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Analysis of a novel coronavirus (2019-nCOV) system with variable Caputo-Fabrizio fractional order

Pratibha Verma, Manoj Kumar*

Department of Mathematics, Motilal Nehru National Institute of Technology Allahabad, Prayagraj 211004, Uttar Pradesh, India

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ABSTRACT

The main aim of this study is to present a new variable fractional-order derivatives for novel coronavirus (2019-nCOV) system with the variable Caputo-Fabrizio in Caputo sense. By using the fixed point theory, we explore the new existence and uniqueness results of the solution for the proposed 2019-nCOV system. The existence result is obtained with the aid of the Krasnoselskii fixed point theorem while the uniqueness of the solution has been investigated by utilizing the Banach fixed point theorem. Furthermore, we study the generalized Hyers-Ulam stability as well as the generalized Hyers-Ulam-Rassias stability and also discuss some more interesting results for the proposed system.

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1. Introduction

Fractional calculus (FC) is a powerful tool discovered in 1695 when Leibniz investigated the derivative of arbitrary order. Many researcher have been studied the application of the fractional derivative and fractional differential equations (FDEs). There are some mathematicians firstly define the fractional derivative, such as Liouville, Grunwald, Letnikov, Riemann, and Caputo. Vital physical, real-world problems phenomena are well presented by FDEs, namely, acoustics, viscoelasticity, electrochemistry, electromagnetics, material science. The Complex system is describing by FC due to its applications [1–5,23–25].

In the city of china Wuhan, many researchers investigate that a variety of critical dynamical problems exhibit fractional-order behavior that may vary with time/space. This phenomenon introduced that variable-order fractional calculus (VO-FC) is a nature to give an effective and more powerful mathematical framework to present the complex problems. The VO fractional derivative is an extension of the standard/arbitrary derivative. It is not stable for characterizing phenomena, for example, when the diffusion process evolves in a porous medium or the external field changes with time. So, the VO fractional diffusion models have been used to study many complex physical problems ranging from science and engineering fields. The applications of the VO fractional derivative in processing of geographical data, signature verification, viscoelasticity, diffusion processes [6,7].

In Wuhan, where was the first time identified, the virus is known as novel corona virus in short COVID-19. It has been declared an epidemic virus because of its nature. In investigations, the researcher found that virus has been transmitted between peoples and animals. The initial symptoms of this infection are fever, cough, and breathing difficulties and next steps, several acute respiratory syndromes, the infection can cause pneumonia, kidney failure, and even death. Researchers are motivated to study the situation and establish concllusions, and based on the current seniority of the crucial time to analyze the virus through mathematical modeling. Many researchers developed the mathematical models for the virus with different situations/conditions and found several models to describe the corresponding situations. The system of ordinary differential equations describes the developed mathematical model for the COVID-19. In the previous work, the authors converted this model into the system of fractional differential equations for more upgrade versions of the odinary order models for more obatin informations of the models. However, in this scenario, this virus is getting more and more attention in the current era. Many researchers are focused on finding the virus's behavior with various conditions and are also interested in obtaining the saturation stage of the virus [8–11]. In present work, we use vari-

* Corresponding author.
E-mail addresses: pratibhaverma@mnnit.ac.in (P. Verma), manoj@mnnit.ac.in (M. Kumar).

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able fractional operators for more effective results getting from the COVID-19 proposed model.

In [12–15], discussed the exact and approximate solutions by using different numerical methods for various types of fractional differential equations (FDEs). They also described the error and convergence analysis. The adopted methods are helpful to obtain the analytical-approximate solution for FDEs. The existence and uniqueness of the fractional-order solution are discussed for the 2019-nCOV models to obtain the batter results compared to ordinary order 2019-nCOV models. For finding the results, many authors had utilized the fixed point theory as a reliable tool. This affirmative approach confirms the existence and uniqueness of the 2019-nCOV models with fractional order. Also, for differential equations, the stability analysis is an important aspect. From several types of stability theory generated in the literature, Ulam-Hyers type stability is one of the most used and interesting types. The Ulam-Hyers-Rassiass stability is a generalized version of the Ulam-Hyers stability. For more general work, we use the Ulam-Hyers-Rassiass stability [16–20]. Motivation from this work, we also inspired to obtain existence, uniqueness, and stability using the fixed point theory and the Ulam-Hyers stabilities for more generalized 2019-nCOV models such as the 2019-nCOV model involve variable fractional order using a non-singular derivative. Here, we adopt the definition for variable factional is known as the variable Caputo-Fabrizio fractional derivative involve the non-singular kernel with Caputo derivative. Michele Caputo and Mauro Fabrizio discuss a fractional derivative operator in 2015 based on the non-sigular kernel and exponential function to overcome the singular kernel's problems. Their fractional derivative operators does not have a singular kernel. They have demonstrated that their derivative operator was suitable for the solution of complex physical problems. Due to non-singular kernel, the derivative provides most appropriate results for modeling of real-world problems [21,22].

This discussion aims to investigate the more generalized framework for the proposed 2019-nCOV system with the variable fractional order using non-singular kernel derivative and study an analysis of the generalized 2019-nCOV system. In the analysis, we establish the new results for the existence and uniqueness of a solution with stability analysis. Additionally, the work is to find out the existence of the position solutions, maximal/minimal solutions for the proposed generalized 2019-nCOV system. This work investigates more reliable and suitable model for a novel coronavirus in real-world problems using VO fractional operators. The order of the model extended as variable fractional order in which we replace integer order with bounded and continuous functions, replacing the different types function as the order of the model, which describes several roles of this virus on our daily life. We find a new reliable model that will be more accurate and useful to analyze the virus. It is well Known that the fractional-order is a particular case of variable fractional order, so the fractional-order 2019-nCOV system is also a specific case of the variable fractional order (VFO) 2019-nCOV system. Our proposed VFO 2019-nCOV system provides a more realistic/suitable situations in the current stage of the virus. The analysis of the proposed VFO system is also interesting to identify the existence, uniqueness of the solution, stability analysis is also required aspect, there exist any positive and maximal/minimal solutions of the proposed model. These all points promote the VFO model as well as suitability on the virus to present the situations that happened through this virus. Before obtaining the numerical solutions, this analysis is important to improve further works for the proposed VFO 2019-nCOV system.

The rest of this paper is structured in the following way. Section 2, we present some fundamental results about variable Caputo-Fabrizio fractional derivative and useful theorems, lemmas. In Section 3, introduces the classical model of the 2019-nCOV system. We describe the proposed 2019-nCOV system with variable Caputo-Fabrizio fractional derivative in Section 4. We obtain the new results existence, uniqueness, stability, and some important theorems for the generalized variable fractional 2019-nCOV system in Section 5, the manuscript is finished by concluding remarks in Section 6.

2. Fundamental results

In this section, we recall useful definitions, results related to the nonsingular kernel variable fractional operator, namely, variable Caputo-Fabrizio fractional derivative involves Caputo derivative, and some useful theorems, lemmas are needed for the study of the main results.

