}$$

Production cost (PC)

When a manufacturer produces a product or a lot of a given quantity, this requires a certain amount of resources per product. The production cost is the unit production cost for each product multiplied by the total number of units produced for each product. Here, the production costs are calculated as the product of production cost per unit Cp_n, production rate P_n, and the total production time. Thus, it can be found as follows:

$$\begin{aligned}\mathrm{PC} &= \sum_{n=1}^{N} C_{P_n} P_n \left(\sum_{i=1}^{M} T_{\mathrm{in}} + T_{0n}\right)\\ &= \sum_{n=1}^{N} \frac{C_{P_n}}{r}\left(\sum_{i=1}^{M} X_{\mathrm{in}} + q_n\right).\end{aligned} \tag{3}$$

Inventory holding cost (IHC)

All produced products are stored in the warehouse at a cost. To calculate the holding cost, first, an average inventory is calculated. The manufacturing system begins producing products and faces production disruption after some production time T_{0n}, and from the start to the time when disruption occurs, the produced quantity is q_n. The system faces disruption for the time period T_{dn}, the setup time is S_{tn}, the production run time for the first cycle is T_{1n}, and the system holds the pre-disruption quantity q_n for the entire time of the production cycle (see Fig. 1). Thus, the total produced quantity is calculated (See Appendix A) as

$$\begin{aligned}&\sum_{n=1}^{N}\left[\frac{1}{2}q_n T_{0n} + q_n(T_{\mathrm{dn}} + S_{\mathrm{tn}} + T_{1n}) + \frac{1}{2}X_{1n}T_{1n} + \frac{1}{2}X_{2n}T_{2n}\right.\\ &\left. + \ldots + \frac{1}{2}X_{\mathrm{Mn}}T_{\mathrm{Mn}}\right]\\ &= \sum_{n=1}^{N}\frac{1}{2}\left[\frac{q_n^2}{\mathrm{rP}_n} + 2q_n(T_{\mathrm{dn}} + S_{\mathrm{tn}}) + \frac{2q_n X_{1n}}{\mathrm{rP}_n} + \sum_{i=1}^{M}\frac{(X_{\mathrm{in}})^2}{\mathrm{rP}_n}\right].\end{aligned}$$

The unit holding cost multiplied by the total inventory in the recovery time window, which is the area under the curve in the recovery time window, is the total inventory holding cost:

$$\mathrm{IHC} = \sum_{n=1}^{N}\frac{1}{2}H_n\left[\frac{q_n^2}{\mathrm{rP}_n} + 2q_n(T_{\mathrm{dn}} + S_{\mathrm{tn}}) + \frac{2q_n X_{1n}}{\mathrm{rP}_n} + \sum_{i=1}^{M}\frac{(X_{\mathrm{in}})^2}{\mathrm{rP}_n}\right]. \tag{4}$$

Inspection cost (IC)

All produced products are inspected, and a fixed cost is incurred to inspect all products. The product inspection cost is

$$IC = \sum_{n=1}^{N} \frac{C_{I_n}}{r}\left(\sum_{i=1}^{M} X_{\text{in}} + q_n\right). \tag{5}$$

Defective products rejection cost (DPRC)

All produced products are inspected and the defective products are separated from the perfect products. Further, all imperfect products are rejected and thrown out of the manufacturing system at a fixed cost. The process reliability is r and the rate of rejection is $(1 - r)$. The total rejection cost is the rejection cost per unit rejected multiplied by the number of units rejected after inspection:

$$\begin{aligned} DPRC &= \sum_{n=1}^{N} C_{R_n}(1-r)P_n\left(\sum_{i=1}^{M} T_{\text{in}} + T_{0n}\right)\\ &= \sum_{n=1}^{N} C_{R_n}\left(\frac{1}{r}-1\right)\left(\sum_{i=1}^{M} X_{\text{in}} + q_n\right). \end{aligned} \tag{6}$$

Interest and depreciation cost (IDC)

The total cost of interest and depreciation equation is considered to be a general power function (for instance, see [11]). It is inversely proportional to the setup cost A_n and directly proportional to the process reliability r. Hence, the cost of interest and depreciation is

$$IDC = \sum_{n=1}^{N} \text{MaA}_n^{-b} r^c. \tag{7}$$

Backorder cost (BC)

The backorder is the quantity that is not fulfilled at the time of order but is delivered at a later date when the required quantity is available. The backorder cost is calculated as the backorder cost per unit multiplied by the backordered number of units and time delay. Hence, the backorder cost (See Appendix B) is

$$\begin{aligned} BC &= \sum_{n=1}^{N} B_n[(X_{1n}+q_n)\text{delay}_1 + \sum_{i=2}^{M} X_{\text{in}}.\ \text{delay}_i]\\ &= \sum_{n=1}^{N} B_n\left[(X_{1n}+q_n)\left(T_{\text{dn}} + \frac{q_n}{\text{rP}_n} + \frac{X_{1n}}{\text{rP}_n} - \frac{X_{1n0}}{\text{rP}_n}\right)\right.\\ &\quad + \sum_{i=2}^{M} X_{\text{in}}\left(T_{\text{dn}} + (i-1)S_{\text{tn}} + \frac{q_n}{\text{rP}_n} + \sum_{j=1}^{i}\frac{X_{\text{jn}}}{\text{rP}_n} - \sum_{j=1}^{i}\frac{X_{\text{jn0}}}{\text{rP}_n}\right.\\ &\quad \left.\left. - \sum_{j=1}^{i-1}\left(\frac{X_{\text{jn0}}}{D_n} - \frac{X_{\text{jn0}}}{\text{rP}_n}\right)\right)\right] \end{aligned} \tag{8}$$

Lost sales cost (LSC)

When the demand quantity is not available at the time of order and the customer will not wait for the stock to be replenished, the lost sales cost is incurred. The lost sales cost is the unit lost sales cost multiplied by the lost sales units. Therefore, the lost sales cost (See Appendix C) is

$$\begin{aligned} LSC &= \sum_{n=1}^{N} L_n\left(\sum_{i=1}^{M} X_{\text{in0}} - \text{rP}_n(T_{0n} + T_{1n} + T_{2n} + \ldots + T_{\text{Mn}})\right)\\ &= \sum_{n=1}^{N} L_n\left(\sum_{i=1}^{M} X_{\text{in0}} - \sum_{i=1}^{M} X_{\text{in}} - q_n\right). \end{aligned} \tag{9}$$

