RESEARCH ARTICLE

Mathematical model for spreading of COVID-19 virus with the Mittag–Leffler kernel

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Abstract

In the Nidovirales order of the Coronaviridae family, where the coronavirus (crown-like spikes on the surface of the virus) causing severe infections like acute lung injury and acute respiratory distress syndrome. The contagion of this virus categorized as severed, which even causes severe damages to human life to harmless such as a common cold. In this manuscript, we discussed the SARS-CoV-2 virus into a system of equations to examine the existence and uniqueness results with the Atangana–Baleanu derivative by using a fixed-point method. Later, we designed a system where we generate numerical results to predict the outcome of virus spreadings all over India.

KEYWORDS

AB-derivative, coronavirus, fixed-point techniques, fractional calculus, mathematical models

1 | INTRODUCTION

The on-going pandemic of COVID-19, viral pneumonia break out in late 2019 in Wuhan, China, which has its spread worldwide across 210 nations named as "SARS-CoV-2." It is raging around the world with an immense toll in terms of human, economic, and social impact. Within a short span, it raises an alert in every country all over the world like a pandemic disease, which urges every nation to forecast their precautionary actions to control and contain the wild spread of the virus as the severity of the disease will harm human life badly. Since the novel coronavirus is new to the world to forecast some impact of the pandemic situation and to build a mitigation plan the similarity effects of Severe Acute Respiratory Syndrome (SARS) and Middle East Respiratory Syndrome epidemics in 2003 and

2009 were used for study and analysis. From the study of the initial spread of COVID-19, many of the mathematical models were came into the act from contributors across the world to determine the severity of the spread. Whenever a contagious disease extends its tributary, it follows certain patterns of spread which widely help us to identify and monitor the dynamics of the disease outbreaks. The method we used to estimate the spread of the disease is a factor that drives us to finalize the measures to get rid of infectious diseases. The outbreak of the disease within the country or state for time is usually nonlinear, which propels us to design the system where we can study those dynamic nonlinear phenomena. By this system, we can able to define the transmission of such contagious disease, which helps us to interpret the remedial measures to stop or contain the spread of contagious disease.

In recent years, fractional differentiation has been drawing increasing attention in the study of social and physical behaviors where scaling power law of fractional order appears universal as an empirical description of such complex phenomena. The fractional-order models are more adequated than the previously used integer-order models because fractional-order derivatives and integrals describe the memory and hereditary properties of different phenomena [1].

Koca [2] investigated the Ebola virus spreading within a particular place of the population by *AB*-derivative. Dokuyucu and Dutta [3] discussed the model for Ebola virus with the Caputo derivative without a singular kernel in the fractional order. Dong et al. [4] derived a model for the granular SEIR epidemic with fractional order. Babaei et al. [5] investigated the impact of therapy treatments on the control of HIV/AIDS spread in prisons, and there is no cure, or some treatment methods are available. Baleanu et al. [6] studied a Rubella disease model with Caputo–Fabrizio derivative. Danane et al. [7] discussed Hepatitis B virus infection with antibody immune response. Bekiros and Kouloumpou [8] described the infectious disease dynamics with the SBDiEM model. Moore et al. [9] studied the Caputo–Fabrizio derivative to the treatment compartment for the HIV/AIDS. Xiao and Chen [10] analyzed a predator–prey model with the disease. Ucar [11] discussed the existence and uniqueness result for a smoking model with nonsingular derivatives. Agarwal and Singh [12] modeled the transmission dynamics of the Nipah virus of fractional order. Khan et al. [13] studied the Caputo–Fabrizio derivative for modeling the dynamics of hepatitis. Veeresha et al. [14] analyzed the numerical solution of Schistosomiasis disease in biological phenomena. Khan et al. [15] used the nonsingular Mittag–Leffler Law for the HIV-TB co-infection model. For more references to the *AB*-derivative, the reader can refer References [16–26].

Atangana [27] constructed a model for the effect of lockdown by proposing new fractal fractional differential and integral operators. Khan and Atangana [28] discussed the novel coronavirus (2019-nCov) with *AB*-derivative. Shaikh et al. [29, 30] described and analyzed the pandemic spreading of COVID-19, the outbreak in India with fractional derivative. Arino and Portet [31] described the spreading of COVID-19 in the population by the SLIAR epidemic model. Abdo et al. [32] studied the existence and stability of the novel coronavirus (COVID-19) model. Fanelli and Piazza [33] analyzed of the spreading of COVID-19 in China, Italy, and France. Memon et al. [34] discussed the epidemiological system using real incidence data from Pakistan. Valentim et al. [35] described a fractional calculus to improve tumor growth models. Luo et al. [36] investigated an epidemic model for pulse vaccination strategy. Gao et al. [37] analyzed the pulse vaccination and saturation incidence of a delayed epidemic model. Veeresha et al. [38, 39] discussed the q-homotopy analysis transform method and fractional natural decomposition method in COVID-19 model, and refer References 40–56 for models.

Here, we investigate the mathematical model for spreading of COVID-19 virus in the world with Atangana–Baleanu fractional derivative. Consider the following systems:

$$
\left.\begin{aligned}
({}^{ABC}_{0}D^\zeta_t)(S_p(t)) &= \Delta_b - \lambda_d S_p - \frac{\alpha_I S_p(I_p + \beta_A A_p)}{N} - \gamma_{QS} S_p Q_p \\
({}^{ABC}_{0}D^\zeta_t)(E_p(t)) &= \frac{\alpha_I S_p(I_p + \beta_A A_p)}{N} + \gamma_{QS} S_p Q_p - (1 - \phi_A)\delta_{RI} E_p - \phi_A \mu_{EI} E_p - \lambda_d E_p \\
({}^{ABC}_{0}D^\zeta_t)(I_p(t)) &= (1 - \phi_A)\delta_{RI} E_p - (\sigma_{RI} + \lambda_d)I_p \\
({}^{ABC}_{0}D^\zeta_t)(A_p(t)) &= \phi_A \mu_{EI} E_p - (\rho_{RA} + \lambda_d)A_p \\
({}^{ABC}_{0}D^\zeta_t)(R_p(t)) &= \sigma_{RI} I_p + \rho_{RA} A_p - \lambda_d R_p \\
({}^{ABC}_{0}D^\zeta_t)(Q_p(t)) &= \kappa_{IQ} I_p + \nu_{AQ} A_p - \eta_Q Q_p \\
\text{with } \ & S_p(0) = S_0, \ \ E_p(0) = E_0, \\
& I_p(0) = I_0, \ \ A_p(0) = A_0, \\
& R_p(0) = R_0, \ \ Q_p(0) = Q_0.
\end{aligned}\right\}
\tag{1.1}
$$

where $0 < \zeta < 1$, N represents the total population of people, $S_p(t)$ represents susceptible people, $E_p(t)$ represents exposed people, $I_p(t)$ represents infected people, $A_p(t)$ represents asymptotically infected people, $R_p(t)$ represents recovered or removed people, $Q_p(t)$ represents people in the reservoir or market (people affected directly by seafood), Δ_b and λ_d represent birth and natural death rate, α_I is disease transmission coefficient, β_A is transmission multiple (asymptomatically infected), γ_{QS} is disease transmission coefficient from seafood places to suspectiable people, ϕ_A is proposition of asymptomatic infection, σ_{RI} is removal rate, ρ_{RA} is recovery rate, η_Q is removing rate of viruses from the seafood market, μ_{EI} is transmission rate of becomes infected, δ_{RI} is transmission rate after completing the incubation period, ν_{AQ} is asymptotically infected directly contributing the virus from market, and κ_{IQ} is infected symptoms of the virus from market.

