

RESEARCH ARTICLE

Optimal control and differential game solutions for social distancing in response to epidemics of infectious diseases on networks

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Summary

In this paper, the problem of social distancing in the spread of infectious diseases in the human network is extended by optimal control and differential game approaches. SEAIR model on simulation network is used. Total costs for both approaches are formulated as objective functions. SEAIR dynamics for group k that contacts with k individuals including susceptible, exposed, asymptotically infected, symptomatically infected and improved or safe individuals is modeled. A novel random model including the concept of social distancing and relative risk of infection using Markov process is proposed. For each group, an aggregate investment is derived and computed using adjoint equations and maximum principle. Results show that for each group, investments in the differential game are less than investments in an optimal control approach. Although individuals' participation in investment for social distancing causes to reduce the epidemic cost, the epidemic cost according to the second approach is too much less than the first approach.

KEYWORDS

differential game, infectious diseases, investment, network, optimal control, social distancing

1 | INTRODUCTION

Social distancing is changes in behavior that is applied to prevent the transmission of the disease. The dynamics of infectious disease such as influenza, smallpox, aids, etc. depending on the contact network. Social distancing practices can stop or reduce the severity of the epidemic. This distancing is applied to allow individuals to change their behavior in order to reduce the risk of disease. The structure of networks in the prevalence of epidemics such as HIV,¹⁻³ SARS,^{4,5} influenza,⁶⁻⁹ etc. is important. The basic reproduction number is dependent on the distribution of the contacts that are known as the distribution of degrees.^{1,10,11} Primary work focuses on the effects of small cycles.¹²⁻¹⁶ Most epidemics such as influenza, HIV, Sars, have expanded.¹⁷⁻¹⁹ It is used network theory to study the dynamics of transmission of infectious diseases in which the nodes represent people, etc.¹⁹ There are two topological and structural properties of the grids which cause the dynamics to spread: Transmissibility^{19,20} and centrality.^{13,21} Recovery of infections increases the number of infectious diseases and the mortality rate of the host.^{22,23} In order to study the best usage of social distancing, we apply control optimal and differential game approaches at a population scale. In game theory, the differential game is a situation of conflict or cooperative where players choose strategies over time.

There are more than one player in the differential game and the payoff to each player depends on the selection paths by all players. Of course, the control problem can be considered as a special case of differential games in which there is only one player. In the differential games, players make their moves over time. In this paper, we find the best investment in social distancing with optimal control and differential game approaches.

Vaccination is considered as one of the most measures to control influenza outbreak. In 2019, Arefin et al²⁴ investigated two types of vaccines; The traditional trivalent (TIV) flu vaccine and the quadrivalent (QIV) vaccine. They showed that people can choose QIV or TIV vaccine or they select none. Kabir et al²⁵ presented a model in which a dual social dilemma situation was exhibited. In this model, the treatment behavior changes on a time scale, and the vaccination uptake evolves on a time scale. A model that joins evolutionary game theory with epidemiological dynamics is presented by Tanaka et al.²⁶ In this model, the policies that encourage vaccination are appraised and in order to suppress the overall social cost of a vaccination program, individual subsidies for vaccinations can be directed to hub agents with priority.

The effect coming from the failure of getting perfect immunity to the individuals who perpetrated vaccination earlier is studied by Alam et al.²⁷ Their new model slows down the spread of the infectious and also facilitates quicker recovery time. In order to evaluate the performance of vaccination-subsidizing policies in the face of an epidemic, Kuga et al²⁸ combined evolutionary game theory and mathematical epidemiology. They investigated how the topology of social networks affects the results. Analyzing the effect of different heterogeneous networks in a population was presented by a two-layer susceptible-infected-recovered/unaware-aware (SIR-UA) epidemic model. In this study, the random network topologies are applied to investigate the impact of awareness on the spread of epidemics.²⁹ The check of this point that a disease transmission network does not necessarily correspond with an information diffusion network was done by Fukuda et al.³⁰ Evolutionary game theory can be in the context of epidemiology. Two approaches, quasi-analytical and multi-agent simulation, can illuminate how an infectious disease diffuses across a social network, which is affected by the public attitude toward vaccination.³¹

Vaccination game and social distancing strategies both aim to control an epidemic and minimize its cost. However since in this paper, we assumed that, one is in the lack of vaccine, the concept called “social distancing” is considered and discussed. Therefore in this paper, social distancing is a measure to control an epidemic as long as the vaccine is produced and is available.

In Section 2, the network dynamic of SEAIR model together with the aggregate investment parameter in social distancing are proposed. In Section 2.3, the related novel SEAIR objective functional for the best investment in social distancing is given. In Section 3, the problem for social distancing and its relative risk is solved by using optimal control and differential game approaches. In Section 4, stability is investigated and in Section 5, results and conclusions are given.

2 | MODEL

It is shown that in contrast to classical compartment models like SIR, a model that can consider the epidemics of infectious diseases on networks is a model that classifying the population as susceptible (S), exposed (E), asymptotically infected (A), symptomatically infected (I) and removed (R). Here, we use SEAIR model on networks to obtain the best aggregate investment, which is the average investment of the corresponding population. In order to determine the best aggregate investment strategy with social distancing, we use two different approaches; optimal control and differential game.

2.1 | SEAIR model

Consider a Susceptible Exposed Asymptotically infected Symptomatically infected Recovered (SEAIR) epidemic model with susceptible (X_1), exposed (X_2), asymptotically infected (X_3), symptomatically infected (X_4), and removed (X_5) states. When an individual is infected with an influenza virus, a short time span between the occurrence of a disease agent and the onset of symptoms, which is called a munition or latency period. This class of individuals who pass through the infected to stricken are called X_2 class. In X_2 class, a significant number of individuals do not reveal the symptoms of illness, but they are capable of transmitting illness. The differential equations describing the SEAIR model are as follows:¹⁹

$$\begin{aligned}\frac{dX_1}{dt} &= -\lambda_1 X_1 X_3 - \lambda_2 X_1 X_4 \\ \frac{dX_2}{dt} &= \lambda_1 X_1 X_3 + \lambda_2 X_1 X_4 - \delta X_2\end{aligned}$$

$$\begin{aligned}
 \frac{dX_3}{dt} &= \gamma\delta X_2 - \alpha_1 X_3 \\
 \frac{dX_4}{dt} &= (1 - \gamma)\delta X_2 - \alpha_2 X_4 \\
 \frac{dX_5}{dt} &= \alpha_1 X_3 + \alpha_2 X_4,
 \end{aligned}
 \tag{1}$$

where the parameters are shown in the Table 1 and are estimated in Table 2.

It is assumed that a safe individual will never be susceptible or infected again, and in this study the births and deaths of natural individuals and migrations will be ignored. In this study, we examine individuals in the network. Each person is marked with one vertex and we show the contact between two individuals with an edge that connects two vertices to each other.