Total cost

The total cost per item (TC) is calculated by combining setup cost, production cost, inventory holding cost, inspection cost, interest and depreciation cost, backorder cost, and lost sales cost, and is given by

$$\begin{aligned} TC &= SC + PC + IHC + IC + DPRC + IDC + BC + LSC\\ &= \sum_{n=1}^{N} A_n M + \sum_{n=1}^{N}\frac{C_{P_n}}{r}\left(\sum_{i=1}^{M} X_{\text{in}} + q_n\right) + \sum_{n=1}^{N}[\frac{1}{2}H_n\{\frac{q_n^2}{\text{rP}_n}\\ &\quad + 2q_n(T_{\text{dn}} + S_{\text{tn}}) + \frac{2q_nX_{1n}}{\text{rP}_n} + \sum_{i=1}^{M}\frac{(X_{\text{in}})^2}{\text{rP}_n}\}] + \sum_{n=1}^{N}\frac{C_{I_n}}{r}\left(\sum_{i=1}^{M} X_{\text{in}} + q_n\right)\\ &\quad + \sum_{n=1}^{Z} C_{R_n}\left(\frac{1}{r}-1\right)\left(\sum_{i=1}^{M} X_{\text{in}} + q_n\right) + \sum_{n=1}^{N}\text{MaA}_n^{-b}r^c\\ &\quad + \sum_{n=1}^{Z} L_n\left(\sum_{i=1}^{M} X_{\text{in0}} - \sum_{i=1}^{M} X_{\text{in}} - q_n\right) + \sum_{n=1}^{Z} B_n[(X_{1n}+q_n)(T_{\text{dn}} + \frac{q_n}{\text{rP}_n}\\ &\quad + \frac{X_{1n}}{\text{rP}_n} - \frac{X_{1n0}}{\text{rP}_n}) + \sum_{i=2}^{M} X_{\text{in}}\cdot\{T_{\text{dn}} + (i-1)S_{\text{tn}} + \frac{q_n}{\text{rP}_n} + \sum_{j=1}^{i}\frac{X_{\text{jn}}}{\text{rP}_n}\\ &\quad - \sum_{j=1}^{i}\frac{X_{\text{jn0}}}{\text{rP}_n} - \sum_{j=1}^{i-1}\left(\frac{X_{\text{jn0}}}{D_n} - \frac{X_{\text{jn0}}}{\text{rP}_n}\right)\}] \end{aligned} \tag{10}$$

Revenues (Rev)

The revenue generated by the acceptable items, during the recovery time window, is calculated as selling price per unit multiplied by the demand quantity.

$$\begin{aligned} Rev &= \sum_{n=1}^{N}\left[m_1 C_{P_n} D_n\left(\sum_{i=1}^{M} T_{\text{in}} + T_{0n} + \text{MS}_{\text{tn}}\right)\right]\\ &= \sum_{n=1}^{N}\left[m_1 C_{P_n} D_n\left\{\sum_{i=1}^{M}\frac{X_{\text{in}}}{\text{rP}_n} + \frac{q_n}{\text{rP}_n} + \text{MS}_{\text{tn}}\right\}\right] \end{aligned} \tag{11}$$

Total profit

Total profit for the system is calculated as,

$$\text{Totalprofit(TP)} = \text{Totalrevenues} - \text{Totalcost} \tag{12}$$

Max. $TP(X_{in})$

$$\begin{aligned} &= \sum_{n=1}^{N}\left[m_1 C_{P_n} D_n\left\{\sum_{i=1}^{M}\frac{X_{\text{in}}}{\text{rP}_n} + \frac{q_n}{\text{rP}_n} + \text{MS}_{\text{tn}}\right\}\right] - \sum_{n=1}^{N} A_n M\\ &- \sum_{n=1}^{N}\frac{C_{P_n}}{r}\left(\sum_{i=1}^{M} X_{\text{in}} + q_n\right) - \sum_{n=1}^{N}[\frac{1}{2}H_n\{\frac{q_n^2}{\text{rP}_n} + 2q_n(T_{\text{dn}} + S_{\text{tn}})\\ &+ \frac{2q_nX_{1n}}{\text{rP}_n} + \sum_{i=1}^{M}\frac{(X_{\text{in}})^2}{\text{rP}_n}\}] - \sum_{n=1}^{N}\frac{C_{I_n}}{r}\left(\sum_{i=1}^{M} X_{\text{in}} + q_n\right)\\ &- \sum_{n=1}^{Z} C_{R_n}\left(\frac{1}{r}-1\right)\left(\sum_{i=1}^{M} X_{\text{in}} + q_n\right) - \sum_{n=1}^{N}\text{MaA}_n^{-b}r^c\\ &- \sum_{n=1}^{Z} L_n\left(\sum_{i=1}^{M} X_{\text{in0}} - \sum_{i=1}^{M} X_{\text{in}} - q_n\right) - \sum_{n=1}^{N} B_n[(X_{1n}+q_n)(T_{\text{dn}} + \frac{q_n}{\text{rP}_n}\\ &+ \frac{X_{1n}}{\text{rP}_n} - \frac{X_{1n0}}{\text{rP}_n}) + \sum_{i=2}^{M} X_{\text{in}}\cdot\{T_{\text{dn}} + (i-1)S_{\text{tn}} + \frac{q_n}{\text{rP}_n} + \sum_{j=1}^{i}\frac{X_{\text{jn}}}{\text{rP}_n}\\ &- \sum_{j=1}^{i}\frac{X_{\text{jn0}}}{\text{rP}_n} - \sum_{j=1}^{i-1}\left(\frac{X_{\text{jn0}}}{D_n} - \frac{X_{\text{jn0}}}{\text{rP}_n}\right)\}] \end{aligned} \tag{13}$$