Definition 1. (Verma and Kumar [3], 4], 5]) The function $\varrho(\theta)$ is differenttiable, the variable Caputo derivative of order $\vartheta(\theta) \in [0,1)$ is given as

$$ {}_{0}^{C}D_{\theta}^{\vartheta(\theta)}\varrho(\theta) = \frac{1}{\Gamma(1-\vartheta(\theta))}\int_0^{\theta}\frac{1}{(\theta-a)^{\vartheta(\theta)}}\varrho'(a)da. \quad (1)$$

Note: The VO fractional operators (derivatives/integral) of $\ell - 1 < \vartheta(\theta) \le \ell$ and $\varrho(\theta)$ is bounded in interval $\theta \in [0,T]$.

Definition 2. (Dua et al. [6], Jia et al. [7]) Let $\varrho(\theta) \in H^1([0,T])$, and $\vartheta(\theta) \in [0,1)$, then the $\vartheta(\theta)$th-order variable Caputo-Fabrizio derivative of $\varrho(\theta)$ in the Caputo sence is defined as

$$ {}_{0}^{CF}D_{\theta}^{\vartheta(\theta)}\varrho(\theta) = \frac{\mathbb{E}(\vartheta(\theta))}{1-\vartheta(\theta)}\int_0^{\theta}\varrho'(a)\exp\left(\frac{-\vartheta(\theta)(\theta-a)}{(1-\vartheta(\theta))}\right)da, \quad (2)$$

Here, $H^1(0,T)$ is a Hilbert space. The integral (2) on right converges and the function $\mathbb{E}(\vartheta(\theta))$ is a normalizing function depending on $\vartheta(\theta)$ such that $\mathbb{E}(0)=\mathbb{E}(1)=1$.

Definition 3. (Dua et al. [6], Jia et al. [7]) The nonsingular kernel type variable fractional integral is defined by

$$ {}_{0}^{CF}J_{\theta}^{\vartheta(\theta)}\varrho(\theta) = \frac{(1-\vartheta(\theta))}{\mathbb{E}(\vartheta(\theta))}\varrho(\theta) + \frac{\vartheta(\theta)}{\mathbb{E}(\vartheta(\theta))}\int_0^{\theta}\varrho(a)da, 0<\vartheta(\theta)\le 1. \quad (3)$$

Lemma 1. *(Dua et al. [6], Jia et al. [7]) Let $\varrho(\theta) \in C([0,T])$, then the solution of the following variable Caputo-Fabrizio fractional differential equation*

$$\left.\begin{array}{l} {}_{0}^{CF}D_{\theta}^{\vartheta(\theta)}\varrho(\theta) = \omega(\theta), \theta \in [0,T], 0<\vartheta(\theta)\le 1, \\ \varrho(0)=\omega_0, \omega_0 \in \mathbb{R}, \end{array}\right\} \quad (4)$$

is given by

$$\varrho(\theta) = \omega_0 + \frac{(1-\vartheta(\theta))}{\mathbb{E}(\vartheta(\theta))}\omega(\theta) + \frac{\vartheta(\theta)}{\mathbb{E}(\vartheta(\theta))}\int_0^{\theta}\omega(a)da. \quad (5)$$

Theorem 1. *(Verma and Kumar [3–5]) Every contraction mapping on a complete metric space has a unique fixed point.*

Theorem 2. *(Verma and Kumar [3–5]) Let Y be a compact metric space. Let $C(Y,\mathbb{R})$ be given the sup norm metric. Then a set $P \subset C(Y)$ is compact iff P is bounded, closed and equicontinous.*

Theorem 3. *(Shera et al. [9]) Let $\eta \subset X$ be a closed, bounded and convex subset of real Banach space X and let Q_1 and Q_2 be operators on η satisfying the following conditions*

(i) $Q_1(\eta') + Q_2(\eta'') \in \eta, \forall \eta', \eta'' \in \eta$,
(ii) Q_1 *is a strict contraction on η, that is, there exists a $r \in [0,1)$ such that $|Q_1(x_1) - Q_1(x_2)| \le r|x_1 - x_2|, \forall x_1, x_2 \in \eta$,*
(iii) Q_2 *is continuous on η and Q_2 is a relatively compact subset of X.*

Then there exist at least one solution $\eta' \in \eta$ such that $Q_1(\eta') + Q_2(\eta') = \eta'$.

3. Classical mathematical system of the proposed 2019-nCOV system (non-fractional order)

In this section we consider the 2019-nCOV system suggested in Gao et al. [10], Tuan et al. [11], describe as

$$\left.\begin{array}{l} \eta_1'(\omega) = a_q - b_q\eta_1(\omega) - c_q\eta_1(\omega)[x(\omega) + \rho\Omega(\omega)] - c_\epsilon\eta_1(\omega)\epsilon(\omega),\\ \eta_2'(\omega) = c_q\eta_1(\omega)[x(\omega) + \rho\Omega(\omega)] + c_\epsilon\eta_1(\omega)\epsilon(\omega) - (1-\xi_q)\zeta_q\eta_2(\omega) - \xi_q\zeta_q'\eta_2(\omega) - b_q\eta_2(\omega),\\ x'(\omega) = (1-\xi_q)\zeta_q\eta_2(\omega) - [\alpha_q + b_q]x(\omega),\\ \Omega'(\omega) = \xi_q\zeta_q'\eta_2(\omega) - [\alpha_q' + b_q]\Omega(\omega),\\ h'(\omega) = \alpha_q x(\omega) + \alpha_q'\Omega(\omega) - b_q h(\omega),\\ \epsilon'(\omega) = \delta[x(\omega) + d\Omega(\omega) - \epsilon(\omega)], \end{array}\right\} \quad (6)$$

Here, $a_q, b_q, c_q, \rho, c_q, \alpha_q, \delta, d$ are nonzero constants, and the initial conditions for the system (6) are given by

$$\eta_1(0) = \gamma_1 > 0, \eta_2(0) = \gamma_2 > 0, x(0) = \gamma_3 > 0, \Omega(0) = \gamma_4 > 0, h(0) = \gamma_5 > 0,$$

$$\zeta_q(0) = \gamma_6 > 0, \zeta_q'(0) = \gamma_7 > 0, \alpha_q'(0) = \gamma_8 > 0, \alpha_q(0) = \gamma_9 > 0, \epsilon(0) = \gamma_{10} > 0,$$

where

η_1: the susceptible people.
η_2: The symbolize exposed people.
x: The symptomatic infected people.
Ω: The asymptomatic infected people.
h: Remove people (Recovered and died people).
a_q: The birth rate.
b_q: The death rate.
c_q: The transmission rate.
ρ: Transfer coefficient.
ϵ: The reservoir (the seafood area).
$\frac{1}{\zeta}$: The incubation period of bat infection.
$\frac{1}{\alpha}$: The infections period of bat infection.

We generalized the 2019-nCOV system [10,11] from an integer order system to variable fractional order system via variable Caputo-Fabrizio fractional derivative with Caputo derivative. In the next section, we present the proposed system (6) as variable fractional order system.