In Section 2, we described about essential results and propositions, which will be important for main problems. In Section 3, we derived the solution for the system of Equation (1.1). In Section 4, we discussed the existence results. Uniqueness results for the above system of equation derived in Section 5. In last section, we illustrate the numerical solutions for Equations (1.1) by using *AB*-derivative in graphical method.

2 | PRELIMINARIES

- The left R–L integral is [1, 57]

$$
({}_0 I^\zeta x)(t) = \frac{1}{\Gamma(\zeta)} \int_0^t (t - s)^{\zeta - 1} x(s)\, ds,
\tag{2.1}
$$

where $\zeta > 0$.

- The left R–L derivative is [1, 57]

$$
({}_0 D^\zeta x)(t) = \frac{d}{dt}\left(\frac{1}{\Gamma(1 - \zeta)} \int_0^t (t - s)^{-\zeta} x(s)\, ds \right),
$$

where $0 < \zeta < 1$.

- The Caputo derivative is [1, 57]

$$
({}^C_0 D^\zeta x)(t) = \frac{1}{\Gamma(1 - \zeta)} \int_0^t (t - s)^{-\zeta} x'(s)\, ds,
$$

where $0 < \zeta < 1$.

- The Caputo *AB*-derivative is [58]

$$\binom{ABC}{0}D^\zeta x)(t) = \frac{B(\zeta)}{1-\zeta} \int_0^t x'(s)E_\zeta\left[-\zeta\frac{(t-s)^\zeta}{1-\zeta}\right] ds,$$

where $\zeta \in [0, 1]$, $x' \in H'(a, b)$, $a \le b$, and $B(\zeta)$ is a normalizing positive function satisfying $B(0) = B(1) = 1$.

- The associative fractional integral of (2.1) is

$$\binom{AB}{0}I^\zeta x)(t) = \frac{1-\zeta}{B(\zeta)}x(t) + \frac{\zeta}{B(\zeta)}(_0I^\zeta x)(t), \tag{2.2}$$

where $_0I^\zeta$ is the left R–L integral given in (2.1).

Proposition 2.1 *For $0 < \zeta < 1$, we conclude that* [59, 60]

$$\binom{AB}{0}I^\zeta(\binom{ABC}{0}D^\zeta x))(t) = x(t) - x(0)E_\zeta(\lambda t^\zeta) - \frac{\zeta}{1-\zeta}x(0)E_{\zeta,\zeta+1}(\lambda t^\zeta)$$
$$= x(t) - x(0).$$

3 | SOLUTION PART

By using *AB*-fractional integral on both side of (1.1)

$$
\begin{aligned}
&{}^{AB}_0I^\zeta(^{ABC}_0D_t^\zeta)(S_p(t)) = {}^{AB}_0I^\zeta\left[\Delta_b - \lambda_d S_p - \frac{\alpha_I S_p(I_p+\beta_A A_p)}{N} - \gamma_{QS}S_pQ_p\right] \\
&{}^{AB}_0I^\zeta(^{ABC}_0D_t^\zeta)(E_p(t)) = {}^{AB}_0I^\zeta\left[\frac{\alpha_I S_p(I_p+\beta_A A_p)}{N} + \gamma_{QS}S_pQ_p - (1-\phi_A)\delta_{RI}E_p - \phi_A\mu_{EI}E_p - \lambda_d E_p\right] \\
&{}^{AB}_0I^\zeta(^{ABC}_0D_t^\zeta)(I_p(t)) = {}^{AB}_0I^\zeta[(1-\phi_A)\delta_{RI}E_p - (\sigma_{RI}+\lambda_d)I_p] \\
&{}^{AB}_0I^\zeta(^{ABC}_0D_t^\zeta)(A_p(t)) = {}^{AB}_0I^\zeta[\phi_A\mu_{EI}E_p - (\rho_{RA}+\lambda_d)A_p] \\
&{}^{AB}_0I^\zeta(^{ABC}_0D_t^\zeta)(R_p(t)) = {}^{AB}_0I^\zeta[\sigma_{RI}I_p + \rho_{RA}A_p - \lambda_d R_p] \\
&{}^{AB}_0I^\zeta(^{ABC}_0D_t^\zeta)(Q_p(t)) = {}^{AB}_0I^\zeta[\kappa_{IQ}I_p + \nu_{AQ}A_p - \eta_Q Q_p].
\end{aligned}
$$

By using proposition, we get

$$
\begin{aligned}
&S_p(t) - S_p(0) = {}^{AB}_0I^\zeta\left[\Delta_b - \lambda_d S_p - \frac{\alpha_I S_p(I_p+\beta_A A_p)}{N} - \gamma_{QS}S_pQ_p\right] \\
&E_p(t) - E_p(0) = {}^{AB}_0I^\zeta\left[\frac{\alpha_I S_p(I_p+\beta_A A_p)}{N} + \gamma_{QS}S_pQ_p - (1-\phi_A)\delta_{RI}E_p - \phi_A\mu_{EI}E_p - \lambda_d E_p\right] \\
&I_p(t) - I_p(0) = {}^{AB}_0I^\zeta[(1-\phi_A)\delta_{RI}E_p - (\sigma_{RI}+\lambda_d)I_p] \\
&A_p(t) - A_p(0) = {}^{AB}_0I^\zeta[\phi_A\mu_{EI}E_p - (\rho_{RA}+\lambda_d)A_p] \\
&R_p(t) - R_p(0) = {}^{AB}_0I^\zeta[\sigma_{RI}I_p + \rho_{RA}A_p - \lambda_d R_p] \\
&Q_p(t) - Q_p(0) = {}^{AB}_0I^\zeta[\kappa_{IQ}I_p + \nu_{AQ}A_p - \eta_Q Q_p].
\end{aligned}
$$

substitute the initial conditions to the above system of equation becomes

$$
\begin{aligned}
&S_p(t) - S_0 = {}^{AB}_0I^\zeta\left[\Delta_b - \lambda_d S_p - \frac{\alpha_I S_p(I_p+\beta_A A_p)}{N} - \gamma_{QS}S_pQ_p\right] \\
&E_p(t) - E_0 = {}^{AB}_0I^\zeta\left[\frac{\alpha_I S_p(I_p+\beta_A A_p)}{N} + \gamma_{QS}S_pQ_p - (1-\phi_A)\delta_{RI}E_p - \phi_A\mu_{EI}E_p - \lambda_d E_p\right] \\
&I_p(t) - I_0 = {}^{AB}_0I^\zeta[(1-\phi_A)\delta_{RI}E_p - (\sigma_{RI}+\lambda_d)I_p] \\
&A_p(t) - A_0 = {}^{AB}_0I^\zeta[\phi_A\mu_{EI}E_p - (\rho_{RA}+\lambda_d)A_p] \\
&R_p(t) - R_0 = {}^{AB}_0I^\zeta[\sigma_{RI}I_p + \rho_{RA}A_p - \lambda_d R_p] \\
&Q_p(t) - Q_0 = {}^{AB}_0I^\zeta[\kappa_{IQ}I_p + \nu_{AQ}A_p - \eta_Q Q_p].
\end{aligned}
\tag{3.1}
$$