The above profit is maximized subject to the following budget, space, demand, capacity, delivery and transportation constraints:

Budget constraint

In real life production systems, the manufacturer has a limit for the available budget. Thus, the manufacturer never exceeds this budget limit in terms of the costs, i.e., The total production cost is less than or equal to the budget limit, i.e.;

$$\sum_{n=1}^{N} C_{P_n}(q_n + X_{\text{in}}) \le W. \tag{14}$$

Space constraint

Every production setup has a limited storage capacity for the inventory, which usually considered as space constraint. The space constraint ensures that the space occupied by all products is less than or equal to the total space available at the production setup as follows:

$$\sum_{n=1}^{N} s_n(q_n + X_{\text{in}}) \le S. \tag{15}$$

Production capacity constraint

When the system functions according to the original production plan without facing any disruption, the production quantity is equal to the desired number. The following constraint ensures the cycle production quantity in an ideal system for all the products::

$$X_{\text{in0}} = Q_n. \tag{16}$$

Recovery cycle production capacity constraint

The following constraints set the production in each cycle of the recovery time window as less than or equal to the production quantity for the ideal system. These two constraints ensures that the production quantity in each cycle of the recovery time window is less than the production quantity in each cycle before the occurrence of the production disruption. This is because of the transportation and delivery requirements.

$$X_{1n} + q_n \le X_{1n0} \tag{17}$$

$$X_{\text{in}} \le X_{\text{in0}};\ i = 2, 3, ...M;\ n = 1, 2. .N \tag{18}$$

The model is developed considering the recovery window production capacity. The production capacity is limited in that the system can never produce more than the capacity during the disruption recovery period. It gives the recovery time window production capacity constraint as

$$\sum_{n=1}^{N}\left[\sum_{i=1}^{M} X_{\text{in}} + q_n\right] \le \sum_{n=1}^{N} \text{rP}_n\left(\sum_{i=1}^{M}\frac{X_{\text{in0}}}{D_n} - \text{MS}_{\text{tn}} - T_{\text{dn}}\right). \tag{19}$$

Process reliability constraint

The process reliability is always less than or equal to one and is only equal to 1 when the production system is considered fully reliable. However, in real-life systems, process reliability is always less than 1. Thus,

$$r \le 1 \tag{20}$$

The system has reliability $\boldsymbol{r}$ which affects the overall rate of production of the item and must be higher than the total demand. As the reliability of the system is less than 1 ($r < 1$), the production rate can be such that the combined rP_n is higher than the demand:

$$\text{rP}_n \ge D_n \tag{21}$$

Demand constraint for recovery time window

The demand constraint during the recovery time windows is

$$\sum_{n=1}^{N}\left(\sum_{i=1}^{M} X_{\text{in}} + q_n\right) \ge \left[\left(\frac{\sum_{i=1}^{M} X_{\text{in}} + q_n}{\text{rP}_n} + \text{MS}_{\text{tn}}\right)D_n - \left(\sum_{i=1}^{M} X_{\text{in0}} - \sum_{i=1}^{M} X_{\text{in}} - \sum_{n=1}^{N} q_n\right)\right]. \tag{22}$$

Non-negativity conditions

The following equations represent the ideal time non-negativity conditions:

$$\sum_{n=1}^{N}\left[\frac{X_{1n} + q_n}{D_n} - \frac{X_{2n}}{\text{rP}_n} - S_{\text{tn}}\right] \ge 0, \tag{23}$$

$$\sum_{n=1}^{N}\left[\frac{X_{\text{in}}}{D_n} - \frac{X_{(i+1)n}}{\text{rP}_n} - S_{\text{tn}}\right] \ge 0.\quad i = 2, 3, ...,M;\quad n = 1, 2, ...,N; \tag{24}$$

The following two equations ensure the delay time non-negativity:

$$T_{\text{dn}} + \sum_{n=1}^{N}\left[\frac{q_n}{\text{rP}_n} + \frac{X_{1n}}{\text{rP}_n} - \frac{X_{1n0}}{\text{rP}_n}\right] \ge 0, \tag{25}$$

$$T_{\text{dn}} + (i-1)S_{\text{tn}} + \sum_{n=1}^{N}\left[\frac{q_n}{\text{rP}_n} + \sum_{j=1}^{i}\frac{X_{\text{jn}}}{\text{rP}_n} - \sum_{j=1}^{i}\frac{X_{\text{jn0}}}{\text{rP}_n} - \sum_{j=1}^{i-1}\left(\frac{X_{\text{jn0}}}{D_n} - \frac{X_{(j+1)n0}}{\text{rP}_n}\right)\right] \ge 0;\quad i = 1, 2, ...,M;\quad n = 1, 2. ..,N. \tag{26}$$

Finally, all the decision variables are non-negative:

$$X_{\text{in}} \ge 0;\quad i = 1, 2, ...,M;\quad n = 1, 2. ..,N. \tag{27}$$

4. Solution methodology

In literature, various analytical techniques have been used to solve small- and medium-sized problems. However, finding the optimal solution in a complex non-linear constrained problems by using the analytical approaches presented in the literature is very difficult or nearly impossible because of heavy computing overheads. Consequently, there is also a need to reduce the time, and enhance the precision and quality of the solutions for the optimization of complex real-life problems. The proposed multi-item recovery model is a complex constrained mathematical model and it cannot be solved with the analytical procedure because of its complexity. However, this mathematical model is solved by using the two global search metaheurisitc techniques, (1) genetic algorithm (GA) and (2) pattern search (PS). These metaheurisitics are not local search-based and usually these are population-based. These techniques were used because of their ability to effectively deal with a large number of parameters in complex optimization problems. To obtain the solution, we coded the model in MATLAB R2015b and solved using the optimization toolbox. We executed this on an Intel Core i5 with a 3.20 GHz CPU and 8GB RAM.

4.1. Solution approach for the production-disruption problem

A single disruption problem given in this model can be solved via the help of search algorithms. To validate the model results, two search techniques a genetic algorithm (GA) and pattern search (PS), were implemented in MATLAB R2015b. GA is a natural genetic-based general purpose optimization technique that mostly explores the given search space [15]. Pattern search is also a search technique for use within a given search space and is easy to apply to nonlinear optimization problems with constraints. We compare the results obtained by both search techniques at the end of the next section.

