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Generalized logistic growth modeling of the COVID-19 pandemic in Asia

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ABSTRACT

Several months into the ongoing novel coronavirus disease 2019 (COVID-19) pandemic, this work provides a simple and direct projection of the outbreak spreading potential and the pandemic cessation dates in Chinese mainland, Iran, the Philippines and Chinese Taiwan, using the generalized logistic model (GLM). The short-term predicted number of cumulative COVID-19 cases matched the confirmed reports of those who were infected across the four countries and regions, and the long-term forecasts were capable to accurately evaluate the spread of the pandemic in Chinese mainland and Chinese Taiwan, where control measures such as social distancing were fully implemented and sustained, suggesting GLM as a valuable tool for characterizing the transmission dynamics process and the trajectory of COVID-19 pandemic along with the impact of interventions.

Introduction

Coronavirus disease 2019 (COVID-19), caused by the novel severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) was identified in Chinese mainland, in November 2019, declared to be a public health emergency of international concern in two months, and recognized as a pandemic on 11th March 2020. As of 1st July 2020, approximately 10.8 million cases of COVID-19 have been reported in 213 countries and territories, resulting in approximately 520,000 deaths. Emerging infectious diseases (EID), which appear in a population for the first time, or that may have existed previously but is rapidly spreading, are possibly the deadliest and continue to challenge human health.

As the world races to find a vaccine or a treatment to combat the pandemic, many concerns arise about the outbreak severity, particularly the potential number of infected people. Hence, it is of a great importance to estimate the outbreak evolution using epidemiology models. Here, the epidemiological dataset of confirmed cases with COVID-19 in Chinese mainland, Iran, the Philippines and Chinese Taiwan was analyzed, using the generalized logistic model (GLM), also known as Richards' model. This empirical function has made many remarkable coincidences with real SARS, Zika and Ebola epidemic data for real-time prediction of outbreak development (Chowell et al., 2017; Heisey, 2004; Smirnova et al., 2016; Zhou & Yan, 2003). Early assessment of the severity of infection and transmissibility can help quantify COVID-19 pandemic potential and anticipate the likely number of infected people by the end of the epidemics.

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Data and methods

Specific countries/regions were selected as denoting different COVID-19 incidence scales. Chinese mainland and Iran, the two major centers of COVID-19 outbreak in eastern and southern Asia, respectively with tens of thousands of cases; the Philippines, a representative of an archipelagic country with thousands of cases, and Chinese Taiwan, with only hundreds of cases. For each country/region, the officially reported data on COVID-19 daily cases from the onset of the outbreak to July 1st, 2020 were collected from governmental or health authorities' websites (Appendix Table 1 Fig. 1A–D). To allow outbreak projection, the data from the early phase of the outbreak (35–40 days) were fitted with the GLM, an extension of the standard logistic or sigmoid functions, as described previously (Richards, 2009; Zhou & Yan, 2003). The originally logistic model was developed by Verhulst in 1838 for biological populations growth modelling (Verhulst, 1838). Half a century ago, an extension of this classic logistic model, allowing for more flexible curvature of the S shape where the growth curve is asymmetrical, was introduced, establishing the Richards curve or generalized logistic model (Richards, 1959). This approach was chosen for its simplicity, minimal number of parameters, and for its ability to capture the true extent of the prevalence of the pandemic. Inspired from population biology, this model assumes an initial exponential growth phase that saturates as the number of cases accumulates owing to sustained control interventions and newly-adopted human behaviors. It enables the evaluation of the cumulative number of COVID-19 cases, represented by *Y(t)*. The dynamics of *Y*, in period *t*, can be expressed as:

$$Y(t) = \frac{K}{\left(1 + e^{-r(t - t_m)}\right)^{\frac{1}{\alpha}}}$$

Where K is the upper asymptote, or the maximum cumulative case incidence and r is the intrinsic growth rate during the exponential phase. t_m is the turning point, the time where maximum number of cases per day occur, is estimated as $t_m = K(1+\alpha)^{-\frac{1}{\alpha}}$. The exponent α measures the deviation from the symmetric classic logistic curve. $\alpha = 1$ illustrates a symmetric distribution centered at t_m; $\alpha > 1$ or $\alpha < 1$ indicates that *Y(t)* grow faster or slower, respectively, than predicted by the model. Not a single new COVID-19 case emerging within 3 consecutive months defines the end of the epidemic (Zhou & Yan, 2003). The basic reproduction number of COVID-19, R_0, the expected number of secondary cases produced by a single infection was estimated as $R_0 = e^{rT}$, where T is the mean serial interval; the time that elapses between onset of symptoms in the primary case and onset of symptoms of the secondary case.

