The Duration of Systole in an Electrocardiogram in Normal Humans and in Patients with Heart Disease*

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RELATIONSHIP OF THE PULSE FREQUENCY (HEART RATE) AND THE DURATION OF THE VENTRICULAR ELECTROCARDIOGRAM (QT INTERVAL) IN NORMAL HUMANS AT REST

All clinicians agree that patients with heart disease may have an uncertain prognosis, with the main reason due to the limited resources to analyze the functionality of the myocardium. Therefore, it is desirable to find abnormalities that are not indirect symptoms of a weakness of the vascular system, but rather, depend more directly on the work of the myocardium.

Graphic methods opened a number of new possibilities that resulted in a better insight into the working myocardium. However, not all possible conclusions derived from these graphical methods have been analyzed in a comparable manner. Arrhythmias and the impairment of the propagation of an electrical pulse are of particular interest. Graphic signs that are associated with an abnormality of the myocardium, especially pulsus alternans and inversion of the T-wave in the electrocardiogram, have been intensively studied.

Graphic recordings and certain interesting features of the electrocardiogram have not been studied intensely. One area of special interest is the duration of the ventricular electrocardiogram (the duration of electrical systole). A detailed study of the duration of electrical systole is promising because recent progress in the physiology of the heart analyzed primarily the time dependency of the working heart. In addition, several publications have identified specific alterations in the duration of the ventricular electrocardiogram of the heart in different forms of heart disease. For example, Steriopulo[18] described elongation of the duration of electrical systole in aortic insufficiency, Fr. Kraus and G. Nicolai[13] identified a similar finding in myodegeneratio cordis (page 265), while H. Straub[20] found a reduction of the duration of the systole in paroxysmal auricular tachycardia. A scientific evaluation of such comments is possible only if it is known what kind of alterations occur in the duration of the ventricular electrocardiogram in normal humans with a healthy heart.

Measuring the duration of the ventricular electrocardiogram in normal humans has been done frequently and the results have been published. However, a systematic study regarding standardization of the measurements has, to my knowledge, not been done. A systematic study will result in basic knowledge by which pathological changes can be evaluated. If one wants to determine the duration of the ventricular electrocardiogram, it is important to know what interval to measure, how to measure it, and the size of the measurement error.

The information to be obtained from recordings of the electrocardiogram relate to the duration of electrical systole that are associated with the mechanical contraction of the ventricle. According to studies by Kahn,[12] Carl Tigerstedt,[22] Garten,[7] and Wiggers,[25] the electrical activity of the ventricle begins 0.02 to 0.045 seconds earlier than the increase in ventricular pressure and ends often 0.03 to 0.05 seconds before the valves of the aorta are closed. Sometimes, the ventricular electrocardiogram ends after the valves of the aorta are closed. An exact agreement of the duration of the electrical and mechanical event has not been detailed in many studies. Wiggers[25] points out that the duration of the electrical and mechanical events can change independently. In experiments with frog hearts, a simultaneous shortening of the duration of the electrocardiogram and a prolongation of the

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mechanical duration of the systole have been observed (G. Ralph Mines,[17] page 220). The electrical and mechanical events are not two expressions of the same routine in the heart muscle, although they are connected to each other. For example, Locke and Rosenheim[16] found that even though an isolated heart stops beating if it is perfused with calcium free solution, it will continue to exhibit a strong rhythmic action potential (this phenomenon has been extensively studied by G. Ralph Mines[17]). Aug. Hoffmann[10] observed the same phenomenon in muscarine-poisoned hearts.

It is important to realize that knowledge about the duration of the mechanical systole cannot be reliably obtained from the duration of the electrical systole. However, in usual conditions the duration of both events may be in good alignment. It is not appropriate to use numerous prior studies describing the duration of the mechanical heart systole in humans as a basis for electrocardiographic measurements. My studies will show that the results of measurements of the recordings from the heart beat and the pulse in general fit the electrocardiographic recordings.

