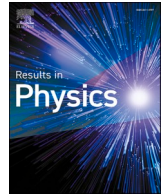




Since January 2020 Elsevier has created a COVID-19 resource centre with free information in English and Mandarin on the novel coronavirus COVID-19. The COVID-19 resource centre is hosted on Elsevier Connect, the company's public news and information website.

Elsevier hereby grants permission to make all its COVID-19-related research that is available on the COVID-19 resource centre - including this research content - immediately available in PubMed Central and other publicly funded repositories, such as the WHO COVID database with rights for unrestricted research re-use and analyses in any form or by any means with acknowledgement of the original source. These permissions are granted for free by Elsevier for as long as the COVID-19 resource centre remains active.



## Factor analysis approach to classify COVID-19 datasets in several regions

Mohammad Reza Mahmoudi<sup>a</sup>, Dumitru Baleanu<sup>b,c</sup>, Shahab S. Band<sup>d</sup>, Amir Mosavi<sup>e,f</sup>

<sup>a</sup> Department of Statistics, Faculty of Science, Fasa University, Fasa, Fars, Iran

<sup>b</sup> Department of Mathematics, Faculty of Art and Sciences, Cankaya University Balgat 06530, Ankara, Turkey

<sup>c</sup> Institute of Space Sciences, Magurele-Bucharest, Romania

<sup>d</sup> Future Technology Research Center, College of Future, National Yunlin University of Science and Technology 123 University Road, Section 3, Douliou, Yunlin 64002, Taiwan, ROC

<sup>e</sup> John von Neumann Faculty of Informatics, Obuda University, 1034 Budapest, Hungary

<sup>f</sup> School of Economics and Business, Norwegian University of Life Sciences, 1430 Ås, Norway

### ARTICLE INFO

#### Keywords:

Correlation  
Factor analysis  
Covid-19  
Coronaviruses

### ABSTRACT

The aim of this research is to investigate the relationships between the counts of cases with Covid-19 and the deaths due to it in seven countries that are severely affected from this pandemic disease. First, the Pearson's correlation is used to determine the relationships among these countries. Then, the factor analysis is applied to categorize these countries based on their relationships.

### Introduction

In the winter months of 2019–2020, another type of coronavirus, Covid-19, has been reported in Wuhan [1]. This virus has severe destructive effects on the respiratory system. From January to now (April 18, 2020), this epidemic has become epidemic all over the world and day by day the cases with Covid-19 and the deaths due to Covid-19 are extremely increasing in most of countries [2–15]. There are many techniques analyze the natural phenomena including artificial intelligence, mathematical and statistical methods such as optimization, deep learning, time series analysis, machine learning, regression modeling, clustering and numerical analysis [16–39]. Since Covid-19 has many impacts on environment, health, society and economy, the study of the rate of spread of this disease and the comparison of its rate in different countries is essential. There are some researches about the classification of Covid-19 datasets [40–43]. These researches are based on time series analysis, principal component analysis and fuzzy clustering.

The aim of this research is to study the relationships between the counts of the cases with Covid-19 and the deaths due to it in seven countries that are severely affected from this pandemic disease. First, the coefficients of correlation are computed to determine the relationships between these countries. Then, the factor analysis is applied to categorize these countries using the counts of cases and deaths.

### Material and method

This section is devoted to study the research's dataset and to and to introduce the factor analysis.

#### Dataset

In this work, the counts of the cases with Covid-19 and the deaths due to it in United States America, United Kingdom, Spain, Italy, Iran, Germany, and France from February 22 to April 18 of 2020, are considered [43,44]. Table 1 summarizes the descriptive statistics of dataset containing the mean and the standard deviation. It can be observed that Iran and United States America have the minimum and the maximum counts of the cases with Covid-19. In addition, Germany and United States America have the minimum and the maximum of the deaths due to Covid-19. The plots for the counts of the cases with Covid-19 and the deaths due to it are also demonstrated in Fig. 1.

The relationships between the rates of the spread of Covid-19 among these countries have been studied using Pearson's coefficient of correlation. As it can be seen in Table 2, all of the values are more than 0.5 and significant, and consequently there are strong positive relationships between the rates of spread of Covid-19 in all of countries.