Parameter estimates for this dynamical system are typically subject to sources of uncertainty arising from noise in the data. In order to estimate the uncertainty of our model estimates and construct the 95% confidence intervals, we used a parametric bootstrap approach to randomly generate multiple samples from the best-fit curve of the empirical distribution of the parameters. For each of the four localities data sets, 800 bootstrap iterations were computed.

The model should conform to several assumptions: first, as the number of tests conducted affects the reported number of daily cases, similar number of individuals are tested daily. Secondly, cases are not imported from outside the country/region. Thirdly, the model does not consider human behavior and is conditional on the assumption that public gatherings are highly limited, allowing the epidemic to follow its natural course. Likewise, implementation of intervention measures, such as enhanced hygiene, isolation, contact tracing, restrictions on social contacts and migration by air or train, are maintained continuously.

Results and discussion

Corresponding to the basic premise of GLM, as of April 10th, the cumulative COVID-19 cases curve of each country/region consists of a single peak of high incidence, resulting in a sigmoid curve with a single turning point (Fig. 1E–H). For all localities, high correlations between observed and predicted incidence were found ($R^2 > 0.99$, p value $< 2.2e^{-16}$).

To evaluate the forecasting performance of the model, the total number of cases on April 10th were estimated based on the observed incidence during the initial stage of the pandemic. A subset data of 35 days was used for Chinese mainland (January 21st to February 24th, 2020), Iran (February 26th to March 31st, 2020) and the Philippines (February 19th to March 24th, 2020). In Chinese Taiwan, due to a smaller scale of cases, dataset of 40 days (February 25th to April 4th, 2020) was required for optimal fit to the model. Forecasting the total number of cases on April 10th, 2020, was done 46, 10, 17 and 6 consecutive days ahead for Chinese mainland, Iran, the Philippines and Chinese Taiwan, respectively. In all cases, the predicted cumulative number of cases was similar to the one officially reported (Fig. 1E–H, Table 1). The predicted number was 81796 for Chinese mainland, 68225 for Iran, 4430 for the Philippines and 425 for Chinese Taiwan, while the observed total number of cases was 81953, 68192, 4195 and 382, respectively [see Table for 95% confidence interval (CI)]. The maximum predicted cumulative incidence, K, was estimated to be 81871 for Chinese mainland, 105547 for Iran, 6367 for the Philippines, and 483 for Chinese Taiwan. The estimated exponential daily growth rate (r) of COVID-19 was 22% in Chinese mainland, 10% in Iran, 18% in the Philippines, and 14% in Chinese Taiwan. These rates suggest that the cumulative number of infected people will double every 4.5, 10, 5 and 7 days in Chinese mainland, Iran, the Philippines and Chinese Taiwan, respectively, if control interventions to contain the pandemic are not implemented. In all countries/regions, the estimated asymmetric parameter α was close to 1, indicating that the differences in COVID-19 cases before and after t_m will be distributed similarly.