Major steps of the proposed solution algorithm for real-time based single-disruption model can be found as follows:

Solution algorithm

1 Input all data for the ideal system and get Q_n by using the Eq. (1).
2 Assign the values $X_{in0} = Q_n$
3 Put $i = 1$ for disruption for all products $i = 1, 2, 3$
4 Input pre-disruption quantity q_n and the disruption period (T_{dn}).
5 Generate initial population for the recovery cycle from the start of the disruption.
6 Solve the proposed model with two, Genetic algorithm and Pattern search, search
7 Update the X_{in} value as a revised lot size from (6) and note down the revised production plan.
8 Stop.

Table 2
Parameter ranges.

Parameters	Range of data
Setup cost (A_n)	[20,200]
Pre-disruption quantity (q_n)	[0,Q_n]
Disruption period (T_{dn})	[0.0001, $\frac{Q_n - q_n}{rP_n}$]
Inventory holding cost (H_n)	[0.5,10]
Lost sales cost (L_n)	[1,100]
Backorder cost (B_n)	[1,100]

5. Numerical experiments

In this section, we inspect results for the single-disruption case. In this study, we define a disruption situation as the coalition of the pre-disruption situation and the duration of the disruption. We consider these parameters as uniform random variables in this study. To test the recommended model within the intervals, over 50 disruption test problems are generated by varying the parameters. We solve the proposed model by both the GA and PS search techniques and analyze and compare the obtained results.

The ranges of the parameters are given in Table 2 and the input data for single disruption problem is given in Table 3.

The parameters used for the GA and PS to execute the model and obtain the solution are as follows

Parameters for GA

- Population size: 200.
- Population type: Double vector.
- Crossover fraction: 0.8.
- Maximum number of generations: 3000.
- Function tolerance: 1e − 8.
- Nonlinear constraint tolerance: 1e − 8.
- Hybrid function: Pattern search.
- All other parameters in the optimization tool box were set as the default.

Parameters for PS

- Maximum number iterations: 100 × Number of variables.
- Polling order: Success.
- X tolerance: 1e − 8.
- Function tolerance: 1e − 8.
- Nonlinear constraint tolerance: 1e − 8.
- Cache tolerance: 1e − 8.
- Search method: Latin Hypercube.
- Maximum function evaluations: 1,000,000.
- All other parameters in the optimization tool box were set as the default.

5.1. Input data for single disruption problem

To examine the results, most of the input data from [39] were used:

Table 3
Input data for single disruption problem.

Test instance	Disruption period (Td_n)	Pre-disruption quantity (q_i)
1	(0.0020, 0.0025, 0.0030)	(750, 800, 950)
2	(0.0052, 0.0060, 0.0072)	(1025, 1225, 1375)
3	(0.0076, 0.0090, 0.0094)	(500, 675, 800)

St = (0.000050, 0.000060, 0.000065) year/setup	C_r = (6, 8, 10) \$/unit
D = *(350000, 400000, 550000* units/year	L = (5, 15, 20) \$/unit
P = (400000, 500000, 600000) units/year	B = (5, 10, 12) \$/unit
T_d = (0.0025, 0.0030, 0.0035) year	S = 250000 meter sq.
s = (1.1, 2.2, 3.3) m.sq./unit	a = 1000
C_I = (0.2, 0.4, 0.6) \$/unit	m = 2.5
q = (750, 800, 950) units	c = 0.75
C_p = (30, 40, 50) \$/unit	r = 0.95
A = (40, 50, 60) \$/setup	b = 0.5
H = (0.8, 1, 1.2) \$/unit	N = 3
W = \$6500000	M = 5

From the above data, $X_{in} = Q_n$ can be easily calculated:

$$Q_1 = \sqrt{\frac{2A_1 rP_1}{H_1}} \approx 6164, \quad Q_2 = \sqrt{\frac{2A_2 rP_2}{H_2}} \approx 6892,$$

$$Q_3 = \sqrt{\frac{2A_3 rP_3}{H_3}} \approx 7550.$$

below the results for each example are given where only the best results out of thirty runs are provided.

Example 1.

In the first numerical example, a disruption problem with the upper and lower bounds is considered. The results for Example 1 are given in Table 4 for GA and for PS.

Example 2.

In the second numerical example, a disruption problem is considered with budget and space constraints, as well as all the upper and lower bounds mentioned in the above mathematical model. The results for Example 2 are given in Table 5 for GA and for PS.

Example 3.

In the third and last numerical example, a disruption problem is considered with all the constraints as well as the upper and lower bounds already mentioned in the above model and the data given above. The results for Example 3 are given in Table 6 for GA and for PS.

5.2. Comparison of results

To judge the consistency, best results of 24 independent runs have been compared for two different search techniques (GA and PS). All the

Table 4
GA results for Example 1. and PS results for Example 1.

Test instance	Product type (i)	X_{i1}	X_{i2}	X_{i3}	X_{i4}	X_{i5}	Total profit (\$)
	1	5414	6164	6164	6164	6164	
1	2	6042	6892	6892	6892	6892	3148460.78
	3	6599	7549	7549	7549	7549	
	1	5139	6164	6164	6164	6164	
2	2	5667	6892	6892	6892	6892	3148065.75
	3	6174	7549	7549	7549	7549	
	1	5664	6164	6164	6164	6164	
3	2	6217	6892	6892	6892	6892	3147491.56
	3	6749	7549	7549	7549	7549	
Test instance	**Product type (i)**	X_{i1}	X_{i2}	X_{i3}	X_{i4}	X_{i5}	**Total profit (\$)**
	1	5414	6164	6164	6164	6164	
1	2	6042	6892	6892	6892	6892	3148460.78
	3	6599	7549	7549	7549	7549	
	1	5139	6164	6164	6164	6164	
2	2	5667	6892	6892	6892	6892	3148065.76
	3	6174	7549	7549	7549	7549	
	1	5664	6164	6164	6164	6164	
3	2	6217	6892	6892	6892	6892	3147491.59
	3	6749	7549	7549	7549	7549	

Table 5
GA results and PS results for Example 2.