Aug. Waller,[23] the discoverer of the electrocardiogram, was the first to measure the duration of the human electrical systole on the electrocardiogram, and he estimated its duration to be about 0.35 seconds. He also noted that this systolic electrical interval fits with the duration of the mechanical systole measured by Donders. The duration of the normal human ventricular electrocardiogram has been measured by several investigators—Einthoven and de Lint,[2] Fr. Kraus and G. Nicolai,[13] Lipschütz,[15] and Ad Hecht.[8] The data vary from 0.3 to 0.4 seconds. However very low values like 0.18 seconds (Lipschütz[15]) or very high values like 0.48 seconds (Kraus and Nicolai[13]) have also been described. Such data can be explained, according to my experience from analyzing hundreds of recordings, by technical differences during the measurement. Kraus and Nicolai[13] found that the electrocardiographic relationship between the pulse frequency and duration of electrical systole is similar to that observed earlier by several investigators (Landois,[14] Edgren,[1] A.H.Garrod,[5] Thurston,[21] F. Stockmann,[19] Fr. Kraus[13]). The absolute duration of the electrical systole is decreased with increased heart rate, but the percentage value of the duration of systole will increase. In other words, with increasing pulse frequency the duration of electrical systole and diastole will decrease, however, the decrease in the duration of electrical diastole is larger than the decrease in electrical systole.

The first goal of a study about the duration of the ventricular electrocardiogram must be to establish a mathematical relationship between the duration of the ventricular electrocardiogram and the pulse frequency (heart rate). In addition, variations in the normal duration of electrical systole have to be determined. The existing studies regarding the duration of electrical systole are not satisfactory to solve this problem. Therefore these studies have to be done first.

To limit variations during measurements as much as possible, it is necessary to apply specific rules during the measurement. Two variables have to be measured. First, the duration of the ventricular electrocardiogram (QT interval), and second, the average time interval between two successive systoles, the pulse period. Each of these two parameters contains special types of variation.

To determine the duration of the ventricular electrocardiogram, the beginning and the end of ventricular systole in the electrocardiogram has to be measured. The beginning can always be determined with high accuracy, if one works the string in a string galvanometer with enough reaction speed. In the beginning, I worked with silver-coated quartz strings, but later I switched to the far better aluminum strings, which are mentioned by Wertheim-Salomonsen[24] (and can be obtained from Heraeus in Hanau). If the ventricular electrocardiogram starts with the Q-wave, measurement should begin with the onset of this wave since this wave is the first part of ventricular systole; if no Q-wave is present, the measurement should begin with the R-wave. The beginning of the ventricular electrocardiogram is more accurate to identify than is necessary (an accuracy of 0.001 seconds is possible).

The end of the electrocardiogram is much more difficult to determine, as pointed out by R. H. Kahn.[12] It is clear that the end or final portion of the T-wave is part of ventricular systole. It is also known that in several electrocardiograms a small elevation, the U-wave, appears after the T-wave. Einthoven,[3] who first discovered the U-wave in a case of myodegeneratio cordis, believed that this wave also originated in the ventricle. Later on, as the U-wave was also found in normal humans, it was thought that the U-wave does not depend on the work of the ventricles (see also for example

H. E. Hering,[9] Aug. Hoffmann,[11] page 85). For me, the U-wave was not part of the ventricular electrocardiogram "sensu strictiori," and I always measured to the end of the T-wave. Often, the end of the T-wave in a recording is a sharp bend that is clearly visible; many times the T-wave terminates very slowly in a slope that gradually approaches the baseline. This pattern does not depend on an insufficient reaction velocity of the string, because a higher reaction velocity does not change the outcome. I always considered the end of the T-wave as that point where the tangent of the curve crosses the baseline. Thus, I measured the maximal length of the ventricular electrocardiogram without the U-wave. In this way it is possible to determine the end of an electrocardiogram in almost all normal recordings and in many pathological recordings with a variation of not more then 0.01 seconds (see below). Especially in pathological conditions, with a high pulse frequency or extended pulse transmission time from the atrium to the ventricle, it is possible that the last part of the T-wave overlaps with the atrial-wave of the following systole. In such cases, a determination of the duration of the ventricular cardiogram is not possible. The difficulties that arise in pathological conditions by an abnormal (negative) shape of the T-wave, or where the T-wave is missing, will be discussed in a future essay, where I will describe this relationship in patients with heart disease. During the measurement, it is important that both the beginning and the end of the ventricular tracing are on the same side of the baseline, because otherwise the wideness of the shadow of the string may lead to errors. I always made my measurements on the positive side of the baseline.