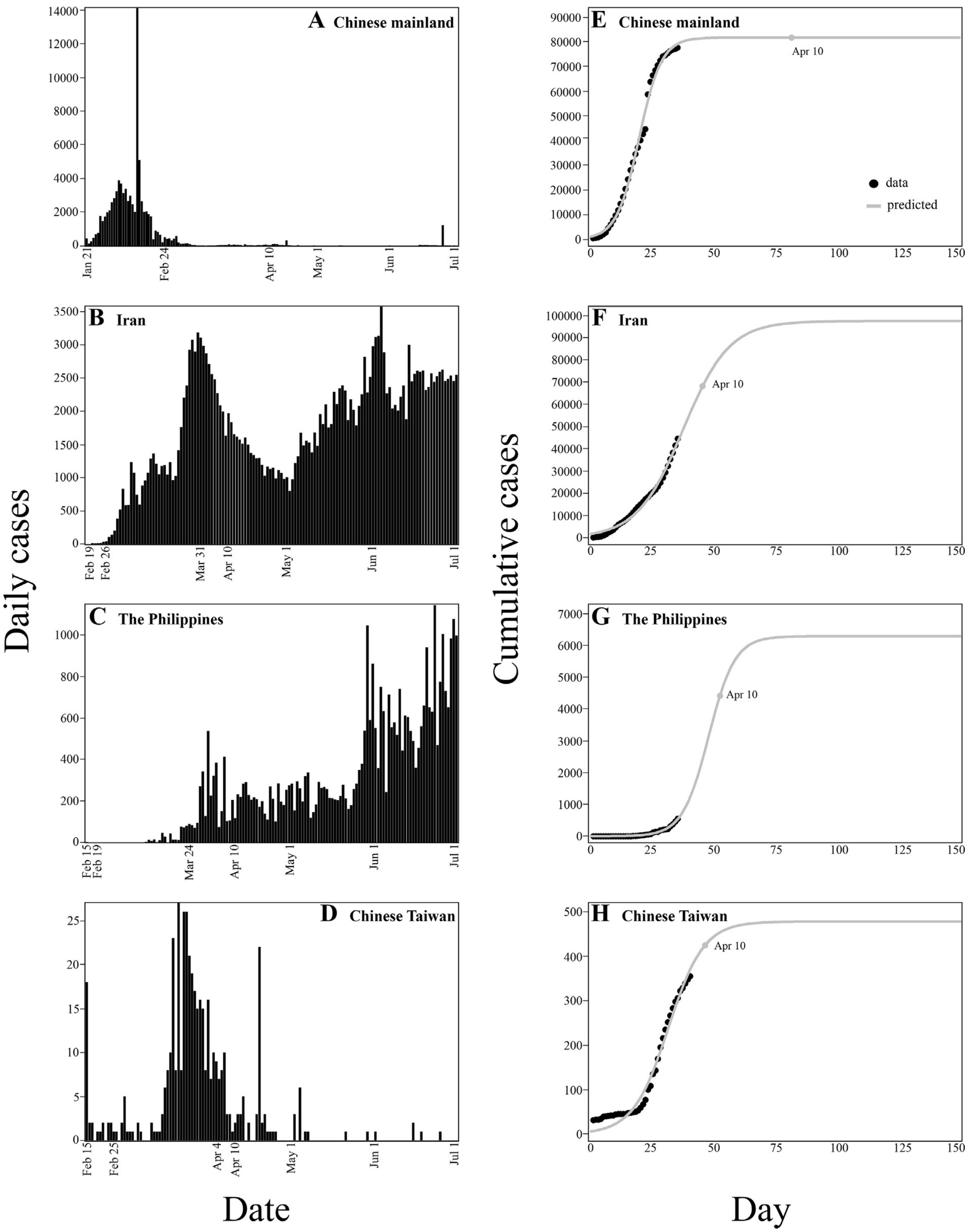

Fig. 1. The generalized logistic growth model-predicted size of the COVID-19 pandemic in Chinese mainland, Iran, the Philippines and Chinese Taiwan. On the left (A-D): the daily number of new confirmed COVID-19 cases. On the right (E-H): the observed (black circles) and the model-fitted and predicted cumulative cases (grey solid line) over time. The grey circle denotes the predicted number of cumulative cases as of 10th April 2020.

Table 1
The generalized logistic growth model estimates of COVID-19 pandemic.

Parameter	Chinese mainland	Iran	The Philippines	Chinese Taiwan
R^2[a]	0.997	0.995	0.995	0.99
α [95% CI[b]]	0.99 [0.97, 1.02]	1 [0.8, 1.6]	0.99 [0.35, 1.63]	0.98 [0.88, 1.13]
r [95% CI[b]]	0.22 [0.21, 0.23]	0.1 [0.09, 0.11]	0.19 [0.18, 0.2]	0.14 [0.13, 0.15]
R_0 [95% CI[b]]	3.59 [3.39, 3.79]	1.86 [1.83, 1.89]	3 [2.8, 3.2]	2.26 [1.86, 2.66]
April 10th projected No. of cases [95% CI[b]]	81796.55 [81796.53, 81796.57]	68225 [65971, 70382]	4430 [4172, 4669]	425 [418, 431]
April 10th observed No. of cases	81953	68192	4195	382
t_m projected [95% CI[b]]	19.4 [19.14, 19.66]	37.8 [33.9, 41.7]	44.7 [39.9, 49.5]	31.3 [29.86, 32.74]
t_m observed	Day 23 (February 12th)	Day 34 (March 30th)	Day 42 (March 31st)	Day 23 (March 20th)
Maximum No. of cases [95% CI[b]]	81871 [80520, 83222]	105547 [69451, 141643]	6367 [0, 22374]	483 [441, 525]
Pandemic cessation day number and date [95% CI[b]]	102 [72, 127] May 1st [April 1st, May 26th]	213 [147, 252] September 25th [July 21st, December 3rd]	131 [119, 157] June 28th [June 16th, July 24th]	129 [80, 181] July 2nd [May 14th, August 23rd]
July 1st observed No. of cases	83534	230211	38511	447

α, the deviation from the symmetric classic logistic curve; r, the intrinsic growth rate during the exponential phase; R_0, the basic reproduction number; t_m, the turning point of the model, the date in the brackets indicates the actual day where the maximum number of cases has occurred.
[a] Pearson's correlation goodness-of-fit of the model, p value $< 2.2e^{-16}$.
[b] CI, confidence interval.