Test instance	Product type (*i*)	X_{i1}	X_{i2}	X_{i3}	X_{i4}	X_{i5}	Total profit ($)
	1	5414	6164	6164	6164	6164	
1	2	6042	6803	6892	6892	6892	3148441.09
	3	6599	7549	7549	7472	7549	
	1	5139	6164	6164	6164	6164	
2	2	5667	6892	6892	6892	6892	3148065.77
	3	6174	7549	7549	7549	7549	
	1	5664	6164	6164	6164	6164	
3	2	6217	6892	6892	6892	6892	3147491.57
	3	6749	7549	7549	7549	7549	
Test instance	**Product type (*i*)**	X_{i1}	X_{i2}	X_{i3}	X_{i4}	X_{i5}	**Total profit ($)**
	1	5414	6164	6164	6164	6164	
1	2	6042	6892	6892	6892	6892	3148460.78
	3	6599	7549	7549	7549	7549	
	1	5139	6164	6164	6164	6164	
2	2	5667	6892	6892	6892	6892	3148065.77
	3	6174	7549	7549	7549	7549	
	1	5664	6164	6164	6164	6164	
3	2	6217	6892	6892	6892	6892	3147491.59
	3	6749	7549	7549	7549	7549	

Table 6
GA results and PS results for Example 3.

Test instance	Product type (*i*)	X_{i1}	X_{i2}	X_{i3}	X_{i4}	X_{i5}	Total profit ($)
	1	5414	6164	6164	6164	6164	
1	2	6042	6891	6892	6892	6892	3148460.78
	3	6599	7548	7533	7549	7549	
	1	5139	6164	6164	6164	6164	
2	2	5667	6892	6892	6892	6892	3148065.76
	3	6174	7549	7549	7549	7549	
	1	5664	6164	6164	6164	6164	
3	2	6217	6892	6892	6892	6892	3147491.55
	3	6749	7549	7549	7549	7549	
Test instance	**Product type (*i*)**	X_{i1}	X_{i2}	X_{i3}	X_{i4}	X_{i5}	**Total profit ($)**
	1	5414	6164	6164	6164	6164	
1	2	6042	6892	6892	6892	6892	3148460.78
	3	6599	7549	7549	7549	7549	
	1	5139	6164	6164	6164	6164	
2	2	5667	6892	6892	6892	6892	3148065.76
	3	6174	7549	7549	7549	7549	
	1	5664	6164	6164	6164	6164	
3	2	6217	6892	6892	6892	6892	3147491.55
	3	6749	7549	7549	7549	7549	

test problems have been generated randomly within the ranges given in Table 2. The obtained average percentage of error or deviation for the two search techniques was 0.00118% and it can be considered as negligible.

To calculate the average value of percentage deviation, following equation was used

$$\text{Averagepercentageerror} = \frac{100}{N} \times \sum \frac{|\text{TotalprofitfromGA} - \text{TotalprofitfromPS}|}{\text{TotalprofitfromPS}} \tag{28}$$

The comparison of results for 24 test runs, obtained from GA and PS, is presented in Table 7. From the comparison of results, one can see that the results are very much consistent for both, GA and PS, search techniques.

Table 7
Comparison of the result between PS and GA for Example 3.

Test instance	Total Profit		Percentage error (%)	Running time (sec.)	
	GA	PS		GA	PS
1	3148243.0	3148460.8	0.0069	238	4
2	3148065.8	3148065.8	0	288	4
3	3147491.6	3147491.6	0	295	4

Table 8
Sensitivity analysis with respect to key parameters for Example 3.

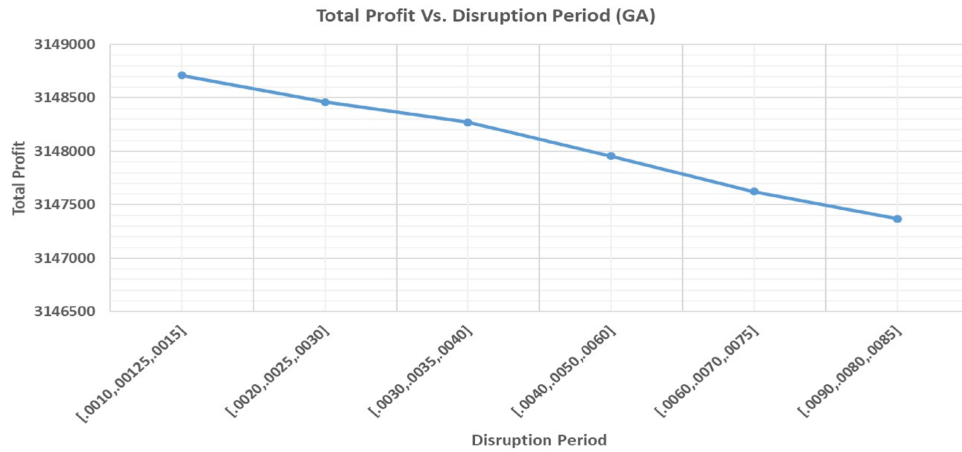
Parameters	Parameter change (%)	Change in total profit (%)
T_{dn}	−50%	+0.008
	−25%	+0.003
	+25%	−0.004
	+50%	−0.008
q_n	−50%	−0.04
	−25%	−0.02
	+25%	+0.02
	+50%	+0.04
B_n	−50%	−0.04
	−25%	−0.02
	+25%	+0.02
	+50%	+0.04
L_n	−50%	+0.66
	−25%	+.033
	+25%	−0.33
	+50%	−0.66

