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From 2SFCA to i2SFCA: integration, derivation and validation

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Abstract

Uneven distributions of population and service providers lead to geographic disparity in access for residents and varying workload for staff in facilities. The former can be captured by spatial accessibility in the traditional two-step floating catchment area (2SFCA) method; and the latter can be measured by potential crowdedness in the newly developed inverted 2SFCA (or i2SFCA) method. Residents-based accessibility and facility crowdedness are two sides of the same coin in examining the geographic variability of resource allocation. This short research note derives the formulations of both methods to solidify their theoretical foundation, and uses a case study to validate both. By doing so, the 2SFCA and i2SFCA are fully integrated into one conceptual framework, derived with extensions to the Huff model, and validated by empirical data.

Keywords

2-Step Floating Catchment Area (2SFCA) method; inverted 2-Step Floating Catchment Area (i2SFCA) method; Huff model; theoretical foundation

1. Popularity of 2SFCA, promise of i2SFCA and missing pieces

Accessibility refers to the relative ease by which services can be reached from a given location. Since its inception (Luo and Wang, 2003), the *2-step floating catchment area (2SFCA)* method has been a popular measure of spatial accessibility. Table 1 outlines its calibration process. Note that it adopts the more general formulation of 2SFCA, termed "*generalized 2SFCA*", by using a generic distance decay function *f(d)* to model the supply-demand interaction (Wang, 2012). The classic 2SFCA uses a fixed catchment area size and is considered a special case of generalized 2SFCA. For the remainder of the paper, the generalized 2SFCA is adopted, and simply referred to as 2SFCA. For clarification, the 2SFCA method measures place accessibility, different from, although built on, the work on individual (such as space-time) accessibility (Kwan 1998).

There are at least three reasons for the popularity of 2SFCA. First, it overcomes the shortcomings of preceding methods that focus on either proximity to the nearest facility or simply supply-demand ratios within "fixed geographical or administrative boundaries" (McGrail, 2012:2). It recognizes and accounts for two realities, i.e., people value access to multiple service providers beyond the closest one and their choices are often not bounded by geopolitical units. Secondly, it strikes a balance in conceptualization of spatial accessibility

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as a simple match ratio of supply and demand versus technical challenges in modeling complex spatial behavior. The former yields an intuitive interpretation of accessibility score (e.g., physicians per 1000 people), and the latter is achieved by generalizing the spatial interaction as a distance decay effect that takes various forms (Wang, 2012). Thirdly, its implementation in GIS is straightforward, and made even more convenient and user friendly by an automated ArcGIS toolkit (Zhu and Wang, 2015).

The newly-proposed *inverted 2SFCA* (abbreviated as "*i2SFCA*") reverses the order of two steps (i.e., switches the variables for supply and demand) in the traditional 2SFCA, and its implementation is also summarized in Table 1. While 2SFCA defines spatial accessibility for residents, i2SFCA estimates "potential crowdedness" at a facility. Taking hospital care as an example, the i2SFCA method measures the potential saturation level for a hospital (e.g., patients per bed) or work load for its staff (e.g., patients per nurse). Some usages of this new measure can be speculated. Does the index help explain stress levels (or turnover rates) of staff or waiting time of clients or patients for service providers? Is it related to their job satisfaction levels? The variability of its value may suggest which facilities experience shortage or surplus in staffing or other capacity measures, and call for adjustment in resource allocation to reduce disparity. All are promising areas for empirical studies to demonstrate its value.

However, several issues need to be resolved so that the 2SFCA and i2SFCA methods can be fully substantiated, comprehended and when necessarily, amended. Can the 2SFCA method be derived theoretically, and also validated empirically, as Wang (2018) did for the i2SFCA method? No literature has done so for the 2SFCA. This research note attempts to fill such voids in order to (1) solidify the theoretical foundation for both methods, (2) clarify the symmetry and connection between them, and (3) pave the way to integrate them in one framework.

2. Deriving the 2SFCA and i2SFCA methods

Readers may refer to Figure 1 for graphic illustration of the derivations of both methods.

This section begins with a brief review of deriving i2SFCA as in Wang (2018). According to Huff (1963) model, residents at location i (denoted by D_i) choose a particular facility j with capacity S_j, among a set of alternatives S_l ($l = 1, 2, \ldots, n$), with a probability such as

$$Prob_{Di\_j} = S_j f(d_{ij}) / \sum_{l=1}^{n} (S_l f(d_{il})) \quad (1)$$

Note that a generic distance decay function *f(d)* is adopted in place of Huff's power (gravity-based) function (Jia et al. 2017).