4. Mathematical system of the proposed 2019-nCOV system based on variable Caputo-Fabrizio (CF) fractional derivative

In this section, we replace the integer order by variable fractional order, namely, variable Caputo-Fabrizio fractional derivative (VCF-FD). We consider the system (6) and describe in the form of the variable CF-FD as follows:

$$\left.\begin{array}{l} {}_0^{CF}D_\omega^{\vartheta(\omega)}\eta_1(\omega) = a_q - b_q\eta_1(\omega) - c_q\eta_1(\omega)[x(\omega) + \rho\Omega(\omega)] - c_\epsilon\eta_1(\omega)\epsilon(\omega),\\ {}_0^{CF}D_\omega^{\vartheta(\omega)}\eta_2(\omega) = c_q\eta_1(\omega)[x(\omega) + \rho\Omega(\omega)] + c_\epsilon\eta_1(\omega)\epsilon(\omega) - (1-\xi_q)\zeta_q\eta_2(\omega) - \xi_q\zeta_q'\eta_2(\omega) - b_q\eta_2(\omega),\\ {}_0^{CF}D_\omega^{\vartheta(\omega)}x(\omega) = (1-\xi_q)\zeta_q\eta_2(\omega) - [\alpha_q + b_q]x(\omega),\\ {}_0^{CF}D_\omega^{\vartheta(\omega)}\Omega(\omega) = \xi_q\zeta_q'\eta_2(\omega) - [\alpha_q' + b_q]\Omega(\omega),\\ {}_0^{CF}D_\omega^{\vartheta(\omega)}h(\omega) = \alpha_q x(\omega) + \alpha_q'\Omega(\omega) - b_q h(\omega),\\ {}_0^{CF}D_\omega^{\vartheta(\omega)}\epsilon(\omega) = \delta[x(\omega) + d\Omega(\omega) - \epsilon(\omega)], \end{array}\right\} \quad (7)$$

and the initial conditions are

$$\eta_1(0) = \gamma_1 > 0, \eta_2(0) = \gamma_2 > 0, x(0) = \gamma_3 > 0, \Omega(0) = \gamma_4 > 0, h(0) = \gamma_5 > 0,$$

$$\zeta_q(0) = \gamma_6 > 0, \zeta_q'(0) = \gamma_7 > 0, \alpha_q'(0) = \gamma_8 > 0, \alpha_q(0) = \gamma_9 > 0, \epsilon(0) = \gamma_1 0 > 0.$$

Where the operator ${}_0^{CF}D_\omega^{\vartheta(\omega)}$ is the variable CF-FD with order $0 < \vartheta(\omega) \le 1$ and $\omega \in [0, \nu]$.

5. Main results of the proposed 2019-nCOV system based on VCF-FD

In this section, we investigate the uniqueness, existence, Hyers-Ulam stabilities, existence of positive solution, existence of maximal and minimal solutions and the continuation theorem are presented in the following subsections:

5.1. Existence and uniqueness of solution

In the present section, we discuss the uniqueness and existence results of the solution with the help of fixed point theory and variable fractional order for the proposed 2019-nCOV system (7). Now, we express the right-side of the system (7), as

$$\left.\begin{array}{l}\Theta_1(\omega,\eta_1(\omega),\eta_2(\omega),x(\omega),\Omega(\omega),h(\omega),\epsilon(\omega)) = a_q - b_q\eta_1(\omega) - c_q\eta_1(\omega)[x(\omega)+\rho\Omega(\omega)] - c_\epsilon\eta_1(\omega)\epsilon(\omega),\\ \Theta_2(\omega,\eta_1(\omega),\eta_2(\omega),x(\omega),\Omega(\omega),h(\omega),\epsilon(\omega)) = c_q\eta_1(\omega)[x(\omega)+\rho\Omega(\omega)] + c_\epsilon\eta_1(\omega)\epsilon(\omega) - (1-\xi_q)\zeta_q\eta_2(\omega) - \xi_q\zeta_q'\eta_2(\omega) - b_q\eta_2(\omega),\\ \Theta_3(\omega,\eta_1(\omega),\eta_2(\omega),x(\omega),\Omega(\omega),h(\omega),\epsilon(\omega)) = (1-\xi_q)\zeta_q\eta_2(\omega) - [\alpha_q+b_q]x(\omega),\\ \Theta_4(\omega,\eta_1(\omega),\eta_2(\omega),x(\omega),\Omega(\omega),h(\omega),\epsilon(\omega)) = \xi_q\zeta_q'\eta_2(\omega) - [\alpha_q'+b_q]\Omega(\omega),\\ \Theta_5(\omega,\eta_1(\omega),\eta_2(\omega),x(\omega),\Omega(\omega),h(\omega),\epsilon(\omega)) = \alpha_q x(\omega) + \alpha_q'\Omega(\omega) - b_q h(\omega),\\ \Theta_6(\omega,\eta_1(\omega),\eta_2(\omega),x(\omega),\Omega(\omega),h(\omega),\epsilon(\omega)) = \delta[x(\omega)+d\Omega(\omega)-\epsilon(\omega)],\end{array}\right\} \tag{8}$$

by using system (8), the generalized system (7) can be presented in the following form

$$\left.\begin{array}{l}{}^{CF}_{0}D^{\vartheta(\omega)}_{\omega}\Theta(\omega) = \chi(\omega,\Theta(\omega)), \omega\in[0,\nu], 0<\vartheta(\omega)\le 1,\\ \Theta(0)=\Theta_0,\end{array}\right\} \tag{9}$$

where the vectors $\Theta(\omega) = (\eta_1(\omega),\eta_2(\omega),x(\omega),\Omega(\omega),h(\omega),\epsilon(\omega))$, $\Theta_0(\omega) = (\eta_{10},\eta_{20},x_0,\Omega_0,h_0,\epsilon_0)$, and

$$\chi(\omega,\Theta(\omega)) = \begin{cases}\Theta_1(\omega,\eta_1(\omega),\eta_2(\omega),x(\omega),\Omega(\omega),h(\omega),\epsilon(\omega)),\\ \Theta_2(\omega,\eta_1(\omega),\eta_2(\omega),x(\omega),\Omega(\omega),h(\omega),\epsilon(\omega)),\\ \Theta_3(\omega,\eta_1(\omega),\eta_2(\omega),x(\omega),\Omega(\omega),h(\omega),\epsilon(\omega)),\\ \Theta_4(\omega,\eta_1(\omega),\eta_2(\omega),x(\omega),\Omega(\omega),h(\omega),\epsilon(\omega)),\\ \Theta_5(\omega,\eta_1(\omega),\eta_2(\omega),x(\omega),\Omega(\omega),h(\omega),\epsilon(\omega)),\\ \Theta_6(\omega,\eta_1(\omega),\eta_2(\omega),x(\omega),\Omega(\omega),h(\omega),\epsilon(\omega)).\end{cases}$$

Using Lemma 1, the Eq. (9), becomes

$$\Theta(\omega) = \Theta_0(\omega) + \frac{(1-\vartheta(\omega))}{\mathbb{E}(\vartheta(\omega))}\chi(\omega,\Theta(\omega)) + \frac{\vartheta(\omega)}{\mathbb{E}(\vartheta(\omega))}\int_0^\omega \chi(a,\Theta(a))da. \tag{10}$$

Let $\beta = C([0,\nu])$ be a Banach space with norm defined as $\|\Theta\| = \max_{\omega\in[0,\nu]}\{|\Theta|, \forall\,\Theta\in\beta\}$ and $\vartheta^* = \min\{\vartheta(\omega), \omega\in[0,\nu]\}$ and $\vartheta^{**} = \max_{\omega\in[0,\nu]}\{\vartheta(\omega), \omega\in[0,\nu]\}$ be the minimum and maximum value of the variable fractional order $\vartheta(\omega)$ on $[0,\nu]$.

