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Study on the mathematical modelling of COVID-19 with Caputo-Fabrizio operator

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ABSTRACT

In this article we study a fractional-order mathematical model describing the spread of the new coronavirus (COVID-19) under the Caputo-Fabrizio sense. Exploiting the approach of fixed point theory, we compute existence as well as uniqueness of the related solution. To investigate the exact solution of our model, we use the Laplace Adomian decomposition method (LADM) and obtain results in terms of infinite series. We then present numerical results to illuminate the efficacy of the new derivative. Compared to the classical order derivatives, our obtained results under the new notion show better results concerning the novel coronavirus model.

1. Introduction

In the last month of the year 2019, a severe outbreak of respiratory disease originated in Wuhan City of Hubei province in China [1]. The causative agent had been identified and isolated from a single patient at the start of January 2020, and the disease was called COVID-19 i.e., the novel coronavirus. The initial source of the spreading of this virus was an animal. However, the ratio of reported cases rose due to the interaction between humans [2]. Recent studies show that the virus has almost reached every corner of the world. Up to December 7, 2020, about 66,243,918 individuals are reported infected with the disease and has caused 1,528,984 deaths. The infection is contagious due to its rapid spread and has been declared a pandemic by the WHO. Symptoms of the virus include fever, fatigue, cough and breathing difficulties, etc. The infection has attracted the attention of researchers due to its severe threat to human life and novel characteristics. Infectious diseases epidemiology has a rich literature and a variety of mathematical models have been developed to study various infectious diseases, see e.g., [3–8].

Several researchers investigated the current pandemic caused by coronavirus with different techniques of analysis [9–17]. Variety of mathematical models exist in the literature regarding the current pandemic which give the future forecast of the disease. Various control strategies have been suggested by number of researchers [18–21]. Recently Khan et al. [22] used a mathematical model for the description of the temporal transmission of the novel coronavirus and presented control strategies to eradicate the infection. Their model is given below

$$\begin{aligned}
\frac{d\mathbf{S}_h(t)}{dt} &= \Pi^* - (\beta_1^*\mathbf{I}_h(t) + d^* + \beta_2^*\mathbf{W}(t))\mathbf{S}_h(t),\\
\frac{d\mathbf{I}_h(t)}{dt} &= (\beta_1^*\mathbf{I}_h(t) + \beta_2^*\mathbf{W}(t))\mathbf{S}_h(t) - (\sigma^* + d_1^* + d^*)\mathbf{I}_h(t),\\
\frac{d\mathbf{R}_h(t)}{dt} &= \sigma^*\mathbf{I}_h(t) - d^*\mathbf{R}_h(t),\\
\frac{d\mathbf{W}(t)}{dt} &= \alpha^*\mathbf{I}_h(t) - \eta^*\mathbf{W}(t).
\end{aligned} \tag{1}$$

In model (1), the letters $\mathbf{S}_h(t)$, $\mathbf{I}_h(t)$ and $\mathbf{R}_h(t)$ are used to respectively denote the susceptible, infected and recovered population, while $\mathbf{W}(t)$ symbolizes the reservoirs compartment. The parameter Π^* represents the newborn rate which is assumed to be susceptible. β_1^* and β_2^* represent the disease transmission rates from the infected individuals and reservoirs respectively to the susceptible population. The symbol d^* is used to denote the natural mortality rate while the death caused by the disease is

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symbolized by d_1^*. Similarly the recovery rate is σ^* and removing rate of the virus is η^*, while the ratio contributed the virus into seafood market is denoted by α^*.

First we show that solutions of the model (1) are bounded. Assume $N(t)$ represents the total population at time t. Taking the temporal derivative of $N(t)$ and exploiting values from the given model, we obtain

$$\frac{dN(t)}{dt} - dN \leq \Pi^*.$$

Solution of this equation under the initial conditions $\mathbf{S}_h(0) \geq 0, \mathbf{I}_h(0) \geq 0, \mathbf{R}_h(0) \geq 0, \mathbf{W}(0) \geq 0$ and $N(0) = N_0$ has the form

$$N(t) \leq \frac{\Pi^*}{d^*} + (N_0 - \frac{\Pi^*}{d^*})e^{-d^*t}.$$

This solution is bounded when t grows without bound.

The possible uniform states (i.e., the disease free (DFE) and the endemic) and the threshold quantity (basic reproductive number) along with detailed qualitative analysis have been addressed for the above model (1) as follows

$$\begin{aligned}
&(\mathbf{S}_{h0}, 0, 0, 0, 0), \text{ where } \frac{\Pi^*}{d^*},\\
&R_0 = \frac{\beta_1 \Pi^*}{d^*(\sigma^* + d^* + d_1^*)} + \frac{\alpha^* \beta_2^* \Pi^*}{\eta^* d^*(\sigma^* + d^* + d_1^*)},\\
&\mathbf{S}_h^* = \frac{\eta^*(\sigma^* + d^* + d_1^*)}{\eta^* \beta_1^* + \alpha^* \beta_2^*},\\
&\mathbf{I}_h^* = \frac{\eta^* d^*(\sigma^* + d^* + d_1^*)(R_0 - 1)}{\sigma^* \beta_1^* \eta^* + \sigma^* \beta_2^* \alpha^* + d^* \beta_1^* \eta^* + d^* \beta_2^* \alpha^* + d_1^* \beta_1^* \eta^* + d_1^* \beta_2^* \alpha^*},\\
&\mathbf{R}_h^* = \frac{\sigma^*}{d^*} I_h^*,\\
&\mathbf{W}^* = \frac{\alpha^* \eta^*(\sigma^* + d^* + d_1^*)(R_0 - 1)}{\sigma^* \beta_1^* \eta^* + \sigma^* \beta_2^* \alpha^* + d^* \beta_1^* \eta^* + d^* \beta_2^* \alpha^* + d_1^* \beta_1^* \eta^* + d_1^* \beta_2^* \alpha^*}.
\end{aligned} \tag{2}$$

Fractional calculus is the extension of integer order calculus.Lebniz drew the attention of the researchers towards fractional order derivatives. Researchers did not frequently work in the field due to the lack of solution for the fractional order differential equations. The field was focused on, once fractional order derivatives and integral were properly defined as the classic models did not explain them properly besides giving errors in it. Fractional order epidemic models explain the phenomenon in a better way while taking even the missing aspects into account. Due to this, several researchers feel attracted to the field and are contributing to epidemiology. However, finding a real life situation to be explained by fractional order model is a daunting task. These models have greater autonomy than the classical ones. These models have the capability to exploit the numerical method in order to work out non integer order nonlinear systems. This situation have encouraged several researchers to exploit fractional order system while making some modifications in the integral order models

Compared with fractional derivatives, the derivatives of integer order do not, in general, investigate the dynamic more accurately. In literature, variety of notions of fractional derivative have been used by number of mathematicians for the description of various dynamical systems. Various valuable definitions have been presented by Caputo, Hadamard, Riemann and Liouville etc. [23,24]. Different approaches, such as iterative and numerical methods, have been used to investigate fractional mathematical models under the usual Caputo derivative sense, see, [25–27].