Consider (for simplicity, we replace),

$$Z_1(t, S_p) = \Delta_b - \lambda_d S_p - \frac{\alpha_I S_p(I_p + \beta_A A_p)}{N} - \gamma_{QS} S_p Q_p$$

$$Z_2(t, E_p) = \frac{\alpha_I S_p(I_p + \beta_A A_p)}{N} + \gamma_{QS} S_p Q_p - (1 - \phi_A)\delta_{RI} E_p - \phi_A \mu_{EI} E_p - \lambda_d E_p$$

$$Z_3(t, I_p) = (1 - \phi_A)\delta_{RI} E_p - (\sigma_{RI} + \lambda_d) I_p$$

$$Z_4(t, A_p) = \phi_A \mu_{EI} E_p - (\rho_{RA} + \lambda_d) A_p$$

$$Z_5(t, R_p) = \sigma_{RI} I_p + \rho_{RA} A_p - \lambda_d R_p$$

$$Z_6(t, Q_p) = \kappa_{IQ} I_p + \nu_{AQ} A_p - \eta_Q Q_p.$$

By using the above consideration in system of Equation (3.1), we get

$$\left.\begin{aligned}
S_p(t) - S_0 &= {}^{AB}_0 I^\zeta Z_1(t, S_p) \\
E_p(t) - E_0 &= {}^{AB}_0 I^\zeta Z_2(t, E_p) \\
I_p(t) - I_0 &= {}^{AB}_0 I^\zeta Z_3(t, I_p) \\
A_p(t) - A_0 &= {}^{AB}_0 I^\zeta Z_4(t, A_p) \\
R_p(t) - R_0 &= {}^{AB}_0 I^\zeta Z_5(t, R_p) \\
Q_p(t) - Q_0 &= {}^{AB}_0 I^\zeta Z_6(t, Q_p).
\end{aligned}\right\} \tag{3.2}$$

Using (2.2) in (3.2), we get

$$\left.\begin{aligned}
S_p(t) - S_0 &= \frac{1-\zeta}{B(\zeta)} Z_1(t, S_p) + \frac{\zeta}{B(\zeta)}{}_0 I^\zeta Z_1(t, S_p) \\
E_p(t) - E_0 &= \frac{1-\zeta}{B(\zeta)} Z_2(t, E_p) + \frac{\zeta}{B(\zeta)}{}_0 I^\zeta Z_2(t, E_p) \\
I_p(t) - I_0 &= \frac{1-\zeta}{B(\zeta)} Z_3(t, I_p) + \frac{\zeta}{B(\zeta)}{}_0 I^\zeta Z_3(t, I_p) \\
A_p(t) - A_0 &= \frac{1-\zeta}{B(\zeta)} Z_4(t, A_p) + \frac{\zeta}{B(\zeta)}{}_0 I^\zeta Z_4(t, A_p) \\
R_p(t) - R_0 &= \frac{1-\zeta}{B(\zeta)} Z_5(t, R_p) + \frac{\zeta}{B(\zeta)}{}_0 I^\zeta Z_5(t, R_p) \\
Q_p(t) - Q_0 &= \frac{1-\zeta}{B(\zeta)} Z_6(t, Q_p) + \frac{\zeta}{B(\zeta)}{}_0 I^\zeta Z_6(t, Q_p).
\end{aligned}\right\} \tag{3.3}$$

The above system of Equations (3.3) is the solution of the systems of Equations (1.1).

$$\left.\begin{aligned}
T(S_p(t)) &= \frac{1-\zeta}{B(\zeta)} Z_1(t, S_p) + \frac{\zeta}{B(\zeta)}{}_0 I^\zeta Z_1(t, S_p) \\
T(E_p(t)) &= \frac{1-\zeta}{B(\zeta)} Z_2(t, E_p) + \frac{\zeta}{B(\zeta)}{}_0 I^\zeta Z_2(t, E_p) \\
T(I_p(t)) &= \frac{1-\zeta}{B(\zeta)} Z_3(t, I_p) + \frac{\zeta}{B(\zeta)}{}_0 I^\zeta Z_3(t, I_p) \\
T(A_p(t)) &= \frac{1-\zeta}{B(\zeta)} Z_4(t, A_p) + \frac{\zeta}{B(\zeta)}{}_0 I^\zeta Z_4(t, A_p) \\
T(R_p(t)) &= \frac{1-\zeta}{B(\zeta)} Z_5(t, R_p) + \frac{\zeta}{B(\zeta)}{}_0 I^\zeta Z_5(t, R_p) \\
T(Q_p(t)) &= \frac{1-\zeta}{B(\zeta)} Z_6(t, Q_p) + \frac{\zeta}{B(\zeta)}{}_0 I^\zeta Z_6(t, Q_p).
\end{aligned}\right\} \tag{3.4}$$

4 | EXISTENCE SOLUTIONS

Theorem 4.1 *The kernels Z_1, Z_2, Z_3, Z_4, Z_5, and Z_6 satisfy the Lipschitz condition and contraction if the following inequality holds*:

$$0 \le a_1 < 1$$
$$0 \le a_2 < 1$$
$$0 \le a_3 < 1$$
$$0 \le a_4 < 1$$
$$0 \le a_5 < 1$$
$$0 \le a_6 < 1.$$

Proof.　Consider $Z_1(t, S_p) = \Delta_b - \lambda_d S_p - \frac{\alpha_I S_p (I_p + \beta_A A_p)}{N} - \gamma_{QS} S_p Q_p$.

Let S_p and S_{p1} be two function, so we have following:

$$
\begin{aligned}
\|Z_1(t, S_p) - Z_1(t, S_{p1})\| = \| & \left[\Delta_b - \lambda_d S_p - \frac{\alpha_I S_p (I_p + \beta_A A_p)}{N} - \gamma_{QS} S_p Q_p \right] \\
& - \left[\Delta_b - \lambda_d S_{p1} - \frac{\alpha_I S_{p1}(I_p + \beta_A A_p)}{N} - \gamma_{QS} S_{p1} Q_p \right] \| \\
\le \| & \left[\Delta_b - \lambda_d S_p - \frac{\alpha_I S_p I_p}{N} - \frac{\alpha_I S_p \beta_A A_p}{N} - \gamma_{QS} S_p Q_p \right] \\
& - \left[\Delta_b - \lambda_d S_{p1} - \frac{\alpha_I S_{p1} I_p}{N} - \frac{\alpha_I S_{p1} \beta_A A_p}{N} - \gamma_{QS} S_{p1} Q_p \right] \| \\
\le \| & \Delta_b - \left[\lambda_d - \frac{\alpha_I I_p}{N} - \frac{\alpha_I \beta_A A_p}{N} - \gamma_{QS} Q_p \right] S_p \\
& - \left(\Delta_b - \left[\lambda_d - \frac{\alpha_I I_p}{N} - \frac{\alpha_I \beta_A A_p}{N} - \gamma_{QS} Q_p \right] S_{p1} \right) \| \\
\le & \|\Delta_b - l S_p - \Delta_b + l S_{p1}\| \\
\le & a_1 \|S_p(t) - S_{p1}(t)\|
\end{aligned}
$$

where $l = \lambda_d - \frac{\alpha_I I_p}{N} - \frac{\alpha_I \beta_A A_p}{N} - \gamma_{QS} Q_p$.

Consider $u = \max_{t \in J} \| S_p(t) \|$, $v = \max_{t \in J} \| E_p(t) \|$, $w = \max_{t \in J} \| I_p(t) \|$, $x = \max_{t \in J} \| A_p(t) \|$, $y = \max_{t \in J} \| R_p(t) \|$, and $z = \max_{t \in J} \| Q_p(t) \|$. Hence, the Lipschitz condition is satisfied for Z_1 and if $0 \le a_1 < 1$, then it is also a contraction for Z_1.