E-mail addresses: [mahmoudi.m.r@fasau.ac.ir](mailto:mahmoudi.m.r@fasau.ac.ir) (M.R. Mahmoudi), [dumitru@cankaya.edu.tr](mailto:dumitru@cankaya.edu.tr) (D. Baleanu), [shamshirbands@yuntech.edu.tw](mailto:shamshirbands@yuntech.edu.tw) (S.S. Band), [amir.mosavi@kvk.uni-obuda.hu](mailto:amir.mosavi@kvk.uni-obuda.hu) (A. Mosavi).

<https://doi.org/10.1016/j.rinp.2021.104071>

Received 24 November 2020; Received in revised form 5 March 2021; Accepted 8 March 2021

Available online 22 March 2021

2211-3797/© 2021 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>).

**Table 1**

The mean and standard deviation for the counts of the cases with Covid-19 and the deaths due to this pandemic disease.

	Country	Mean ± Standard Deviation
Cases	United States America	11900.8 ± 13327.5
	Spain	3187.6 ± 3016.8
	Italy	2922.6 ± 2056.1
	Germany	2329.2 ± 2245.2
	France	1851.5 ± 1876.0
	United Kingdom	1842.1 ± 2207.7
	Iran	1294.5 ± 921.5
Deaths	United States America	628.0 ± 1013.4
	Italy	385.5 ± 309.5
	Spain	330.1 ± 340.5
	France	316.6 ± 447.3
	United Kingdom	247.1 ± 342.0
	Iran	80.7 ± 55.6
	Germany	69.7 ± 94.9

*Principles of factor analysis*

Factor analysis (FA) as a popular multivariate statistical technique transforms some dependent features into some other features called factors such that the first factors of this transformation have the main information of the first dataset [45]. In other words, FA is used to convert a dataset with high dimensions to a dataset with lower dimensions, by considering minimum factors such that the dimension of the converted dataset is decreased. FA focuses on the correlations of variables such that the variables in a factor are highly correlated with each other and the variables in different factors are highly uncorrelated with each other. In applications, the number of the main factors in FA is usually considered as the number of the eigen-values of the correlation's matrix with the values larger than one. To investigate the suitability of FA, the Kaiser-Meyer-Olkin (KMO) index is used. The KMO greater than 0.8 verifies the accuracy of FA.

Assume  $X = (X_1, \dots, X_p)^T$  is a random vector. Denote

$$\mu = E(X) = (\mu_1, \dots, \mu_p)^T$$

and

$$\Sigma = Var(X) = \begin{bmatrix} \sigma_{11} & \dots & \sigma_{1p} \\ \dots & \dots & \dots \\ \sigma_{p1} & \dots & \sigma_{pp} \end{bmatrix}$$

as the mean vector and covariance matrix of  $X$ .

The equation of factor analysis with  $m$  factors ( $m \leq p$ ) is presented by

$$X - \mu = LF + \epsilon,$$

such that

$$L = \begin{bmatrix} l_{11} & \dots & l_{1m} \\ \dots & \dots & \dots \\ l_{p1} & \dots & l_{pm} \end{bmatrix},$$

$$F = (F_1, \dots, F_m)^T,$$

and

$$\epsilon = (\epsilon_1, \dots, \epsilon_p)^T.$$

The matrix  $L$  and the vectors  $F$  and  $\Psi$  are called the factor loading matrix, the factors and errors, respectively.

This model can be rewritten by

$$X_i - \mu_i = \sum_{j=1}^m l_{ij}F_j + \epsilon_i, i = 1, \dots, p,$$

such that  $l_{ij}$  is named as the loading of  $X_i$  on the factor  $F_j$ .

In orthogonal factor analysis, we have

$$Cov(X, F) = L,$$

and

$$\Sigma = LL^T + \Psi,$$

where

$$\Psi = Var(\epsilon)$$

Consequently,

$$Var(X_i) = \sum_{j=1}^m l_{ij}^2 + \Psi_i,$$

and

$$Cov(X_i, X_j) = \sum_{k=1}^m l_{ik}l_{jk}.$$

$\sum_{j=1}^m l_{ij}^2$  is determines the proportion of  $Var(X_i)$  that can be explained by the factors  $F_1, \dots, F_m$ .

The main aim of factor analysis is to find the values of the loadings. To compute the matrices  $L$  and  $\Psi$ , different approaches such as principal component and maximum likelihood can be applied. Principal component approach uses eigen-values and eigen-vectors to decompose the matrix  $\Sigma$  to find the matrix  $L$ . Maximum likelihood approach computes the likelihood and then optimize it to find the matrices  $L$  and  $\Psi$ .