According to the model, the earliest time for the current COVID-19 pandemic to cease, was evaluated to occur after 102 days (May 1st) in Chinese mainland, 213 days (September 25th) in Iran, 131 days (June 28th) in the Philippines, and 129 days (July 2nd) in Chinese Taiwan. A mean serial interval of 5.8 days (He et al., 2020) was used to calculate R_0, the basic reproduction number of COVID-19 infections. R_0 estimates were 3.59, 1.86, 3 and 2.26 in Chinese mainland, Iran, the Philippines and Chinese Taiwan, respectively, similar to those published previously (Zhao et al., 2020) and to that of SARS (Liu et al., 2020). These early stages R_0 are likely to decrease with control measurements policies continuation. Indeed, predicting COVID-19 dynamics based on its initial growth phase, revealed that the turning point of each country has occurred closer to the lower limit of the 95% CI and even earlier than expected, due to the control measurements effectiveness. The turning point was estimated as day 19.4 [19.14, 19.66] in Chinese mainland, 37.8 [33.9, 41.7] in Iran, 44.7 [39.9, 49.5] in the Philippines and 31.3 [29.86, 32.74] in Chinese Taiwan, whereas the observed highest number of daily cases in Chinese mainland (14108), Iran (3186), the Philippines (538) and Chinese Taiwan (27), as of April 10th, has occurred on day 23, 34, 42, and 23, respectively (Table 1).

Forecasting COVID-19 pandemic is challenging in the context of an outbreak caused by novel pathogen for which its natural history and modes of transmission are unknown. Since the GLM is trained on the existing data and is designed to fit the development of epidemic curves, rather than EID estimation only, it could provide a good fit to the limited available COVID-19 epidemiological data to characterize the transmission dynamics process and the trajectory of COVID-19 pandemic along with the impact of interventions (Wang et al., 2012). The primary goal of this prediction, made on 10th April 2020, during the height of the public panic and uncertainty, is to evaluate when the pandemic might be brought under control based on its initial growth phase and is conditional to intervention measures continuation. Changes of the current policies or human behavior may affect the actual contact rate and the subsequent development of the epidemic. Additionally, testing kits deficiency can lead to poor diagnosis and incomplete data, which may implicate the model robustness. As of 1st July 2020, almost three months after this prediction, the officially reported number of cumulative cases was 83534 for Chinese mainland, 230211 for Iran, 38511 for the Philippines, and 447 for Chinese Taiwan. While the short-term predictions matched the reports of the total infected people across the four countries/regions, the long-term maximum predicted number of cumulative COVID-19 cases matched the confirmed reports of those who were infected only in Chinese mainland and Chinese Taiwan.

Exploring the governmental policies dynamics in Iran and the Philippines suggests that substantial changes in policies and human behavior have influenced the model robustness. One major change in the Philippines, is the total number of overseas Filipino workers (OFWs) that have returned to the country during the pandemic. During February and March, approximately 850–980 land-based and seafarers from cruise ships Filipino workers have returned to the Philippines. In April alone, more than 17250 Filipinos repatriates were reported by the department of health Philippines (DOH) and the department of foreign affairs (DPA), almost 18-fold change, compared with the number of Filipino repatriates during the previous months of the pandemic, on which this prediction is based on. The increased number of OFWs and the growing probably of COVID-19 infected people among them controvert one of the crucial assumptions of the model which states that new cases are not imported from abroad, due to the model inability to assess imported cases of COVID-19 and their onwards transmission. Furthermore, significant differences in COVID-19 testing rates were observed in the Philippines, contrasting the model assumption that the testing policy is somewhat consistent over time. Up to April 19th, DOH reported that it had performed a total of approximately 53000 individual tests for COVID-19. On May 2nd, additional 3000 tests per day were added, and two days later DOH reported that the testing capacity of laboratories for COVID-19 in the country had reached the 5000-mark, recording 5264 tests per day. Starting May 15th, the country can accommodate up to 9200 tests daily. Whereas a total of 53000 tests for COVID-19 were performed during the first months of the pandemic, starting mid-May, a similar number of tests were performed in just 5–6 days, leading to considerable increase in the number of detected COVID-19 cases in the Philippines.