The aforesaid derivative contains a singular kernel which causes complications in fractional order derivatives. Caputo and Fabrizio reported a novel idea about the fractional order derivative which is based upon non-singular kernel [25,28,29]. Some important results have been listed based upon the Caputo-Fabrizio fractional integral. These studies show that the above mentioned operator is in fact the functions's fractional average with fractional integral in the sense of Riemann-Liouville. It has been reported that the above mentioned derivative has significant applications in thermal and material sciences [30–32]. LADM is a very useful technique used for the analytical approximate solution of several non-linear problems. This technique has been well exploited for the problems based on the fractional as well as classic differential equations [33–36]. Nevertheless, the above mentioned technique is barely used for fractional order differential equations having the non-singular kernel [37]. Fractional calculus is a developing area and has been widely used for the extension of classic epidemiological models. Remarkable work in this regard includes the approach of [38–43]. In this article, we investigate a fractional version of the novel coronavirus model (1) and use the notion of fractional Caputo-Fabrizio derivative [28].

Currently, idea of the fractional Caputo-Fabrizio derivative has been used by number of researchers to investigate variety of problems in biological and physical sciences, see for instance, [44–49]. The idea of CF derivative has been repeatedly used to describe the dynamics of various communicable infections [50–53]. Stability, existence and uniqueness of various mathematical models have been investigated using the CF derivatives [54–56]. In many situations, the fractional order models are more beneficial to describe the dynamics of various real world problems in a better way. The reason is that fractional order derivatives have greater degree of freedom compared with classical integer order derivatives. The new type non-singular derivatives of fractional order have been found more suitable to study thermal problems instead of ordinary derivatives.

Inspired from the above stated literature, we investigate the series type solution of coronavirus mathematical model (1) via Caputo-Fabrizio fractional derivative. We consider the (1) having fractional-order q describing the situation lies between two integer values. The result is achieved by having the whole density of every compartment converging quickly at low orders. We refer the reader, for further detail study of fractional calculus and applications to study [57–64].

Fractional extension of the model (1) has the following form:

$$\begin{aligned}
{}^{CF}\mathcal{D}_t^q \mathbf{S}_h(t) &= \Pi^{*q} - (\beta_1^{*q}\mathbf{I}_h(t) + \beta_2^{*q}\mathbf{W}(t) + d^{*q})\mathbf{S}(t),\\
{}^{CF}\mathcal{D}_t^q \mathbf{I}_h(t) &= (\beta_1^{*q}\mathbf{I}_h(t) + \beta_2^{*q}\mathbf{W}(t) + d^{*q})\mathbf{S}(t)\\
&\quad -(\sigma^{*q} + d^{*q} + d_1^{*q})\mathbf{I}_h(t),\\
{}^{CF}\mathcal{D}_t^q \mathbf{R}_h(t) &= \sigma^{*q}\mathbf{I}_h(t) - d^{*q}\mathbf{R}_h(t),\\
{}^{CF}\mathcal{D}_t^q \mathbf{W}(t) &= \alpha^{*q}\mathbf{I}_h(t) - \eta^{*q}\mathbf{W}(t).
\end{aligned} \tag{3}$$

We will investigate the model under the underlying initial conditions

$$\mathbf{S}_0 = \mathbf{S}(0),\ \mathbf{I}_0 = \mathbf{I}(0),\ \mathbf{R}_0 = \mathbf{R}(0),\ \mathbf{W}_0 = \mathbf{W}(0).$$

Our current manuscript has the following organization. We briefly revise basic definitions and notions used in the article in Section 2. Section 4 is devoted to study the existence as well as uniqueness of the concerned solution with the aid of fixed point theory approach. In section (5), using the Adomian decomposition technique along with Laplace integral transform, we investigate analytical solution of the model under investigation. In section 6, with the help of available data, we perform numerical simulations for the support and verification of our analytical findings.

2. Preliminaries

In the following we recall some useful results.

Definition 1. [29] Let $g \in H^1[0,T], T>0, q \in (0,1)$, then the Caputo-Fabrizio fractional derivative can be defined as

$$^{CF}\mathcal{D}_t^q[g(t)] = \frac{\mathcal{M}(q)}{1-q}\int_0^t g'(x)\exp\left[-q\frac{t-\rho}{1-q}\right]dx, \quad (4)$$

where $\mathcal{M}(q)$ denotes the normalization function and $\mathcal{M}(0)=\mathcal{M}(1)=1$. When $g \in H^1[0,T]$ then the CFFD can be recalled as

$$^{CF}\mathcal{D}_t^q[g(t)] = \frac{\mathcal{M}(q)}{1-q}\int_0^t \Big(g(t)-g(x)\Big)\exp\left[-q\frac{t-x}{1-q}\right]dx, \quad (5)$$

Definition 2. [29] The Caputo-Fabrizio fractional integral with $q(0,1)$ can be defined as for a function g

$$^{CF}\mathcal{I}_t^q[g(t)] = \frac{1-q}{\mathcal{N}(q)} + \frac{q}{\mathcal{N}(q)}\int_0^t g(x)dx, \quad t \geq 0. \quad (6)$$

Definition 3. [37] The general relation for the Laplace transform of CFFD can be defined as

$$\mathcal{L}[^{CF}\mathbf{D}_t^q g(t)] = \frac{s\mathcal{L}[g(t)]-g(0)}{s+q(1-s)}, \quad s \geq 0. \quad (7)$$

3. Non-negative Solution

Suppose $\Psi = \{(\mathbf{S}_h,\mathbf{I}_h,\mathbf{R}_h) \in \mathbf{R}_3^+ : \mathbf{S}_h+\mathbf{I}_h+\mathbf{R}_h \leq \frac{\Pi^*}{d^*}, \mathbf{W} \in \mathbf{R}^+ : \mathbf{W} \leq \frac{\alpha^*}{\eta^*}\frac{\Pi^*}{d^*}\}$, we show that the closed set Ψ is the region of the feasibility of system (3).