The accuracy of the determination of the systole depends mostly, of course, on the accuracy of the time registration during the recordings. I have a photokymograph with a full apparatus for photographic glass-plates that uses the working principles of an Atwood fall-machine. The velocities of the plates move very smoothly and regularly, because of a cone immersed in an oil-filled cylinder; the velocity is changeable over a wide range by a valve at the cone. This apparatus is build by the Society for Scientific Instruments in Cambridge. On the moving plates, the time is photographically registered as the ordinate by the principle of S. Garten.[6] This time registration apparatus works by the principle of LaCour, where between the poles of two small electromagnets the electric current is interrupted by a regularly swinging tuning fork (this apparatus is similar to that one constructed by Bull in the Mayer-Institute). The time between two ordinates is 0.04 seconds. However, the velocity of the plates is very regular and can be determined with an accuracy of 0.001 seconds, a higher accuracy compared to the time to determine the duration of the electrocardiogram from the recordings. The normal velocity of the plates was 25 millimeter per second and 0.01 seconds were conveniently measurable.

On every plate (12 × 18 cm) about seven seconds were registered, which means that every electrocardiographic recording contained several systoles (frequently 6 to 14). The duration of all systoles on a graph was measured, and the average was calculated. If I used the average error from the determination of the duration of one systole, calculated by a Gauss standard method, then I obtain the following data (from lead II of 10 normal humans).

	Calculated Average Variation of a Single Duration of Systole (seconds)	**Calculated Average Variation of the Mean from All Systoles Obtained from One Graph (seconds)**
1	0.006	0.002
2	0.006	0.003
3	0.000	0.000
4	0.003	0.001
5	0.004	0.002
6	0.008	0.004
7	0.008	0.003
8	0.010	0.005
9	0.009	0.003
10	0.006	0.004

The average variation of the average duration of a ventricular electrocardiogram does not exceed 0.005 seconds. This means the third decimal number should be calculated and variations of more than 0.015 seconds for a single measurement will almost never be found.

One problem in the determination of the electrocardiographic duration of a systole remains. Einthoven[4] observed that the identified peaks in an electrocardiogram in different leads are often not matched to the identical phase of a heart period, and thus, the total duration of the ventricular electrocardiogram will be different in different leads. A possible explanation of this phenomenon is that the graphs are projections of vectors (the action potentials of the heart muscle). The biggest differences that I found for the duration of ventricular systoles in Einthoven's electrocardiographic recordings of 3 normal subjects at various pulse frequencies are:

		Period of Pulse (Reciprocal Value of the Pulse Frequency (seconds)	Duration of the Ventricular Electrocardiogram (seconds)
	Lead. I	0.628	0.321
A	Lead. II	0.622	0.336
	Lead.III	0.607	0.348
	Lead. I	0.890	0.360
S	Lead. II	0.887	0.367
	Lead.III	0.912	0.358
	Lead. I	0.842	0.344
N	Lead. II	0.858	0.344
	Lead.III	0.850	0.328

Variations up to 0.027 seconds in the ventricular electrocardiogram are possible and are bigger than the error of the measurement. To avoid a tremendous amount of material, I always focused on the duration of the right hand-left foot lead.

After describing the way to measure the duration of ventricular systole, it is necessary to describe the determination of the pulse frequency. The pulse frequency is determined from the pulse period, the time from the beginning of one systole to the beginning of the next. As mentioned, the electrical beginning of electrical systole is always clearly visible, and the duration of the pulse period is more accurately determined than the duration of a systole. However, a different source of variations could be disturbing. The pulse is almost never regular in mathematical terms; durations of pulse periods are changing, for example, during different phases of respiration. Therefore the pulse period must be presented as an average of the measured periods (often between 6 and 14 heart beats). The following table presents measurements in 10 normal subjects and shows that although the error of the measurement of a single duration of a period becomes meaningless, the average variation (if calculated by Gauss) of an average duration of the pulse period becomes bigger than the average variation in the duration of electrical systole. This is a consequence of an irregularity of the normal working heart.

	Calculated Average Variations of a Single Duration of a Pulse Period (seconds)	Calculated Variation of the Average Duration of a Pulse Period (seconds)
1	0.016	0.006
2	0.024	0.009
3	0.037	0.018
4	0.007	0.003
5	0.046	0.019
6	0.034	0.014
7	0.020	0.009
8	0.042	0.019
9	0.013	0.004
10	0.084	0.017

The mean error of the average value of the duration of a pulse period varied between 0.003 and 0.019 seconds. The third decimal is therefore meaningless, but will be written in the tables to follow.