6. Sensitivity analysis

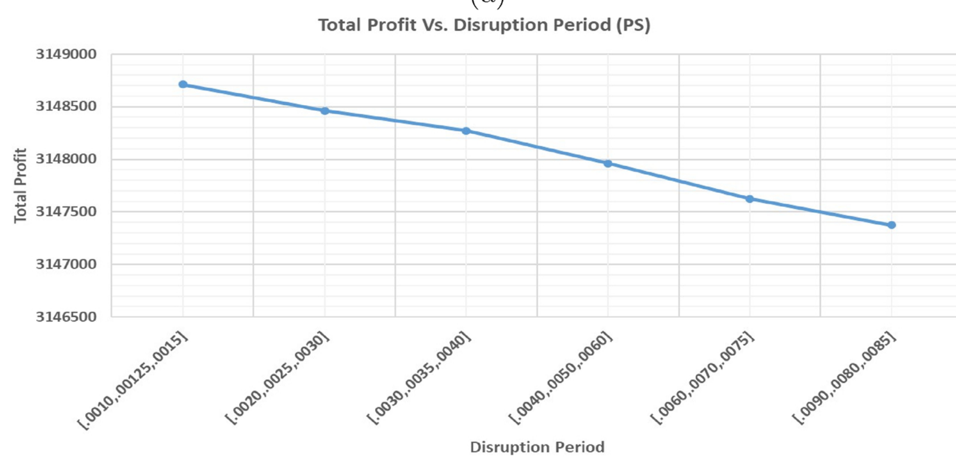
In this part, sensitivity analysis for Test instance 1 of example 3 are given. A sensitivity analysis is performed for each of the five key parameters T_{dn}, q_n, B_n, and L_n. The pattern search technique was used for the sensitivity analysis. The total effect of the parameter changes −50%, −25%, +25%, and +50% are calculated, and details are given in Table 8.

For characterizing the impact, the sensitivity analysis is performed different parameters. From the sensitivity analysis table, one can see that the total profit decreases with the increase of the disruption time T_{dn} and unit lost sales cost L_n. The total profit increases in the reverse case for disruption time and unit lost sales cost. However, profit is increasing with the increase of the pre-disruption quantity q_n and unit backorder cost B_n and vice versa. Overall profit is more sensitive for the two parameters, the unit lost sales cost L_n and the reliability of the system r, and it changes values very quickly with a smaller change in the parametric value.

Figs. 2, 3, and 4 show the changes in the total profit with respect to disruption period, pre-disruption quantity, backorder cost, and lost sales cost. For each analysis, only one parameter is changed and the remaining parameters are kept constant. Authors make these analyses for both the search techniques, GA and PS. Fig. 2 shows the changes in the total profit with the disruption period. The total profit decreases with the increase in the disruption period. The profit decreases linearly when the disruption period is higher than 0.0035 and the authors depict it in Fig. 2. Fig. 3 presents the impact of pre-disruption quantity on the profitability of the system. The total profit increases with the increase in pre-disruption quantity and increment are more when the pre-disruption quantity is closer to the production quantity X_{in0} for the ideal cycle. Fig. 4shows the increment in lost sales cost has a strong impact on the profit and causes a linear decrease in overall profit for the production system. The trend is the same for both GA and PS.



(a)



(b)

Fig. 2. Changes in total profit with respect to disruption period T_{dn} (a: For GA & b: For PS).

6.1. Managerial insights

This production disruption model can be applied to the production systems where the production environment is imperfect and producing multiple items at the same setup. More importantly, in multiple item production systems, the manufacturer must consider the proposed or suitable type of storage space limit and budget limitations over different costs and investments. The managers of any production industry are always anxious about breakdowns (especially random breakdown) due to defective production. Any disruption may occur within a production setup and it can cause defective production or production with low quality. Therefore, the management aims to control the breakdown anyhow. To do this, they generally consider corrective maintenance or preventive maintenance or both, but all these maintenance policies are too costly and it is difficult to maintain and based on literature, only disruptions are available in a single-stage production system. Nowadays, many complex systems are made, where disruptions may occur anytime. The outcomes of this model will benefit industries to maximize profit. Managers can predict the amount of disruption affected products; they can predict how much time the production process will be suspended due to production such that they can commit with other players for selling their products. Finally, by the outcomes, the industry can make proper scheduling of throughput of finished products and the arrival rate of raw material for any multi-product production system. Authors assumed the lot size is fixed throughout the planning horizon. However, the lot size may be split to meet the transportation capacities i.e. truck capacity. We also assumed the demand is known and fixed. While in practical situations the demand may fluctuate in different conditions. Therefore, management should take care of these parameters while applying the proposed approach.

The numerical problems are used to develop the methods of making recovery plans after the production disruption occurs which can present managers with some examples to solve the disruption problems in real environment. In a real operation of production systems, there are a number of variables and constants. These numerical are surely not to analyze all possible situations, but they can

- demonstrate the effectiveness of the methods,
- indicate how to make the recovery plans after the production disruption occurs,
- present the advantages of the solution methodology developed for the production system, and
- provides a comparison of results and time taken to solve the problem by GA and PS search techniques.