Lemma 1. *The closed set Ψ is positively invariant with respect to fractional system (3).*

Proof. We add the first three equation of system (3) to obtain total population

$$^{CF}\mathcal{D}_t^q\mathbf{N}_h(t) = \Pi^{*q} - d^{*q}\mathbf{N}_h(t),$$

where $\mathbf{N}_h(t) = \mathbf{S}_h(t)+\mathbf{I}_h(t)+\mathbf{R}_h(t)$. By using the Laplace transform, we have

$$\mathbf{N}_h(t) = \mathbf{N}_h(0)E_q(-d^{*q}t^q) + \int_0^t \Pi^{*q}x^{q-1}E_{q,q}(-d^{*q}x^q)dx.$$

The term $\mathbf{N}_h(0)$, in above equation, is the size of initial human population. After performing a simple algebra, one may write

$$\begin{aligned}\mathbf{N}_h(t) &= \mathbf{N}_h(0)E_q(-d^{*q}t^q) + \int_0^t \Pi^{*q}x^{q-1}\sum_{j=0}^{\infty}\frac{(-1)^j d^{*q}x^{jq}}{\Gamma(jq+q)}dx,\\ &= \frac{\Pi^{*q}}{d^{*q}} + E_q(-d^{*q}t^q)\Big(\mathbf{N}_h(0) - \frac{\Pi^{*q}}{d^{*q}}\Big).\end{aligned}$$

Thus if $\mathbf{N}_h(0) \leq \frac{\Pi^{*q}}{d^{*q}}$ then for $t>0$, $\mathbf{N}_h(t) \leq \frac{\Pi^{*q}}{d^{*q}}$. Similarly, for $\mathbf{W}$, we have

$$\begin{aligned}^{CF}\mathcal{D}_t^q\mathbf{W}(t) &= \aleph^{*q}\mathbf{I}_h(t) - \eta^{*q}\mathbf{W}(t),\\ &\leq \aleph^{*q}\mathbf{N}_h(t) - \eta^{*q}\mathbf{W}(t).\end{aligned}$$

One then obtains

$$\mathbf{W}(t) \leq \frac{\alpha^{*q}\frac{\Pi^{*q}}{d^{*q}}}{\eta^{*q}} + E_q(-d^{*q}t^q)\Big(\mathbf{W}(0) - \frac{\alpha^{*q}\frac{\Pi^{*q}}{d^{*q}}}{\eta^{*q}}\Big).$$

So if $\mathbf{W}(0) \leq \frac{\alpha^{*q}\frac{\Pi^{*q}}{d^{*q}}}{\eta^{*q}}$, then $\mathbf{W}(t) \leq \frac{\alpha^{*q}\frac{\Pi^{*q}}{d^{*q}}}{\eta^{*q}}$. Consequently, the closed set Ψ is positively invariant with respect to fractional model (3). □

4. Results regarding the existence and uniqueness for the model

It is important to know whether solution of a mathematical model exists or not. This problem can be answered with the aid of fixed point theory approach. We find the concerned solution's uniqueness and existence. This goal is obtained using the Banach fixed point theorem. Applying Picard's operator technique to the proposed model (3), we determine the existence and uniqueness results. Let us define

$$\begin{aligned}&^{CF}\mathcal{D}_t^q\mathbf{S}_h(t) = \mathcal{G}_1(t,\mathbf{S}_h(t)) = \Pi^{*q} - (\beta_1^{*q}\mathbf{I}_h(t) + \beta_2^{*q}\mathbf{W}(t) + d^{*q})\mathbf{S}(t),\\ &^{CF}\mathcal{D}_t^q\mathbf{I}_h(t) = \mathcal{G}_2(t,\mathbf{I}_h(t)) = (\beta_1^{*q}\mathbf{I}_h(t) + \beta_2^{*q}\mathbf{W}(t) + d^{*q})\mathbf{S}(t)\\ &\qquad -(\sigma^{*q} + d^{*q} + d_1^{*q})\mathbf{I}_h(t),\\ &^{CF}\mathcal{D}_t^q\mathbf{R}_h(t) = \mathcal{G}_3(t,\mathbf{R}_h(t)) = \sigma^{*q}\mathbf{I}_h(t) - d^{*q}\mathbf{R}_h(t),\\ &^{CF}\mathcal{D}_t^q\mathbf{W}(t) = \mathcal{G}_4(t,\mathbf{W}(t)) = \alpha^{*q}\mathbf{I}_h(t) - \eta^{*q}\mathbf{W}(t)\end{aligned} \quad (8)$$

We further set

$$\mathcal{Q}_j = \sup_{C[d,b_j]}\left\|\mathcal{G}_1(t,\mathbf{S}_h(t),\mathbf{I}_h(t),\mathbf{R}_h(t),\mathbf{W}(t))\right\|, \quad \text{for} \quad j=1,2,3,4, \quad (9)$$

where

$$\begin{aligned}C[d,b_j] &= [t-d,t+d]\times[u-c_j,u+c_j]\\ &= D\times D_j, \quad \text{for} \quad j=1,2,3,4.\end{aligned} \quad (10)$$

Next, using Banach fixed point theorem for showing the existence and uniqueness of the solution, we define the norm on $C[d,b_j]$ for $j=1,2,3,4$ as

$$\|\mathbf{H}\|_\infty = \sup_{t\in[t-d,t+b]}|\Phi(t)|. \quad (11)$$

Applying $^{CF}\mathcal{I}^q$ to all of the equations in model (3), in view of (8), we obtain,

$$\begin{aligned}\mathbf{S}_h(t) &= \mathbf{S}_h(0) +^{CF}\mathcal{I}^q\Big[\mathcal{G}_1(t,\mathbf{S}_h(t),\mathbf{I}_h(t),\mathbf{R}_h(t),\mathbf{W}(t))\Big],\\ \mathbf{I}_h(t) &= \mathbf{I}_h(0) +^{CF}\mathcal{I}^q\Big[\mathcal{G}_2(t,\mathbf{S}_h(t),\mathbf{I}_h(t),\mathbf{R}_h(t),\mathbf{W}(t))\Big],\\ \mathbf{R}_h(t) &= \mathbf{R}_h(0) +^{CF}\mathcal{I}^q\Big[\mathcal{G}_3(t,\mathbf{S}_h(t),\mathbf{I}_h(t),\mathbf{R}_h(t),\mathbf{W}(t))\Big],\\ \mathbf{W}(t) &= \mathbf{W}(0) +^{CF}\mathcal{I}^q\Big[\mathcal{G}_4(t,\mathbf{S}_h(t),\mathbf{I}_h(t),\mathbf{R}_h(t),\mathbf{W}(t))\Big],\end{aligned} \quad (12)$$

