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A Simple, Interpretable Conversion from Pearson’s Correlation to Cohen’s d for Continuous Exposures

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To the Editor:

Meta-analysts often must convert effect sizes reported on different scales to a common scale for analysis.¹ In particular, it is common to convert Pearson’s correlation, r , computed between an exposure X and an outcome Y to Cohen’s d (also called the “standardized mean difference”), which is the difference in expected Y for a fixed contrast in X , standardized by the standard deviation of Y conditional on X . Letting N denote the total sample size, the standard conversion¹ from r to d is:

$$d = \frac{2r}{\sqrt{1-r^2}} \tag{1.1}$$
$$\widehat{SE}(d) = \frac{2}{\sqrt{(N-1)(1-r^2)}}$$

An important, yet infrequently discussed, point is that this conversion was derived for a Pearson correlation computed between a binary exposure X and a continuous outcome Y , also called a “point-biserial” correlation.²⁻⁴ Note that when X represents a dichotomization of a truly continuous underlying exposure, a special approach³ is required to estimate the correlation between the underlying, continuous exposure and Y ; one cannot simply apply the standard Pearson’s correlation formula to the observed, dichotomized X . Stated otherwise, the point-biserial correlation does *not* consistently estimate the Pearson correlation that would have been obtained using the underlying continuous variable.

Despite the standard conversion’s origins in the binary-exposure setting, meta-analysts in practice often unknowingly apply Equation (1.1) to obtain Cohen’s d from correlations and regression results computed using a continuous X . In fact, a widely referenced textbook on meta-analysis describes Equation (1.1) without stipulating that it is only known to apply for the point-biserial case.¹ Even if Equation (1.1) can be used for correlations computed using a continuous X , its interpretation is unclear: that is, the interpretation of Cohen’s d depends

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on the choice of “groups” in X whose means are compared, but because Equation (1.1) applies for a correlation in which X is already binary, it is not clear which “groups” of X are created when the conversion is instead applied to a correlation using a continuous X .

To allow direct computation of Cohen’s d from Pearson’s r or simple linear regression, we provide a similar conversion and approximate standard error that apply when X is continuous. The resulting effect size represents the average increase in the standardized Y associated with an increase in X of Δ units. To preserve the sign of the effect size, Δ should be set to be positive regardless of the sign of r . Letting s_x denote the sample standard deviation of X , the conversion is:

$$d = \frac{r\Delta}{s_x\sqrt{1-r^2}}$$

$$\widehat{\text{SE}}(d) = |d| \sqrt{\frac{1}{r^2(N-3)} + \frac{1}{2(N-1)}} \quad (1.2)$$

Derivations of these estimates of d and its standard error are provided in the eAppendix, <http://links.lww.com/EDE/B601>. The standard error estimate assumes that X is approximately normal and that N is large. If the standard deviation of X is known rather than estimated, then the term $\frac{1}{2(N-1)}$ should be omitted. As a potential practical limitation, some papers to be meta-analyzed might not report s_x , in which case the meta-analyst might need to substitute an estimate from, for example, a comparable second study or a subsample of the study used to estimate r . In this case, the N in the term $\frac{1}{2(N-1)}$ should be replaced with the size of the second sample used to estimate s_x (see Supplement, <http://links.lww.com/EDE/B601>). The conversion is easy to calculate manually or using the function `r_to_d` in the R package `MetaUtility`.

Comparing Equations (1.1) and (1.2) clarifies the meaning of the “Cohen’s d ” that results from unknowingly applying Equation (1.1) to a correlation computed with a continuous X . Specifically, the result coincides with the effect size associated with an increase in X of two standard deviations. (However, even with $\Delta = 2s_x$, the standard error estimates in Equations (1.1) and (1.2) will, in general, still not coincide.) In many applications, this may represent a rather extreme contrast: for example, if X is normal, then a two-standard-deviation contrast with the reference level set to the mean would involve comparison to the 97.7th quantile of X . Alternatively, a two-standard deviation contrast from one standard deviation below the mean to one standard deviation above is a comparison of the 15.8th quantile to the 81.1th quantile of X . Additionally, the absolute size of a two-standard-deviation contrast in X may differ substantially across study populations and may therefore be challenging to interpret in practice.⁵ Thus, it is perhaps preferable, when possible, to instead fix a specific, scientifically meaningful contrast of interest, Δ , which is held constant across all meta-analyzed studies, and then to apply the proposed conversion in Equation (1.2). The meta-analytic pooled estimate would then correspond to a well-defined contrast in X of Δ units, rather than to a contrast whose size may vary arbitrarily across studies.