To proceed further, we assume the following hypotheses to obtain our main results:

[$\mathcal{M}_1$]: There exist constants $\mathbf{G}_\chi, \mathbf{H}_\chi > 0$, and $k\in[0,1)$ such that

$$|\chi(\omega,\Theta(\omega))| \le \mathbf{G}_\chi|\Theta|^k + \mathbf{H}_\chi.$$

[$\mathcal{M}_2$]: There exists constants $\mathcal{N}_\chi > 0$, such that

$$|\chi(\omega,\Theta'(\omega)) - \chi(\omega,\Theta''(\omega))| \le \mathcal{N}_\chi|\Theta'(\omega) - \Theta''(\omega)|.$$

Now, we convert the system (9) into fixed point problem, we define operator $Q:\beta\to\beta$ as

$$Q(\Theta(\omega)) = \Theta_0(\omega) + \frac{(1-\vartheta(\omega))}{\mathbb{E}(\vartheta(\omega))}\chi(\omega,\Theta(\omega)) + \frac{\vartheta(\omega)}{\mathbb{E}(\vartheta(\omega))}\int_0^\omega \chi(a,\Theta(a))da. \tag{11}$$

Let us present two operators, from the operator (11) such that $Q(\Theta(\omega)) = Q_1(\Theta(\omega)) + Q_2(\Theta(\omega))$, where

$$Q_1(\Theta(\omega)) = \Theta_0(\omega) + \frac{(1-\vartheta(\omega))}{\mathbb{E}(\vartheta(\omega))}\chi(\omega,\Theta(\omega)), \tag{12}$$

$$Q_2(\Theta(\omega)) = \frac{\vartheta(\omega)}{\mathbb{E}(\vartheta(\omega))}\int_0^\omega \chi(a,\Theta(a))da. \tag{13}$$

Theorem 4. *Assume that the hypothesis [$\mathcal{M}_2$] hold and there exists a constant $\mathcal{B}>0$ such that*

$$\mathcal{B} = \left[\frac{(1-\vartheta^*)\mathcal{N}_\chi}{\mathbb{E}(\vartheta^*)} + \frac{\vartheta^*\mathcal{N}_\chi\nu}{\mathbb{E}(\vartheta^*)}\right] < 1, \tag{14}$$

then Q has unique fixed point for the system (9) on β.

Proof. Les us consider $\Theta',\Theta''\in\beta$, then

$$\begin{aligned}\|Q\Theta' - Q\Theta''\| &\le \|Q_1\Theta' - Q_1\Theta''\| + \|Q_2\Theta' - Q_2\Theta''\|\\ &\le \frac{(1-\vartheta(\omega))}{\mathbb{E}(\vartheta(\omega))}\max_{\omega\in[0,\nu]}|\chi(\omega,\Theta'(\omega)) - \chi(\omega,\Theta''(\omega))|\\ &+ \frac{\vartheta(\omega)}{\mathbb{E}(\vartheta(\omega))}\max_{\omega\in[0,\nu]}\left|\int_0^\omega\chi(a,\Theta'(a))da - \int_0^\omega\chi(a,\Theta''(a))da\right|\\ &\le \left[\frac{(1-\vartheta(\omega))\mathcal{N}_\chi}{\mathbb{E}(\vartheta(\omega))} + \frac{\vartheta(\omega)\mathcal{N}_\chi\omega}{\mathbb{E}(\vartheta(\omega))}\right]\max_{\omega\in[0,\nu]}|\Theta' - \Theta''|\\ &\le \left[\frac{(1-\vartheta^*)\mathcal{N}_\chi}{\mathbb{E}(\vartheta^*)} + \frac{\vartheta^*\mathcal{N}_\chi\nu}{\mathbb{E}(\vartheta^*)}\right]\|\Theta' - \Theta''\|\\ &\le \mathcal{B}\|\Theta' - \Theta''\|.\end{aligned}$$

Since $\mathcal{B} = \left[\frac{(1-\vartheta^*)\mathcal{N}_\chi}{\mathbb{E}(\vartheta^*)} + \frac{\vartheta^*\mathcal{N}_\chi\nu}{\mathbb{E}(\vartheta^*)}\right] < 1.$

This impels that the operator Q has unique fixed point, by the Banach fixed point theorem. Consequently, the system (9) has unique solution. □

Theorem 5. *Assume that the hypotheses* [$\mathcal{M}_1$]-[$\mathcal{M}_2$] *holds and if* $0 < \frac{(1-\vartheta^*)\mathcal{N}_\chi}{\mathbb{E}(\vartheta^*)} < 1$, *then the system (9) has at least one solution.*

Proof. Firstly, we present the operator Q is contraction. Let $\Theta \in \mathcal{T}$, where $\mathcal{T} = \{\Theta \in \beta : \|\Theta\| \le w, w > 0\}$ is closed convex set, then

$$\begin{aligned}&\|Q_1(\Theta'(\omega)) - Q_1(\Theta''(\omega))\|\\&= \frac{(1-\vartheta(\omega))}{\mathbb{E}(\vartheta(\omega))} \max_{\omega\in[0,\nu]} |\chi(\omega, \Theta'(\omega))) - \chi(\omega, \Theta''(\omega))|\\&\le \frac{(1-\vartheta^*)}{\mathbb{E}(\vartheta^*)}\mathcal{N}_\chi \|\Theta'(\omega) - \Theta''(\omega)\|.\end{aligned}$$

Hence Q_1 is contraction. Further to prove that the second consider operator Q_2 is compact and continuous, for any $\Theta \in \mathcal{T}$, then Q_2 is contraction as χ is continuous, then

$$\begin{aligned}\|Q_2(\Theta(\omega))\| &= \max_{\omega\in[0,\nu]} \left|\frac{\vartheta(\omega)}{\mathbb{E}(\vartheta(\omega))} \int_0^\omega \chi(a, \Theta(a))da\right|\\&\le \frac{|\vartheta(\omega)|}{|\mathbb{E}(\vartheta(\omega))|} \int_0^\omega |\chi(a, \Theta(a))|da\\&\le \frac{\vartheta^*\nu}{\mathbb{E}(\vartheta^*)}[\mathbf{G}_\chi|\Theta|^k + \mathbf{H}_\chi].\end{aligned}$$

Which shows that Q_2 is bounded. Next, let $\omega_1 > \omega_2 \in [0, \nu]$, such that

$$\begin{aligned}&\|Q_2(\Theta(\omega_1)) - Q_2(\Theta(\omega_2))\|\\&= \frac{\vartheta^*}{\mathbb{E}(\vartheta^*)} \max_{\omega\in[0,\nu]} \left|\int_0^{\omega_1} \chi(a, \Theta(a))da - \int_0^{\omega_2} \chi(a, \Theta(a))da\right|\\&\le \frac{\vartheta^*[\mathbf{G}_\chi|\Theta|^k + \mathbf{H}_\chi]}{\mathbb{E}(\vartheta^*)}|\omega_1 - \omega_2|.\end{aligned}$$

This implies that $\|Q_2(\Theta(\omega_1)) - Q_2(\Theta(\omega_2))\| \to 0$ as $\omega_1 \to \omega_2$.