Multiplying the probability $Prob_{Di\_j}$ by the demand size (e.g., population) D_i at i yields the number of visitors (customers, patients, etc.) from residential location i to facility j, or service volume, denoted by F_{ij}. It is projected as

$$F_{ij} = D_i Prob_{Di\_j}$$

Therefore, the total estimated number of visitors to facility j, denoted by V_j, is

$$V_j = \sum_{i=1}^{m} F_{ij} \tag{2}$$

Finally, normalizing V_j by its capacity S_j yields

$$C_j = V_j / S_j = \sum_{i=1}^{m} F_{ij} / S_j = \left(\sum_{i=1}^{m} D_i \mathrm{Pr}ob_{Di\_j} \right) / S_j = \sum_{i=1}^{m} \left[D_i f(d_{ij}) / \sum_{l=1}^{n} (S_l f(d_{il}) \right] \tag{3}$$

Note that the term S_j is eliminated in both the numerator and denominator for simplification in Equ.(3). C_j is the final formulation of i2SFCA, termed "potential crowdedness" (e.g., patients per bed in a hospital, clients per clerk, etc.). A higher C_j value corresponds to a service facility being more crowded.

The derivation of 2SFCA imitates the above process for deriving i2SFCA. Similar to the notion of Huff model but beginning with a focus on facility location S_j, the proportion of visitors from residential location D_i, denoted by $Prob_{i\_Sj}$, out of alternative origins (other residential locations) D_k ($k = 1, 2, \ldots, m$), is

$$Prob_{i\_Sj} = D_i f(d_{ij}) / \sum_{k=1}^{m} (D_k f(d_{kj})) \tag{4}$$

Note the symmetry between Equ. (1) and (4), where demand size D and facility capacity S switch with each other. In other words, it is the Huff model upside down. While the Huff model predicts that the probability visiting a facility out of alternative destinations is proportional to the facility's size and reversely related to distance, the argument made here is that the probability coming from a residential area out of alternative origins is proportional to the residential area's population size and reversely related to distance. One may relate this symmetry to the derivation of Garin-Lowry Model, where "the proportion of service employment in an area owing to the influence of population in another area out of its impacts on all areas" is symmetric to "the proportion of population in an area owing to the influence of employment in another area out of its impacts on all areas" (Wang, 2015: 221).

With the total capacity at facility j defined as S_j, the portion dedicated (allocated, attributable) for serving the visitors from i out of all demand locations is

$$T_{ij} = S_j Prob_{i\_Sj}$$

If a facility j uses capacity T_{ij} for serving residential location i, summing up these capacities across all facilities ($j = 1, 2, \ldots, n$) with their respective portions dedicated for i yields total resource commanded by demand location i, denoted as W_i:

$$W_i = \sum_{j=1}^{n} T_{ij} = \sum_{j=1}^{n} (S_j Prob_{i\_Sj}) = \sum_{j=1}^{n} [S_j D_i f(d_{ij}) / \sum_{k=1}^{m} (D_k f(d_{kj})] \tag{5}$$

Finally, normalizing the total resource W_i available for i by its demand size D_i yields

$$A_i = W_i / D_i = \sum_{j=1}^{n} [S_j f(d_{ij}) / \sum_{k=1}^{m} (D_k f(d_{kj})] \tag{6}$$

where D_i is eliminated in both the numerator and denominator for simplification. Equ.(6) is the traditional 2SFCA, which is now derived.

The derivations for both the 2SFCA and i2SFCA methods are important for several reasons:

- Foremost, the proofs strengthen the theoretical foundation for the methods and add critical credentials to their scientific rigor. After Michael Goodchild (per his letter to the author in 2002) read an earlier version of the paper by Luo and Wang (2003) that proposed the 2SFCA method, he pointed out its key weakness for lack of a theoretical footing and thus no justification for its advantages over other accessibility measures. The proof finally comes, though almost two decades later.
- Secondly, the processes clarify the precise interpretations for both accessibility and crowdedness measures. They are supply-versus-demand ratios, and the supply-demand interactions are discounted by a distance decay effect. Both focus on an element of "estimated", "projected", or "predicted" size of demand or supply. In 2SFCA, it is estimated supply per demand volume; whereas in i2SFCA, it is estimated demand per supply capacity. Indeed, one important property of 2SFCA is that the weighted mean of accessibility equals the ratio of total supply (S) to total demand (D) in a study area (Wang 2015:110–111). A similar property applies to i2SFCA, i.e., the weighted mean of crowdedness equals the ratio of total demand to total supply in a study area.
- Thirdly, the derivations illuminate what the two methods really intend to capture and the value of preserving their simplicity. As Wang (2018:253) pointed out, barring special circumstances, it is unnecessary to introduce "additional complications such as '3SFCA' (e.g., Wan, Zou, and Sternberg 2012; Chu et al. 2016)."

3. Validating 2SFCA and i2SFCA in a case study

A case study illustrates briefly how to implement the 2SFCA and i2SFCA, and more importantly, validates both methods. Readers may refer to Jia et al. (2017) for more details on the study area, data sources and related data processing issues. Based on the 2011 State

Inpatient Databases (SID) of Florida (AHRQ 2011) and other public accessible sources, the following data sets are prepared:

1. 268 hospitals with capacity defined as staffed bed size (S_j),
2. 983 zip code areas with demand approximated as population (D_i), and
3. volume of patient flow (G_{ij}) and travel time between (1) and (2) (d_{ij}).