It is assumed that the population is divided into n distinct groups of sizes $(k = 1, 2, \dots, n)N_k$ such that each individual is in group k , he (she) will have k contacts per day. Let, the size of the population is N ($N = N_1 + N_2 + \dots + N_n$) thus the probability that an individual has k contacts will be $P_k = \frac{N_k}{N}$, which is called the degree distributions of the network. It is shown in Empirical studies that many real networks have scale-free degree distributions $p(k) \approx k^{-\gamma}$ with $2 \leq \gamma \leq 3$ and epidemic threshold is not shown in epidemic model.¹⁸ If $X_{1_k}, X_{2_k}, X_{3_k}, X_{4_k}$ and represent the number of susceptible, exposed, asymptotically infected, symptomatically infected and recovered individuals belonging to group k (where $X_{1_k} + X_{2_k} + X_{3_k} + X_{4_k} + X_{5_k} = N_k$), then the following system of differential equations is presented as follows.²⁰

$$\begin{aligned}
 \frac{dX_{1_k}(t)}{dt} &= -\lambda_1 k X_{1_k}(t) \Theta^{X_{3_k}}(t) - \lambda_2 k X_{1_k}(t) \Theta^{X_{4_k}}(t) \\
 \frac{dX_{2_k}(t)}{dt} &= \lambda_1 k X_{1_k}(t) \Theta^{X_{3_k}}(t) + \lambda_2 k X_{1_k}(t) \Theta^{X_{4_k}}(t) - \delta X_{2_k}(t) \\
 \frac{dX_{3_k}(t)}{dt} &= \gamma \delta X_{2_k}(t) - \alpha_1 X_{3_k}(t) \\
 \frac{dX_{4_k}(t)}{dt} &= (1 - \gamma) \delta X_{2_k}(t) - \alpha_2 X_{4_k}(t) \\
 \frac{dX_{5_k}(t)}{dt} &= \alpha_1 X_{3_k}(t) + \alpha_2 X_{4_k}(t),
 \end{aligned}
 \tag{2}$$

TABLE 1 Parameters of the SEAIR model

Parameters	Description
λ_1	Transmission coefficient between community S_k and I_j
λ_2	Transmission coefficient between community S_k and A_j
δ	Rate of becoming infectious after latency
γ	Rate of becoming asymptotically infected
$1 - \gamma$	Rate of becoming symptomatically infected
α_1	Recovery rate of asymptotically infected
α_2	Recovery rate of symptomatically infected

TABLE 2 Parameters estimated from the observed data in China³⁶

Parameters	Description
λ_1	0.01
λ_2	0.188
δ	0.4
γ	0.85
$1 - \gamma$	0.15
α_1	0.141
α_2	0.141

where $\Theta^{X_4}(t) = \frac{\sum_{i=1}^n iX_{4_i}}{n}$ and $\Theta^{X_3}(t) = \frac{\sum_{i=1}^n iX_{3_i}}{n}$ and basic reproduction number $R_0 = \lambda_2 X_1(0) \left(\frac{\gamma}{\alpha_2} + \frac{\lambda_1(1-\gamma)}{\lambda_2 \alpha_1} \right)$. Also,

$$\frac{dX_{1_k}}{dt} + \frac{dX_{2_k}}{dt} + \frac{dX_{3_k}}{dt} + \frac{dX_{4_k}}{dt} + \frac{dX_{5_k}}{dt} = 0$$

$X_{1_k} + X_{2_k} + X_{3_k} + X_{4_k} + X_{5_k} = N_k$ is constant. Also, for system (2), we consider the following boundary conditions:

$$X_{1_k}(0) = X_{1_{k0}}, X_{3_k}(0) = X_{3_{k0}}, X_{4_k}(0) = X_{4_{k0}}, X_{2_k}(0) = X_{5_k}(0) = 0$$

2.2 | SEAIR model and the aggregate investment in social distancing

We are going to use the aggregate investment (control) in this network model. \bar{c}_{s_k} is defined as the aggregate investment of susceptible individuals in social distancing belonging to the group k . The overbar notation is used to indicate that the aggregate investment \bar{c}_{s_k} is an average investment aggregated over all people in the population. c_I is the daily cost of the infection. We first apply the following change to the variable:

$$\bar{c}_k = \frac{\bar{c}_{s_k}}{c_I}$$

As a result, the relative risk for the group k according to \bar{c}_k is defined as:

$$\sigma_k(\bar{c}_k) = \frac{1}{1 + m_k \bar{c}_k}, \tag{3}$$

m_k , is maximum efficiency of social distancing for group k . The maximum efficiency of social distancing will be the maximum number of susceptible people who separate themselves from the population before the cost of this distancing increases than the cost of the disease.

The relative risk graph for the different values of m_k is given in Figure 1. In order to work with dimensionless equations, we should take: (let that, $\lambda_1 = \lambda_2 = \lambda$, $\alpha_1 = \alpha_2 = \alpha$):

$$S_k = \frac{\lambda}{\alpha} X_{1_k}, E = \frac{\lambda \delta}{\alpha^2} X_{2_k}, A_k = \frac{\lambda}{\alpha} X_{2_k}, I_k = \frac{\lambda}{\alpha} X_{1_k},$$

$$R_k = \frac{\lambda}{\alpha} X_{5_k}, \Theta^A = \frac{\lambda}{\alpha} \Theta^{X_3}, \Theta^I = \frac{\lambda}{\alpha} \Theta^{X_4}, \hat{t} = at.$$

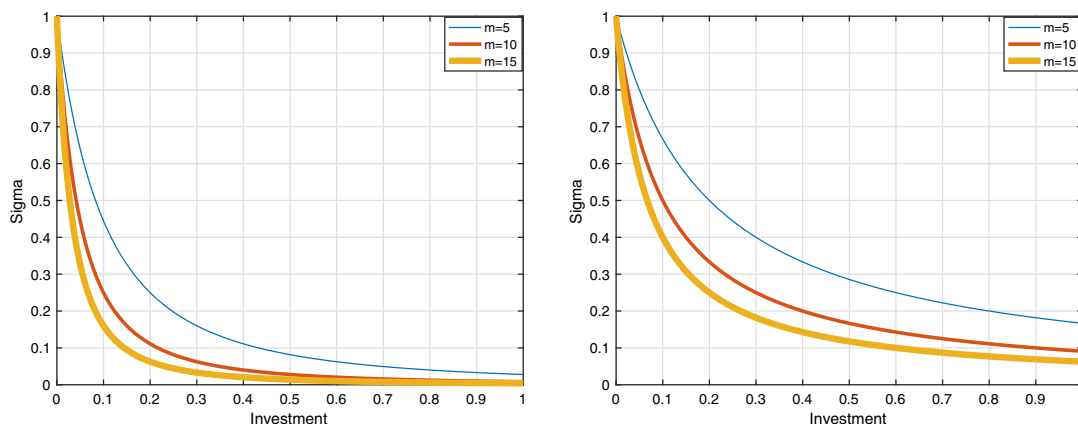


FIGURE 1 The graphs are related to the relative risk of infection. By increasing the maximum efficiency, the relative risk decreases. The right graph shows the relative risk of infection for each group according to just its investment strategy but the left graph shows the relative risk of infection for each group according to both its investment strategy and others groups investment. The results show that the relative risk of infection in the right graph is less than the relative risk of infection in the left graph [Colour figure can be viewed at wileyonlinelibrary.com]

hereafter, it will drop the hat-notation so the dimensionless parameters will be worked. The dynamics of this network model for an epidemic with respect to $\sigma_k(\bar{c}_k)$ is as follows:

$$\begin{aligned}\frac{dS_k(t)}{dt} &= -\sigma_k(\bar{c}_k)kS_k(t)\Theta^A(t) - \sigma_k(\bar{c}_k)kS_k(t)\Theta^I(t) \\ \frac{dE_k(t)}{dt} &= \sigma_k(\bar{c}_k)kS_k(t)\Theta^A(t) + \sigma_k(\bar{c}_k)kS_k(t)\Theta^I(t) - \sigma_k(\bar{c}_k)E_k(t) \\ \frac{dA_k(t)}{dt} &= \gamma\sigma_k(\bar{c}_k)E_k(t) - A_k(t) \\ \frac{dI_k(t)}{dt} &= (1 - \gamma)\sigma_k(\bar{c}_k)E_k(t) - I_k(t) \\ \frac{dR_k(t)}{dt} &= A_k(t) + I_k(t).\end{aligned}\quad (4)$$

2.3 | Cost functional

Consider the problem of minimizing the performance index J . The whole cost of the epidemic for the community, J , is the sum of the direct costs together with the indirect costs of any economic repercussions from the epidemic. It is considered just considered direct costs of the epidemic (the daily costs from infection, daily investments for social distancing, and the costs for vaccination). Mathematically,³²

$$\begin{aligned}J &= -\frac{c_V}{c_I}S(t_f)e^{-ht_f} - \frac{(A(t_f) + I(t_f))e^{-ht_f}}{1 + h} \\ &\quad - \int_0^{t_f} [(\bar{c}(S + E) + (A + I))]e^{-ht} dt,\end{aligned}\quad (5)$$

where c_v is the cost of vaccination per person, and h is the discount rate.