Hence, the operator Q_2 is equicontinuous. So, by the Theorem 2, Q_2 is compact. Thus, the corresponding system has at least one solution. □

5.2. Stability analysis

In this section, we present the Ulam types stabilities of the proposed system (9). Before proceed the further process, we discuss some definitions and notions as given by

Definition 4. The proposed system (9) is Ulam-Hyers stable if for any $\epsilon > 0$ and let $\Theta \in \beta$ be any solution of the inequality

$$\|{}_0^{CF}D_\omega^{\vartheta(\omega)}\Theta(\omega) - \chi(\omega, \Theta(\omega))\| \le \epsilon, \omega \in [0, \nu], \tag{15}$$

there exists unique solution Θ' of the system (9) with $\mathbb{J}_k > 0$ such that

$$\|\Theta(\omega) - \Theta'(\omega)\| \le \mathbb{J}_k\epsilon, \omega \in [0, \nu]. \tag{16}$$

Further, the system (9) will be generalized Ulam-Hyers stable if there exists $\Phi \in C(\mathbb{R}, \mathbb{R})$ with $\Phi(0) = 0$, for any solution Θ of the Eq. (15) and Θ' be unique solution of (9) such that

$$\|\Theta - \Theta'\| \le \Phi(\epsilon), \tag{17}$$

then the system (9) is generalized Ulam-Hyers stable.

Remark 1. If there exists $\mathcal{G} \in C([0, \nu], \mathbb{R})$, the $\Theta \in \beta$ satisfies inequality (15) if

(i) $|\mathcal{G}(\omega)| \le \epsilon, \forall\omega \in [0, \nu]$,
(ii) ${}_0^{CF}D_\omega^{\vartheta(\omega)}\Theta(\omega) = \chi(\omega, \Theta(\omega)) + \mathcal{G}(\omega), \forall\omega \in [0, \nu]$.

Let us consider the corresponding system (9) as

$$\left.\begin{aligned}&{}_0^{CF}D_\omega^{\vartheta(\omega)}\Theta(\omega) = \chi(\omega, \Theta(\omega)) + \mathcal{G}(\omega),\\&\Theta(0) = \Theta_0.\end{aligned}\right\} \tag{18}$$

We need the following result for further analysis.

Lemma 2. *The following inequality hold to the problem (18).*

$$|\Theta(\omega) - \mathcal{Q}\Theta(\omega)| \le \left[\frac{(1-\vartheta^*)}{\mathbb{E}(\vartheta^*)} + \frac{\vartheta^*\nu}{\mathbb{E}(\vartheta^*)}\right]\epsilon.$$

Proof. With the help of Lemma 1, the solution of the system (18) is given by

$$\Theta(\omega) = \Theta_0 + {}_0^{CF}J_\omega^{\vartheta(\omega)}(\chi(\omega, \Theta(\omega))) + {}_0^{CF}J_\omega^{\vartheta(\omega)}(\mathcal{G}(\omega)).$$

Using the operator (11), we have

$$\begin{aligned}|\Theta(\omega) - \mathcal{Q}\Theta(\omega)| &\le \left[\frac{(1-\vartheta(\omega))}{\mathbb{E}(\vartheta(\omega))}|\mathcal{G}(\omega)| + \frac{\vartheta(\omega)}{\mathbb{E}(\vartheta(\omega))}\int_0^\omega |\mathcal{G}(\omega)|da\right]\\&\le \left[\frac{(1-\vartheta^*)}{\mathbb{E}(\vartheta^*)} + \frac{\vartheta^*\nu}{\mathbb{E}(\vartheta^*)}\right]\epsilon.\end{aligned}$$

□

Definition 5. The proposed system (9) is Ulam-Hyers-Rassias stable for $\Psi \in C([0, \nu], \mathbb{R})$, if for $\epsilon > 0$ and $\Theta \in \beta$ be any solution of inequality

$$\|{}_0^{CF}D_\omega^{\vartheta(\omega)}\Theta(\omega) - \chi(\omega, \Theta(\omega))\| \le \Psi(\omega)\epsilon, \omega \in [0, \nu], \tag{19}$$

there exists unique solution Θ' of the system (9) with $\mathbb{J}_k > 0$ such that

$$\|\Theta(\omega) - \Theta'(\omega)\| \le \mathbb{J}_k\Psi(\omega)\epsilon, \omega \in [0, \nu]. \tag{20}$$

Further, for $\Psi \in C([0, \nu], \mathbb{R})$ if there exists $\mathcal{J}_{k,\Psi}$ and $\epsilon > 0$, for any solution Θ of the Eq. (19) and Θ' be unique solution of (9) such that

$$\|\Theta - \Theta'\| \le \mathcal{J}_{k,\Psi}\Psi(\omega), \omega \in [0, \nu], \tag{21}$$

then the system (9) is generalized Ulam-Hyers-Rassias stable.

Remark 2. If there exists $\mathcal{G} \in C([0, \nu], \mathbb{R})$, the $\Theta \in \beta$ satisfies inequality (19) if

(i) $|\mathcal{G}(\omega)| \le \epsilon\Psi(\omega), \forall\omega \in [0, \nu]$,
(ii) ${}_0^{CF}D_\omega^{\vartheta(\omega)}\Theta(\omega) = \chi(\omega, \Theta(\omega)) + \mathcal{G}(\omega), \forall\omega \in [0, \nu]$.

Let us consider the corresponding system (9) as

$$\left.\begin{aligned}&{}_0^{CF}D_\omega^{\vartheta(\omega)}\Theta(\omega) = \chi(\omega, \Theta(\omega)) + \mathcal{G}(\omega),\\&\Theta(0) = \Theta_0.\end{aligned}\right\} \tag{22}$$

We need the following result for further analysis.

Lemma 3. *The following inequality hold to the problem (9).*

$$|\Theta(\omega) - \mathcal{Q}\Theta(\omega)| \le \left[\frac{(1-\vartheta^*)}{\mathbb{E}(\vartheta^*)} + \frac{\vartheta^*\nu}{\mathbb{E}(\vartheta^*)}\right]\Psi(\omega)\epsilon.$$

Proof. With the help of Lemma 1, the solution of the system (9) is given by

$$\Theta(\omega) = \Theta_0 + {}_0^{CF}J_\omega^{\vartheta(\omega)}(\chi(\omega, \Theta(\omega))) + {}_0^{CF}J_\omega^{\vartheta(\omega)}(\mathcal{G}(\omega)).$$

Using the operator (11), we have

$$\begin{aligned}|\Theta(\omega) - \mathcal{Q}\Theta(\omega)| &\le \left[\frac{(1-\vartheta(\omega))}{\mathbb{E}(\vartheta(\omega))}|\mathcal{G}(\omega)| + \frac{\vartheta(\omega)}{\mathbb{E}(\vartheta(\omega))}\int_0^\omega |\mathcal{G}(\omega)|da\right]\\&\le \left[\frac{(1-\vartheta^*)}{\mathbb{E}(\vartheta^*)} + \frac{\vartheta^*\nu}{\mathbb{E}(\vartheta^*)}\right]\Psi(\omega)\epsilon.\end{aligned}$$

□

Theorem 6. *Under the hypotheses of Lemma 2, the solution of the system (9) is Ulam-Hyers stable and also generalized Ulam-Hyers stable if* $\left[\frac{(1-\vartheta^*)\mathcal{N}_\chi}{\mathbb{E}(\vartheta^*)} + \frac{\mathcal{N}_\chi\nu}{\mathbb{E}(\vartheta^*)}\right] \le 1$.