Similarly, the other kernels satisfy the Lipschitz condition as follows:

$$\|Z_2(t, E_p) - Z_2(t, E_{p1})\| \le a_2 \|E_p(t) - E_{p1}(t)\|$$
$$\|Z_3(t, I_p) - Z_3(t, I_{p1})\| \le a_3 \|I_p(t) - I_{p1}(t)\|$$
$$\|Z_4(t, A_p) - Z_4(t, A_{p1})\| \le a_4 \|A_p(t) - A_{p1}(t)\|$$
$$\|Z_5(t, R_p) - Z_5(t, R_{p1})\| \le a_5 \|R_p(t) - R_{p1}(t)\|$$
$$\|Z_6(t, Q_p) - Z_6(t, Q_{p1})\| \le a_6 \|Q_p(t) - Q_{p1}(t)\|. \qquad \blacksquare$$

Theorem 4.2 *Let $\widetilde{M} \subset H$ be bounded and $l, m, n, k, p, q > 0$ such that*

$$\|S_p(t_2) - S_p(t_1)\| \le C_1 \|t_2 - t_1\|$$
$$\|E_p(t_2) - E_p(t_1)\| \le C_2 \|t_2 - t_1\|$$

$$\|I_p(t_2) - I_p(t_1)\| \le C_3 \|t_2 - t_1\|$$

$$\|A_p(t_2) - A_p(t_1)\| \le C_4 \|t_2 - t_1\|$$

$$\|R_p(t_2) - R_p(t_1)\| \le C_5 \|t_2 - t_1\|$$

$$\|Q_p(t_2) - Q_p(t_1)\| \le C_6 \|t_2 - t_1\|$$

where $S_p, E_p, T_p, A_p, R_p, Q_p \in \widetilde{M}$. Then $\overline{T(\widetilde{M})}$ is compact.

Proof. Let

$$W_1 = \max_{0 \le t \le 1} \max_{0 \le S_p < l} Z_1(t, S_p(t)),$$

$$W_2 = \max_{0 \le t \le 1} \max_{0 \le E_p < m} Z_2(t, E_p(t)),$$

$$W_3 = \max_{0 \le t \le 1} \max_{0 \le I_p < n} Z_3(t, I_p(t)),$$

$$W_4 = \max_{0 \le t \le 1} \max_{0 \le A_p < k} Z_4(t, A_p(t)),$$

$$W_5 = \max_{0 \le t \le 1} \max_{0 \le R_p < p} Z_5(t, R_p(t)),$$

$$W_6 = \max_{0 \le t \le 1} \max_{0 \le Q_p < q} Z_6(t, Q_p(t)),$$

$\exists$ a constants l, m, n, k, p, $q > 0$ such that $\|S_p(t)\| < l$, $\|E_p(t)\| < m$, $\|I_p(t)\| < n$, $\|A_p(t)\| < k$, $\|R_p(t)\| < p$, $\|Q_p(t)\| < q$. For all $S_p, E_p, I_p, A_p, R_p, Q_p \in \widetilde{M}$, we get

$$\|TS_p(t)\| = \|\frac{1-\zeta}{B(\zeta)} Z_1(t, S_p) + \frac{\zeta}{B(\zeta)} {}_0 I^\zeta Z_1(t, S_p)\|$$

$$\le \frac{1-\zeta}{B(\zeta)} \|Z_1(t, S_p)\| + \frac{\zeta}{B(\zeta)} \|\frac{1}{\Gamma(\zeta)} \int_0^t (t-s)^{(\zeta-1)} Z_1(s, S_p) ds\|$$

$$\le \frac{1-\zeta}{B(\zeta)} \|Z_1(t, S_p)\| + \frac{\zeta}{B(\zeta)} \frac{(t-0)^\zeta}{\zeta \Gamma(\zeta)} \|Z_1(s, S_p) ds\|$$

$$\le \frac{1-\zeta}{B(\zeta)} \|Z_1(t, S_p)\| + \frac{(t-0)^\zeta}{B(\zeta)\Gamma(\zeta)} \|Z_1(s, S_p) ds\|$$

$$\le \left(\frac{1-\zeta}{B(\zeta)} + \frac{(t-0)^\zeta}{B(\zeta)\Gamma(\zeta)} \right) \|Z_1(s, S_p) ds\|$$

$$\le \left(\frac{1-\zeta}{B(\zeta)} + \frac{(t-0)^\zeta}{B(\zeta)\Gamma(\zeta)} \right) W_1.$$

Similarly,

$$\|TE_p(t)\| \le \left(\frac{1-\zeta}{B(\zeta)} + \frac{(t-0)^\zeta}{B(\zeta)\Gamma(\zeta)} \right) W_2$$

$$\|TI_p(t)\| \le \left(\frac{1-\zeta}{B(\zeta)} + \frac{(t-0)^\zeta}{B(\zeta)\Gamma(\zeta)} \right) W_3$$

$$\|TA_p(t)\| \le \left(\frac{1-\zeta}{B(\zeta)} + \frac{(t-0)^\zeta}{B(\zeta)\Gamma(\zeta)} \right) W_4$$

$$\|TR_p(t)\| \le \left(\frac{1-\zeta}{B(\zeta)} + \frac{(t-0)^\zeta}{B(\zeta)\Gamma(\zeta)} \right) W_5$$

$$\|TQ_p(t)\| \le \left(\frac{1-\zeta}{B(\zeta)} + \frac{(t-0)^\zeta}{B(\zeta)\Gamma(\zeta)} \right) W_6.$$

Consequently, $T(\widetilde{M})$ is bounded.

Next, to examine $t_1 < t_2$ and $S_\mathrm{p}(t), E_\mathrm{p}(t), I_\mathrm{p}(t), A_\mathrm{p}(t), R_\mathrm{p}(t), Q_\mathrm{p}(t) \in \widetilde{M}$ and then for every $\varepsilon > 0$ if $\|t_2 - t_1\| < \delta$, we get

$$
\begin{aligned}
\|TS_\mathrm{p}(t_2) - TS_\mathrm{p}(t_1)\| = \| & \frac{1-\zeta}{B(\zeta)}[Z_1(t_2, S_\mathrm{p}(t_2)) - Z_1(t_1, S_\mathrm{p}(t_1))] \\
& + \frac{\zeta}{B(\zeta)}{}_0 I^\zeta [Z_1(t_2, S_\mathrm{p}(t_2)) - Z_1(t_1, S_\mathrm{p}(t_1))]\| \\
\leq & \frac{1-\zeta}{B(\zeta)}\|Z_1(t_2, S_\mathrm{p}(t_2)) - Z_1(t_1, S_\mathrm{p}(t_1))\| \\
& + \frac{\zeta}{B(\zeta)}\left[\int_0^{t_2}(t_2-s)^{(\zeta-1)}Z_1(s, S_\mathrm{p}(s))\,ds - \int_0^{t_1}(t_1-s)^{(\zeta-1)}Z_1(s, S_\mathrm{p}(s))\,ds\right] \\
\leq & \frac{1-\zeta}{B(\zeta)}\|Z_1(t_2, S_\mathrm{p}(t_2)) - Z_1(t_1, S_\mathrm{p}(t_1))\| \\
& + \frac{\zeta}{B(\zeta)}\|Z_1(t_2, S_\mathrm{p}(t_2)) - Z_1(t_1, S_\mathrm{p}(t_1))\|\frac{(t_2-t_1)^\zeta}{\zeta\Gamma(\zeta)} \\
\leq & \left(\frac{1-\zeta}{B(\zeta)} + \frac{(t_2-t_1)^\zeta}{B(\zeta)\Gamma(\zeta)}\right)\|Z_1(t_2, S_\mathrm{p}(t_2)) - Z_1(t_1, S_\mathrm{p}(t_1))\|.
\end{aligned}
\tag{4.1}
$$