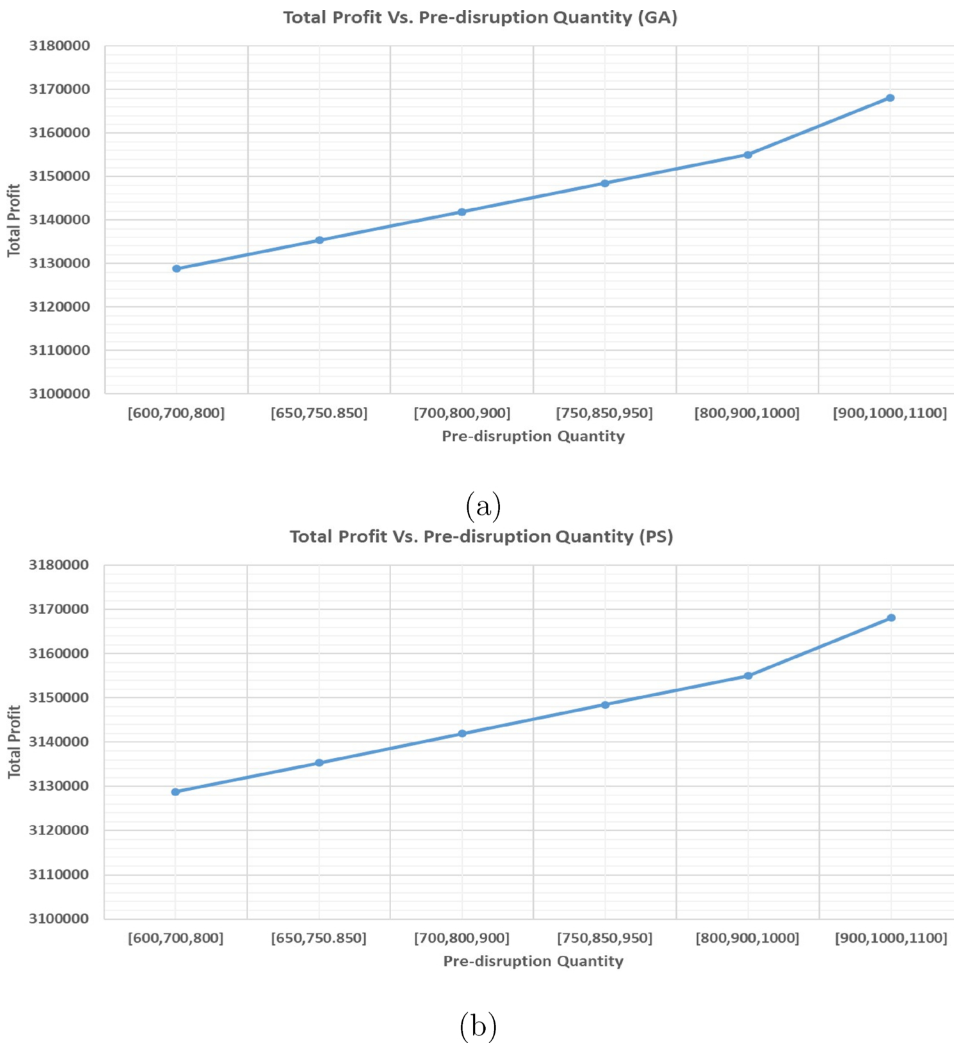


Fig. 3. Changes in total profit with respect to pre-disruption quantity q_n (a: For GA & b: For PS).

7. Conclusions

In production systems, disruption can occur at any time within the production run-time. Without a recovery plan for such disruptions, the organization may face huge financial and reputation losses. Furthermore, imperfect production lines are very common in real-life and greatly impact a company's profit and loss. The aim of this study was to develop a recovery plan for a disrupted single-stage multi-product imperfect production system. A mathematical model with constraints was developed for single-stage multi-product imperfect production systems. Space and budget constraints were considered, based on real life experiences. For implementation in the real-time recovery planning of a disrupted production system, a few examples were provided. These examples were solved using the pattern search (PS) and genetic algorithm (GA) solution approaches. Similar results were obtained by both solution approaches. In addition to this, sensitivity analyses were performed to show the impact on total profit; these were performed while changing one parameter and keeping all others constant. This model only considers single production disruption or multi-disruption without the effect of the previous disruption, which is a major limitation of this model. Immediate extension of this model is possible by considering multiple dependent production disruptions. Further, this model can be extended to different directions related to production or supply chain. The first extension of this model can be for the fuzzy or uncertain environment, parameter like demand can be considered as a triangular or trapezoidal fuzzy number. Several researchers considered different parameters as fuzzy for inventory models [47], and it would be a new direction if fuzzy parameters are combined with production disruption problem. The fuzzy nature of parameters will have a noticeable impact on the system and profit. Secondly, this model can be extended with the probabilistic proportion for defective products and all imperfect products reworked in the same setup or with separate setup, see for reference [18]. Another possible extension can be, multi-stage production with stochastic lead time demand [20]. Few other future research directions include considering the case of multiple shipments in a supply chain [10] with multi-stage imperfect production and random defective rate [54].

Conflicts of interest

The authors declare no conflicts of interest.

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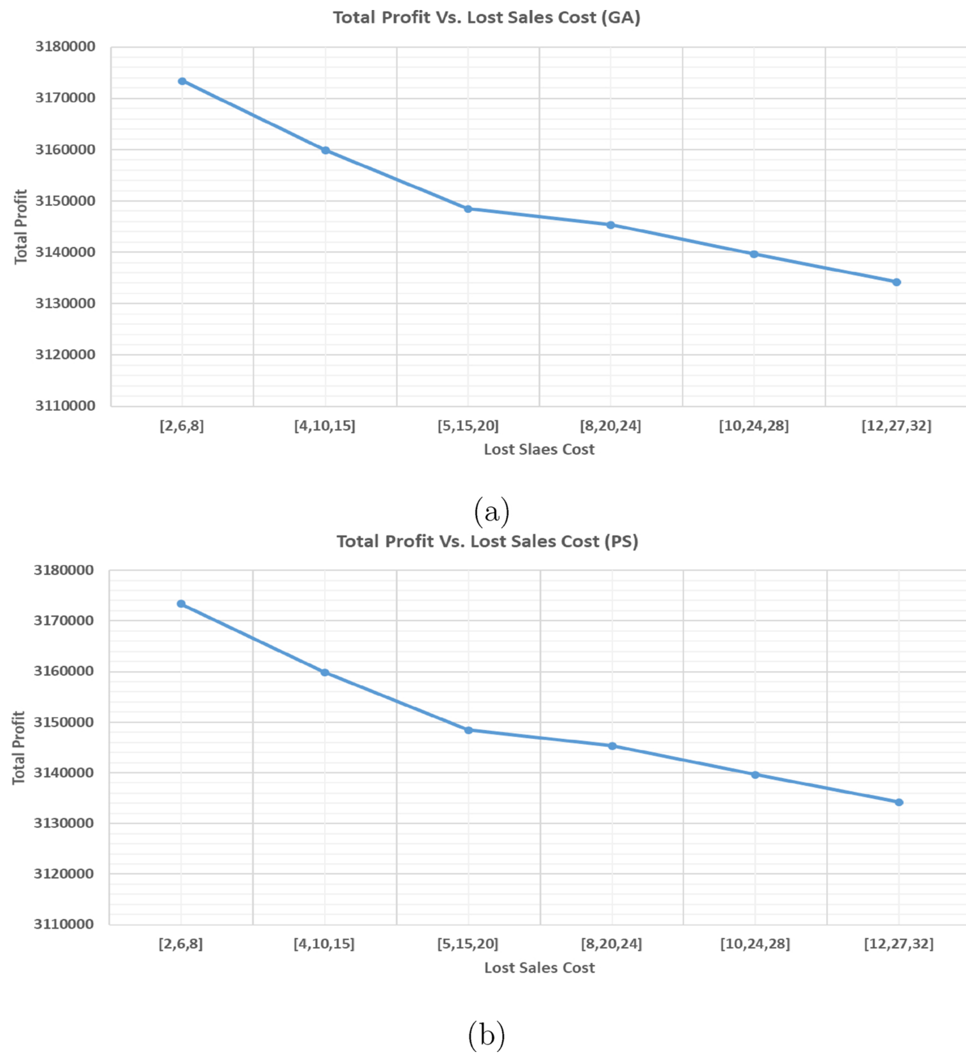