By evaluating the right hand side of (12) and writing it in the form given below, we have

$$\begin{aligned}\mathcal{Y}(t) &= \mathcal{Y}_0(t) + [\Omega(t,\mathcal{Y}(t)) - \Omega_0(t)]\frac{1-q}{\mathcal{N}(q)}\\ &\quad + \frac{q}{\mathcal{N}(q)}\int_0^t \Omega(x,\mathcal{Y}(x))dx,\end{aligned} \quad (13)$$

where

$$\begin{cases}\mathcal{Y}(t) = (\mathbf{S}_h(t),\mathbf{I}_h(t),\mathbf{R}_h(t),\mathbf{W}(t))^T,\\ \mathcal{Y}_0(t) = (\mathbf{S}_h(0),\mathbf{I}_h(0),\mathbf{R}_h(0),\mathbf{W}(0))^T,\\ \Omega(t,\mathcal{Y}(t)) = (\mathcal{G}_j(t,\mathbf{S}_h(t),\mathbf{I}_h(t),\mathbf{R}_h(t),\mathbf{W}(t)))^T, \quad j=1,2,3,4.\end{cases} \quad (14)$$

Let us define the Picard's operator as

$$\mathbf{A}: C(\mathbf{V},\mathbf{V}_1,\mathbf{V}_2,\mathbf{V}_3,\mathbf{V}_4) \to C(\mathbf{V},\mathbf{V}_1,\mathbf{V}_2,\mathbf{V}_3,\mathbf{V}_4). \quad (15)$$

With the help of (13) and (14), the operator in (15) is defined as

$$\begin{aligned}\mathbf{A}\mathcal{Y}(t) &= \mathcal{Y}_0(t) + [\Omega(t,\mathcal{Y}(t)) - \Omega_0(t)]\frac{1-q}{\mathcal{N}(q)}\\ &\quad + \frac{q}{\mathcal{N}(q)}\int_0^t \Omega(x,\mathcal{Y}(x))dx.\end{aligned} \quad (16)$$

Assuming that the model under investigation satisfies

$$\|\mathcal{Y}\| \leq \max\{d_1,d_2,d_3,d_4\}, \quad (17)$$

one may be able to write

$$\begin{aligned}\|\mathbf{A}\mathcal{Y}-\mathcal{Y}_0(t)\| &= \sup_{t\in D}\left|\Omega(t,\mathcal{Y}(t))\frac{1-q}{\mathcal{N}(q)}+\frac{q}{\mathcal{N}(q)}\int_0^t\Omega(x,\mathcal{Y}(x))dx\right|, \leq \frac{1-q}{\mathcal{N}(q)}\sup_{t\in D}|\Omega(t,\mathcal{Y}(t))|+\frac{q}{\mathcal{N}(q)}\sup_{t\in D}\int_0^t|\Omega(x,\mathcal{Y}(x))|dx,\\ &\leq \frac{1+t_0}{\mathcal{N}(q)}\mathcal{M}, \quad \mathcal{M}=\max\{\mathcal{M}_j\} \quad \text{for} \quad j=1,2,...,7,\\ &< \mathcal{M}d \leq \max\{d_1,d_2,d_3,d_4,d_5,d_6,d_7\}=\overline{d}, \quad t_0=\sup\{|t|: t\in D\}.\end{aligned} \tag{18}$$

In Eq. (18) we have defined $d=\frac{1+t_0}{\mathcal{N}(q)}$. It follows that

$$d<\frac{\overline{d}}{\mathcal{M}}.$$

Furthermore to evaluate the equality given by

$$\|\mathbf{A}\mathcal{Y}_1-\mathbf{A}\mathcal{Y}_2\|=\sup_{t\in D}|\mathcal{Y}_1-\mathcal{Y}_2|, \tag{19}$$

we make use of (13) and can write

$$\begin{aligned}\|\mathbf{A}\mathcal{Y}_1-\mathbf{A}\mathcal{Y}_2\| &= \sup_{t\in D}\left|\frac{1-q}{\mathcal{N}(q)}(\Omega(t,\mathcal{Y}_1(t)))-\Omega(t,\mathcal{Y}_2(t))+\frac{q}{\mathcal{N}(q)}\int_0^t(\Omega(x,\mathcal{Y}_1(x)))-\Omega(x,\mathcal{Y}_2(x))dx\right|,\\ &\leq \frac{1-q}{\mathcal{N}(q)}k\sup_{t\in D}|\mathcal{Y}_1(t)-\mathcal{Y}_2(t)|+\frac{qk}{\mathcal{N}(q)}\sup_{t\in D}\int_0^t|\mathcal{Y}_1(x)-\mathcal{Y}_2(x)|dx, \quad \text{with} \quad k<1,\\ &\leq \left[\frac{1+t_0}{\mathcal{N}(q)}\right]k\|\mathcal{Y}_1-\mathcal{Y}_2\|, \leq dk\|\mathcal{Y}_1-\mathcal{Y}_2\|.\end{aligned} \tag{20}$$

Since Ω is a contraction, consequently $dk<1$ and thus one may deduce that the operator $\mathbf{A}$ is also a contraction. This implies the uniqueness of solution of the system (12).

5. Analytical solution of model (3)

In the following, we compute series solution of our model. Applying Laplace transform on both side of model (3) and exploiting the normalization value onwards $\mathcal{M}(q)=1$, one may write

$$\begin{aligned}\mathcal{L}[\mathbf{S}_h(t)] &= \frac{\mathbf{S}(0)}{s}+\frac{s+q(1-s)}{s}\mathcal{L}[\Pi^{*q}-(\beta_1^{*q}\mathbf{I}_h(t)+\beta_2^{*q}\mathbf{W}(t)+d^{*q})\mathbf{S}_h(t)],\\ \mathcal{L}[\mathbf{I}_h(t)] &= \frac{\mathbf{I}_h(0)}{s}+\frac{s+q(1-s)}{s}\mathcal{L}[(\beta_1^{*q}\mathbf{I}_h(t)+\beta_2^{*q}\mathbf{W}(t)+d^{*q})\mathbf{S}_h(t)-(\sigma^{*q}+d^{*q}+d_1^{*q})\mathbf{I}_h(t)],\\ \mathcal{L}[\mathbf{R}_h(t)] &= \frac{\mathbf{R}_h(0)}{s}+\frac{s+q(1-s)}{s}\mathcal{L}[\sigma^{*q}\mathbf{I}_h(t)-d^{*q}\mathbf{R}_h(t)],\\ \mathcal{L}[\mathbf{W}(t)] &= \frac{\mathbf{W}(0)}{s}+\frac{s+q(1-s)}{s}\mathcal{L}[\alpha^{*q}\mathbf{I}_h(t)-\eta^{*q}\mathbf{W}(t)].\end{aligned} \tag{21}$$