Note that demand in each zip code (D_i) here is defined by population, not patient volume as in Wang (2018). This revision is significant as population data is much more accessible from the census than patient data. More importantly, the accessibility and crowdedness measures derived from this demand definition better reflect the potential (predicted or estimated) nature of both measures, and enhance the value of designed validation in the case study. Aggregating the volume of patient flow, G_{ij}, by zip code area i yields the actual number of total patients generated by i, and aggregating it by hospital j yields the actual number of total patients discharged by j. Both are used for validations of 2SFCA and i2SFCA, respectively, as shown in the vertical axis in Figures 4a and 4b, correspondingly.

A challenge in implementing the 2SFCA and i2SFCA methods in any empirical study is how to define "the best fitting analytical function and related parameters" (Wang, 2012: 1107) for the distance decay effect. The best practice is to use real-world data reflecting individual travel behaviors. A simple spatial interaction model is written as

$$G_{ij} = aD_iS_jf\left(d_{ij}\right) \tag{7}$$

where both demand D_i and supply S_j assume a unitary elasticity (exponent = 1) for simplicity, consistent with the Huff model.

Rearranging Equ.(7) and taking logarithms on both sides yield (Wang, 2015:33)

$$\ln\left(G_{ij}/\left(D_iS_j\right)\right) = lna + lnf\left(d_{ij}\right)$$

Between the popular choices for $f(d_{ij})$ (power vs. exponential function), the case study suggests the power function with a higher fitting power, where β=1.3.

One may follow Table 1 to implement the 2SFCA and i2SFCA step by step. Here, an automated ArcGIS toolkit developed by Zhu and Wang (2015) is used for implementing both. As shown in Figure 2, it uses the tool, "Generalized 2SFCA (w External Distance Table)", under the toolkit "Accessibility." The interface reads the supply layer and associated info, the demand layer and associated info, and the distance matrix between them; defines the distance decay function and associated parameter; and outputs the result in a table. Calibrating i2SFCA uses the same toolkit and data, and the only difference is to switch the data inputs for the demand and supply layers and their associated fields. The results are shown in Figures 3a–3b, for accessibility across ZIP code areas and for potential crowdedness in hospitals, respectively.

Recall the accessibility measure by 2SFCA in Equ.(6), A_i is the normalized total resource W_i available for i by its population D_i, or $A_i = W_i/D_i$. In other words, $W_i = A_iD_i$ is total accessibility for population at *i*. How good is the 2SFCA-derived accessibility? One way to validate it is to examine whether W_i is a good predictor of actual volume of patients generated by ZIP code area *i*. In other words, the 2SFCA intends to estimate how conveniently residents seek hospital care (in the context of the case study) given their locations and the hospitals' locations.

Figure 4(a) shows that the two are highly correlated (with $R^2 = 0.68$ or a correlation coefficient of 0.82). Therefore, the 2SFCA method is largely validated in the case study. That is to say, merely considering the locations and sizes of residents and hospitals (and the transportation impedance between them), the 2SFCA method explains 2/3 of the variability of hospitalizations. Many other factors (e.g., demographic-socioeconomic attributes of individuals, insurance or lack of it, personal preference) may help explain the remaining variance. In short, the validation shows how close are the 2SFCA-estimated potential accessibility and the revealed accessibility.

Similarly, the i2SFCA-defined potential crowdedness, C_j in Equ.(3), is the estimated patients per bed, or $C_j = V_j/S_j$. Therefore, estimated total patients in a hospital *j* is $V_j = C_jS_j$, or termed "total crowdedness." How good is this estimate? Figure 4(b) shows that total crowdedness and actual number of patients discharged by hospitals are correlated with $R^2 = 0.62$ (or a correlation coefficient of 0.78)[i]. This validates that the i2SFCA-defined potential crowdedness is a good predictor of actual patients cared by individual facilities. In other words, given the spatial patterns of population and hospitals, the i2SFCA method predicts 62% of variability of discharged patient volume (without accounting for many complex factors such as quality, reputation, history, efficiency and management style of hospitals). Or simply, the i2SFCA-derived potential crowdedness captures the majority of actual crowdedness of facilities.

4. Concluding comments

In summary, residents accessibility measured by 2SFCA and facility crowdedness by i2SFCA are two aspects in gauging the geographic variability of resource allocation. The two methods capture similar traits of disparity in surplus of a resource in some areas and scarcity in other areas, i.e., two sides of the same coin. However, they have their distinctive emphases for different purposes and are most likely to differ in scales. In our case study, the number of hospitals is far fewer than that of the ZIP code areas, and in the meantime one ZIP code area may contain multiple hospitals. Even when the scales for demand and supply locations happen to be the same, their values are not reciprocal to each other. However, as stated previously, the weighted mean of accessibility by 2SFCA across the whole study area (S/D) is indeed the reciprocal of the weighted mean of crowdedness by the i2SFCA (D/S) in the same study area.