Similarly,

$$
\left.
\begin{aligned}
\|TE_\mathrm{p}(t_2) - TE_\mathrm{p}(t_1)\| &\leq \left(\frac{1-\zeta}{B(\zeta)} + \frac{(t_2-t_1)^\zeta}{B(\zeta)\Gamma(\zeta)}\right)\|Z_2(t_2, E_\mathrm{p}(t_2)) - Z_2(t_1, E_\mathrm{p}(t_1))\| \\
\|TI_\mathrm{p}(t_2) - TI_\mathrm{p}(t_1)\| &\leq \left(\frac{1-\zeta}{B(\zeta)} + \frac{(t_2-t_1)^\zeta}{B(\zeta)\Gamma(\zeta)}\right)\|Z_3(t_2, I_\mathrm{p}(t_2)) - Z_3(t_1, I_\mathrm{p}(t_1))\| \\
\|TA_\mathrm{p}(t_2) - TA_\mathrm{p}(t_1)\| &\leq \left(\frac{1-\zeta}{B(\zeta)} + \frac{(t_2-t_1)^\zeta}{B(\zeta)\Gamma(\zeta)}\right)\|Z_4(t_2, A_\mathrm{p}(t_2)) - Z_4(t_1, A_\mathrm{p}(t_1))\| \\
\|TR_\mathrm{p}(t_2) - TR_\mathrm{p}(t_1)\| &\leq \left(\frac{1-\zeta}{B(\zeta)} + \frac{(t_2-t_1)^\zeta}{B(\zeta)\Gamma(\zeta)}\right)\|Z_5(t_2, R_\mathrm{p}(t_2)) - Z_5(t_1, R_\mathrm{p}(t_1))\| \\
\|TQ_\mathrm{p}(t_2) - TQ_\mathrm{p}(t_1)\| &\leq \left(\frac{1-\zeta}{B(\zeta)} + \frac{(t_2-t_1)^\zeta}{B(\zeta)\Gamma(\zeta)}\right)\|Z_6(t_2, Q_\mathrm{p}(t_2)) - Z_6(t_1, Q_\mathrm{p}(t_1))\|.
\end{aligned}
\right\}
\tag{4.2}
$$

Consider

$$
\begin{aligned}
\|Z_3(t_2, I_\mathrm{p}(t_2)) - Z_3(t_1, I_\mathrm{p}(t_1))\| = \|& [(1-\phi_\mathrm{A})\delta_\mathrm{RI}E_\mathrm{p} - (\sigma_\mathrm{RI} + \lambda_\mathrm{d})I_\mathrm{p}(t_2)] \\
& - [(1-\phi_\mathrm{A})\delta_\mathrm{RI}E_\mathrm{p} - (\sigma_\mathrm{RI} + \lambda_\mathrm{d})I_\mathrm{p}(t_1)]\| \\
\leq & (\sigma_\mathrm{RI} + \lambda_\mathrm{d})\|I_\mathrm{p}(t_2) - I_\mathrm{p}(t_1)\| \\
\leq & C_3\|t_2 - t_1\|.
\end{aligned}
\tag{4.3}
$$

Similarly,

$$
\left.
\begin{aligned}
\|Z_1(t_2, S_\mathrm{p}(t_2)) - Z_1(t_1, S_\mathrm{p}(t_1))\| &\leq C_1\|t_2 - t_1\| \\
\|Z_2(t_2, E_\mathrm{p}(t_2)) - Z_2(t_1, E_\mathrm{p}(t_1))\| &\leq C_2\|t_2 - t_1\| \\
\|Z_4(t_2, A_\mathrm{p}(t_2)) - Z_4(t_1, A_\mathrm{p}(t_1))\| &\leq C_4\|t_2 - t_1\| \\
\|Z_5(t_2, R_\mathrm{p}(t_2)) - Z_5(t_1, R_\mathrm{p}(t_1))\| &\leq C_5\|t_2 - t_1\| \\
\|Z_6(t_2, Q_\mathrm{p}(t_2)) - Z_6(t_1, Q_\mathrm{p}(t_1))\| &\leq C_6\|t_2 - t_1\|.
\end{aligned}
\right\}
\tag{4.4}
$$

Now, substitute the systems (4.3) and (4.4) in Equations (4.1) and (4.2), we get

$$\left.\begin{aligned}
\|TS_p(t_2) - TS_p(t_1)\| &\leq \left(\frac{1-\zeta}{B(\zeta)} + \frac{(t_2-t_1)^\zeta}{B(\zeta)\Gamma(\zeta)}\right) C_1 \|t_2 - t_1\| \\
\|TE_p(t_2) - TE_p(t_1)\| &\leq \left(\frac{1-\zeta}{B(\zeta)} + \frac{(t_2-t_1)^\zeta}{B(\zeta)\Gamma(\zeta)}\right) C_2 \|t_2 - t_1\| \\
\|TI_p(t_2) - TI_p(t_1)\| &\leq \left(\frac{1-\zeta}{B(\zeta)} + \frac{(t_2-t_1)^\zeta}{B(\zeta)\Gamma(\zeta)}\right) C_3 \|t_2 - t_1\| \\
\|TA_p(t_2) - TA_p(t_1)\| &\leq \left(\frac{1-\zeta}{B(\zeta)} + \frac{(t_2-t_1)^\zeta}{B(\zeta)\Gamma(\zeta)}\right) C_4 \|t_2 - t_1\| \\
\|TR_p(t_2) - TR_p(t_1)\| &\leq \left(\frac{1-\zeta}{B(\zeta)} + \frac{(t_2-t_1)^\zeta}{B(\zeta)\Gamma(\zeta)}\right) C_5 \|t_2 - t_1\| \\
\|TQ_p(t_2) - TQ_p(t_1)\| &\leq \left(\frac{1-\zeta}{B(\zeta)} + \frac{(t_2-t_1)^\zeta}{B(\zeta)\Gamma(\zeta)}\right) C_6 \|t_2 - t_1\|.
\end{aligned}\right\}$$

that is, $\|TS_p(t_2) - TS_p(t_1)\| \to 0$ as $t_2 \to t_1$. Similarly, for all $T(\widetilde{M}) \to 0$ as $t_2 \to t_1$. Thus $T(\widetilde{M})$ is equicontinous.

According to Arezla–Azscoli theorem, "Let Ω be compact Hausdroff Metric space. Then $\widetilde{M} \in \underline{C(\Omega)}$ is relatively compact iff $\widetilde{M}$ is uniformly bounded and uniformly continuous," $T(\widetilde{M})$ is compact. ∎

5 | UNIQUENESS SOLUTIONS

In an earlier section, we expressed the existence solution for SARS-CoV-2 spreading predict with *AB*-derivative by using a fixed-point method. Now, we proceed uniqueness results of the system (3.4) with initial conditions.

$$\begin{aligned}
\|T(S_{p_1}(t)) - T(S_{p_2}(t))\| &= \|\frac{1-\zeta}{B(\zeta)}[Z_1(t, S_{p_1}(t)) - Z_1(t, S_{p_2}(t))] \\
&\quad + \frac{\zeta}{B(\zeta)} {}_0I^\zeta[Z_1(t, S_{p_1}(t)) - Z_1(t, S_{p_2}(t))]\| \\
&\leq \frac{1-\zeta}{B(\zeta)} \|Z_1(t, S_{p_1}(t)) - Z_1(t, S_{p_2}(t))\| \\
&\quad + \frac{\zeta}{B(\zeta)} \|Z_1(t, S_{p_1}(t)) - Z_1(t, S_{p_2}(t))\| \frac{(t-0)^\zeta}{\zeta\Gamma(\zeta)} \\
&\leq \left(\frac{1-\zeta}{B(\zeta)} + \frac{(t-0)^\zeta}{B(\zeta)\Gamma(\zeta)}\right) \|Z_1(t, S_{p_1}(t)) - Z_1(t, S_{p_2}(t))\| \\
&\leq \left(\frac{1-\zeta}{B(\zeta)} + \frac{(t-0)^\zeta}{B(\zeta)\Gamma(\zeta)}\right) a_1 \|S_{p_1}(t) - S_{p_2}(t)\|.
\end{aligned}$$