To determine the relationship between the pulse frequency (heart rate) and the duration of the ventricular electrocardiogram (QT interval), two different approaches are possible. First, the pulse frequency could be altered in different ways (for example by muscle work or drugs) and the relationship could be studied in electrographic recordings. A number of my experiments performed in this way led to the conclusion that this approach should not be used since the duration of electrical systole may change for the same reasons that caused a change of the pulse frequency. The results of this study will be published in a later essay. Second, it was possible to determine the duration of electrical systole in an electrocardiogram on humans with different pulse frequencies at similar external conditions. I tried this way successfully and will describe the results as follows.

The normal participants were examined in the same external conditions as I used in exams of patients with heart disease in the hospital of the university. This was also done to obtain comparable data for latter studies about pathological conditions. Persons sat quietly in an armchair and were connected to the baths with the electrodes of the string galvanometer. They were resting here at least for 15 minutes before the electrographic recordings were made. All persons were examined with a stethoscope, their body weight and height were determined. I was as critical as possible to make sure that only normal healthy subjects were studied. Individuals who were sick during the past month (those who had a cured sore throat or had a cold) were excluded. Hospitalized patients were not among the participants. Also excluded were those who knew that they had earlier acute rheumatism, scarlet fever, diphtheria, or syphilis. Lastly, subjects were excluded if the stethoscope revealed an enlarged heart or heart murmurs. In this way it became difficult to find older people who were suitable for these studies. However, a few were found.

A total of 50 persons were examined. These were 35 subjects between 20 and 40 years of age, 19 men and 16 women. Because I also wished to study very high pulse frequencies, which occur at rest in children, I examined 9 healthy children, 6 boys and 3 girls. None of them was younger then 2 years of age, because such young children have difficulty sitting quietly for 15 minutes. Only 6 subjects were older than 40 years of age, 3 men and 3 women, and a few of them were very old. All together, 28 healthy male and 22 healthy female individuals were examined.

In most individuals, only one recording was made. However, I obtained several recordings in a few subjects. This led to problems, because on these recording the pulse frequency was most of the time somewhat different. A simple calculation of the average duration of the pulse period and the systole would lead to mistakes, because the pulse period and the duration of the systole are not directly proportional to each other. I chose for these persons those recordings where the pulse frequency was the lowest.

The results are summarized in the Main Table. The persons are organized by the pulse frequency, and the gender, age, weight, height, pulse frequency, average duration of the pulse period, and the duration of the ventricular electrocardiogram are provided. The data in the last columns are discussed below.

When the results are presented graphically (Figure 1), the duration of the ventricular electrocardiogram (QT interval) lengthens slightly with longer pulse periods (slower heart rate). For example, at the lowest pulse frequency studied (52 beats per minute and a pulse period of 1.15 seconds) the duration of the ventricular electrocardiogram (QT interval) lasts 0.40 seconds. With a doubling of the pulse frequency (104 pulses per minute and a pulse period of 0.575 seconds) the ventricular cardiogram shortens to 0.315 seconds, a 21%, reduction while the pulse period is shortened by 50%.

The goal for these studies is, as mentioned above, to identify the variations of the duration of the ventricular electrocardiogram for a given pulse frequency in normal subjects; only then will it be possible to compare pathological with normal findings. The simplest way to achieve this goal is by a mathematical analysis of the obtained data, expressing the relationship between the duration of the ventricular electrocardiogram (QT) and the pulse frequency (heart rate) as a mathematical function.

To find an empiric equation of the relationship between these two variables, I limited the equation to no more then two constants. Since the variability provides a better fit of the results with a smaller average error, the application of complicated equations involving several constants was avoided.

If the duration of the pulse period is denoted with p, the duration of the ventricular electrocardiogram with s, the constants with x and y, then the results are best fit by the equation $s = \mathrm{x} * p^{\mathrm{y}}$, as found by mathematical analysis.

In the equation given as $s = x * p^y$, the actual value of x and y can be calculated by the method of least squares of the 50 obtained results by using the Gauss method. x and y should have such values that the sum of the squares of the variations, which exist between the observed data and those calculated by the equation, becomes a minimum. This can be done the following way.