When the loading values are estimated, we can consider loading plots. Loading plots can be used to

- Study the correlations between variables
- Categorize and Classify the variables
- Detect the number of factors

In loading plot, the angle between two variables ( $\theta$ ) determines the correlation ( $r$ ) between them.  $\theta = 90^\circ$  verifies that two variables are uncorrelated ( $r = 0$ ). The cases  $\theta = 0^\circ$  and  $\theta = 180^\circ$  refer to exact positive and negative linear relationship, respectively.

**Results**

This section reports the results of FA approach to classify the countries based on research's variables. It should be noted that the number of the main factors in FA was considered as the number of the eigen-values of the correlation's matrix with the values larger than one. Moreover, the KMO values were more than 0.8 that verify the accuracy of FA

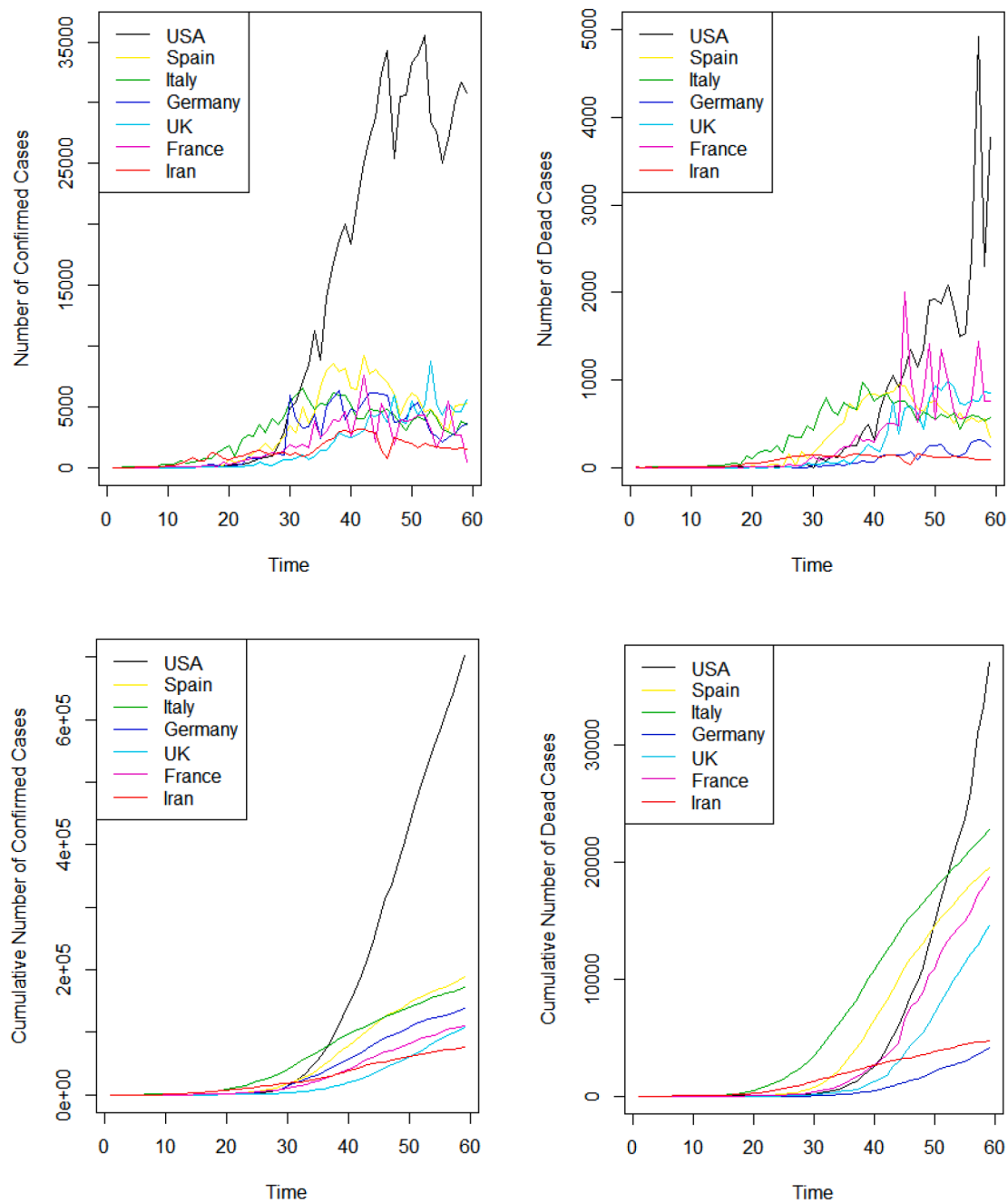


Fig. 1. Counts of the cases (Left side of Top), counts of the deaths (Right side of Top), cumulative counts of the cases (Left side of Bottom), and cumulative counts of the deaths (Right side of Bottom).

approach.