The total cost of the epidemic for group k , J_k ,

$$\begin{aligned}J_k &= -\frac{c_V}{c_I}S_k(t_f)e^{-ht_f} - \frac{(A_k(t_f) + I_k(t_f))e^{-ht_f}}{1 + h} \\ &\quad - \int_0^{t_f} [(\bar{c}_k(S_k + E_k) + (A_k + I_k))]e^{-ht} dt.\end{aligned}\quad (6)$$

3 | BEST INVESTMENT IN SOCIAL DISTANCING

In order to determine the best strategy of investment in social distancing, we consider two approaches, optimal control approach and differential game approach. The reason for using an optimal control approach in this work is analyzing the solution which is obtained from the differential game approach.

3.1 | Optimal control approach

According to optimal control approach, we determine the best investment of each group with regardless of the investment of other groups. The above system is written in the form of the matrix and we have:

$$\begin{pmatrix} \frac{dS_k(t)}{dt} \\ \frac{dE_k(t)}{dt} \\ \frac{dA_k(t)}{dt} \\ \frac{dI_k(t)}{dt} \\ \frac{dR_k(t)}{dt} \end{pmatrix} = \begin{pmatrix} -\sigma_k(\bar{c}_k)k\Theta^A(t) - \sigma_k(\bar{c}_k)k\Theta^I(t) & 0 & 0 & 0 & 0 \\ \sigma_k(\bar{c}_k)k\Theta^A(t) + \sigma_k(\bar{c}_k)k\Theta^I(t) & -\sigma_k(\bar{c}_k) & 0 & 0 & 0 \\ 0 & \gamma\sigma_k(\bar{c}_k) & -1 & 0 & 0 \\ 0 & (1 - \gamma)\sigma_k(\bar{c}_k) & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} S_k(t) \\ E_k(t) \\ A_k(t) \\ I_k(t) \\ R_k(t) \end{pmatrix}.$$

As a result, the transmission rate matrix, Q , is as follows:

$$Q = \begin{pmatrix} -\sigma_k(\bar{c}_k)k\Theta^A(t) - \sigma_k(\bar{c}_k)k\Theta^I(t) & 0 & 0 & 0 & 0 \\ \sigma_k(\bar{c}_k)k\Theta^A(t) + \sigma_k(\bar{c}_k)k\Theta^I(t) & -\sigma_k(\bar{c}_k) & 0 & 0 & 0 \\ 0 & \gamma\sigma_k(\bar{c}_k) & -1 & 0 & 0 \\ 0 & (1-\gamma)\sigma_k(\bar{c}_k) & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}. \quad (7)$$

Considering the The Markov process, as follows:

$$\dot{P} = Q(t, c)P,$$

where $P(t)$ is probabilities that people are in which states and c is the daily investment. The expected present values (V) are calculated (The expected present value is average value one expects after accounting for the probabilities of all future events, and discounting future costs relative to immediate costs) because of the exact time spent and the precise payoff are not predicted. As follows:

$$-\dot{V} = (Q^T - h)V - v, \quad (8)$$

where, h is discount rate and $v = (-\bar{c}_S, -\bar{c}_E, -\bar{c}_A, -\bar{c}_I, -\bar{c}_R)$. As result, we have for the group k ; $v_k = (-\bar{c}_{S_k}, -\bar{c}_{E_k}, -\bar{c}_{A_k}, -\bar{c}_{I_k}, -\bar{c}_{R_k})$. In this problem $v_k = (-\bar{c}_k, -\bar{c}_k, -\bar{c}_k, -1, 0)$. Let $h = 0$.³³ As a result,

$$\begin{aligned} -\frac{dV_{S_k}(t)}{dt} &= (-\sigma_k(\bar{c}_k)k\Theta^A(t) - \sigma_k(\bar{c}_k)k\Theta^I(t))V_{S_k}(t) \\ &\quad + (\sigma_k(\bar{c}_k)k\Theta^A(t) + \sigma_k(\bar{c}_k)k\Theta^I(t))V_{E_k}(t) - \bar{c}_k \\ -\frac{dV_{E_k}(t)}{dt} &= -\sigma_k(\bar{c}_k)V_{E_k}(t) + \gamma\sigma_k(\bar{c}_k)V_{A_k}(t) + (1-\gamma)\sigma_k(\bar{c}_k)V_{I_k}(t) - \bar{c}_k \\ -\frac{dV_{A_k}(t)}{dt} &= -V_{A_k}(t) + V_{R_k}(t) - \bar{c}_k \\ -\frac{dV_{I_k}(t)}{dt} &= -V_{I_k}(t) + V_{R_k}(t) - 1 \\ -\frac{dV_{R_k}(t)}{dt} &= 0. \end{aligned} \quad (9)$$

By solving system (9), we have:

$$\begin{aligned} V_{E_k}(t) &= \frac{-(\gamma\bar{c}_k\sigma(\bar{c}_k) + (1-\gamma)\sigma(\bar{c}_k) + \bar{c}_k)}{\sigma(\bar{c}_k)}, \\ V_{A_k}(t) &= -\bar{c}_k, V_{I_k}(t) = -1, V_{R_k}(t) = 0. \end{aligned}$$

then

$$\begin{aligned} -\frac{dV_{S_k}(t)}{dt} &= -k\sigma_k(\bar{c}_k)(\Theta^A(t) + \Theta^I(t))(V_{S_k}(t) \\ &\quad + \frac{(\gamma\bar{c}_k\sigma(\bar{c}_k) + (1-\gamma)\sigma(\bar{c}_k) + \bar{c}_k)}{\sigma(\bar{c}_k)}) - \bar{c}_k. \end{aligned} \quad (10)$$

Now, we have reduced following system:

$$\begin{aligned} -\frac{dV_{S_k}(t)}{dt} &= -k\sigma_k(\bar{c}_k)(\Theta^A(t) + \Theta^I(t))(V_{S_k}(t) \\ &\quad + \frac{(\gamma\bar{c}_k\sigma(\bar{c}_k) + (1-\gamma)\sigma(\bar{c}_k) + \bar{c}_k)}{\sigma(\bar{c}_k)}) - \bar{c}_k \end{aligned}$$

$$\begin{aligned}
 \frac{dS_k(t)}{dt} &= -\sigma_k(\bar{c}_k)kS_k(t)\Theta^A(t) - \sigma_k(\bar{c}_k)kS_k(t)\Theta^I(t) \\
 \frac{dE_k(t)}{dt} &= \sigma_k(\bar{c}_k)kS_k(t)\Theta^A(t) + \sigma_k(\bar{c}_k)kS_k(t)\Theta^I(t) \\
 &- \sigma_k(\bar{c}_k)E_k(t) \\
 \frac{dA_k(t)}{dt} &= \gamma\sigma_k(\bar{c}_k)E_k(t) - A_k(t) \\
 \frac{dI_k(t)}{dt} &= (1 - \gamma)\sigma_k(\bar{c}_k)E_k(t) - I_k(t). \\
 S_k(0) &= S_{k_0}, I_k(0) = I_{k_0}, A_k(0) = A_k(0), \\
 E_k(0) &= R_k(0) = 0, V_S(t_f) = 0.
 \end{aligned}
 \tag{11}$$

$-V_{S_k}(0)$ will be the expected total cost of the epidemic to group k .

A Nash equilibrium strategy for a population game is a one that is the best response, even when other people are utilizing the same strategy. If Nash equilibrium strategy shows a Nash equilibrium of any subgame of the primary game, it will be a subgame perfect equilibrium. This means that if we consider any smaller game that consisted of the main game, players' behavior would represent a Nash equilibrium of that smaller game. We won solve the problem of ruling out finite-time blowup of the Hamilton-Jacobi equation and appointing existence and uniqueness of subgame perfect equilibria. But the analyses support the guess that the strategies calculated are the unique subgame perfect equilibria. The equilibria can be calculated using the general methods of Isaacs.³⁴ The kernel idea is to use a greedy algorithm, in any step in the game, find the best investment strategy in social distancing that maximizes V_S .