The implications of the two methods for public policy are interconnected. Take hospital care as an example. Any adjustment made to distribution of residents, hospitals, or the

[i]Note that R^2 here is 0.62, slightly lower than 0.65 in Figure 7 of Wang (2018). As stated previously, this study defines demand by population instead of patient volume, and the derived crowdedness measure better reflects its essence of "potential" with absence of any knowledge of actual patient visitation data. The reduction in its prediction power from 0.65 to 0.62 is negligible.

transportation connection between them will change both accessibility for residents and crowdedness for hospitals. However, the formulation of a decision or policy based on one method may have a different emphasis or target than the other. For instance, one may formulate a planning problem to minimize accessibility across residents by adjusting facility location and/or allocation. The direct outcome is on reduced disparity in hospital care access for residents. Another planning problem can be formulated to minimize the variability of crowdedness across hospitals by allocating available new resources. The direct result becomes trimming the gap in work load for staff.

5. Data and codes availability statement

The follow data and program files are including in one ZIP file IJGIS_Data.zip (available for download at https://doi.org/10.6084/m9.figshare.11944263.v1):

- Geodatabase IJGIS_FL.gdb with two features (Hospitals and ZIP_Code_Area) and table (OD_AllFlow_Time). The feature "Hospitals" contains fields OBJECTID and NUMBEDS. The feature "ZIP_Code_Area" contains fields ZONEID and Popu. The O-D travel time table "OD_AllFlow_Time" contains fields PatientZIP_ZoneID (corresponding to ZONEID in the feature ZIP_Code_Area), Hospital_ObjectID (corresponding to OBJECTID in the feature Hospitals), and Total_Minutes (travel time in minutes). All feature, table and their associated field names are self-explanatory.
- ArcGIS toolkit program file Accessibility.tbx, with four associated Python scripts grouped under folder Scripts. As stated previously, this case study uses the 4th tool "Generalized 2SFCA (w External Distance Table)", with interface shown in Figure 2. For installing or accessing an ArcGIS toolkit, refer to Wang (2015, p.90).

The above data files are sufficient to replicate the implementations of 2SFCA and i2SFCA methods.

Deriving the best fitting distance decay function and its parameter would require the actual patient volumes from zip code areas to hospitals. Validating the two methods would require the total number of patients generated from each zip code and discharged by each hospital. Per the requirement of data user agreement with the AHRQ (2011), such data cannot be disclosed. For readers interested in designing labs for classroom instructions, contact the author for "mocked" data (i.e., data that preserve the general pattern of the original data and contain a significant randomized component).

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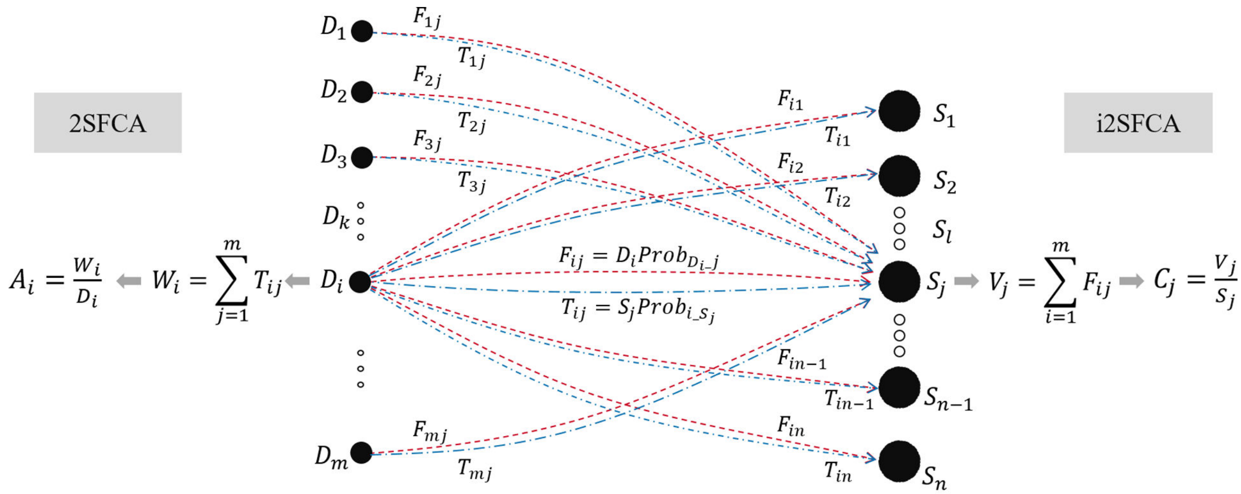


D_i, S_j represent demand location i and supply location j, respectively

F_{ij} represents projected flow from demand location i to supply location j

T_{ij} represents estimated capacity at supply location j allocated for serving demand location i

Other links F_{kl} and T_{kl} between $D_k (k \neq i)$ and $S_l (l \neq j)$ are not shown

Figure 1.