In the following, we find solutions for each class of the model under consideration in the series form as

$$\mathbf{S}(t)=\sum_{q=0}^{\infty}\mathbf{S}_q(t),\ \mathbf{I}(t)=\sum_{q=0}^{\infty}\mathbf{I}_q(t),\ \mathbf{R}(t)=\sum_{q=0}^{\infty}\mathbf{R}_q(t),\ \mathbf{W}(t)=\sum_{q=0}^{\infty}\mathbf{W}_q(t). \tag{22}$$

The nonlinear terms $\mathbf{I}_h(t)\mathbf{S}(t), \mathbf{W}(t)\mathbf{S}(t)$ can be decomposed by using the Adomian polynomials technique as

$$\mathbf{I}(t)\mathbf{S}(t)=\sum_{q=0}^{\infty}\mathbf{X}_q(\mathbf{I},\mathbf{S}), \quad \mathbf{W}(t)\mathbf{S}(t)=\sum_{q=0}^{\infty}\mathbf{Y}_q(\mathbf{W},\mathbf{S}), \tag{23}$$

where the Adomian polynomial $\mathbf{X}_q(\mathbf{I},\mathbf{S}), \mathbf{Y}_q(\mathbf{W},\mathbf{S})$ can be written as

$$\begin{aligned}\mathbf{X}_q(\mathbf{I},\mathbf{S}) &= \frac{1}{q!}\frac{d^q}{d\lambda^q}\left[\sum_{k=0}^{q}\lambda^k\mathbf{I}_k(t)\sum_{k=0}^{q}\lambda^k\mathbf{S}_k(t)\right]_{\lambda=0}\\ \mathbf{Y}_q(\mathbf{W},\mathbf{S}) &= \frac{1}{q!}\frac{d^q}{d\lambda^q}\left[\sum_{k=0}^{q}\lambda^k\mathbf{W}_k(t)\sum_{k=0}^{q}\lambda^k\mathbf{S}_k(t)\right]_{\lambda=0}\end{aligned}$$

Using of Eqs. (22) and (23), the system (21) takes the following form

$$\begin{aligned}
&\mathcal{L}\Big[\textstyle\sum_{q=0}^{\infty}\mathbf{S}_q(t)\Big]=\frac{\mathbf{S}(0)}{s}+\frac{s+q(1-s)}{s}\mathcal{L}\Big[\Pi^{*q}-\beta_1^{*q}\sum_{q=0}^{\infty}\mathbf{X}_q(t)-\beta_2^{*q}\sum_{q=0}^{\infty}\mathbf{Y}_q(t)-d^{*q}\sum_{q=0}^{\infty}\mathbf{S}_q(t)\Big],\\
&\mathcal{L}\Big[\textstyle\sum_{q=0}^{\infty}\mathbf{I}_q(t)\Big]=\frac{\mathbf{I}(0)}{s}+\frac{s+q(1-s)}{s}\mathcal{L}\Big[\beta_1^{*q}\sum_{q=0}^{\infty}\mathbf{X}_q(t)+\beta_2^{*q}\sum_{q=0}^{\infty}\mathbf{Y}_q(t)+d^{*q}\sum_{q=0}^{\infty}\mathbf{S}_q(t)-(\sigma^{*q}+d^{*q}+d_1^{*q})\sum_{q=0}^{\infty}\mathbf{I}_q(t)\Big],\\
&\mathcal{L}\Big[\textstyle\sum_{q=0}^{\infty}\mathbf{R}_q(t)\Big]=\frac{\mathbf{R}(0)}{s}+\frac{s+q(1-s)}{s}\mathcal{L}\Big[\sigma^{*q}\sum_{q=0}^{\infty}\mathbf{I}_q(t)-d^{*q}\sum_{q=0}^{\infty}\mathbf{R}_q(t)\Big],\\
&\mathcal{L}\Big[\textstyle\sum_{q=0}^{\infty}\mathbf{W}_q(t)\Big]=\frac{\mathbf{W}(0)}{s}+\frac{s+q(1-s)}{s}\mathcal{L}\Big[\alpha^{*q}\sum_{q=0}^{\infty}\mathbf{I}_q(t)-\eta^{*q}\sum_{q=0}^{\infty}\mathbf{W}_q(t)\Big].
\end{aligned}\tag{24}$$

Equating like terms on both side of (24), we can write

$$\begin{aligned}
&\mathcal{L}[\mathbf{S}_0(t)]=\frac{\mathbf{S}_0}{s},\quad \mathcal{L}[\mathbf{I}_0(t)]=\frac{\mathbf{I}_0}{s},\quad \mathcal{L}[\mathbf{R}_0(t)]=\frac{\mathbf{R}_0}{s},\quad \mathcal{L}[\mathbf{W}_0(t)]=\frac{\mathbf{W}_0}{s},\\
&\mathcal{L}[\mathbf{S}_1(t)]=\frac{s+q(1-s)}{s}\mathcal{L}\Big[\Pi^{*q}-\beta_1^{*q}\mathbf{X}_0(t)-\beta_2^{*q}\mathbf{Y}_0(t)-d^{*q}\mathbf{S}_0(t)\Big],\\
&\mathcal{L}[\mathbf{I}_1(t)]=\frac{s+q(1-s)}{s}\mathcal{L}\Big[\beta_1^{*q}\mathbf{X}_0(t)+\beta_2^{*q}\mathbf{Y}_0(t)+d^{*q}\mathbf{S}_0(t)-(\sigma^{*q}+d^{*q}+d_1^{*q})\mathbf{I}_0(t)\Big],\\
&\mathcal{L}[\mathbf{R}_1(t)]=\frac{s+q(1-s)}{s}\mathcal{L}\Big[\sigma^{*q}\mathbf{I}_0(t)-d^{*q}\mathbf{R}_0(t)\Big],\\
&\mathcal{L}[\mathbf{W}_1(t)]=\frac{s+q(1-s)}{s}\mathcal{L}\Big[\alpha^{*q}\mathbf{I}_0(t)-\eta^{*q}\mathbf{W}_0(t)\Big],\\
&\qquad\vdots\\
&\mathcal{L}[\mathbf{S}_{q+1}(t)]=\frac{s+q(1-s)}{s}\mathcal{L}\Big[\Pi^{*q}-\beta_1^{*q}\mathbf{X}_q(t)-\beta_2^{*q}\mathbf{Y}_q(t)-d^{*q}\mathbf{S}_q(t)\Big],\\
&\mathcal{L}[\mathbf{I}_{q+1}(t)]=\frac{s+q(1-s)}{s}\mathcal{L}\Big[\beta_1^{*q}\mathbf{X}_q(t)+\beta_2^{*q}\mathbf{Y}_q(t)+d^{*q}\mathbf{S}_q(t)-(\sigma^{*q}+d^{*q}+d_1^{*q})\mathbf{I}_q(t)\Big],\\
&\mathcal{L}[\mathbf{R}_{q+1}(t)]=\frac{s+q(1-s)}{s}\mathcal{L}\Big[\sigma^{*q}\mathbf{I}_q(t)-d^{*q}\mathbf{R}_q(t)\Big],\\
&\mathcal{L}[\mathbf{W}_{q+1}(t)]=\frac{s+q(1-s)}{s}\mathcal{L}\Big[\alpha^{*q}\mathbf{I}_q(t)-\eta^{*q}\mathbf{W}_q(t)\Big].
\end{aligned}\tag{25}$$