Proof. Consider $\Theta \in \beta$ be any solution and $\Theta' \in \beta$ be unique solution of the system (9), then we have

$$|\Theta(\omega)-\Theta'(\omega)| = |\Theta(\omega)-\mathcal{Q}\Theta(\omega)| + |\mathcal{Q}\Theta(\omega)-\Theta'(\omega)|$$
$$\leq \left[\frac{(1-\vartheta^*)}{\mathbb{E}(\vartheta^*)}+\frac{\vartheta^*\nu}{\mathbb{E}(\vartheta^*)}\right]\epsilon + \left[\frac{(1-\vartheta^*)\mathcal{N}_\chi}{\mathbb{E}(\vartheta^*)}+\frac{\mathcal{N}_\chi\nu}{\mathbb{E}(\vartheta^*)}\right]|\Theta(\omega)-\Theta'(\omega)|$$
$$\leq \frac{\left[\frac{(1-\vartheta^*)}{\mathbb{E}(\vartheta^*)}+\frac{\vartheta^*\nu}{\mathbb{E}(\vartheta^*)}\right]}{1-\left[\frac{(1-\vartheta^*)\mathcal{N}_\chi}{\mathbb{E}(\vartheta^*)}+\frac{\mathcal{N}_\chi\nu}{\mathbb{E}(\vartheta^*)}\right]}\epsilon.$$

Thus, the proposed model (9) is Ulam-Hyers stable as well as generalized Ulam-Hyers stable. □

Theorem 7. *Under the hypotheses of Lemma 3, the solution of the system (9) is Ulam-Hyers-Rassias stable and also generalized Ulam-Hyers-Rassias stable if* $\left[\frac{(1-\vartheta^*)\mathcal{N}_\chi}{\mathbb{E}(\vartheta^*)}+\frac{\mathcal{N}_\chi\nu}{\mathbb{E}(\vartheta^*)}\right]\leq 1$.

Proof. Consider $\Theta \in \beta$ be any solution and Θ' be unique solution of the system (9), then we have

$$|\Theta(\omega)-\Theta'(\omega)| = |\Theta(\omega)-\mathcal{Q}\Theta(\omega)| + |\mathcal{Q}\Theta(\omega)-\Theta'(\omega)|$$
$$\leq \left[\frac{(1-\vartheta^*)}{\mathbb{E}(\vartheta^*)}+\frac{\vartheta^*\nu}{\mathbb{E}(\vartheta^*)}\right]\Psi(\omega)\epsilon + \left[\frac{(1-\vartheta^*)\mathcal{N}_\chi}{\mathbb{E}(\vartheta^*)}+\frac{\mathcal{N}_\chi\nu}{\mathbb{E}(\vartheta^*)}\right]|\Theta(\omega)-\Theta'(\omega)|$$
$$\leq \frac{\left[\frac{(1-\vartheta^*)}{\mathbb{E}(\vartheta^*)}+\frac{\vartheta^*\nu}{\mathbb{E}(\vartheta^*)}\right]}{1-\left[\frac{(1-\vartheta^*)\mathcal{N}_\chi}{\mathbb{E}(\vartheta^*)}+\frac{\mathcal{N}_\chi\nu}{\mathbb{E}(\vartheta^*)}\right]}\Psi(\omega)\epsilon.$$

Thus, the proposed system (9) is Ulam-Hyers-Rassias stable as well as generalized Ulam-Hyers-Rassias stable. □

5.3. Positive solutions theorems

In this section, we study the existence of positive, continuous solution for the proposed system (9).

For further analysis we use following assumptions for the system (9).

[V_1]: The function $\chi(\omega,\Theta(\omega)) : [0,\nu]\times\mathbb{R}^6 \to \mathbb{R}^6$ is a continuous function.

[V_2]: There exist two different positive constant $\mathcal{M}_1$ and $\mathcal{M}_2$ such that $\mathcal{M}_1 \leq \chi(\omega,\Theta(\omega)) \leq \mathcal{M}_2$.

[V_3]: $\Lambda = \left(\frac{(1-\vartheta^*)}{\mathbb{E}(\vartheta^*)}+\frac{\vartheta^*\nu}{\mathbb{E}(\vartheta^*)}\right)$.

Let $\mathcal{V}\subset\beta$ be a cone define by $\mathcal{V}=\{\Theta\in\beta : \Theta(\omega)\geq 0, 0\leq\omega\leq\nu\}$. The $(\beta,\mathcal{V})$ forms an ordered Banach space. Let $\mathcal{Q}:\mathcal{V}\to\mathcal{V}$ be the operator defined as in Eq. (11), then we have the following lemma.

Lemma 4. *Let assume the hypotheses [V_1]-[V_2] be satisfied. Then $\mathcal{Q}$ is completely continuous.*

Proof. The operator $\mathcal{Q}$ is a bounded. We next to prove that $\mathcal{Q}:\mathcal{V}\to\mathcal{V}$ is continuous. Let $\Theta\in\mathcal{V}$, where $\|\Theta\|\leq d$. Suppose $\mathcal{R}=\{\Theta'\in\mathcal{V} : \|\Theta-\Theta'\|\leq g_1\}$. Then $\|\Theta'\|\leq d+g_1 := g, \forall\Theta'\in\mathcal{R}$. Since χ is continuous on $[0,\nu]\times[0,g]$, then it is uniformly continuous on $[0,\nu]\times[0,g]$.

Thus, for $\epsilon>0$, there exists $\gamma>0\,(\gamma<g_1)$ such that $\|\chi(\omega,\Theta(\omega))-\chi(\omega,\Theta'(\omega))\|\leq\frac{\epsilon}{\Lambda}$, for $\|\Theta-\Theta'\|\leq\gamma, \omega\in[0,\nu]$. If $\|\Theta-\Theta'\|\leq\gamma$ then $\Theta'\in\mathcal{R}$ and $\|\Theta'\|\leq g$. Let us consider $\Theta'\in\mathcal{R}\subset\mathcal{V}, \|\Theta'\|\leq\gamma$, similarly $\|\Theta\|\leq\gamma$. Finally, we have $\|\mathcal{Q}\Theta-\mathcal{Q}\Theta'\|\leq\epsilon$, hence $\mathcal{Q}$ is continuous. Then, $\mathcal{Q}$ has a fixed point (see Theorem 5). □

Then we have the following analysis.