Iran also has seen a rapid surge in the numbers of COVID-19 cases in recent weeks. In Iran, during the initial outbreak, the highest daily the number of reported cases (t_m) reached a high of 3186 on March 30th, when it became one of the worst affected countries outside Chinese mainland. In mid-April, as the number of reported cases decreased to approximately 1000 a day, and since it cannot economically afford to continue restrictions, Iran began to ease its restrictions: shopping malls, bazaars, public parks and recreation areas reopened and travel resumed between different provinces. In May, the government allowed restaurants, cafes, museums, historical sites, all mosques and major shia religious shrines to reopen. Our forecast assumes that existing control measures and levels of social distancing will remain in place during the prediction period, but here, with the considerable easing of restrictions, there was a clearly increased social interaction and the daily the number of reported cases reached a second high of 3574 on June 4th.

Taken together, the study demonstrates that governmental policies and human behavior would have a great effect on containing SARS-CoV-2 and on forward trajectories of COVID-19 with accuracy. The retrospective analysis of SARS-CoV-2 dynamics in Chinese mainland, Iran, the Philippines and Chinese Taiwan, using limited data in the early period of the pandemic found that the GLM has proved useful to generate short-term and long-term forecasts of the trajectory of the pandemic in countries/regions representing a range of COVID-19 incidence scales. The model highlights the significance of fully implemented and continued social distancing measures in containing the virus. Moreover, implementing social distancing policies at an early stage, to reduce the reproduction number (R_0) of the virus, will provide more time for the adaptation of health care systems to the current pandemic and for the development of medicinal solutions. The model also demonstrates that substantial relaxation of the early stage policies in the lack of medicinal and pharmaceutical interventions will probably result in pandemic reemergence. Even though this prediction is conditional to intervention measures continuation, forecasting epidemic size and peak time may help clarify what the future holds, and could be useful to make long-term strategic decisions regarding the distribution of testing and treatment facilities that may be required, and may be helpful to assess the extent of protective and medical equipment needed for the near future.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix

Appendix Table 1
Daily number of new reported COVID-19 cases in Chinese mainland, Iran, the Philippines and Chinese Taiwan.

Date (yyyy-mm-dd)	Chinese mainland	Iran	The Philippines	Chinese Taiwan
2020-21-01	440			
2020-22-01	131			
2020-23-01	259			
2020-24-01	457			
2020-25-01	688			
2020-26-01	769			
2020-27-01	1771			
2020-28-01	1459			
2020-29-01	1737			
2020-30-01	1981			
2020-31-01	2099			
2020-01-02	2589			
2020-02-02	2825			
2020-03-02	3235			
2020-04-02	3884			
2020-05-02	3694			
2020-06-02	3143			

Appendix Table 1 (*continued*)

Date (yyyy-mm-dd)	Chinese mainland	Iran	The Philippines	Chinese Taiwan
2020-07-02	3385			
2020-08-02	2652			
2020-09-02	2973			
2020-10-02	2467			
2020-11-02	2015			
2020-12-02	14108			
2020-13-02	5090			
2020-14-02	2641			
2020-15-02	2008		3	18
2020-16-02	2048			2
2020-17-02	1888			2
2020-18-02	1749			
2020-19-02	382	2		1
2020-20-02	898	3		1
2020-21-02	823	13		2
2020-22-02	648	11		
2020-23-02	214	14		2
2020-24-02	508	18		2
2020-25-02	406	34		1
2020-26-02	433	44		1
2020-27-02	327	106		
2020-28-02	427	143		2
2020-29-02	573	205		5
2020-01-03	202	385		1
2020-02-03	125	523		1
2020-03-03	119	835		1
2020-04-03	139	586		
2020-05-03	143	591		2
2020-06-03	99	1234	2	1
2020-07-03	44	1076	1	
2020-08-03	40	743	4	
2020-09-03	19	595	14	
2020-10-03	24	881	9	2
2020-11-03	15	958	16	1
2020-12-03	20	1075	3	1
2020-13-03	11	1289	12	1
2020-14-03	20	1365	47	3
2020-15-03	16	1209	29	6
2020-16-03	21	1053	2	8
2020-17-03	13	1178	45	10
2020-18-03	34	1192	15	23
2020-19-03	39	1046	15	8
2020-20-03	41	1237	13	27
2020-21-03	46	966	77	8
2020-22-03	39	1028	73	26
2020-23-03	78	1411	82	26
2020-24-03	47	1762	90	21
2020-25-03	67	2206	84	19
2020-26-03	55	2389	71	17
2020-27-03	54	2926	96	15
2020-28-03	45	3076	272	16
2020-29-03	0	2901	343	15
2020-30-03	79	3186	128	8
2020-31-03	36	3110	538	16
2020-01-04	35	2988	227	7
2020-02-04	31	2875	322	10
2020-03-04	19	2715	385	9
2020-04-04	30	2560	76	7
2020-05-04	39	2483	152	8
2020-06-04	32	2274	414	10
2020-07-04	62	2089	104	3
2020-08-04	63	1997	106	3
2020-09-04	42	1634	206	1
2020-10-04	46	1972	119	2
2020-11-04	99	1837	233	3
2020-12-04	108	1657	220	3
2020-13-04	89	1617	284	5
2020-14-04	46	1574	291	
2020-15-04	46	1512	230	2