Now, we compute the Laplace transform of (25), and obtain

$$\begin{aligned}
&\mathbf{S}_0=\mathbf{N}_1,\quad \mathbf{I}_0=\mathbf{N}_2,\quad \mathbf{R}_0=\mathbf{N}_3,\quad \mathbf{W}_0=\mathbf{N}_4,\\
&\mathbf{S}_1=\Big[\Pi^{*q}-\beta_1^{*q}\mathbf{N}_1\mathbf{N}_2-\beta_2^{*q}\mathbf{N}_1\mathbf{N}_4-d^{*q}\mathbf{N}_1\Big](1+q(t-1)),\\
&\mathbf{I}_1=\Big[\beta_1^{*q}\mathbf{N}_1\mathbf{N}_2+\beta_2^{*q}\mathbf{N}_1\mathbf{N}_4+d^{*q}\mathbf{N}_1-(\sigma^{*q}+d^{*q}+d_1^{*q})\mathbf{N}_2\Big](1+q(t-1)),\\
&\mathbf{R}_1=\Big[\sigma^{*q}\mathbf{N}_2-d^{*q}\mathbf{N}_3\Big](1+q(t-1)),\\
&\mathbf{W}_1=\Big[\alpha^{*q}\mathbf{N}_2-\eta^{*q}\mathbf{N}_4\Big](1+q(t-1)),\\
&\mathbf{S}_2=\Pi^{*q}(1+q(t-1))-\Big(\beta_1^{*q}(\mathbf{N}_2s_{11}+\mathbf{N}_1i_{11})+\beta_2^{*q}(\mathbf{N}_4s_{11}\mathbf{N}_1w_{11})+d^{*q}s_{11}\Big)\\
&\qquad\times\Big(\tfrac{1}{2}q^2t^2-2q^2t+2qt+(q-1)^2\Big),\\
&\mathbf{I}_2=\Big(\beta_1^{*q}(\mathbf{N}_2s_{11}+\mathbf{N}_1i_{11})+\beta_2^{*q}(\mathbf{N}_4s_{11}\mathbf{N}_1w_{11})+d^{*q}s_{11}-(\sigma^{*q}+d^{*q}+d_1^{*q})i_{11}\Big)\\
&\qquad\times\Big(\tfrac{1}{2}q^2t^2-2q^2t+2qt+(q-1)^2\Big),\\
&\mathbf{R}_2=\Big(\sigma^{*q}i_{11}-d^{*q}r_{11}\Big)\Big(\tfrac{1}{2}q^2t^2-2q^2t+2qt+(q-1)^2\Big),\\
&\mathbf{W}_2=\Big(\alpha^{*q}i_{11}-\eta^{*q}w_{11}\Big)\Big(\tfrac{1}{2}q^2t^2-2q^2t+2qt+(q-1)^2\Big).
\end{aligned}\tag{26}$$

Table 1
Parametric values for our model (1).

Parameter	Value
Π^{*q}	0.093907997
β_1^{*q}	0.00005
β_2^{*q}	0.0000000123
d^{*q}	0.009567816
σ^{*q}	0.09871
d_1^{*q}	0.00404720925
α^{*q}	0.0398
η^{*q}	0.01

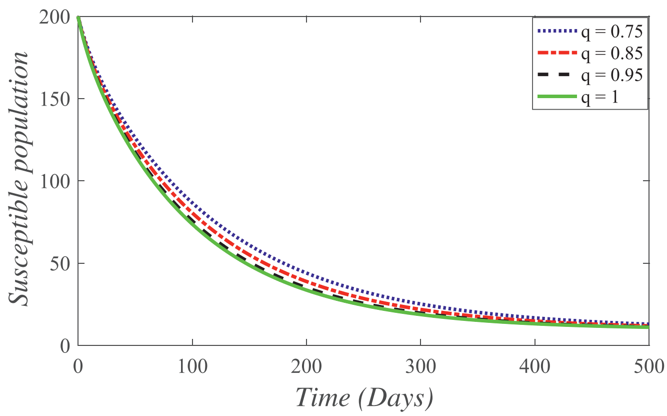

Fig. 1. Plot of the compartment $\mathbf{S}(t)$ in the model under investigation (3) at different arbitrary fractional orders.

Table 2
Parametric values for our model (1).

Parameter	Value
Π^{*q}	0.93907997
β_1^{*q}	0.0005
β_2^{*q}	0.0000123
d^{*q}	0.009567816
σ^{*q}	0.009871
d_1^{*q}	0.00404720925
α^{*q}	0.398
η^{*q}	0.01

Following the same steps, further terms of the series solution can be determined. The unknown values in the above Eq. (26) are calculated as

$$\begin{aligned} s_{11} &= \Pi^{*q} - \beta_1^{*q}\mathbf{N}_1\mathbf{N}_2 - \beta_2^{*q}\mathbf{N}_1\mathbf{N}_4 - d^{*q}\mathbf{N}_1, \\ i_{11} &= \beta_1^{*q}\mathbf{N}_1\mathbf{N}_2 + \beta_2^{*q}\mathbf{N}_1\mathbf{N}_4 + d^{*q}\mathbf{N}_1 - (\sigma^{*q} + d^{*q} + d_1^{*q})\mathbf{N}_2, \\ r_{11} &= \sigma^{*q}\mathbf{N}_2 - d^{*q}\mathbf{N}_3, \\ w_{11} &= \alpha^{*q}\mathbf{N}_2 - \eta^{*q}\mathbf{N}_4. \end{aligned} \tag{27}$$

6. Numerical Simulations and Discussion

This particular part of our article is specified to execute numerical simulations to verify the analytical findings in the above sections. First we present the semi-analytical solution by the aid of Laplace Adomian decomposition method to the first three terms of each compartment in (3) along with the data in Table 1 with different order derivatives. The results have been depicted in Figs. 1–4. Moving on the same line we consider values form Table 2 and plot our results in Figs. 5–8.