We need only study the reduced system

$$\begin{aligned}
 \bar{c}_k^* &= \arg \max_{\bar{c}_k} -k\sigma_k(\bar{c}_k)(\Theta^A(t) + \Theta^I(t))(V_{S_k}(t) \\
 &+ \frac{(\gamma\bar{c}_k\sigma(\bar{c}_k) + (1 - \gamma)\sigma(\bar{c}_k) + \bar{c}_k)}{\sigma(\bar{c}_k)}) - \bar{c}_k.
 \end{aligned}
 \tag{12}$$

As a result, the best investment in social distancing is as follows:

$$\begin{cases}
 \bar{c}_k^* = 0 : -k(\Theta^A(t) + \Theta^I(t))(\sigma'_{k}(0)V_{S_k}(t) \\
 + \gamma\sigma_k(0) + \sigma'_{k}(0) + \gamma\sigma'_{k}(0) + 1) \leq 1 \\
 -k(\Theta^A(t) + \Theta^I(t))(\sigma'_{k}(\bar{c}_k^*)V_{S_k}(t) + \gamma\sigma_k(\bar{c}_k^*) \\
 + \sigma'_{k}(\bar{c}_k^*) + \gamma\sigma'_{k}(\bar{c}_k^*) + 1) = 1 : o.w
 \end{cases}
 \tag{13}$$

$$\bar{c}_k^* = \begin{cases}
 0 : -k(\Theta^A(t) + \Theta^I(t))(\sigma'_{k}(0)V_{S_k}(t) \\
 + \gamma\sigma_k(0) + \sigma'_{k}(0) + \gamma\sigma'_{k}(0) + 1) \leq 1 \\
 \sqrt{\frac{k(\Theta^A(t) + \Theta^I(t))(m_k V_{S_k}(t) + m_k(1 - \gamma) + \gamma)}{1 + k(\Theta^A(t) + \Theta^I(t))}} - 1 \\
 : o.w
 \end{cases}
 \tag{14}$$

3.2 | Differential game approach

We assume that we have n groups and we want to consider a model which makes a game between groups. We find what the best aggregate investment of the group k with respect to the aggregate investments of other groups is. More precisely, we find the best aggregate investment of each group with respect to the aggregate investments of other groups. The dynamics of this network model for an epidemic with respect to relative risk is as follows:

$$\begin{aligned}
 \frac{dS_k(t)}{dt} &= -\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k)kS_k(t)\Theta^A(t) \\
 &- \sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k)kS_k(t)\Theta^I(t)
 \end{aligned}$$

$$\begin{aligned}
 \frac{dE_k(t)}{dt} &= \sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k)kS_k(t)\Theta^A(t) \\
 &\quad + \sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k)kS_k(t)\Theta^I(t) - \sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k)E_k(t) \\
 \frac{dA_k(t)}{dt} &= \gamma\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k)E_k(t) - A_k(t) \\
 \frac{dI_k(t)}{dt} &= (1 - \gamma)\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k)E_k(t) - I_k(t) \\
 \frac{dR_k(t)}{dt} &= A_k(t) + I_k(t),
 \end{aligned} \tag{15}$$

where

$$\sigma_{-k}(\bar{c}_{-k}) = \frac{1}{1 + \sum_{\substack{i=1 \\ i \neq k}}^n m_i \bar{c}_i}. \tag{16}$$

The transmission rate matrix Q , is as follows:

$$Q = \begin{pmatrix} -\sigma_{-k}(\bar{c}_{-k})(\sigma_k(\bar{c}_k)k\Theta^A(t) + \sigma_k(\bar{c}_k)k\Theta^I(t)) & 0 & 0 & 0 & 0 \\ \sigma_{-k}(\bar{c}_{-k})(\sigma_k(\bar{c}_k)k\Theta^A(t) + \sigma_k(\bar{c}_k)k\Theta^I(t)) & -\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k) & 0 & 0 & 0 \\ 0 & \gamma\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k) & -1 & 0 & 0 \\ 0 & (1 - \gamma)\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k) & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}.$$

Now, given the adjoint equations evolving according to the Markov process, we have:

$$-\dot{V} = (Q^T - h)V - v,$$

where $v_k = (-\bar{c}_k, -\bar{c}_k, -\bar{c}_k, -1, 0)$. As a result,

$$\begin{aligned}
 -\frac{dV_{S_k}(t)}{dt} &= (-\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k)k\Theta^A(t) - \sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k)k\Theta^I(t))V_{S_k}(t) \\
 &\quad + (\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k)k\Theta^A(t) + \sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k)k\Theta^I(t))V_{E_k}(t) - \bar{c}_k \\
 -\frac{dV_{E_k}(t)}{dt} &= -\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k)V_{E_k}(t) + \gamma\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k)V_{A_k}(t) \\
 &\quad + (1 - \gamma)\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k)V_{I_k}(t) - \bar{c}_k \\
 -\frac{dV_{A_k}(t)}{dt} &= -V_{A_k}(t) + V_{R_k}(t) - \bar{c}_k \\
 -\frac{dV_{I_k}(t)}{dt} &= -V_{I_k}(t) + V_{R_k}(t) - 1 \\
 -\frac{dV_{R_k}(t)}{dt} &= 0.
 \end{aligned} \tag{17}$$

Let $h = 0$. By solving system (17) we have:

$$\begin{aligned}
 V_{E_k}(t) &= \frac{-(\gamma\bar{c}_k\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k) + (1 - \gamma)\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k) + \bar{c}_k)}{\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k)}, \\
 V_{A_k}(t) &= -\bar{c}_k, V_{I_k}(t) = -1, V_{R_k}(t) = 0,
 \end{aligned} \tag{18}$$

then

$$\begin{aligned}
 -\frac{dV_{S_k}(t)}{dt} &= -k\sigma_k(\bar{c}_k)\sigma_{-k}(\bar{c}_{-k})(\Theta^A(t) + \Theta^I(t))(V_{S_k}(t) \\
 &+ \frac{(\gamma\bar{c}_k\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k) + (1-\gamma)\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k) + \bar{c}_k)}{\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k)}) - \bar{c}_k.
 \end{aligned}
 \tag{19}$$

Now, we have reduced following system:

$$\begin{aligned}
 -\frac{dV_{S_k}(t)}{dt} &= -k\sigma_k(\bar{c}_k)\sigma_{-k}(\bar{c}_{-k})(\Theta^A(t) + \Theta^I(t))(V_{S_k}(t) \\
 &+ \frac{(\gamma\bar{c}_k\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k) + (1-\gamma)\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k) + \bar{c}_k)}{\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k)}) - \bar{c}_k \\
 \frac{dS_k(t)}{dt} &= -\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k)kS_k(t)\Theta^A(t) - \sigma_k(\bar{c}_k)kS_k(t)\Theta^I(t) \\
 \frac{dE_k(t)}{dt} &= \sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k)kS_k(t)\Theta^A(t) + \sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k)kS_k(t)\Theta^I(t) \\
 &- \sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k)E_k(t) \\
 \frac{dA_k(t)}{dt} &= \gamma\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k)E_k(t) - A_k(t) \\
 \frac{dI_k(t)}{dt} &= (1-\gamma)\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k)E_k(t) - I_k(t).
 \end{aligned}
 \tag{20}$$

$$S_k(0) = S_{k_0}, I_k(0) = I_{k_0}, A_k(0) = A_{k_0}, E_k(0) = R_k(0) = 0, V_S(t_f) = 0$$

Most of the equilibria we calculate are obtained numerically. First, $I(t) = I_0e^{-t}$, $A(t) = A_0e^{-t}$. Now according to the maximum principle, we have:

$$\begin{aligned}
 \bar{c}_k^* &= \arg \max_{\bar{c}_k} -k\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k)(\Theta^A(t) + \Theta^I(t))(V_{S_k}(t) \\
 &+ \frac{(\gamma\bar{c}_k\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k) + (1-\gamma)\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k) + \bar{c}_k)}{\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k)}) - \bar{c}_k.
 \end{aligned}
 \tag{21}$$