The standard conversion is alternatively sometimes described in terms of the contrast that arises from dichotomizing X at a given threshold,¹ yet in fact, the conversion often substantially overestimates the contrast produced by dichotomization, even at extreme thresholds of X . For example, we simulated bivariate normal data (1×10^5 observations) where $X \sim N(0,1)$ and $Y \sim N(X,1)$, such that $r = 0.70$. The standard conversion estimates $d = 2.0$ (Figure, dashed red line). We also calculated the true two-group Cohen's d arising from dichotomizing X at various thresholds in $[-2,2]$ (Figure, solid black curve). The Figure shows that the "Cohen's d " from the standard conversion is 47% larger than the true two-group d arising from dichotomization at the mean and still overestimates the true two-group d for extreme dichotomization thresholds near -2 or 2 . For example, for dichotomization at $X = 2$ (i.e., the 97.7th percentile), the standard conversion still overestimates the true two-group Cohen's d by 14%.

In summary, when approximating Cohen's d from Pearson's r or simple linear regression with a continuous X , we caution against using conversions derived for a binary X . We provide a straightforward conversion designed to accommodate the case of a continuous X through specification of a fixed contrast in X ; we believe its use in meta-analysis would enable more precisely interpretable and scientifically meaningful effect sizes.

Supplementary Material

Refer to Web version on PubMed Central for supplementary material.

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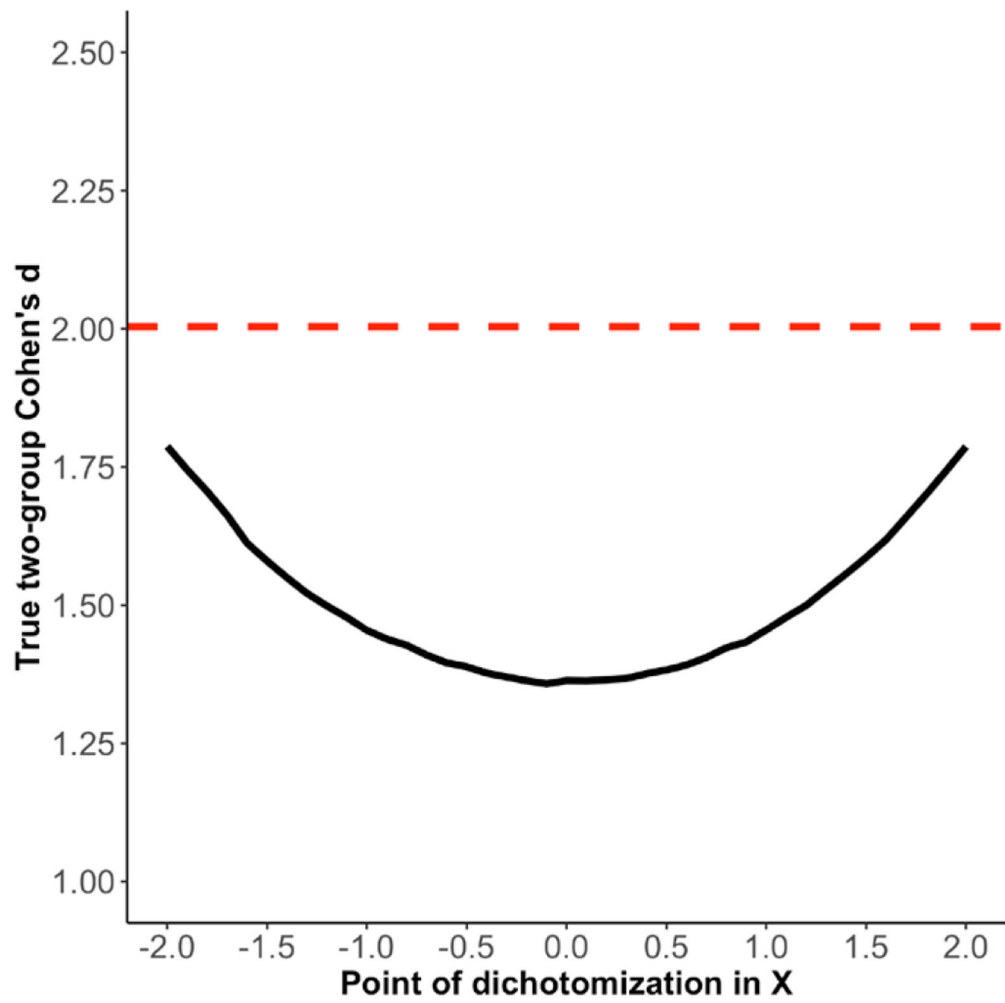


FIGURE. True two-group Cohen's d corresponding to dichotomizing a standard normal X at varying points (solid black curve) versus "Cohen's d " calculated from the standard conversion in Equation (1.1) (dashed red line).