Fig. 1 shows the susceptible class and have been simulated for time 0 – 500, where one can observe rapid decrease in the class.

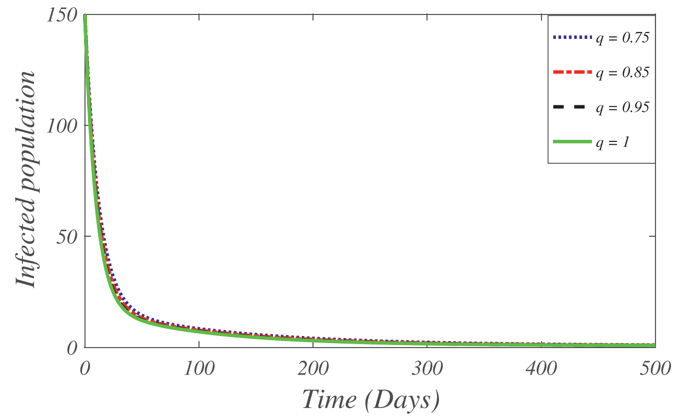

Fig. 2. Plot of the compartment $\mathbf{I}(t)$ in the model under investigation (3) at different arbitrary fractional orders.

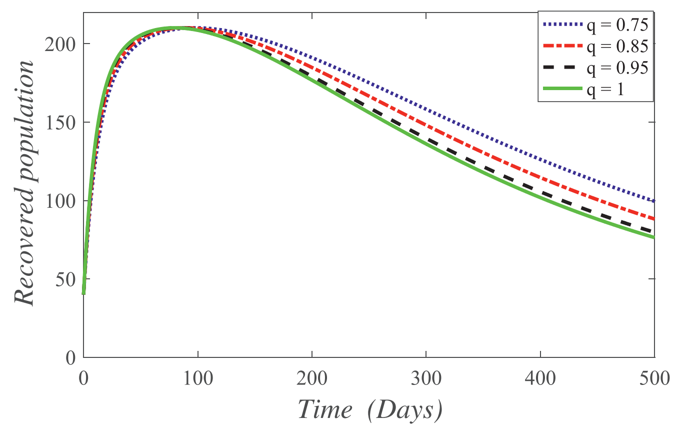

Fig. 3. Plot of the compartment $\mathbf{R}(t)$ in the model under investigation (3) at different arbitrary fractional orders.

The solutions are convergent and become stable with the course of time for different arbitrary order of q.

Similarly Fig. 2 shows the integral curves for the infected class. Similar to the susceptible class, one can observe rapid decrease in the infected class at different fractional orders. This class shows stability and convergence with the passage of time.

Fig. 3 is the recovered class at various fractional order of q. In contrast to the above two classes, this class shows rapid increase at the beginning showing that more people have been infected and hospitalized. Withe the passage of time this growth decreases indicating that following the precautionary measures a quick recovery occurs and the compartment becomes stable.

Fig. 4 is the reservoir class showing a small increase in the initial stage and reach the peak point afterwards. Decrease in the class can be observed after short interval of time and the solution becomes stable later on.

In Figs. 5–8 all the four classes have been plotted for time t in range 0 – 500, considering data of Table 2.

Fig. 5 is the susceptible class showing rapid decrease in the beginning and then a gradually increase for short interval of time to attain the convergence and stability with the passage of time for different arbitrary order of q.

Fig. 6 represents the infected class which shows a small increase up-to the peak value and then a rapid decrease at different fractional orders occurred, predicting the controlling of the pandemic. This class also shows stability and convergence with the passage of time.

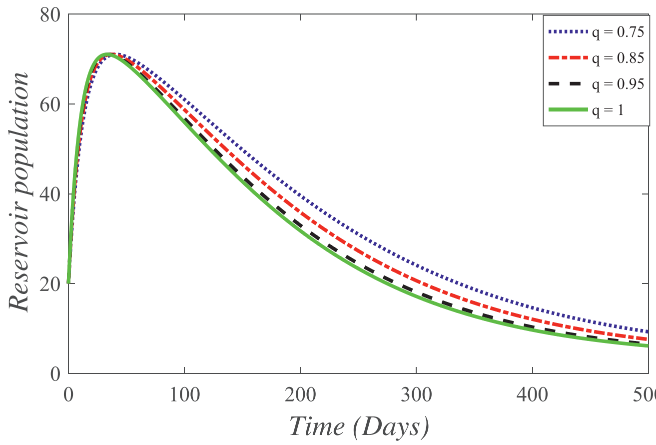

Fig. 4. Plot of the compartment $\mathbf{W}(t)$ in the model under investigation (3) at different arbitrary fractional orders.

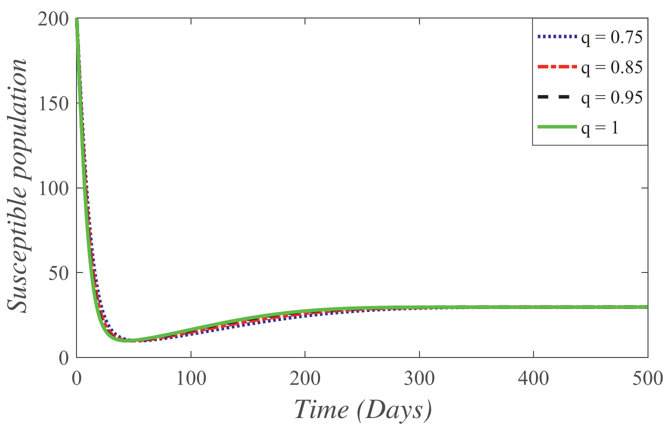

Fig. 5. Plot of the compartment $\mathbf{S}(t)$ in the model under investigation (3) at different arbitrary fractional orders.