As a result, the best investment in social distancing is as follows:

$$\begin{cases}
 \bar{c}_k^* = 0 : -k(\Theta^A(t) + \Theta^I(t))(\sigma_{-k}(\bar{c}_{-k})\sigma'_k(0)V_{S_k}(t) + \gamma\sigma_{-k}(\bar{c}_{-k})\sigma_k(0) \\
 + \sigma_{-k}(\bar{c}_{-k})\sigma'_k(0) + \gamma\sigma_{-k}(\bar{c}_{-k})\sigma'_k(0) + 1) \leq 1 \\
 -k(\Theta^A(t) + \Theta^I(t))(\sigma_{-k}(\bar{c}_{-k})\sigma'_k(0)V_{S_k}(t) + \gamma\sigma_{-k}(\bar{c}_{-k})\sigma_k(0) \\
 + \sigma_{-k}(\bar{c}_{-k})\sigma'_k(0) + \gamma\sigma_{-k}(\bar{c}_{-k})\sigma'_k(0) + 1) = 1 : o.w
 \end{cases}
 \tag{22}$$

$$\bar{c}_k^* = \begin{cases}
 0 : -k(\Theta^A(t) + \Theta^I(t))(\sigma_{-k}(\bar{c}_{-k})\sigma'_k(0)V_{S_k}(t) + \gamma\sigma_{-k}(\bar{c}_{-k})\sigma_k(0) \\
 + \sigma_{-k}(\bar{c}_{-k})\sigma'_k(0) + \gamma\sigma_{-k}(\bar{c}_{-k})\sigma'_k(0) + 1) \leq 1 \\
 \sqrt{\frac{k\sigma_{-k}(\bar{c}_{-k})(\Theta^A(t) + \Theta^I(t))m_k V_{S_k}(t) + m_k(1-\gamma) + \gamma}{1+k(\Theta^A(t) + \Theta^I(t))}} - 1 \\
 m_k : o.w
 \end{cases}$$

4 | STABILITY ANALYSIS IN SEAIR MODEL

We consider SEAIR epidemic model and apply the stability analysis theory. In order to know behavior of the system (9) that is in optimal control, we set in the form of

$$\begin{aligned}
 f_1(V_{S_k}, V_{E_k}, V_{A_k}, V_{I_k}, V_{R_k}) &= -\sigma_k(\bar{c}_k)k\Theta^A(t) - \sigma_k(\bar{c}_k)k\Theta^I(t)V_{S_k}(t) \\
 &+ (\sigma_k(\bar{c}_k)k\Theta^A(t) + \sigma_k(\bar{c}_k)k\Theta^I(t))V_{E_k}(t) - \bar{c}_k \\
 f_2(V_{S_k}, V_{E_k}, V_{A_k}, V_{I_k}, V_{R_k}) &= -\sigma_k(\bar{c}_k)V_{E_k}(t) + \gamma\sigma_k(\bar{c}_k)V_{A_k}(t)
 \end{aligned}$$

$$\begin{aligned}
 &+(1-\gamma)\sigma_k(\bar{c}_k)V_{I_k}(t) - \bar{c}_k \\
 f_3(V_{S_k}, V_{E_k}, V_{A_k}, V_{I_k}, V_{R_k}) &= -V_{A_k}(t) + V_{R_k}(t) - \bar{c}_k \\
 f_4(V_{S_k}, V_{E_k}, V_{A_k}, V_{I_k}, V_{R_k}) &= -V_{I_k}(t) + V_{R_k}(t) - 1 \\
 f_5(V_{S_k}, V_{E_k}, V_{A_k}, V_{I_k}, V_{R_k}) &= 0.
 \end{aligned}$$

To investigate the stability, we consider the Jacobian matrix $\frac{\partial(f_1, f_2, f_3, f_4, f_5)}{\partial(V_{S_k}, V_{E_k}, V_{A_k}, V_{I_k}, V_{R_k})}$ as follows (let, $p = \sigma_k(\bar{c}_k)k(\Theta^A(t) + \Theta^I(t))$):

$$\frac{\partial(f_1, f_2, f_3, f_4, f_5)}{\partial(V_{S_k}, V_{E_k}, V_{A_k}, V_{I_k}, V_{R_k})} = \begin{pmatrix} -p & p & 0 & 0 & 0 \\ 0 & -\sigma_k(\bar{c}_k) & \gamma\sigma_k(\bar{c}_k) & (1-\gamma)\sigma_k(\bar{c}_k) & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

The characteristic equation is,

$$\lambda^5 + \alpha_1\lambda^4 + \alpha_2\lambda^3 + \alpha_3\lambda^2 + \alpha_4\lambda = 0,$$

where,

$$\begin{aligned}
 \alpha_1 &= p + \sigma_k(\bar{c}_k) + 2 \\
 \alpha_2 &= p\sigma_k(\bar{c}_k) + 2p + 2\sigma_k(\bar{c}_k) + 1 \\
 \alpha_3 &= 2p\sigma_k(\bar{c}_k) + p + \sigma_k(\bar{c}_k) \\
 \alpha_4 &= p\sigma_k(\bar{c}_k).
 \end{aligned}$$

For the asymptotic stability we use Samuelson conditions³⁵ and since for different groups the conditions are satisfied, by Samuelson conditions the system (9) in optimal control approach is asymptotically stable.

Now, we investigate the stability in differential game approach and consider the system (17) and have

$$\begin{aligned}
 g_1(V_{S_k}, V_{E_k}, V_{A_k}, V_{I_k}, V_{R_k}) &= -\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k)k(\Theta^A(t) + \Theta^I(t))V_{S_k}(t) \\
 &+ \sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k)k(\Theta^A(t) + \Theta^I(t))V_{E_k}(t) - \bar{c}_k \\
 g_2(V_{S_k}, V_{E_k}, V_{A_k}, V_{I_k}, V_{R_k}) &= -\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k)V_{E_k}(t) + \gamma\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k)V_{A_k}(t) \\
 &+(1-\gamma)\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k)V_{I_k}(t) - \bar{c}_k \\
 g_3(V_{S_k}, V_{E_k}, V_{A_k}, V_{I_k}, V_{R_k}) &= -V_{A_k}(t) + V_{R_k}(t) - \bar{c}_k \\
 g_4(V_{S_k}, V_{E_k}, V_{A_k}, V_{I_k}, V_{R_k}) &= -V_{I_k}(t) + V_{R_k}(t) - 1 \\
 g_5(V_{S_k}, V_{E_k}, V_{A_k}, V_{I_k}, V_{R_k}) &= 0.
 \end{aligned}$$

The Jacobian matrix is as follows: (let, $q = \sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k)k(\Theta^A(t) + \Theta^I(t))$)

$$\frac{\partial(g_1, g_2, g_3, g_4, g_5)}{\partial(V_{S_k}, V_{E_k}, V_{A_k}, V_{I_k}, V_{R_k})} = \begin{pmatrix} -q & q & 0 & 0 & 0 \\ 0 & -\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k) & \gamma\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k) & (1-\gamma)\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k) & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

thus, the characteristic equation is,

$$\lambda^5 + \beta_1 \lambda^4 + \beta_2 \lambda^3 + \beta_3 \lambda^2 + \beta_4 \lambda = 0,$$

where,

$$\begin{aligned} \beta_1 &= q + \sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k) + 2 \\ \beta_2 &= q\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k) + 2q + 2\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k) + 1 \\ \beta_3 &= 2q\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k) + q + \sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k) \\ \beta_4 &= q\sigma_{-k}(\bar{c}_{-k})\sigma_k(\bar{c}_k), \end{aligned}$$

since for different groups the conditions are satisfied, by Samuelson conditions³⁵ the system (17) in differential game is asymptotically stable.

5 | RESULTS AND DISCUSSION

In the lack of vaccine (for the real-world example see coronaviruses epidemic and the current pandemic of covid-19) it makes sense that social distancing considers in mathematical simulations. More specifically, the SEAIR model is used on simulation network and the optimal control approach is used to determine the best investment in each group regardless of other groups. The differential game approach shows a Nash equilibrium of any subgame of the primary game. The average investment in social distancing will decrease when the investments that are calculated on average will have fewer values, it will occur when much more people want to make the investment. As a result, if the number of susceptible individuals that they distance in the network increases, their average investment reduce.