Theorem 8. *Let assume the hypotheses [V_1]-[V_2] hold. Then (9) has at least one positive solution.*

Proof. Let $\mathcal{U}_1=\{\Theta\in\beta : \|\Theta\|\leq|\Theta_0(\omega)|+\mathcal{M}_1\Lambda\}$ and $\mathcal{U}_2=\{\Theta\in\beta : \|\Theta\|\leq|\Theta_0(\omega)|+\mathcal{M}_2\Lambda\}$. For $\Theta\in\mathcal{V}\cap\partial\mathcal{U}_2$, we have $0\leq\Theta(\omega)\leq\Lambda\mathcal{M}_2, \omega\in[0,\nu]$. Since $\chi(\omega,\Theta(\omega))\leq\mathcal{M}_2$, we have

$$Q(\Theta(\omega)) = \Theta_0(\omega)+\frac{(1-\vartheta(\omega))}{\mathbb{E}(\vartheta(\omega))}\chi(\omega,\Theta(\omega)) + \frac{\vartheta(\omega)}{\mathbb{E}(\vartheta(\omega))}\int_0^\omega\chi(a,\Theta(a))da$$
$$\leq|\Theta_0(\omega)|+\left(\frac{(1-\vartheta^*)}{\mathbb{E}(\vartheta^*)}+\frac{\vartheta^*\nu}{\mathbb{E}(\vartheta^*)}\right)\mathcal{M}_2$$
$$\leq|\Theta_0(\omega)|+\Lambda\mathcal{M}_2.$$

Hence, this implies that $\|Q(\Theta(\omega))\|\leq\|\Theta\|$. For $\Theta\in\mathcal{V}\cap\partial\mathcal{U}_1$, we obtain $0\leq\Theta(\omega)\leq|\Theta_0(\omega)|+\Lambda\mathcal{M}_1, \omega\in[0,\nu]$. Since $\chi(\omega,\Theta(\omega))\geq\mathcal{M}_1$, we have $\|Q(\Theta(\omega))\|\geq|\Theta_0(\omega)|+\Lambda\mathcal{M}_1=\|\Theta\|$. This means that the Eq. (9) has a positive solution (see theorem 1.2 in Ibrahim and Momani [20]). The operator Q ha a fixed point in $\mathcal{V}\cap(\bar{\mathcal{U}_2}\setminus\mathcal{U}_1)$. Which implies that the system (9) has a positive solution. □

Theorem 9. *Let $\chi(\omega,\Theta(\omega)) : [0,\nu]\times\mathbb{R}^6\to\mathbb{R}^6$ is a continuous and non-decreasing for each $\omega\in[0,\nu]$. Let there exist φ_0, ϖ_0 satisfying ${}_0^{CF}D_\omega^{\vartheta(\omega)}\varphi_0\leq\varphi_0$, ${}_0^{CF}D_\omega^{\vartheta(\omega)}\varpi_0\geq\varpi_0$ and $0\leq\varphi_0\leq\varpi_0\leq\omega\leq\nu$. The system (9) has a positive solution.*

Proof. Let $\varphi,\varpi\in\mathcal{V}$ such that $\varphi\leq\varpi$, then we have

$$Q(\varphi(\omega)) = \varphi_0(\omega)+\frac{(1-\vartheta(\omega))}{\mathbb{E}(\vartheta(\omega))}\chi(\omega,\varphi(\omega)) + \frac{\vartheta(\omega)}{\mathbb{E}(\vartheta(\omega))}\int_0^\omega\chi(a,\varphi(a))da$$
$$\leq\varpi_0(\omega)+\frac{(1-\vartheta(\omega))}{\mathbb{E}(\vartheta(\theta))}\chi(\omega,\varpi(\omega)) + \frac{\vartheta(\omega)}{\mathbb{E}(\vartheta(\omega))}\int_0^\omega\chi(a,\varpi(a))da = Q(\varpi(\omega)).$$

Thus, $Q(\varphi(\omega))\leq Q(\varpi(\omega)), \forall\omega$, then there exist φ_0,ϖ_0 such that $0\leq\varphi_0\leq\varpi_0$ with $Q(\varphi_0(\omega))\leq\varphi_0(\omega), Q(\varpi_0(\omega))\geq\varpi_0(\omega)$, from theorem 1.3 in Ibrahim and Momani [20]. The operator Q is compact and has a fixed point in ordered Banach space $\langle\varphi,\varpi\rangle$. Thus $Q:\langle\varphi_0,\varpi_0\rangle\to\langle\varphi_0,\varpi_0\rangle$ is compact. Q has a fixed point $\varkappa\in\langle\varphi,\varpi\rangle$. Thus, the system (9) has a positive solution. □

In the following Corollaries, we assume that the function $\chi(\omega,\Theta(\omega)) : [0,\nu]\times\mathbb{R}^+\times\mathbb{R}^+\times\mathbb{R}^+\times\mathbb{R}^+\mathbb{R}^+\times\mathbb{R}^+\to\mathbb{R}^+\times\mathbb{R}^+\times\mathbb{R}^+\times\mathbb{R}^+\times\mathbb{R}^+\times\mathbb{R}^+$ be continuous, non-decreasing and have a existing limit as $\Theta\to\infty$. The 2019-nCOV variable fractional oreder system (9) has a positive solution (see Theorem 9).

Corollary 1. *Let $\chi(\omega,\Theta(\omega)) : [0,\nu]\times\mathbb{R}^+\times\mathbb{R}^+\mathbb{R}^+\times\mathbb{R}^+\mathbb{R}^+\times\mathbb{R}^+\to\mathbb{R}^+\times\mathbb{R}^+\times\mathbb{R}^+\times\mathbb{R}^+\times\mathbb{R}^+\times\mathbb{R}^+$ be continuous and non-decreasing in $[0,\nu]$. If $0<\lim_{\Theta\to\infty}\chi(\omega,\Theta(\omega))<\infty, \omega\in[0,\nu]$, then the system (9) has a positive solution.*

Corollary 2. *Let $\chi(\omega,\Theta(\omega)) : [0,\nu]\times\mathbb{R}^+\times\mathbb{R}^+\times\mathbb{R}^+\times\mathbb{R}^+\times\mathbb{R}^+\times\mathbb{R}^+\to\mathbb{R}^+\times\mathbb{R}^+\times\mathbb{R}^+\times\mathbb{R}^+\mathbb{R}^+\times\mathbb{R}^+$ be continuous and non-decreasing in $[0,\nu]$. If $0<\lim_{\|\Theta\|\to\infty}\max_{\omega\in[0,\nu]}\frac{\chi(\omega,\Theta(\omega))}{\|\Theta\|}<\infty, \omega\in[0,\nu]$, then the 2019-nCOV variable fractional order system (9) has a positive solution.*

The following Corollaries are generalized Corollaries for the generalized system (9).

Corollary 3. *There exist constants* $h_1, h_2 > 0, a \in (0,1]$ *such that* $\chi(\omega, \Theta(\omega)) = h_1\Theta(\omega) + h_2^a$, *then the generalized system (9) has a positive solution.*

Corollary 4. *The function Let* $\chi(\omega, \Theta(\omega)) : [0, \nu] \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+\mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+$ *be continuous and* $|\chi(\omega, \Theta_1(\omega)) - \chi(\omega, \Theta_2(\omega))| \le \alpha|\Theta_1 - \Theta_2|, \forall \Theta_1(\omega), \Theta_2(\omega) \in R^+$ *such that* $\left[\frac{(1-\vartheta^*)\mathcal{N}_\chi}{\mathbb{E}(\vartheta^*)} + \frac{\vartheta^*\mathcal{N}_\chi \nu}{\mathbb{E}(\vartheta^*)}\right] < 1$, *then the generalized system (9) has a positive solution (See Theorem 4).*

Corollary 5. *Assume that there exist continuous function* g_1 *and* g_2 *such that* $0 < g_1(\omega) \le \chi(\omega, \Theta(\omega)) \le g_2(\omega), (\omega, \Theta(\omega)) \in [0, \nu] \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+$, *then the generalized system (9) has a positive solution* $\Theta \in \beta$. □

5.4. Existence of maximal and minimal solutions

In this subsection, we study results for maximal and minimal solution of the system (9).