Similarly,

$$\|T(E_{p_1}(t)) - T(E_{p_2}(t))\| \leq \left(\frac{1-\zeta}{B(\zeta)} + \frac{(t-0)^\zeta}{B(\zeta)\Gamma(\zeta)}\right) a_2 \|E_{p_1}(t) - E_{p_2}(t)\|$$

$$\|T(I_{p_1}(t)) - T(I_{p_2}(t))\| \leq \left(\frac{1-\zeta}{B(\zeta)} + \frac{(t-0)^\zeta}{B(\zeta)\Gamma(\zeta)}\right) a_3 \|I_{p_1}(t) - I_{p_2}(t)\|$$

$$\|T(A_{p_1}(t)) - T(A_{p_2}(t))\| \leq \left(\frac{1-\zeta}{B(\zeta)} + \frac{(t-0)^\zeta}{B(\zeta)\Gamma(\zeta)}\right) a_4 \|A_{p_1}(t) - A_{p_2}(t)\|$$
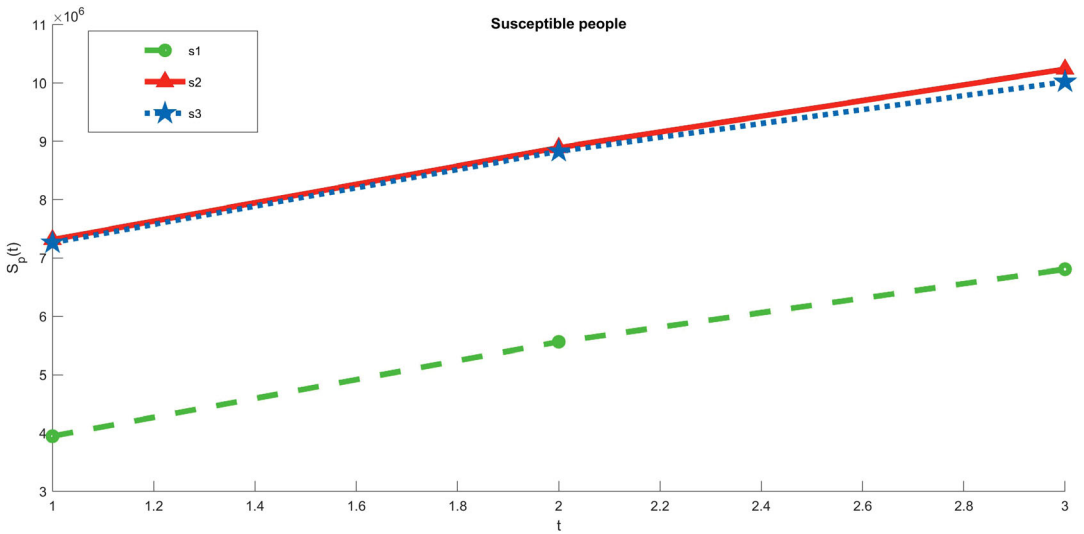
FIGURE 1 Numerical results of the susceptible people $S_p(t)$ with respect to time t when n varies from 1 to 3 [Color figure can be viewed at wileyonlinelibrary.com]

$$\|T(R_{P_1}(t)) - T(R_{P_2}(t))\| \le \left(\frac{1-\zeta}{B(\zeta)} + \frac{(t-0)^\zeta}{B(\zeta)\Gamma(\zeta)} \right) a_5 \|R_{P_1}(t) - R_{P_2}(t)\|$$

$$\|T(Q_{P_1}(t)) - T(Q_{P_2}(t))\| \le \left(\frac{1-\zeta}{B(\zeta)} + \frac{(t-0)^\zeta}{B(\zeta)\Gamma(\zeta)} \right) a_6 \|Q_{P_1}(t) - Q_{P_2}(t)\|.$$

Therefore, if the following conditions holds:

$$\left(\frac{1-\zeta}{B(\zeta)} + \frac{(t-0)^\zeta}{B(\zeta)\Gamma(\zeta)} \right) a_1 < 1$$

$$\left(\frac{1-\zeta}{B(\zeta)} + \frac{(t-0)^\zeta}{B(\zeta)\Gamma(\zeta)} \right) a_2 < 1$$

$$\left(\frac{1-\zeta}{B(\zeta)} + \frac{(t-0)^\zeta}{B(\zeta)\Gamma(\zeta)} \right) a_3 < 1$$

$$\left(\frac{1-\zeta}{B(\zeta)} + \frac{(t-0)^\zeta}{B(\zeta)\Gamma(\zeta)} \right) a_4 < 1$$

$$\left(\frac{1-\zeta}{B(\zeta)} + \frac{(t-0)^\zeta}{B(\zeta)\Gamma(\zeta)} \right) a_5 < 1$$

$$\left(\frac{1-\zeta}{B(\zeta)} + \frac{(t-0)^\zeta}{B(\zeta)\Gamma(\zeta)} \right) a_6 < 1.$$

$\therefore T$ is a contraction. These certain that the model has a unique solution.