According to Figure 1, the results have shown that by increasing the aggregate investment in social distancing, the relative risk decreases. These investments are made by individuals who are susceptible, so there should be an incentive for these individuals to make these investments. Also, this figure shows by increasing the number of susceptible individuals that they are distancing (m), the relative risk decreases. So these individuals want to know what the best investment is that they not to get infected. The two approaches are considered here. In the first approach, which is the optimal control approach, the aggregate investment of each group is obtained regardless of the investment of other groups. In the second approach, which is the game theory approach, the investment of each group in social distancing is obtained with respect to the investment of other groups. Figures 2 and 3 show that social distancing reduces the peak and prolongs the epidemic

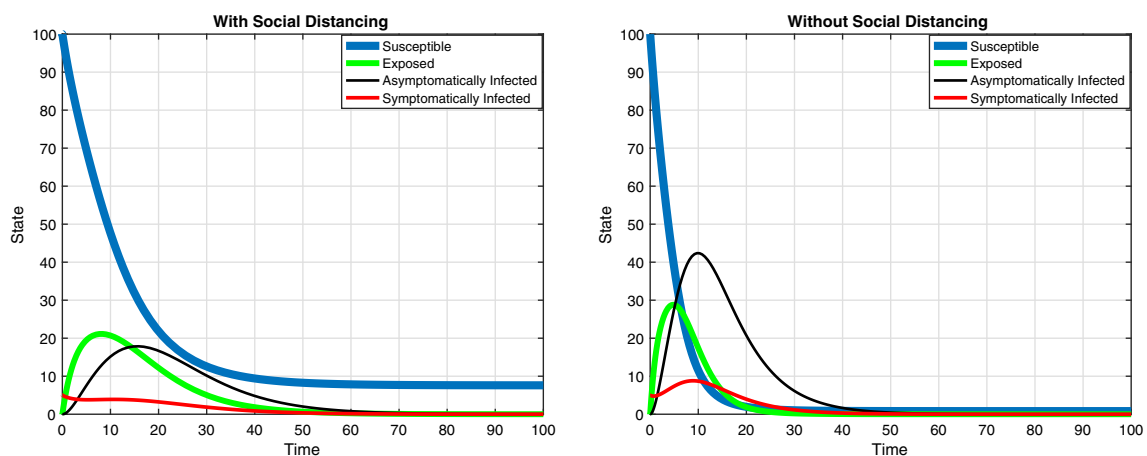


FIGURE 2 The graphs are related to the states of the system with and without social distancing with optimal control approach, $S_0 = 100$, $m = 60$. It is understood that social distancing reduces the peak and prolongs the epidemic by comparing two graphs [Colour figure can be viewed at wileyonlinelibrary.com]

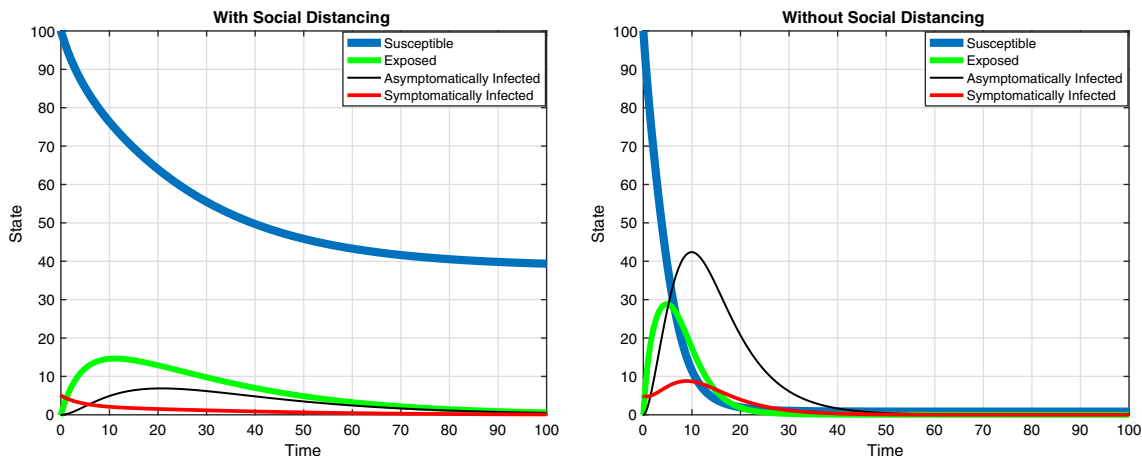


FIGURE 3 The graphs are related to the states of the system with and without social distancing with differential game approach, $S_0 = 100$, $m = 60$. It is understood that social distancing reduces the peak and prolongs the epidemic by comparing two graphs [Colour figure can be viewed at wileyonlinelibrary.com]

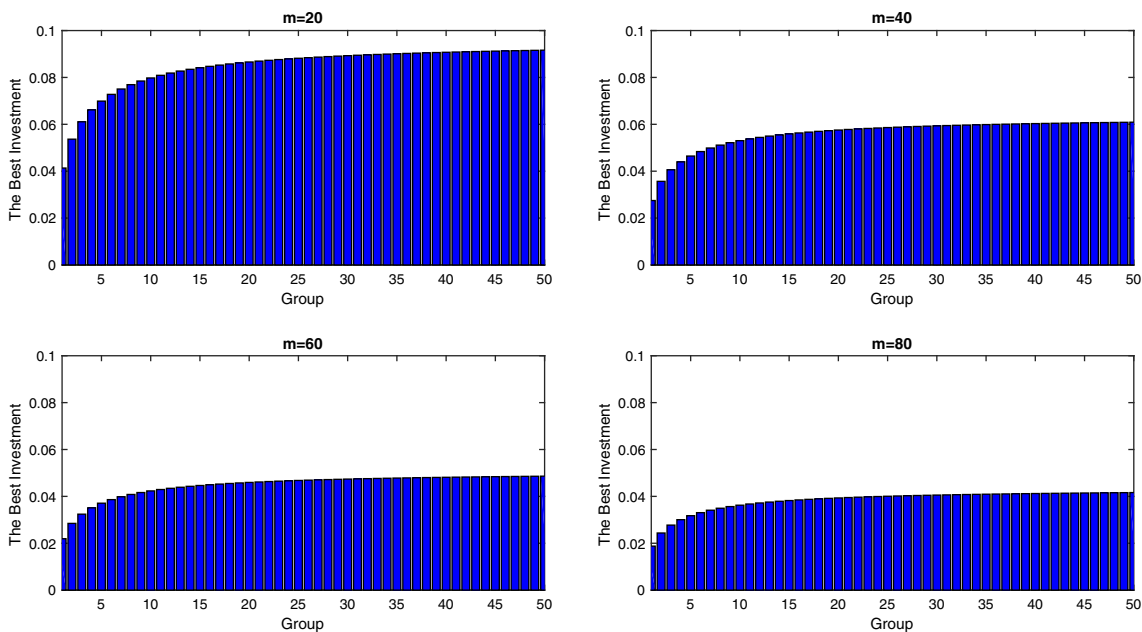


FIGURE 4 The graphs are related to the best investment for different groups with optimal control approach. According to the graphs, by increasing the maximum efficiency m , the best investment is decreased [Colour figure can be viewed at wileyonlinelibrary.com]

by comparing two graphs. According to analyses done in the above figures, it can be seen by increasing the number of contacts, the aggregate investment in social distancing increases. These analyses also show that the maximum efficiency of social distancing (m) plays an important role in the best investment. In Figures 4 and 5 observe that as m increases, the investment in social distancing decreases. If the number of infected individuals increases then the best investment, in both approaches is increased too (see Figures 6 and 7). Figures 8 and 9 shows that with going on the duration of the epidemic, the optimal investment is reduced. A remarkable point which is seen in above figures is that with regard to the two approaches, the aggregate optimal investments for each group in the second approach are less than the aggregate optimal investments for each group in the first approach. This shows if we want to reduce the investment for each group, we should use the differential game theory approach and divide the individuals into separate groups according to their contacts together and then for each group obtain the best aggregate investment with respect to the aggregate investments for other groups.