Illustrating the derivations of 2SFCA and i2SFCA

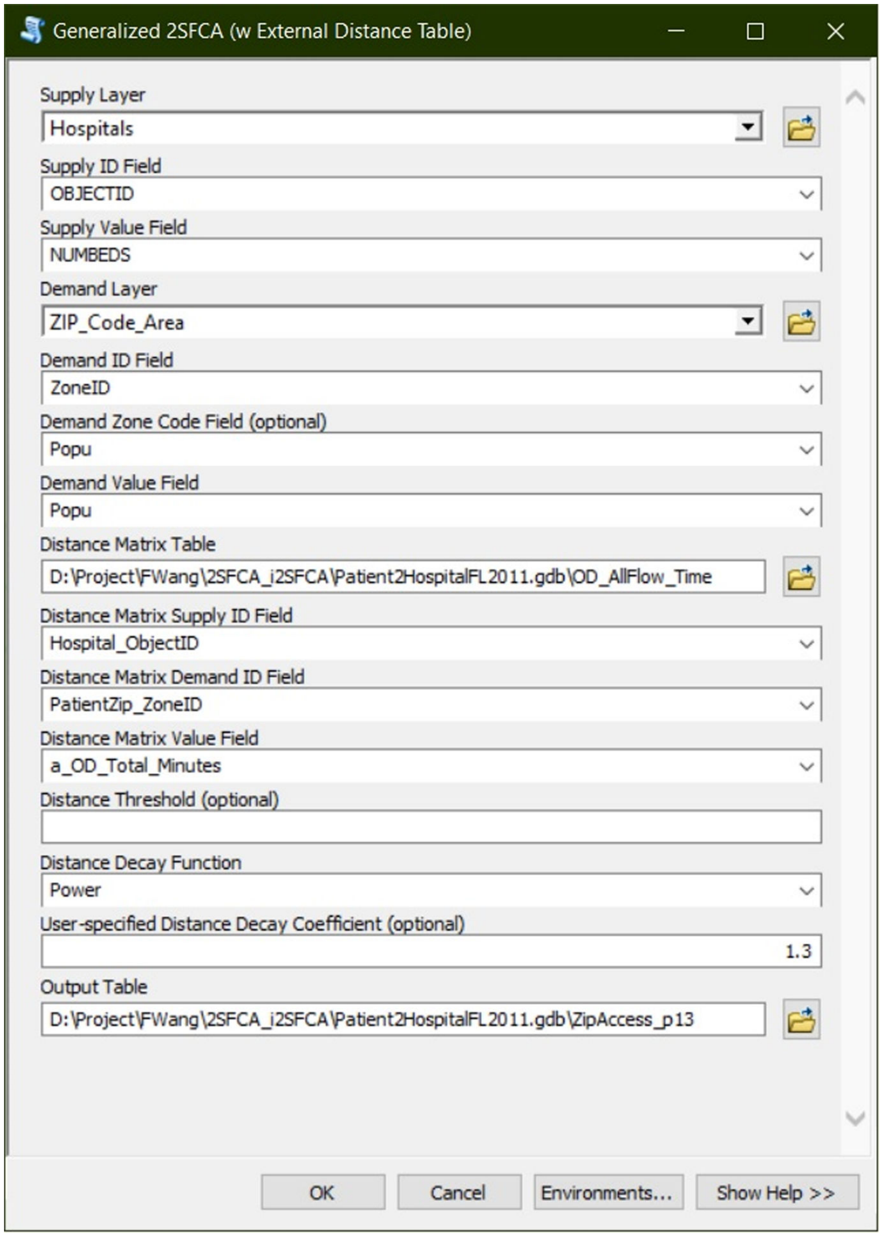
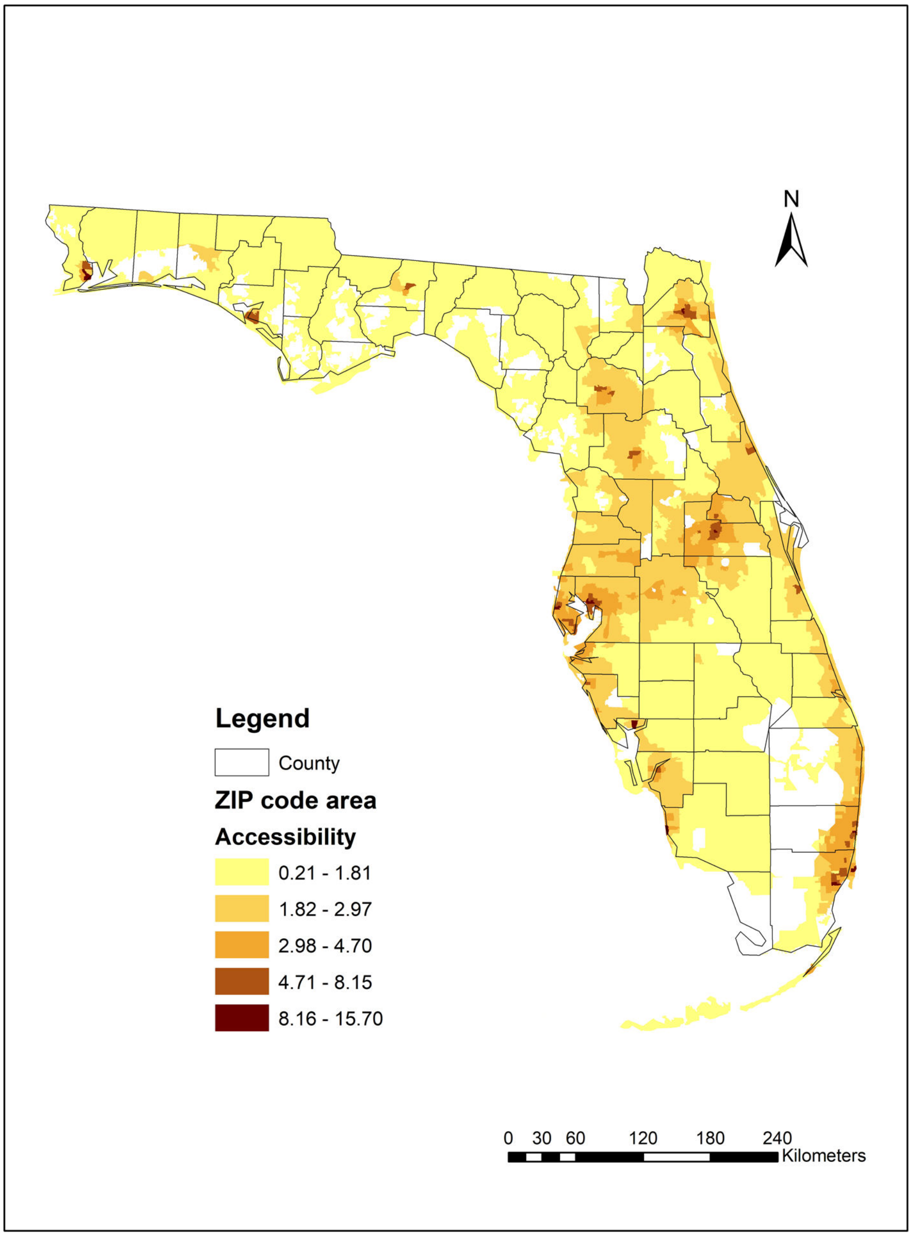