(*continued on next page*)

Appendix Table 1 (*continued*)

Date (yyyy-mm-dd)	Chinese mainland	Iran	The Philippines	Chinese Taiwan
2020-16-04	26	1606	207	
2020-17-04	325	1499	218	
2020-18-04	27	1374	209	3
2020-19-04	16	1343	172	22
2020-20-04	12	1294	200	2
2020-21-04	11	1297	140	3
2020-22-04	30	1194	114	1
2020-23-04	10	1030	271	1
2020-24-04	6	1168	211	1
2020-25-04	12	1134	102	1
2020-26-04	11	1153	285	
2020-27-04	3	991	198	
2020-28-04	6	1112	181	
2020-29-04	22	1073	254	
2020-30-04	4	983	276	
2020-01-05	12	1006	284	
2020-02-05	1	802	156	3
2020-03-05	2	976	295	
2020-04-05	3	1223	262	6
2020-05-05	1	1323	199	
2020-06-05	2	1680	320	1
2020-07-05	2	1485	339	1
2020-08-05	1	1556	120	
2020-09-05	1	1529	147	
2020-10-05	14	1383	184	
2020-11-05	17	1683	292	
2020-12-05	1	1481	264	
2020-13-05	7	1958	268	
2020-14-05	3	1808	258	
2020-15-05	4	2102	215	
2020-16-05	8	1757	214	
2020-17-05	6	1806	208	
2020-18-05	7	2294	205	
2020-19-05	6	2111	224	
2020-20-05	5	2346	279	
2020-21-05	2	2392	213	1
2020-22-05	4	2311	163	
2020-23-05		1869	180	
2020-24-05	3	2180	258	
2020-25-05	11	2023	284	
2020-26-05	7	1787	350	
2020-27-05	1	2080	380	
2020-28-05	2	2258	539	
2020-29-05		2819	1046	1
2020-30-05	4	2282	590	
2020-31-05	2	2516	862	
2020-01-06	16	2979	552	1
2020-02-06	4	3117	359	
2020-03-06		3134	751	
2020-04-06	1	3574	634	
2020-05-06	5	2886	244	
2020-06-06	3	2269	714	
2020-07-06	6	2364	555	
2020-08-06	4	2043	579	
2020-09-06	3	2095	518	
2020-10-06	3	2011	740	
2020-11-06	11	2218	443	
2020-12-06	7	2389	612	
2020-13-06	11	1882	605	
2020-14-06	57	3000	538	
2020-15-06	49	2449	490	2
2020-16-06	40	2563	361	
2020-17-06	44	2612	457	
2020-18-06	28	2596	561	1
2020-19-06	32	2615	660	
2020-20-06	27	2322	941	
2020-21-06	26	2368	652	
2020-22-06	18	2573	630	
2020-23-06	22	2445	1143	

Appendix Table 1 (*continued*)

Date (yyyy-mm-dd)	Chinese mainland	Iran	The Philippines	Chinese Taiwan
2020-24-06	12	2531	470	
2020-25-06	19	2595	774	1
2020-26-06	13	2628	1004	
2020-27-06	21	2456	730	
2020-28-06	17	2489	652	
2020-29-06	12	2536	983	
2020-30-06	19	2457	1076	
2020-01-07	3	2549	977	

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