FIGURE 5 The graphs are related to the best investment with differential game approach for different groups. $m = m_1 = m_2 = \dots = m_{1000} = 60$. If the number of infected individuals increases then the best investment is increased too [Colour figure can be viewed at wileyonlinelibrary.com]

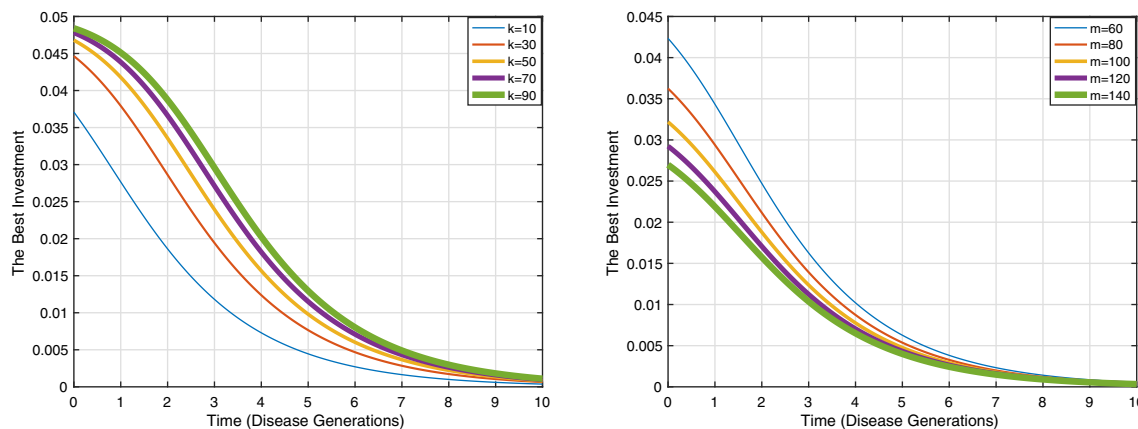
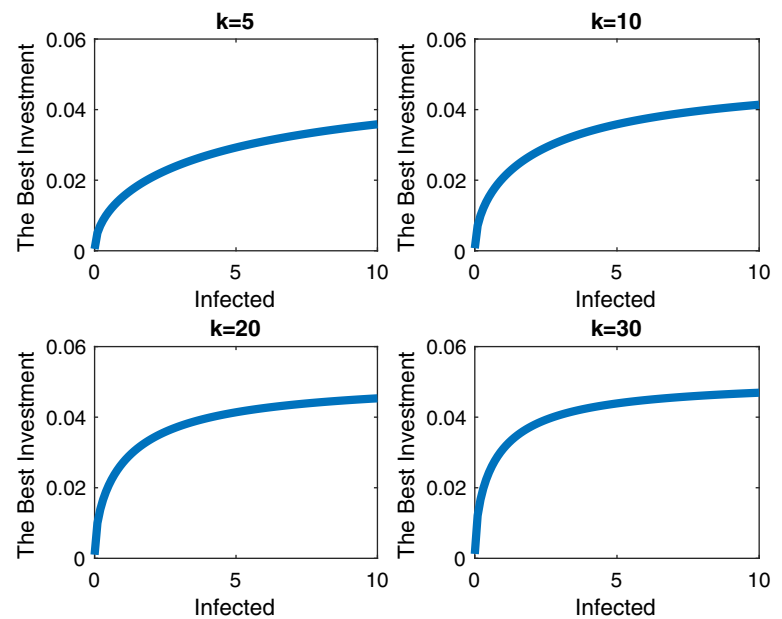


FIGURE 6 The graphs are related to the best investment in term of time with optimal control approach. The results show that by spending time, the best investment is decreased. The left graph is depicted for different the maximum efficiencies of social distancing, $k = 20$. The best investment in social distancing is decreased by increasing the maximum efficiencies of social distancing (m). The right graph is depicted for different ($m = 60$). By increasing the number of contacts, the best investment in social distancing is increased [Colour figure can be viewed at wileyonlinelibrary.com]

Epidemic cost and saving in cost per person with both optimal control approach and differential game approach are shown in Figures 10 and 11. The results show that by increasing the basic reproduction number, epidemic cost per person increases. The results show that social distancing is most beneficial on the network with low contacts. If vaccination and other measures are not available then optimal social distancing never recovers more than 14% of the cost of infection.

In this study, it has been shown that if we deal with a human network and want to control or stop the epidemic, we should impulse individuals to distance. That is, we should motivate individuals in the network to participate in this investment. Two key parameters are considered in this study: the maximum efficiency of social distancing and basic reproduction number. The maximum efficiency of social distancing is the maximum number of susceptible individuals who separate themselves from the population before the cost of social distancing increases from the cost of the disease. By increasing this maximum efficiency, the epidemic cost is reduced. That means that if more individuals are more likely to participate in the distancing, the cost of the epidemic is reduced. Another key parameter is the basic reproduction number. Increasing the basic reproduction number increases the cost of an epidemic. The suggestion in this study is to motivate individuals in the network to participate in distancing. It is shown that the basic reproduction number varies in

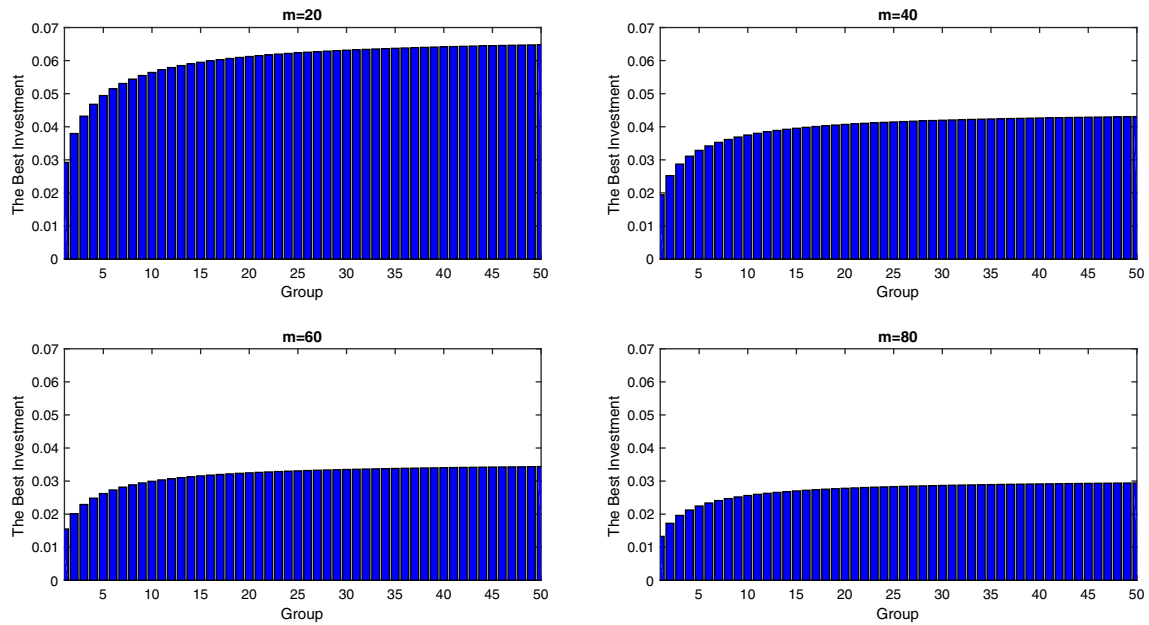


FIGURE 7 The graphs are related to the best investment with optimal control approach for different groups.