*Counts of cases with Covid-19*

The results of FA technique to categorize the research countries, in basis of the counts of the cases with Covid-19, are provided in Fig. 2. The outputs demonstrate the statistical differences between the relationships among the countries and we can categorize the countries into following classes:

- First class: Iran, France, Spain, Germany, Italy.
- Second class: United Kingdom, United States America.

*Counts of the deaths due to Covid-19*

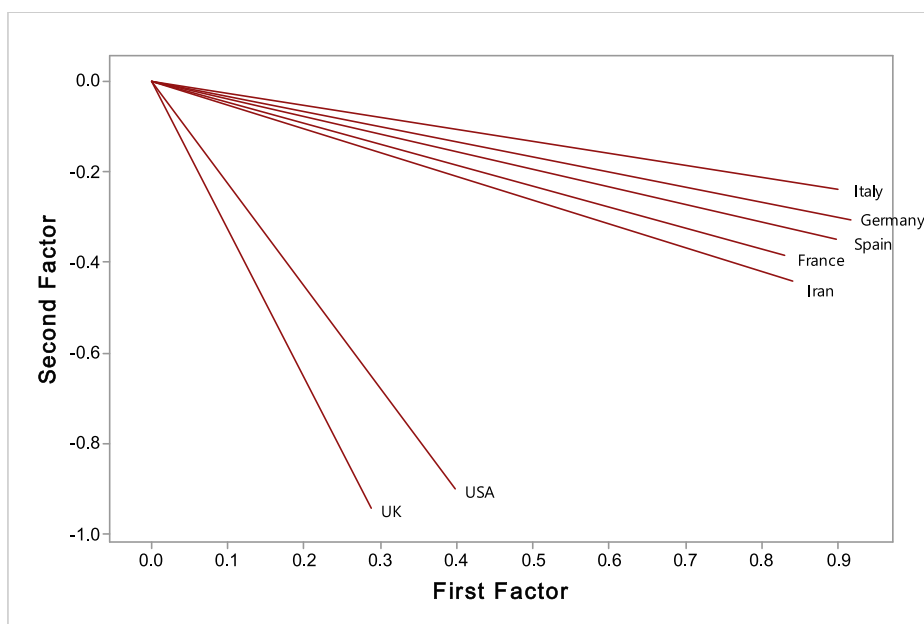
The results of FA technique to categorize the research countries, in basis of the counts of the deaths due to Covid-19, are provided in Fig. 3. The outputs demonstrate the statistical differences between the relationships among the countries and we can categorize the countries into following classes:

- First class: France, United Kingdom, Germany and United States America.
- Second class: Iran, Italy and Spain.

**Table 2**  
Pearson's coefficient of correlation between the rates of spread of Covid-19.

		United States America	Spain	Italy	Germany	United Kingdom	France	Iran
Patients	France	1	0.586	0.850	0.766	0.851	0.856	0.673
	Germany	0.586	1	0.654	0.514	0.562	0.550	0.942
	Iran	0.850	0.654	1	0.848	0.884	0.866	0.718
	Italy	0.766	0.514	0.848	1	0.842	0.879	0.567
	Spain	0.851	0.562	0.884	0.842	1	0.929	0.666
	United Kingdom	0.856	0.550	0.866	0.879	0.929	1	0.654
	United States America	0.673	0.942	0.718	0.567	0.666	0.654	1
Deaths	France	1	0.802	0.621	0.677	0.815	0.804	0.716
	Germany	0.802	1	0.565	0.629	0.764	0.927	0.885
	Iran	0.621	0.565	1	0.934	0.794	0.555	0.449
	Italy	0.677	0.629	0.934	1	0.892	0.626	0.508
	Spain	0.815	0.764	0.794	0.892	1	0.743	0.627
	United Kingdom	0.804	0.927	0.555	0.626	0.743	1	0.889
	United States America	0.716	0.885	0.449	0.508	0.627	0.889	1

\* p-value < 0.001.