The equation $s = x * p^y$ is convertible into $\log s = \log x + y * \log p$, because as mentioned above, s and p are measured with the same average error. The logarithm of x and y should be estimated in a way that $(\log s_1 - \log x - y * \log p_1)^2 + (\log s_2 - \log x - y * \log p_2)^2 + \cdots + (\log s_{50} - \log x - y * \log p_{50})^2$ will become a minimum. Differentiation to $\log x$ makes

$\Sigma(\log s - \log x - y * \log p = 0$, to y : $\Sigma \log p (\log s - \log x - y * \log p) = 0$. These equations will lead to:

$$\log \mathrm{x} = \frac{\Sigma \log \mathrm{p} * \Sigma(\log \mathrm{p} * \log \mathrm{s}) - \Sigma \log \mathrm{s} * \Sigma(\log \mathrm{p})^2}{\Sigma(\log \mathrm{p})^2 - 50 * \Sigma(\log \mathrm{p})^2}$$

$$\mathrm{y} = \frac{\Sigma \log \mathrm{p} * \Sigma \log \mathrm{s} - 50 * \Sigma(\log \mathrm{p} * \Sigma \mathrm{s})}{\Sigma(\log \mathrm{p})^2 + 50 * \Sigma(\log \mathrm{p})^2}$$

The 50 values of p and s are listed in the main table. The calculations are normalized to 1/100 of a

Main Table

Gender	Age Years	Weight Kg	Height cm	Heart Rate bpm	Averaged Duration of Pulse Period Seconds	Averaged Duration of Ventricle ECG Seconds	Duration of Ventricle ECG Calculated from Pulse Period Seconds	Difference Between Measured and Calculated Values Seconds
1 Man	23	65.5	169	51	1.158	0.402	0.400	+0.002
2 “	27	75	182	52	1.155	0.400	0.400	0
3 “	27	69.5	179	55	1.094	0.398	0.393	+0.005
4 Woman	23	68	169	55	1.084	0.420	0.392	+0.028
5 Man	22	63.5	168	60	1.008	0.350	0.382	−0.032
6 “	21	79	180	62	0.968	0.352	0.377	−0.025
7 “	24	70	172	63	0.958	0.390	0.376	+0.014
8 Woman	37	62	154	67	0.904	0.363	0.369	−0.006
9 Man	21	72	181	67	0.900	0.344	0.368	−0.024
10 “	21	60	169	67	0.892	0.402	0.367	+0.035
11 Woman	27	63	157	67	0.890	0.376	0.366	+0.010
12 Man	29			68	0.887	0.367	0.366	+0.001
13 Woman	23	60	165	69	0.872	0.380	0.365	+0.015
14 Man	21	77	175	69	0.868	0.360	0.364	−0.004
15 Woman	21	56	168	72	0.833	0.360	0.359	+0.001
16 “	27	56	174	72.5	0.827	0.373	0.358	+0.015
17 Man	32	62	170	73	0.823	0.344	0.358	−0.014
18 “	32	70	168	75	0.800	0.345	0.354	−0.009
19 Boy	12	37.5	146	75	0.798	0.325	0.354	−0.029
20 Man	22	84	175	75	0.798	0.364	0.354	+0.010
21 Woman	45	68	165	76	0.793	0.356	0.353	+0.003
22 Man	21	73	179	76	0.792	0.356	0.353	+0.003
23 “	71	73	168	76	0.790	0.340	0.353	−0.013
24 “	32	70	172	76	0.787	0.370	0.352	+0.018
25 “	18	57	166	79	0.763	0.352	0.348	+0.004
26 Woman	20	75	170	80	0.753	0.352	0.347	+0.005
27 “	21	59	164	80	0.750	0.348	0.346	+0.002
28 “	23	53	158	83	0.719	0.328	0.342	−0.014
29 “	21	69	166	86	0.699	0.348	0.338	+0.010
30 “	22	63	163	88	0.680	0.340	0.337	+0.003
31 “	27	66	166	88.5	0.676	0.332	0.334	−0.002
32 “	29	70	170	89	0.671	0.327	0.334	−0.007
33 Man	31	62	165	90	0.668	0.337	0.333	+0.004
34 Woman	31	63	162	91	0.657	0.360	0.331	+0.029
35 Man	72	58.5	175	91	0.656	0.333	0.331	+0.002
36 Woman	30	66	169	92	0.652	0.334	0.331	+0.003
37 Boy	9	24	131	94	0.640	0.322	0.329	−0.007
38 Man	28	78	180	96	0.626	0.321	0.326	−0.005
39 Woman	40	57	155	96	0.626	0.350	0.326	+0.024
40 “	45			96	0.625	0.323	0.326	−0.003
41 “	30	62	160	100	0.598	0.311	0.321	−0.010
42 Man	21	68	179	101	0.594	0.312	0.320	−0.008
43 Boy	13	40	150	103	0.583	0.340	0.319	+0.021
44 Girl	4	16	100	104	0.576	0.308	0.317	−0.009
45 Boy	7	24.5	124	105	0.573	0.317	0.316	+0.001
46 Man	25	70	175	114	0.527	0.302	0.308	−0.006
47 Girl	2	15	94	122	0.491	0.284	0.301	−0.017
48 Boy	15	35	137	124	0.481	0.312	0.299	+0.013
49 Man	81	87	157	125	0.480	0.273	0.299	−0.026
50 Girl	3	15	95	135	0.443	0.280	0.290	−0.010