$m = m_1 = m_2 = \dots = m_{1000} = 60$. If the number of infected individuals increases then the best investment is increased too [Colour figure can be viewed at wileyonlinelibrary.com]

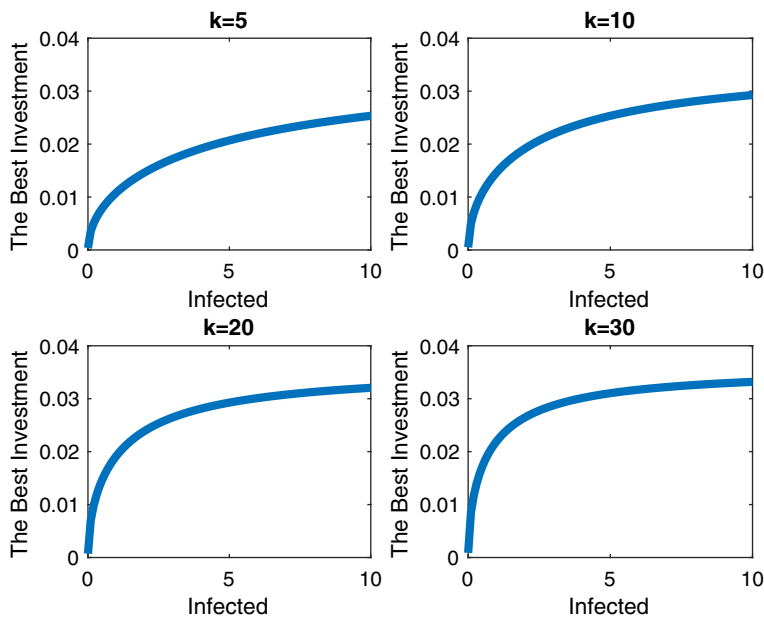


FIGURE 8 The graphs are related to the best investment with differential game approach for different groups. By increasing the number of infected individuals, the best investment is increased too [Colour figure can be viewed at wileyonlinelibrary.com]

different groups. This means that social distancing are more beneficial when the basic reproduction number tends to a specific number that must be calculated for each group. In an optimal control approach, for example, for a group with five contacts, social distancing is more beneficial for a basic reproduction number about three. According to this approach, for a group with the number of fourteen contacts, this number is about two. Moreover, in differential game approach, for example, for a group with a contact number of five, social distancing is more useful for basic reproduction number about five, while for a group with the number of fourteen contacts, this number is about six. The results show that for each individual, the best investment strategy in social distancing and epidemic cost in the differential game approach are less than the optimal control approach.

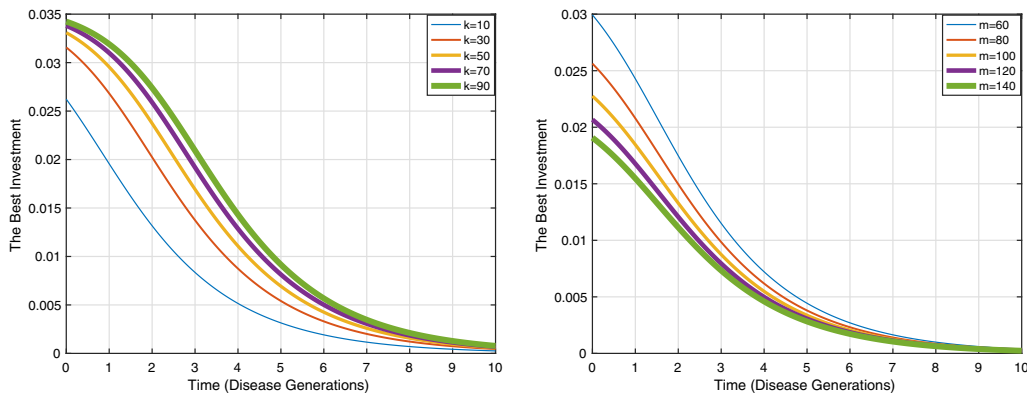


FIGURE 9 The graphs are related to the best investment in term of time with differential game approach. The results show that by spending time, the best investment is decreased. The left graph is depicted for different the maximum efficiencies of social distancing, $k = 10$. By increasing m , the best investment in social distancing is decreased. The right graph is depicted for different groups, $m = 60$. By increasing the number of contacts, the best investment in social distancing is increased [Colour figure can be viewed at wileyonlinelibrary.com]

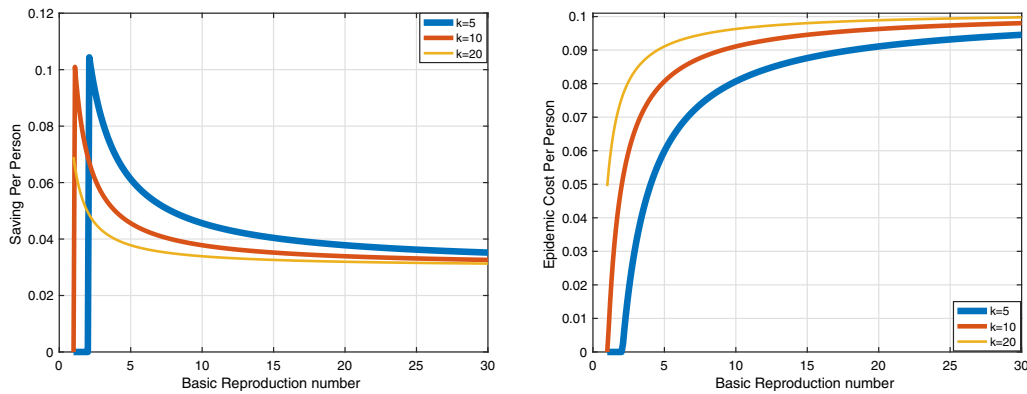


FIGURE 10 The graphs are related to epidemic cost per person and saving in cost per person with an optimal control approach. By increasing the number of contacts, epidemic cost per person is increased. The results show that for the number of low contacts, there is a peak point in the graph ($k = 1, 2, \dots, 10$). But the graph is strictly decreasing for the number of more contacts ($k = 11, 12, \dots$). According to (b), by increasing the number of contacts, saving in cost per person is decreased. These graphs are depicted for $m = 100$ [Colour figure can be viewed at wileyonlinelibrary.com]

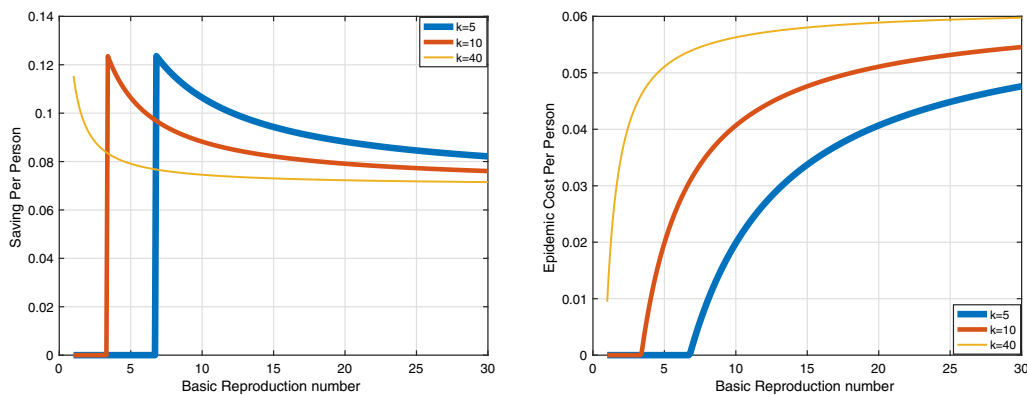


FIGURE 11 The graphs are related to epidemic cost per person and saving in cost per person with a differential game approach. By increasing the number of contacts, epidemic cost per person is increased. The results show that for the number of low contacts, there is a peak point in the graph ($k = 1, 2, \dots, 35$). But the graph is strictly decreasing for the number of more contacts ($k = 35, 36, \dots$). According to (b), by increasing the number of contacts, saving in cost per person is decreased. These graphs are depicted for $m = 100$ [Colour figure can be viewed at wileyonlinelibrary.com]

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SUPPORTING INFORMATION

Additional supporting information may be found online in the Supporting Information section at the end of this article.

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