**Fig. 2.** FA technique to categorize the countries in basis of the counts of the cases with Covid-19.

*Cumulative counts of the cases with Covid-19*

The results of FA technique to categorize the research countries, in basis of the cumulative counts of the cases with Covid-19, are provided in Fig. 4. The outputs demonstrate the statistical differences between the relationships among the countries and we can categorize the countries into following classes:

- First class: France, Spain, Germany, Iran and Italy.
- Second class: United Kingdom and United States America.

*Cumulative counts of the deaths due to Covid-19*

The results of FA technique to categorize the research countries, in basis of the cumulative counts of the deaths due to Covid-19, are provided in Fig. 5. The outputs demonstrate the statistical differences between the relationships among the countries and we can categorize the countries into following classes:

- First class: France, United Kingdom, Germany and United States America.
- Second class: Iran, Italy and Spain.

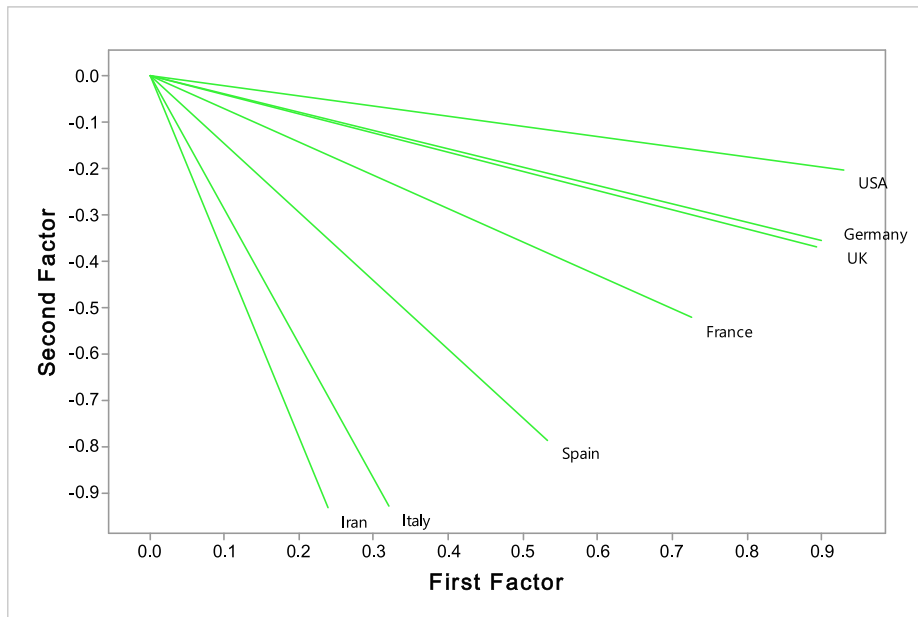


Fig. 3. FA technique to categorize the countries in basis of the counts of the deaths due to Covid-19.

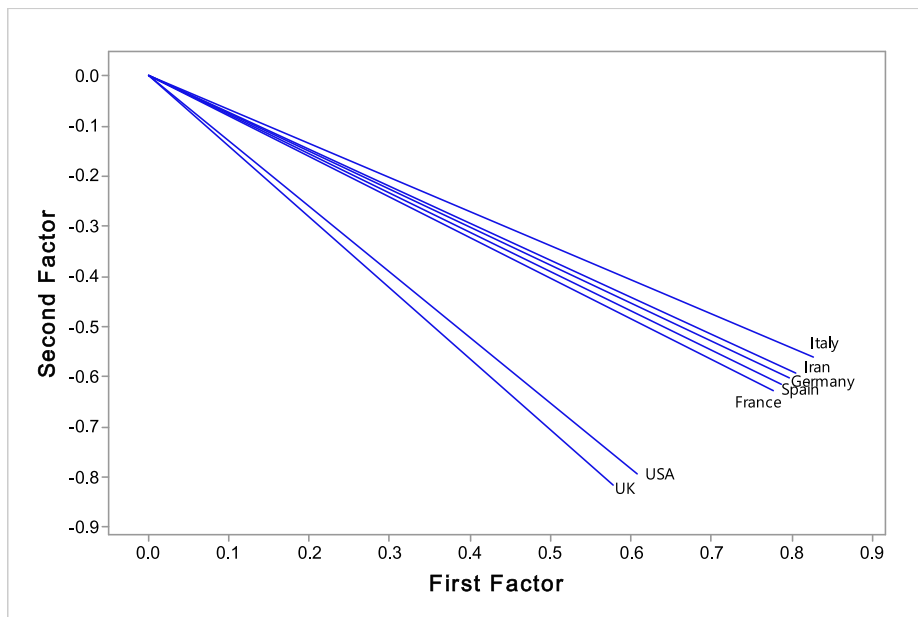


Fig. 4. FA technique to categorize the countries in basis of the cumulative counts of the cases with Covid-19.