Figure 2.

Implementing the Generalized 2SFCA in an automated ArcGIS Toolkit for the case study


N
Legend
County
ZIP code area
Accessibility
0.21 - 1.81
1.82 - 2.97
2.98 - 4.70
4.71 - 8.15
8.16 - 15.70
0 30 60 120 180 240
Kilometers

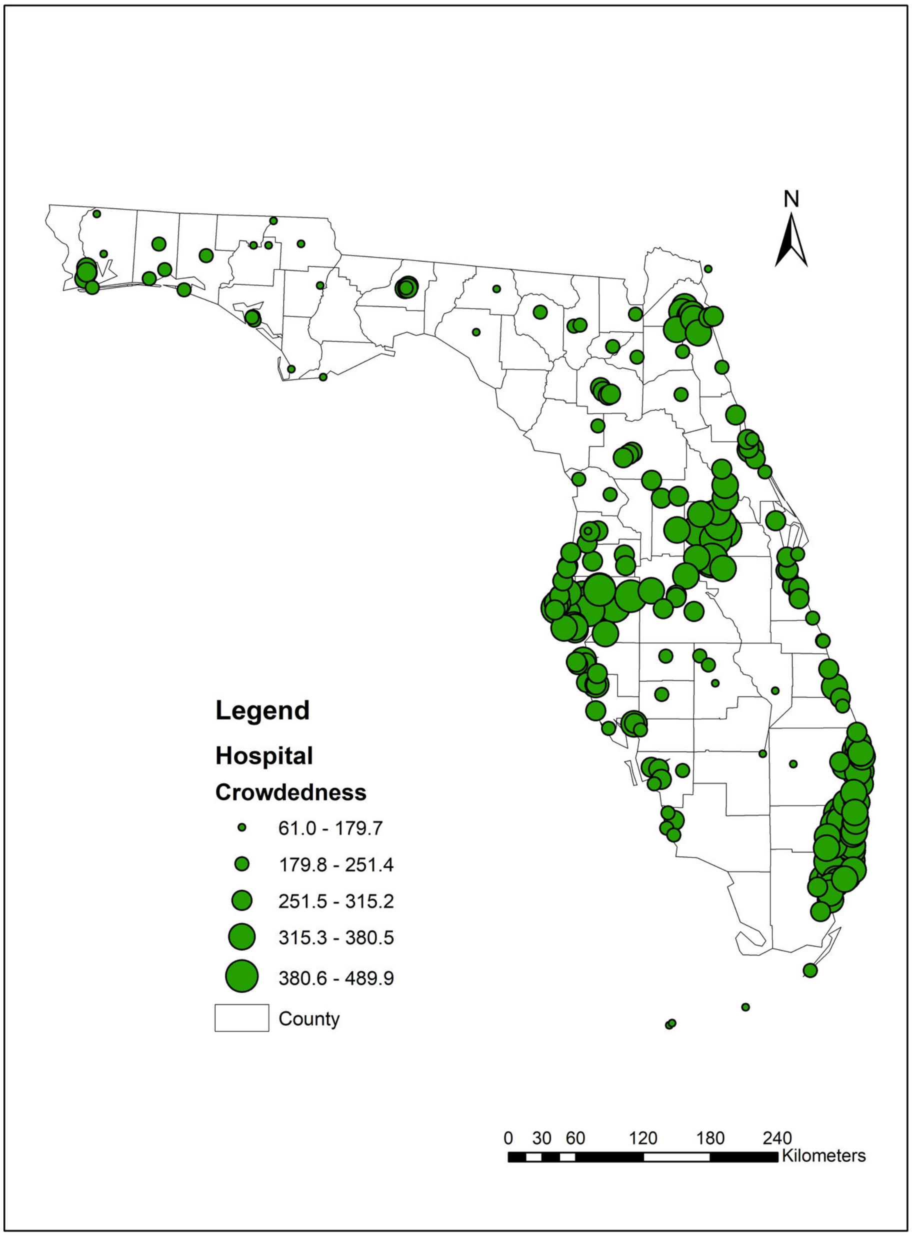
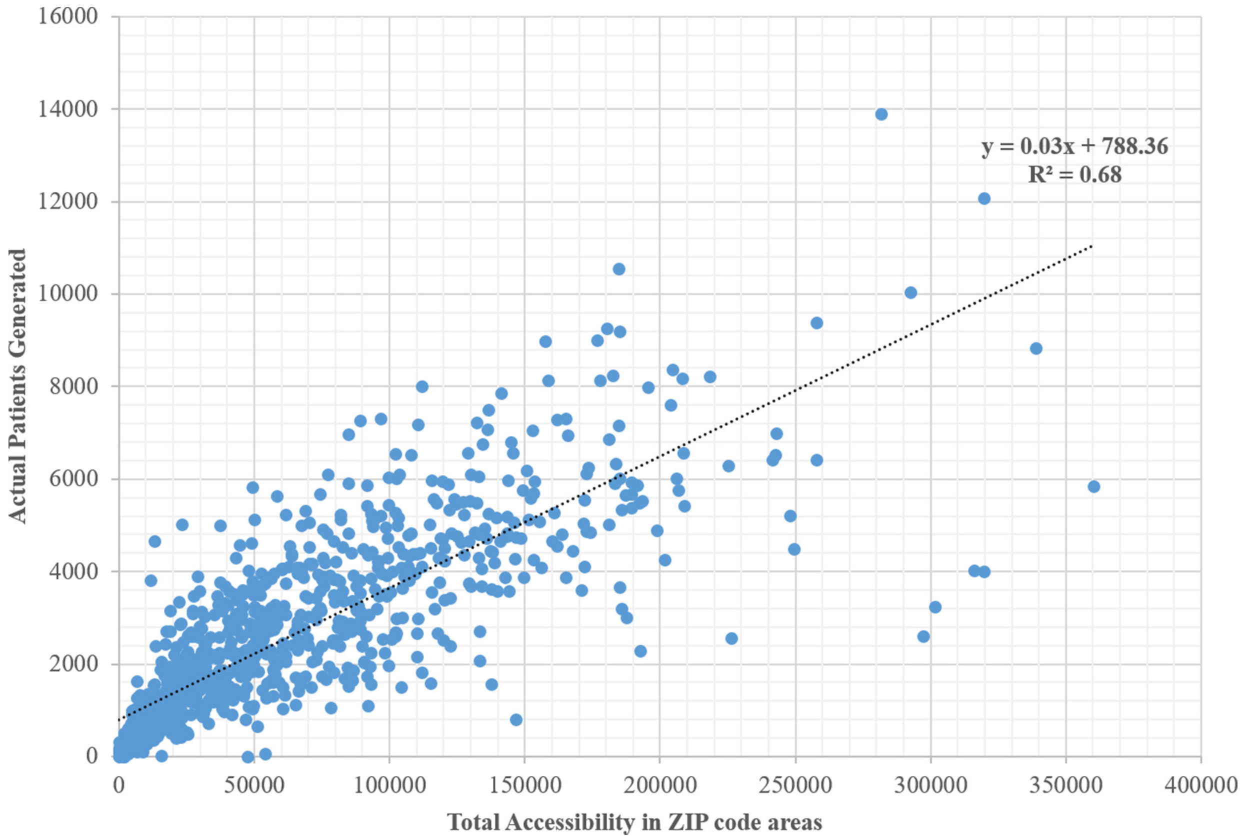


Figure 3.