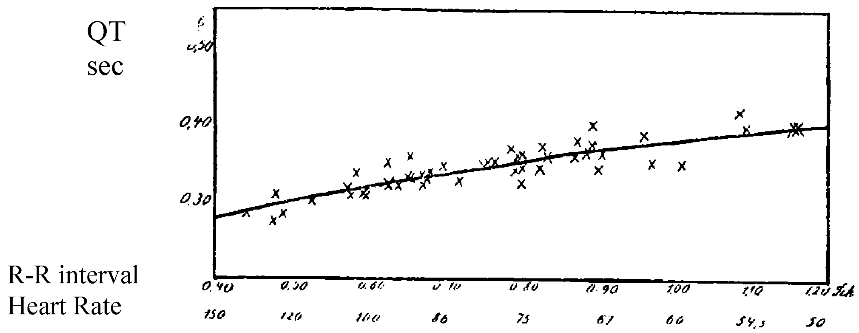

Figure 1.

second. The result of this tiresome calculation is:

$$\log x = \frac{93.41129 * 143.7335 - 76.84152 * 175.0086}{93.41129^2 - 50 * 175.0086} = 0.87211$$

$$x = 7.4492$$

$$y = \frac{93.41129 * 143.7335 - 50 * 143.7335}{93.41129^2 + 50 * 175.0086} = 0.3558$$

The relationship between s and p is therefore $s = 7.4492 * p^{0.3558}$, if s and p are given as 1/100 seconds. It becomes immediately clear that the value $(p)^{0.3558}$ is exchangeable with $\sqrt[3]{p}$ without any significant error, if at the same time the value of x(7.4492) is changed accordingly. In conclusion: $x * \sqrt[3]{p} = 7.4492 * (p)^{0.3558}$; this is $x = 7.4492 * (p)^{0.0225}$, and with an average pulse frequency of p = 0.80 (in 1/100 seconds = 80) will lead to x = 8.22.

The derived relationship is:

$$s = 8.22\sqrt[3]{p}$$

where s is the duration of the ventricular electrocardiogram and p the pulse period, both values as 1/100 seconds. This equation shows that in a normal, resting subject, the duration of electrical systole in an electrocardiogram is proportional to the cube root of the duration of the pulse period *(in translation:* $QT = 8.22\sqrt[3]{RR}$).

In the main table, values for s for every measured number of the pulse period, calculated by using this equation, are listed in the next to last column. The last column lists the errors between the observed and the calculated values of s. From these 50 errors the average error can be calculated by the Gauss method. The sum of these 50 error squares is 108.89 (1/100 seconds), and the average error is $\sqrt{[108.95/(50-2)]} = 1.5063$ or 0.015 seconds for a single error determination. This means that if the observed data vary from the calculated data by an average error of 0.015 seconds, only variations of more then three times of the averaged error, i.e of 0.045 seconds are outside the normal range. The biggest single variation of the measured duration of electrical systole among the 50 normal persons was 0.035 seconds, a value significantly less than the calculated limit of 0.045 seconds.

It remains to be investigated how the variations of the observed and the calculated data in the duration of an electrocardiogram are distributed among the different participants. For example, are all positive variations limited to a special group or is there another regular distribution of the variations.