6 | NUMERICAL RESULTS

Here, we discussed the model of SARS-CoV-2 with *AB*-derivative. Consider the parametric values from the literature [29] in Table 1. Assume 35% of total initial population $N = S_0 + E_0 + I_0 + A_0 + R_0 + Q_0 = 481, 747, 192$, where $E_0 = 1, 724, 266$; $I_0 = 745$; $A_0 = 413$; $R_0 = 66$, susceptible case can be discovered as $S_0 = N - (E_0 + I_0 + A_0) - R_0$ and $Q_0 = 10,00,000$. The birth rate
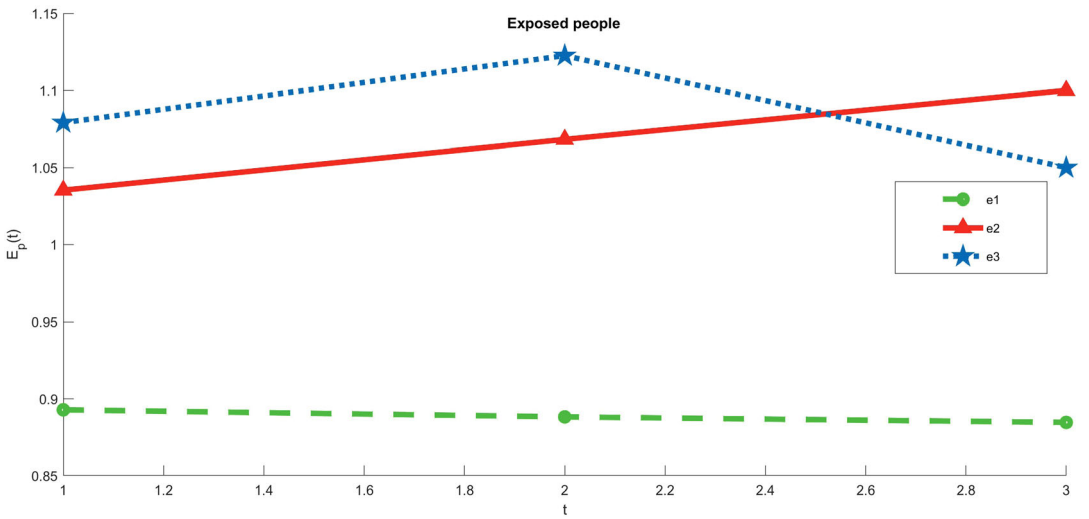
FIGURE 2 Numerical results of the exposed people $E_p(t)$ with respect to time t when n varies from 1 to 3 [Color figure can be viewed at wileyonlinelibrary.com]
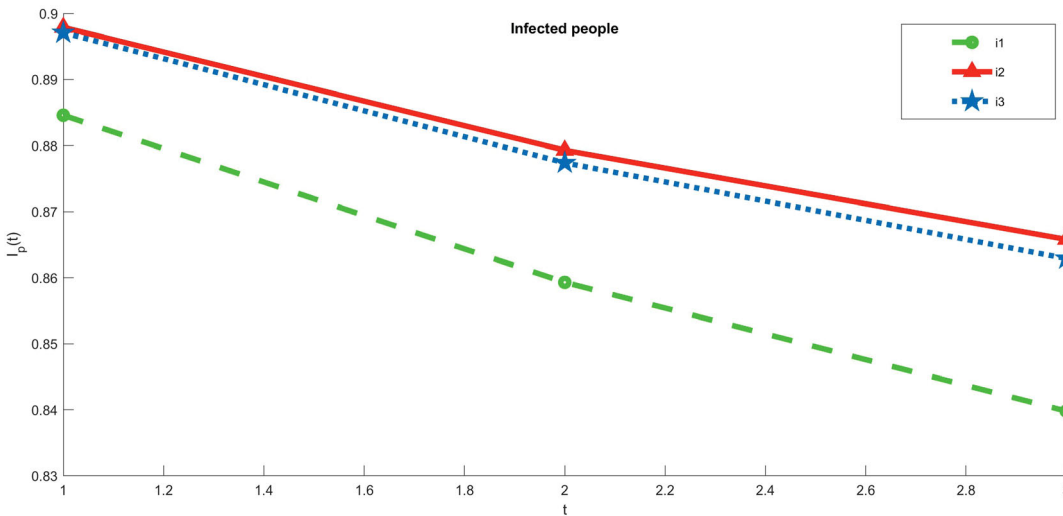

FIGURE 3 Numerical results of the infected people $I_p(t)$ with respect to time t when n varies from 1 to 3 [Color figure can be viewed at wileyonlinelibrary.com]

$\Delta_b = \lambda_d \times N$ and natural death rate $\lambda_d = \frac{1}{69.50}$ (life expectancy in India is 69.50 per year 2019) are derived in the absence of infection.

Consider the fractional order $\zeta = 0.5$ and using the above parametric values, n varies from 1 to 3 in Equation (3.3). The range of n values varies according to the time and the above approximate values to identify the SARS-CoV-2 viruses growth in people simultaneously in Figures 1–6.

$$S_{P_n}(t) = 1 + 3465807.135 - 0.0072 * S_{P_{n-1}}(t) - 2.5947e - 10 * S_{P_{n-1}}(t) * I_{P_{n-1}}(t)$$
$$- 1.5423e - 10 * S_{P_{n-1}}(t) * A_{P_{n-1}}(t) - 6.1550e - 09 * S_{P_{n-1}}(t) * Q_{P_{n-1}}(t)$$
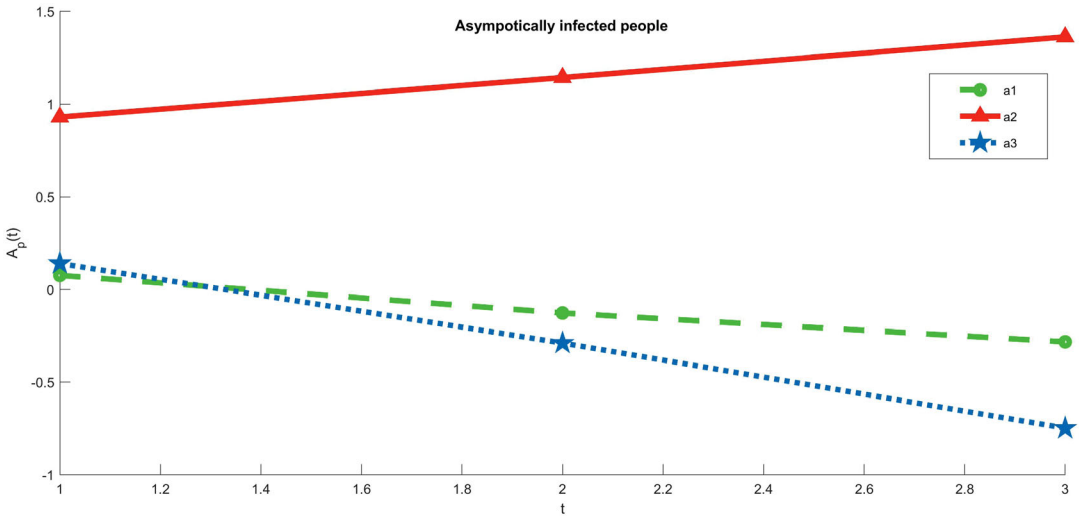$$+ \frac{1955408.38556}{0.5} * t^{0.5} - \frac{0.00406224}{0.5} * t^{0.5} * S_{P_{n-1}}(t)$$

FIGURE 4 Numerical results of the asymptotically infected people $A_p(t)$ with respect to time t when n varies from 1 to 3 [Color figure can be viewed at wileyonlinelibrary.com]
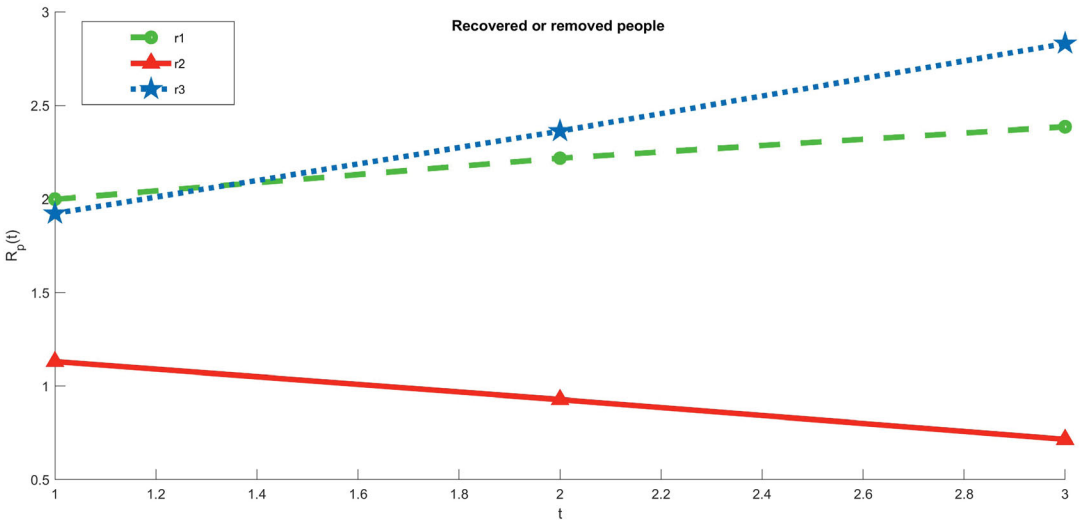