Definition 6. Let $\mathcal{M}_1$ be a solution of the generalized system (9) in $[0, \nu]$. Then $\mathcal{M}_1$ is said to be maximal solution of the Eq. (9), if for every $\Theta(\omega)$ of (9) existing on $[0, \nu]$. The inequality $\Theta(\omega) \le \mathcal{M}_1(\omega), \theta \in [0, \nu]$, holds.

Definition 7. Let $\mathcal{M}_2$ be a solution of the generalized system (9) in $[0, \nu]$. Then $\mathcal{M}_2$ is said to be minimal solution of the Eq. (9), if for every $\Theta(\omega)$ of (9) existing on $[0, \nu]$. The inequality $\Theta(\omega) \ge \mathcal{M}_2(\theta), \theta \in [0, \nu]$, holds.

Theorem 10. *Let* $\chi(\omega, \Theta(\omega)) : [0, \nu] \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+$ *are continuous and non-decreasing function in* Θ *and there exist two constants* $p_1, p_2 > 0$ $(p_1 < p_2)$ *such that*

$$\frac{p_1}{\Theta_0 + \Lambda(\chi(\omega, \Theta(\omega)))} < 1 < \frac{p_2}{\Theta_0 + \Lambda(\chi(\omega, \Theta(\omega))}.$$

Thus, there exist a maximal and minimal solution the generalized system (9) on $[0, \nu]$.

Proof. The fractional integral equation (FIE) of the generalized system (9), we have

$$\Theta(\omega) = \Theta_0 + {}_0^{CF}J_\omega^{\vartheta(\omega)}(\chi(\omega, \Theta(\omega)). \tag{23}$$

Consider the new FIE from the Eq. (23) is

$$\Theta(\omega) = \mathcal{Z} + \Theta_0 + {}_0^{CF}J_\omega^{\vartheta(\omega)}(\chi(\omega, \Theta(\omega))), \omega \in [0, \nu], \mathcal{Z} > 0. \tag{24}$$

Then $\Theta(\omega)$ is given in the Eq. (24) is a solution of the generalized system (9) in (p_1, p_2), $\theta \in [0, \nu]$, for some constants $p_1, p_2 > 0$ such that

$$\frac{p_1}{\mathcal{Z} + \Theta_0 + \Lambda(\chi(\omega, \Theta(\omega))} < 1 < \frac{p_2}{\mathcal{Z} + \Theta_0 + \Lambda(\chi(\omega, \Theta(\omega))}.$$

Now, let $0 < \mathcal{Z}_2 < \mathcal{Z}_1 \le \mathcal{Z}$. Then we have $\Theta_{\mathcal{Z}_2}(0) < \Theta_{\mathcal{Z}_1}(0)$. Now, we can prove that

$$\Theta_{\mathcal{Z}_2}(\omega) \le \Theta_{\mathcal{Z}_1}(\omega), \forall \omega \in [0, \nu].$$

Assume that it is not true. Then there exists a ν_1 such that $\Theta_{\mathcal{Z}_2}(\omega_1) = \Theta_{\mathcal{Z}_1}(\omega_1)$ and $\Theta_{\mathcal{Z}_2}(\omega) < \Theta_{\mathcal{Z}_1}(\omega), \forall\, \omega \in [0, \omega_1)$.

Since, it is given that the function $\chi(\omega, \Theta(\omega))$ is monotonic non-decreasing in Θ, it follows that $\chi(\omega, \Theta_{\mathcal{Z}_2}(\theta)) \le \chi(\omega, \Theta_{\mathcal{Z}_1}(\theta))$. Consequently, using the Eq. (24), we have

$$\begin{aligned} \Theta_{\mathcal{Z}_2}(\omega_1) &= \mathcal{Z}_2 + \Theta_0 + {}_0^{CF}J_{\omega_1}^{\vartheta(\omega_1)}(\chi(\omega_1, \Theta_{\mathcal{Z}_2}(\omega_1)) \\ &< \mathcal{Z}_1 + \Theta_0 + {}_0^{CF}J_{\omega_1}^{\vartheta(\omega_1)}(\chi(\omega_1, \Theta_{\mathcal{Z}_1}(\omega_1)) \\ &= \Theta_{\mathcal{Z}_1}(\omega_1). \end{aligned} \tag{25}$$

Which is contradiction for $\Theta_{\mathcal{Z}_2}(\omega_1) = \Theta_{\mathcal{Z}_1}(\omega_1)$. Hence the above inequality $\Theta_{\mathcal{Z}_2}(\omega) < \Theta_{\mathcal{Z}_1}(\omega)$ is true. This implies, there exists a decreasing sequence $\mathcal{Z}_n$ such that $\mathcal{Z}_n \to 0$ as $n \to \infty$ and $\lim_{n\to\infty} \Theta_{\mathcal{Z}_n}(\omega)$ exists uniformly in $[0, \nu]$. Let us denote the limiting value is $\mathcal{M}_1$. Clearly, by the uniform continuity of the functions $\chi(\omega, \Theta(\omega)) : [0, \nu] \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+ \times \mathbb{R}^+\mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+$. Then the equation

$$\Theta_{\mathcal{Z}_n}(\omega) = \mathcal{Z} + \Theta_0 + {}_0^{CF}J_\omega^{\vartheta(\omega)}(\chi(\omega, \Theta_{\mathcal{Z}_n}(\omega))), \tag{26}$$

which yields that $\mathcal{M}_1$ is a solution of the generalized system (9). To show that $\mathcal{M}_1$ is maximal solution of the generalized system (9), let $\Theta(\omega)$ be any solution of the generalized system (9). Then, we have

$$\Theta(\omega) < \mathcal{Z} + \Theta_0 + {}_0^{CF}J_\omega^{\vartheta(\omega)}(\chi(\omega, \Theta(\omega)) = \Theta_{\mathcal{Z}}(\omega) \tag{27}$$

Here, the maximal solution is unique, it is obvious that $\Theta_{\mathcal{Z}}(\omega)$ tends to $\mathcal{M}_1(\omega)$ uniformly exists in $[0, \nu]$ as $\mathcal{Z} \to 0$, which implies that the existence of maximal solution for the generalized system (9). Similarly, we can show that the minimal solution with similar argument holds. □

6. Conclusion

In this work, we generalized the proposed 2019-nCOV system by replacing the integer-order with variable Caputo-Fabrizio fractional order in Caputo sense. We found the new existence and uniqueness conditions of solution 2019-nCOV system with variable Caputo-Fabrizio fractional-order via fixed point theory. Furthermore, we study the stability of the generalized 2019-nCOV system by using Hyers-Ulam stabilities. Additionally, we investigated some important results for the proposed generalized 2019-nCOV system involving variable Caputo-Fabrizio fractional order.

Credit author statement

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Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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