Fig. 4. Changes in total profit with respect to lost sales cost (a: For GA & b: For PS).

Appendix A

Total inventory holding cost:

$$\sum_{n=1}^{N} H_n\left[\frac{1}{2}q_n T_{0n} + q_n(T_{\mathrm{dn}} + S_{\mathrm{tn}} + T_{n1}) + \frac{1}{2}X_{1n}T_{1n} + \frac{1}{2}X_{2n}T_{2n} + \ldots + \frac{1}{2}X_{\mathrm{Mn}}T_{\mathrm{Mn}}\right]$$

$$= \sum_{n=1}^{N}\frac{1}{2}H_n\left[q_n\frac{q_n}{\mathrm{rq}_n} + 2q_n\left(T_{\mathrm{dn}} + S_{\mathrm{tn}} + \frac{X_{1n}}{\mathrm{rP}_n}\right) + X_{1n}\frac{X_{1n}}{\mathrm{rP}_n} + X_{2n}\frac{X_{2n}}{\mathrm{rP}_n} + \ldots + X_{\mathrm{Mn}}\frac{X_{\mathrm{Mn}}}{\mathrm{rP}_n}\right]$$

$$= \sum_{n=1}^{N}\frac{1}{2}H_n\left[\frac{q_n^2}{\mathrm{rP}_n} + 2q_n\left(T_{\mathrm{dn}} + S_{\mathrm{tn}} + \frac{X_{1n}}{\mathrm{rP}_n}\right) + X_{1n}\frac{X_{1n}}{\mathrm{rP}_n} + X_{2n}\frac{X_{2n}}{\mathrm{rP}_n} + \ldots + X_{\mathrm{Mn}}\frac{X_{\mathrm{Mn}}}{\mathrm{rP}_n}\right]$$

$$= \sum_{n=1}^{N}\frac{1}{2}H_n\left[\frac{q_n^2}{\mathrm{rP}_n} + 2q_n(T_{\mathrm{dn}} + S_{\mathrm{tn}}) + \frac{2q_nX_{1n}}{\mathrm{rP}_n} + \sum_{i=1} M\frac{(X_{\mathrm{in}})^2}{\mathrm{rP}_n}\right]$$

Appendix B

The backorder cost is calculated as

$$\sum_{n=1}^{N} B_n[(X_{1n}+q_n)\text{delay}_1 + \sum_{i=1}^{M} X_{\text{in}} \times \text{delay}_i]$$

$$= \sum_{n=1}^{N} B_n \left[(X_{1n}+q_n)\left[T_{\text{dn}} + \frac{q_n}{\text{rP}_n} + \frac{X_{1n}}{\text{rP}_n} - \frac{X_{1n0}}{\text{rP}_n}\right]\right.$$

$$\left. + \sum_{i=2}^{M} X_{\text{in}}\left[T_{\text{dn}} + (i-1)S_{\text{tn}} + \frac{q_n}{\text{rP}_n} + \sum_{j=1}^{i}\frac{X_{\text{jn}}}{\text{rp}_n} - \sum_{j=1}^{i}\frac{X_{\text{jn0}}}{\text{rp}_n} - \sum_{j=1}^{i-1}(u_{\text{jn}})\right]\right]$$

$$= \sum_{n=1}^{N} B\left[(X_{1n}+q_n)\left[T_{\text{dn}} + \frac{q_n}{\text{rP}_n} + \frac{X_{1n}}{\text{rP}_n} - \frac{X_{1n0}}{\text{rP}_n}\right] + \sum_{i=2}^{M} X_{\text{in}}\left[T_{\text{dn}} + (i-1)S_{\text{tn}}\right.\right.$$

$$+ \frac{q_n}{\text{rP}_n} + \sum_{j=1}^{i}\frac{X_{\text{jn}}}{\text{rp}_n}$$

$$\left.\left. - \sum_{j=1}^{i}\frac{X_{\text{jn0}}}{\text{rp}_n} - \sum_{j=1}^{i-1}\left(\frac{X_{\text{jn0}}}{D_n} - \frac{X_{\text{jn0}}}{\text{rP}_n}\right)\right]\right]$$

Appendix C

The lost sales cost is calculated as

$$= \sum_{n=1}^{N}\left[L\sum_{i=1}^{M} X_{\text{in0}} - L_n\text{rP}_n(T_{0n} + T_{1n} + T_{2n} + \ldots + T_{\text{Mn}})\right]$$

$$= \sum_{n=1}^{N}\left[L_n\sum_{i=1}^{M} X_{\text{in0}} - L_n\text{rP}_n\left(\frac{q_n}{\text{rP}_n} + \frac{X_{1n}}{\text{rP}_n} + \frac{X_{2n}}{\text{rP}_n} + \ldots + \frac{X_{\text{Mn}}}{\text{rP}_n}\right)\right]$$

$$= \sum_{n=1}^{N} L_n\left(\sum_{i=1}^{M} X_{\text{in0}} - \sum_{i=1}^{M} X_{\text{in}} - q_n\right)$$

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