FIGURE 5 Numerical results of the recovered or removed people $R_p(t)$ with respect to time t when n varies from 1 to 3 [Color figure can be viewed at wileyonlinelibrary.com]

$$- \frac{1.4639e-10}{0.5} * t^{0.5} * S_{p_{n-1}}(t) * I_{p_{n-1}}(t) - \frac{8.7017e-11}{0.5} * t^{0.5} * S_{p_{n-1}}(t) * A_{p_{n-1}}(t)$$
$$- \frac{3.4727e-09}{0.5} * t^{0.5} * S_{p_{n-1}}(t) * Q_{p_{n-1}}(t)$$

$$E_{p_n}(t) = 1 + 2.5947e-10 * S_{p_{n-1}}(t) * I_{p_{n-1}}(t) + 1.5423e-10 * S_{p_{n-1}}(t) * A_{p_{n-1}}(t)$$
$$+ 6.1550e-09 * S_{p_{n-1}}(t) * Q_{p_{n-1}}(t) - 0.0096 * E_{p_{n-1}}(t)$$
$$+ \frac{1.46329e-10}{0.5} * t^0.5 * S_{p_{n-1}}(t) * I_{p_{n-1}}(t) + \frac{8.7017e-11}{0.5} * t^0.5 * S_{p_{n-1}}(t) * A_{p_{n-1}}(t)$$
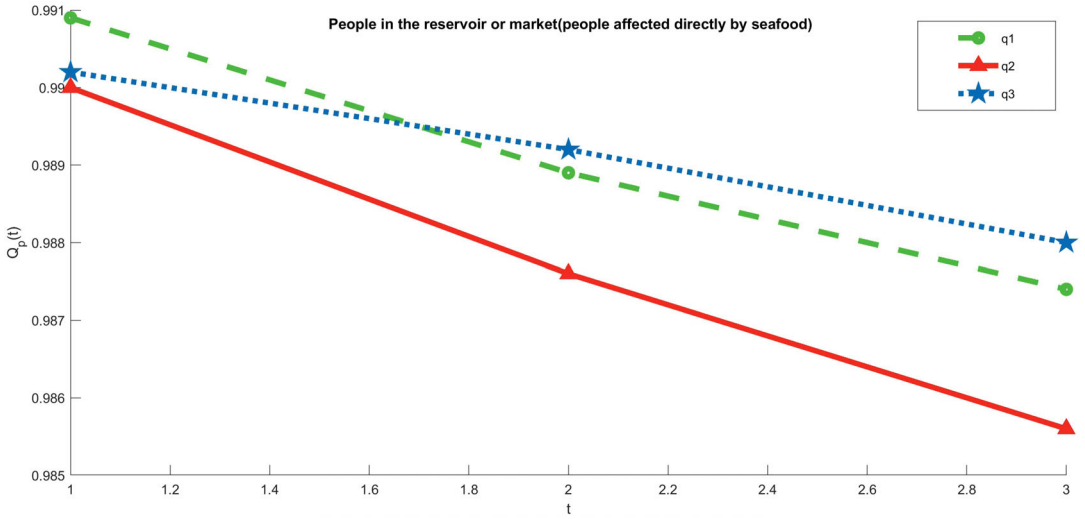
FIGURE 6 Numerical results of the people in the reservoir (affected by seafood) $Q_p(t)$ with respect to time t when n varies from 1 to 3 [Color figure can be viewed at wileyonlinelibrary.com]

TABLE 1 Approximate values of parameter implemented in the SARS-CoV-2 (COVID-19 model) (1.1)

S. No.	Notation with values	Parametric representation
1.	$\Delta_b = 6.9316 \times 10^6$	Birth rate
2.	$\alpha_I = 2.5 \times 10^{-1}$	Contiguous rate
3.	$\lambda_d = 1.44 \times 10^{-2}$	Natural death rate
4.	$\beta_A = 5.944 \times 10^{-1}$	Transmission multiple rate (asymptomatically infected)
5.	$\delta_{RI} = 4.7876 \times 10^{-3}$	After incubation period R_p from A_p
6.	$\mu_{EI} = 5 \times 10^{-2}$	Incubation period of infected I_p from E_p
7.	$\phi_A = 1.243 \times 10^{-2}$	proposition of asymptomatic infected A_p
8.	$\gamma_{QS} = 1.231 \times 10^{-8}$	Transmission coefficient from market to S_p
9.	$\sigma_{RI} = 9.871 \times 10^{-2}$	Removal rate from I_p
10.	$\rho_{RA} = 8.543 \times 10^{-2}$	Recovery rate from A_p
11.	$\kappa_{IQ} = 3.98 \times 10^{-4}$	Infected I_p from Q_p
12.	$\nu_{AQ} = 1 \times 10^{-3}$	Asymptomatic infected A_p from Q_p
13.	$\eta_Q = 1 \times 10^{-2}$	Removing virus rate from Q_t

$$+ \frac{3.4727e - 09}{0.5} * t^0.5 * S_{p_{n-1}}(t) * Q_{p_{n-1}}(t) - \frac{0.0056}{0.5} * t^0.5 * E_{p_{n-1}}(t),$$

$$I_{p_n}(t) = 1 + 0.0024 * E_{p_{n-1}}(t) - 0.0566 * I_{p_{n-1}}(t) + \frac{0.0013}{0.5} * t^0.5 * E_{p_{n-1}}(t)$$
$$- \frac{0.0319}{0.5} * t^0.5 * A_{p_{n-1}}(t),$$

$$A_{p_n}(t) = 1 + 3.1075e - 04 * E_{p_{n-1}}(t) - 0.4344 * A_{p_{n-1}}(t) + \frac{1.7533e - 04}{0.5} * t^0.5 * E_{p_{n-1}}(t)$$
$$- \frac{0.2451}{0.5} * t^0.5 * A_{p_{n-1}}(t),$$

$$R_{\mathrm{p}_n}(t) = 1 + 0.0494 * I_{\mathrm{p}_{n-1}}(t) + 0.4272 * A_{\mathrm{p}_{n-1}}(t) - 0.0072 * R_{\mathrm{p}_{n-1}}(t) + \frac{0.0278}{0.5} * t^{0.5} * I_{\mathrm{p}_{n-1}}(t)$$
$$+ \frac{0.241}{0.5} * t^{0.5} * A_{\mathrm{p}_{n-1}}(t) - \frac{0.0041}{0.5} * t^{0.5} * R_{\mathrm{p}_{n-1}}(t),$$

$$Q_{\mathrm{p}_n}(t) = 1 + 1.9900e - 04 * I_{\mathrm{p}_{n-1}}(t) + 5.000e - 04 * A_{\mathrm{p}_{n-1}}(t) - 0.005 * Q_{\mathrm{p}_{n-1}}(t)$$
$$+ \frac{1.1228e - 04}{0.5} * t^{0.5} * I_{\mathrm{p}_{n-1}}(t) + \frac{2.8210e - 04}{0.5} * t^{0.5} * A_{\mathrm{p}_{n-1}}(t) - \frac{0.0028}{0.5} * t^{0.5} * Q_{\mathrm{p}_{n-1}}(t).$$

7 | CONCLUSION

In this article, we studied a SARS-CoV-2 nonlinear infection-spreading model among various phenomena concerning time by using the Atangana–Baleanu derivative. We derived the solutions, existence results, and unique solutions for the system of equations by the fixed-point method. By using the Mittag–Leffler Kernel with an approximate value, the numerical results will be calculated and plotted in each stage of peoples affected or recovered from SARS-Cov-2 in graphically. We obtained the approximate value of those stages to predict the outcome of SARS-CoV-2 spreading in the country or state or province. In future, by utilizing the *AB*-derivative we can predict the infections of pandemic disease like SARS-CoV-2 (COVID-19).

CONFLICT OF INTEREST
The authors declare that they have no competing interests.

AUTHORS CONTRIBUTIONS
All the authors contributed equally and they read and approved the final manuscript for publication.

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