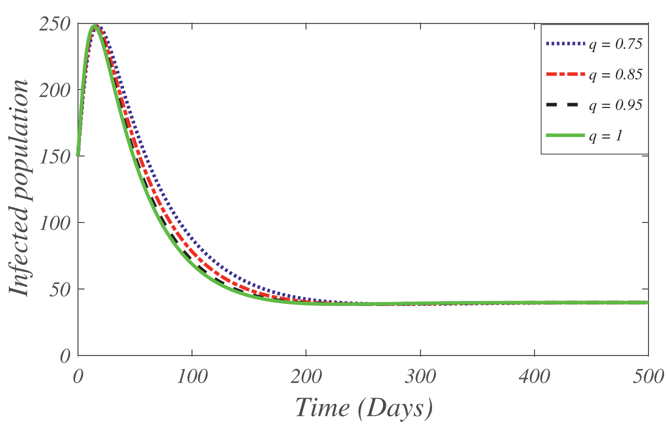

Fig. 6. Plot of the compartment $\mathbf{I}(t)$ in the model under investigation (3) at different arbitrary fractional orders.

Fig. 7 is the recovered class at various fractional order of q. After some time the recovery cases reaches the peak and then becomes stable.

Fig. 8 is the reservoir class showing a very large increases in the initial stages and reaching the peak point. This shows that more people may be exposed to the disease in the coming days if strong precautionary measures are not adopted. After that, the reservoir class decreases for short interval of time and then becomes stable.

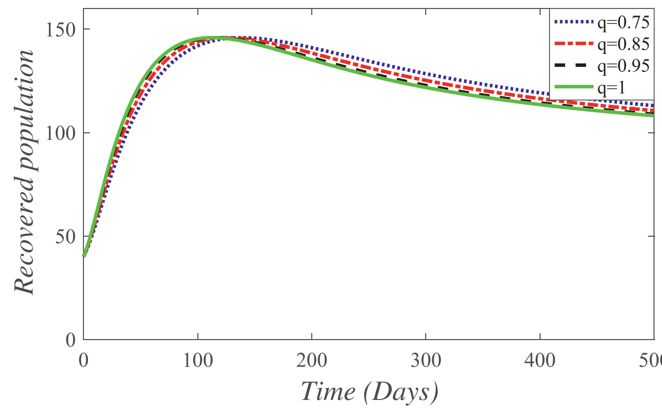

Fig. 7. Plot of the compartment $\mathbf{R}(t)$ in the model under investigation (3) at different arbitrary fractional orders.

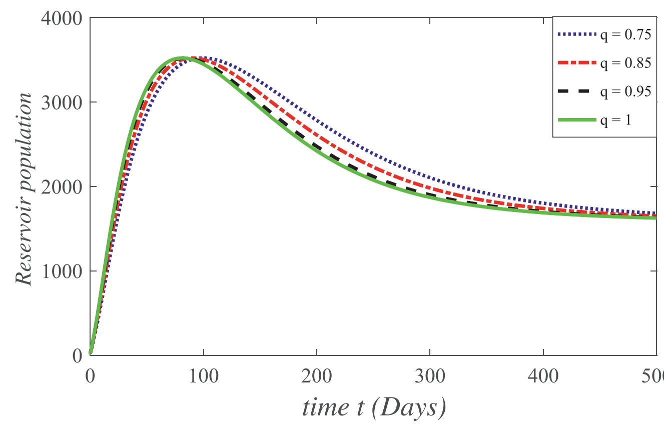

Fig. 8. Plot of the compartment $\mathbf{W}(t)$ in the model under investigation (3) at different arbitrary fractional orders.

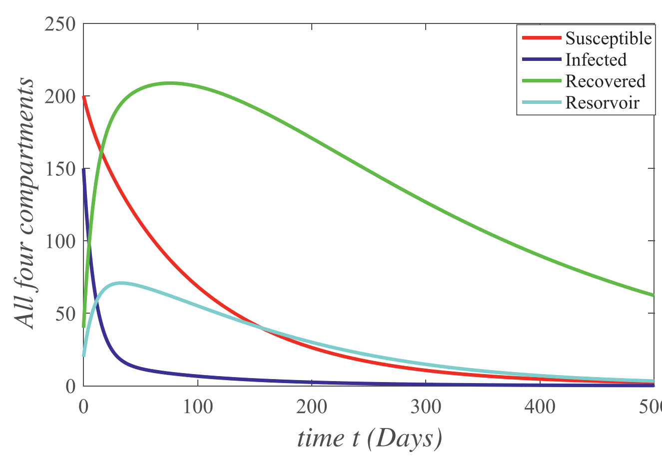

Fig. 9. Plot of compartments in our fractional Covid-19 model (3) for integer order of $q = 1$ for the first three terms of the iterative series.

Fig. 9 represents all the four compartments of model 3 at integer order $q = 1$ considering the data of Table 2. It can be easily concluded that derivatives with fractional order provides better accuracy compared with the traditional derivatives of integer order.

7. Conclusions

In this manuscript, we investigated a mathematical model for coronavirus COVID-19 under CF fractional order derivative. The equilibria and the basic reproductive number along with the qualitative analysis have been discussed for the proposed model. With the aid of fixed point theory approach, the existence as well as uniqueness of the concerned solution have been investigated. By exploiting the Laplace Adomian decomposition technique, an approximate solution the model under consideration has been investigated. We have performed numerical simulations for our model and discussed briefly for various values of the fractional order derivatives. Our graphical representations reflect that the fractional order derivative of CF type provide more realistic analysis than the classical integer-order COVID-19 model.

Optimal control strategies are helpful to the health department for the future prediction of an infectious disease. Optimal control can be used for our model under consideration. Work on this issue is under process and will be reported in a future publication.

Declaration of Competing Interest

All authors have no conflict of interest regarding the publication of this paper.

CRediT authorship contribution statement

Mati ur Rahman: Conceptualization, Formal analysis, Investigation, Methodology, Resources, Software, Writing - original draft, Writing - review & editing. **Saeed Ahmad:** Conceptualization, Formal analysis, Investigation, Methodology, Resources, Software, Writing - original draft, Writing - review & editing. **R.T. Matoog:** Conceptualization, Formal analysis, Funding acquisition, Investigation, Methodology, Project administration, Resources, Software, Validation, Visualization, Supervision, Writing - original draft, Writing - review & editing. **Nawal A. Alshehri:** Conceptualization, Formal analysis, Investigation, Methodology, Resources, Software, Writing - original draft, Writing - review & editing. **Tahir Khan:** Conceptualization, Formal analysis, Investigation, Methodology, Resources, Software, Writing - original draft, Writing - review & editing.

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