An important question is whether the equation $s = 8.22\sqrt[3]{p}$ is valid for all pulse frequencies. Are some pulse frequencies (high, middle, low) associated with larger variations in s, or do some of the variations in s have a special tendency in one direction or another i.e., are most of the s-values too long [positive error] or too short [negative error]? To investigate this, the following table was calculated so that the 50 persons were divided in 5 equally large groups: the first group consists of the 10 lowest pulse frequencies, the second group the 10 next lowest, and so on. For each group the measured value of electrical systole is presented, where the systoles are longer than calculated (positive error) or shorter than calculated (negative error). Within every group the sum of all positive errors and all negative errors (not error-squared) is presented.

		Positive Error		Negative Error	
	Frequency	**n**	**Sum**	**N**	**Sum**
Group 1	51–67	5	+84	5	−87
Group 2	67–75	6	+52	4	−56
Group 3	75–88	8	+48	2	−27
Group 4	88.5–96	5	+62	5	−24
Group 5	100–135	3	+35	7	−86
	Total n	27	+281	23	−280

The distribution of the errors among the different frequencies is as regular as one could wish for. Only a minor overhang of the positive errors at the middle and the negative errors at high frequencies appears. It is doubtful whether these errors can be improved by applying a more complicated equation.

In the following table, the distribution of the errors among the different age groups is summarized.

Persons		Positive Error		Negative Error	
n	Age	N	Sum	n	Sum
6	Less then 20 years	4	+39	5	−72
35	20–40 years	20	+213	15	−166
6	Over 40 years	3	+29	3	−42
Total n: 50		27	+281	23	−260

Again, there is a very even distribution of the errors. Tables organized by body height or the weight of the participants led to the same result.

The conclusion from all the data is that the equation $s = 8.22\sqrt[3]{p}$ with an average error of 0.015 second, adequately describes the relationship between the duration of the ventricular electrocardiogram and the pulse period (and therefore also the pulse frequency) among the 50 examined normal persons at rest. The equation appears to be valid over a wide age range (2 to 87 years) and pulse frequencies (51 to 135 per minute). Lower or higher pulse frequencies in resting humans are found only in pathological conditions. About this I will talk in a future essay.

Instead of the equation, the graph in Figure 1 can, of course, be used, too. However, it is important to remember that variations of more than 3 times the average error for the duration of the ventricular electrocardiogram, that is more than 0.045 seconds in resting humans, can occur in pathological conditions.

It is very interesting that A. H. Garrod[5] in 1871 and E. Thurston[21] in 1876 measured duration of systole using a mechanical radial curve, and they described the relationship of the duration of systole to the pulse frequency with a nearly identical equation (but in a more complicated way). Garrod[5] concluded that the duration of systole, measured on the radial curve, is proportional to the cube root of the curve from the heart tip (Herzspitzenkurve), and the square root of the pulse frequency. However, he investigated only 15 persons, and his small amount of data material was not mathematically analyzed.

The present empirical equation is based only the expression of 50 observation. This fact does not lower its value for the identification of pathological variations. I can't predict if this equation will also have physiological impact, but value of the equation will be determined by appropriate experiments.

The influence of nerves and activity on the duration of electrical systole, independent of the pulse frequency, will be described in a future essay.

CONCLUSION

1. Electrographic instruments and the defined measurements allowed determination of the average duration of the ventricular electrocardiogram in the normal human heart. The Gauss error was 0.005 seconds, and variations in the average duration of the pulse period (the reciprocal of the pulse frequency) did not exceed 0.019 seconds.
2. The duration of the ventricular electrocardiogram is somewhat variable in different leads. Therefore, the duration of electrical systole was always measured in the right hand-left foot lead.
3. The duration of the ventricular electrocardiogram was determined at rest in 50 normal male and female subjects who had a wide range of ages.
4. From the obtained data it was possible by mathematical analysis to hypothesize a relationship between the duration of the ventricular electrocardiogram (*s*) and the duration of the pulse period (*p*), which can be expressed in the following standard equation:

   $s = 8.22\sqrt[3]{p}$ *(in translation:* $QT = 8.22\sqrt[3]{RR}$)
   (where *s* and p are expressed in 1/100 seconds)
5. The measured data of s fit those calculated from the standard equation using an average (Gauss) error of 0.015 seconds for a single duration.
6. The variations between the observed and calculated values of *s* are very evenly distributed among the observed data and are not influenced by the pulse frequency, age, or size of the individual.

7. Variations of the observed duration of electrical systole, calculated by using a standard equation, when larger than three times of the average error, that is more than 0.045 seconds, are considered to be pathological.

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