**Conclusion**

Since Covid-19 has many impacts on environment, health, society and economy, the study of the rate of spread of this disease and the comparison of its rate in different countries is essential. The aim of this research was to study the cases with Covid-19 and the deaths due to this pandemic disease in seven countries that are severely affected from this

pandemic disease. The cases and the deaths in United States America, United Kingdom, Spain, Italy, Iran, Germany, and France from February 22 to April 18 of 2020, were considered. First, the coefficients of correlation were computed to determine the relationships among these countries. The outputs showed that there were strong positive relationships between the rates of spread in all of countries. Then, the factor analysis was applied to categorize the countries in basis of the

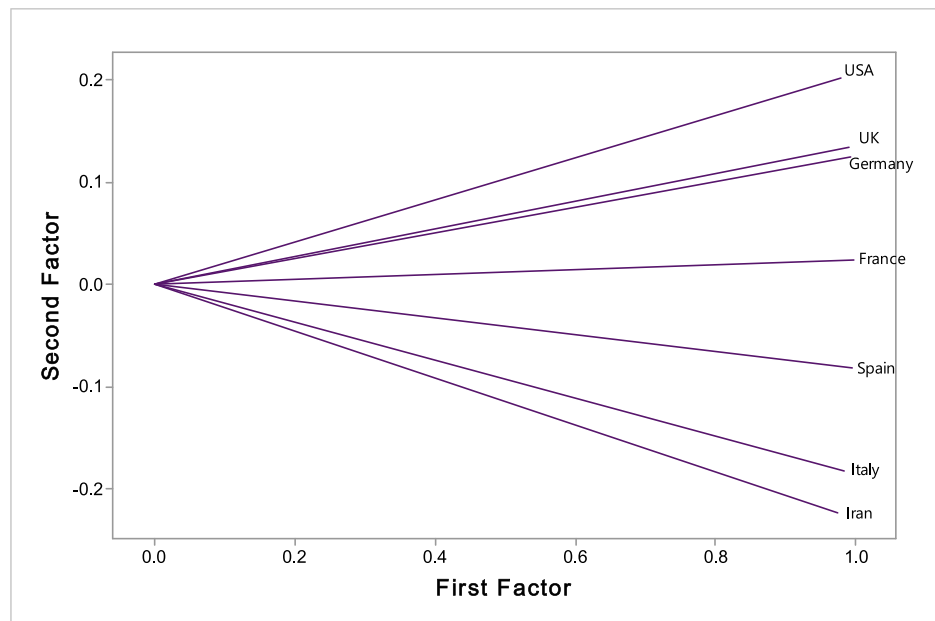


Fig. 5. FA technique to categorize the countries in basis of the cumulative counts of the deaths due to Covid-19.

counts and deaths. For the cases with Covid-19, United Kingdom and United States America were similarly distributed to each other and were differently distributed from other countries. Also, for the deaths, Iran, Italy and Spain were similarly distributed to each other and were differently distributed from other countries. For future works, the authors suggest classifying the Covid-19 datasets of more regions based on FA technique, or apply this technique to classify the regions for other epidemic or pandemic diseases.

#### CRedit authorship contribution statement

**Mohammad Reza Mahmoudi:** Conceptualization, Investigation, Data curation, Validation, Methodology, Software, Writing - original draft. **Dumitru Baleanu:** Conceptualization, Supervision, Visualization, Writing - review editing. **Shahab S. Band:** Validation, Visualization, Software, Writing - review editing. **Amir Mosavi:** Visualization, Writing - review editing.

#### Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

#### References

- [1] Lu R, Zhao X, Li J, Niu P, Yang B, Wu H, et al. Genomic characterisation and epidemiology of 2019 novel coronavirus: implications for virus origins and receptor binding. *The Lancet* 2020;395(10224):565–74.
- [2] Burke RM, Midgley CM, Dratch A, et al. Active Monitoring of Persons Exposed to Patients with Confirmed COVID-19 — United States, January–February 2020. *MMWR Morb Mortal Wkly Rep* 2020;69:245–6. <https://doi.org/10.15585/mmwr.mm6909e1>.
- [3] Hunter DJ. Covid-19 and the stiff upper lip—the pandemic response in the united kingdom. *N Engl J Med* 2020.
- [4] Razai MS, Doerholt K, Ladhani S, Oakeshott P. Coronavirus disease 2019 (covid-19): a guide for UK GPs. *BMJ* 2020;6:368.
- [5] Lillie PJ, Samson A, Li A, Adams K, Capstick R, Barlow GD, et al. Novel coronavirus disease (Covid-19): the first two patients in the UK with person to person transmission. *J Infect* 2020.
- [6] Legido-Quigley H, Mateos-García JT, Campos VR, Gea-Sánchez M, Muntaner C, McKee M. The resilience of the Spanish health system against the COVID-19 pandemic. *Lancet Public Health* 2020.
- [7] Lazzarini M, Putoto G. COVID-19 in Italy: momentous decisions and many uncertainties. *Lancet Global Health* 2020.
- [8] Onder G, Rezza G, Brusaferro S. Case-fatality rate and characteristics of patients dying in relation to COVID-19 in Italy. *JAMA* 2020.
- [9] Remuzzi A, Remuzzi G. COVID-19 and Italy: what next? *Lancet* 2020.
- [10] Takian A, Raoofi A, Kazempour-Ardebili S. COVID-19 battle during the toughest sanctions against Iran. *Lancet (London, England)* 2020;395(10229):1035.
- [11] Rothe C, Schunk M, Sothmann P, Bretzel G, Froeschl G, Wallrauch C, et al. Transmission of 2019-nCoV infection from an asymptomatic contact in Germany. *N Engl J Med* 2020;382(10):970–1.
- [12] Amrane S, Tissot-Dupont H, Doudier B, Eldin C, Hocquart M, Mailhe M, et al. A respiratory virus snapshot. *Travel Med Infect Dis* 2020;2020:101632.
- [13] Stoecklin SB, Rolland P, Silue Y, Mailles A, Campese C, Simondon A, et al. First cases of coronavirus disease 2019 (COVID-19) in France: surveillance, investigations and control measures, January 2020. *Eurosurveillance* 2020;25(6):2000094.
- [14] Gautret P, Lagier JC, Parola P, Meddeb L, Mailhe M, Doudier B, et al. Hydroxychloroquine and azithromycin as a treatment of COVID-19: results of an open-label non-randomized clinical trial. *Int J Antimicrob Agents* 2020;20:105949.
- [15] Fanelli D, Piazza F. Analysis and forecast of COVID-19 spreading in China, Italy and France. *Chaos, Solitons Fractals* 2020;134:109761.
- [16] Haghbin H, Mahmoudi MR, Shishebor Z. Large sample inference on the ratio of two independent binomial proportions. *J Math Ext* 2011;5(1):87–95.
- [17] Mahmoudi MR, Mahmoodi M. Inference on the ratio of correlations of two independent populations. *J Math Ext* 2014;7(4):71–82.
- [18] Mahmoudi MR, Mahmoodi M. Inference on the ratio of variances of two independent populations. *J Math Ext* 2014;7(2):83–91.
- [19] Mahmoudi MR, Nematollahi AR, Soltani AR. On the detection and estimation of the simple harmonizable processes. *Iran J Sci Technol (Sciences)* 2015;39(2):239–42.
- [20] Mahmoudi MR, Mahmoudi M, Nahavandi E. Testing the difference between two independent regression models. *Commun Stat Theory Methods* 2016;45(21):6284–9.
- [21] Nematollahi AR, Soltani AR, Mahmoudi MR. Periodically correlated modeling by means of the periodograms asymptotic distributions. *Stat Pap* 2017;58(4):1267–78.
- [22] Mahmoudi MR, Maleki M. A new method to detect periodically correlated structure. *Comput Stat* 2017;32(4):1569–81.
- [23] Zarei AR, Mahmoudi MR. Evaluation of changes in RD1st index effected by different Potential Evapotranspiration calculation methods. *Water Resour Manag* 2017;31(15):4981–99.
- [24] Bahrami M, Amiri MJ, Mahmoudi MR, Koochaki S. Modeling caffeine adsorption by multi-walled carbon nanotubes using multiple polynomial regression with interaction effects. *J Water Health* 2017;15(4):526–35.
- [25] Mahmoudi MR, Maleki M, Pak A. Testing the difference between two independent time series models. *Iran J Sci Technol A (Sciences)* 2017;41:665–9.
- [26] Mahmoudi MR, Behboodiani J, Maleki M. Inference on the ratio of means in two independent populations. *J Stat Theory Appl* 2017;16(3):366–74.
- [27] Mahmoudi MR, Heydari MH, Avazzadeh Z. On the asymptotic distribution for the periodograms of almost periodically correlated (cyclostationary) processes. *Digital Signal Process* 2018;81:186–97.
- [28] Mahmoudi MR. On comparing two dependent linear and nonlinear regression models. *J Test Eval* 2018;47(1):449–58.
- [29] Mahmoudi MR, Maleki M, Pak A. Testing the equality of two independent regression models. *Commun Stat Theory Methods* 2018;47(12):2919–26.