(a) 2SFCA-derived hospital accessibility across ZIP code areas in Florida 2011, (b) i2SFCA-derived potential crowdedness for hospitals in Florida 2011


y = 0.03x + 788.36
R² = 0.68
Actual Patients Generated
Total Accessibility in ZIP code areas
0
2000
4000
6000
8000
10000
12000
14000
16000
0
50000
100000
150000
200000
250000
300000
350000
400000

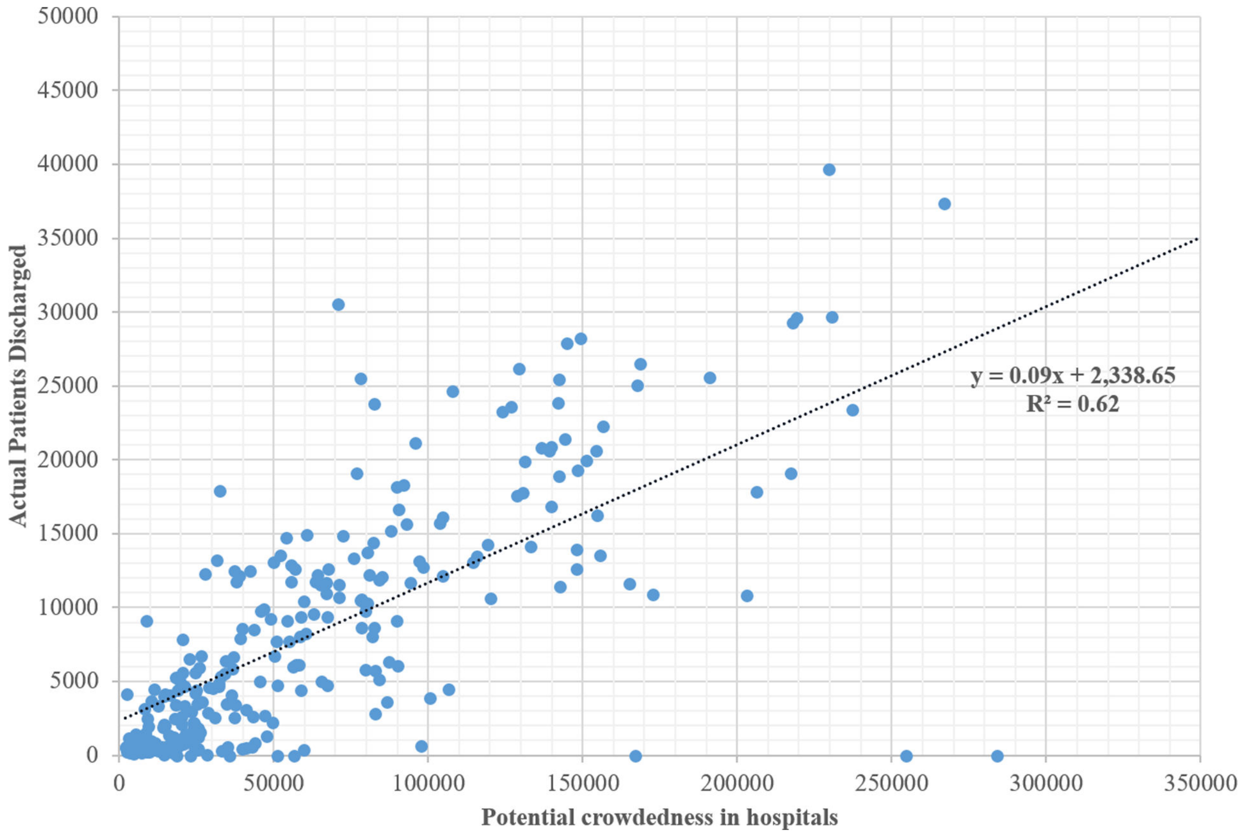


Figure 4.
Validating (a) 2SFCA, and (b) i2SFCA

Table 1.

Implementing 2SFCA versus i2SFCA

	2SFCA	i2SFCA
Objective	Measuring spatial accessibility of service by residents at location (i)	Measuring potential crowdedness at a facility location (j)
Step 1	For each supply location j, sum up surrounding demands D_k, discounted by distance decay function $f(d_{kj})$, across all demand locations k (=1, 2, …, m), and compute the supply to demand ratio R_j: $R_j = S_j / \sum_{k=1}^{m} (D_k f(d_{kj}))$	For each demand location i, sum up supplies S_l, discounted by distance decay function $f(d_{il})$, across all supply locations l (=1, 2, …, n), and compute the demand to supply ratio r_i: $r_i = D_i / \sum_{l=1}^{n} (S_l f(d_{il}))$
Step 2	For each demand location i, sum up ratios R_j, discounted by distance decay function $f(d_{ij})$, across all supply locations j (=1, 2, …, n), to obtain the accessibility A_i at demand location i: $A_i = \sum_{j=1}^{n} [R_j f(d_{ij})] = \sum_{j=1}^{n} [S_j f(d_{ij}) / \sum_{k=1}^{m} (D_k f(d_{kj}))]$	For each supply location j, sum up ratios r_i, discounted by distance decay function $f(d_{ij})$, across all demand locations i (=1, 2, …, m), to obtain the crowdedness C_j at supply location j: $C_j = \sum_{i=1}^{m} [r_i f(d_{ij})] = \sum_{i=1}^{m} [D_i f(d_{ij}) / \sum_{l=1}^{n} (S_l f(d_{il}))]$
Property	Weighted mean of accessibility (using the demand amount as weight) is equal to the ratio of total supply to total demand in the study area	Weighted mean of crowdedness (using the supply capacity as weight) is equal to the ratio of total demand to total supply in the study area
	Weighted means of accessibility and crowdedness are reciprocal of each other	

(Revised from Wang (2018: 254)