- [30] Heydari MH, Avazzadeh Z, Mahmoudi MR. Chebyshev cardinal wavelets for nonlinear stochastic differential equations driven with variable-order fractional Brownian motion. *Chaos, Solitons Fractals* 2019;124:1105–24.
- [31] Ji-jun P, Mahmoudi MR, Baleanu D, Maleki M. On comparing and classifying several independent linear and non-linear regression models with symmetric errors. *Symmetry* 2019;11(6):820.
- [32] Mahmoudi MR, Heydari MH, Avazzadeh Z. Testing the difference between spectral densities of two independent periodically correlated (cyclostationary) time series models. *Commun Stat Theory Methods* 2019;48(9):2320–8.
- [33] Mahmoudi MR, Heydari MH, Roohi R. A new method to compare the spectral densities of two independent periodically correlated time series. *Math Comput Simulat* 2019;160:103–10.
- [34] Mahmoudi MR, Nasirzadeh R, Mohammadi M. On the ratio of two independent skewnesses. *Commun Stat-Theor Methods* 2019;48(7):1721–7.
- [35] Mahmoudi MR, Heydari MH, Pho KH. Fuzzy clustering to classify several regression models with fractional Brownian motion errors. *Alexandria Eng J* 2020; 59(4):2811–8.
- [36] Mahmoudi MR, Baleanu D, Tuan BA, Pho KH. A novel method to detect almost cyclostationary structure. *Alexandria Eng J* 2020;59(4):2339–46.
- [37] Zhou R, Mahmoudi MR, Mohammed SNQ, Pho KH. Testing the equality of the spectral densities of several uncorrelated almost cyclostationary processes. *Alexandria Eng J* 2020 [Article in Press].
- [38] Mahmoudi MR, Maleki M, Borodin K, Pho KH, Baleanu D. On comparing and clustering the spectral densities of several almost cyclostationary processes. *Alexandria Eng J* 2020;59(4):2555–65.
- [39] Mahmoudi MR, Heydari MH, Avazzadeh Z, Pho KH. Goodness of fit test for almost cyclostationary processes. *Digital Signal Process* 2020;96:102597.
- [40] Mahmoudi MR, Baleanu D, Mansor Z, Tuan BA, Pho KH. Fuzzy clustering method to compare the spread rate of Covid-19 in the high risks countries. *Chaos Solitons Fractals* 2020;140:110230.
- [41] Maleki M, Mahmoudi MR, Heydari MH, Pho KH. Modeling and forecasting the spread and death rate of coronavirus (COVID-19) in the world using time series models. *Chaos Solitons Fractals* 2020;140:110151.
- [42] Maleki M, Mahmoudi MR, Wraith D, Pho KH. Time series modelling to forecast the confirmed and recovered cases of COVID-19. *Travel Med Infect Dis* 2020;101742.
- [43] Mahmoudi MR, Heydari MH, Qasem SN, Mosavi A, Band SS. Principal component analysis to study the relations between the spread rates of COVID-19 in high risks countries. *Alexandria Eng J* 2021;60(1):457–64.
- [44] Salehi M, Arashi M, Bekker A, Ferreira J, Chen DG, Esmaeili F, et al. A synergetic R-shiny portal for modeling and tracking of COVID-19 data. *Front Public Health* 2021;8:623624.
- [45] Johnson RA, Wichern D. *Multivariate Analysis*. Ltd: John Wiley & Sons; 2002.