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ORIGINAL RESEARCH

Cryptocurrency volatility forecasting: What can we learn from the first wave of the COVID-19 outbreak?

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Abstract

This study aims to examine the issue of cryptocurrency volatility modelling and forecasting based on high-frequency data. More specifically, this study assesses whether crisis periods, particularly the coronavirus disease pandemic, influence the dynamic of cryptocurrency volatility. We investigate the four main cryptocurrency markets (Bitcoin, Ethereum Classic, Ethereum, and Ripple) from April 2018 to June 2020. The realized volatility measure is computed and decomposed to various components (continuous versus discontinuous, positive and negative semi-variances, and signed jumps). A variety of heterogeneous autoregressive (HAR) models are developed including these components, thereby enabling assessment of different assumptions (including persistence and asymmetric dynamic) of modelling and volatility forecasting based on in-sample and out-of-sample forecasting strategies, respectively. Our results reveal three main findings. First, the extended HAR model that includes the positive and negative jumps appears to be the best model for predicting future volatility for both crisis and non-crisis periods. Second, during the crisis period, only the negative jump component is statistically significant. Third, in terms of volatility forecasting, the results show that the extended HAR model that includes positive and negative semi-variances outperform the other models.

Keywords In-sample forecasting · Realized volatility · Out-of-sample forecasting · Semi-variances · signed jump

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1 Introduction

The cryptocurrency market has gained traction among media, policymakers, investors, academicians, and regulators, especially since the “bubble” experienced in 2017. This market has also been a source of criticism and debate regarding its specific stylized facts, notably revived after the contradictory positions of popular Nobel laureates Joseph Stiglitz and Robert Shiller. The former suggested that Bitcoin ought to be outlawed whereas the latter highlighted that investor interest in the cryptocurrency market was due to its anti-government, anti-regulation fee. From an academic perspective, there is a huge debate regarding whether cryptocurrency might be considered a currency or an asset (Yuneline, 2019; White et al., 2020; among others) as the prices have exhibited a significant surge since 2017, accompanied by high volatility. For example, the Bitcoin price remained below $1000 before February 2017; however, it reached up to $20,000 in December 2017, subsequently declining to around $8000 in February 2018. It rose again in May 2018 to reach $13,000 and fell rapidly to around $3000 in December 2018. During 2019, Bitcoin prices averaged at around $7000. Since 2020 and during the coronavirus disease (COVID-19) pandemic, another indication of a price bubble has been observed in the Bitcoin market, as is the prices have been around $24,000 (the highest since its introduction) in December 2020.

This surge in Bitcoin prices, observed during the COVID-19 pandemic, has been accompanied by a significant amount of literature investigating whether cryptocurrencies, particularly the bitcoin, might serve as a refuge during a period of turmoil, such as the ongoing health crisis (Huynh et al., 2020; Paule-Vianez et al., 2020; Thampanya et al., 2020; Mnif et al., 2020; Madani et al., 2021, among others). In this context, it might be important to analyse and propose further insights in terms of volatility modelling and cryptocurrency market forecasting, especially during crisis periods such as the COVID-19 pandemic, allowing investors and hedgers to minimize risks through portfolio diversification and develop appropriate hedging positions, and assist policymakers in formulating regulatory policies by refining the asset prices volatility prediction for risk assessment. This study aims to contribute to the literature on cryptocurrency market volatility modelling and forecasting, particularly during crisis periods.

The empirical literature related to cryptocurrency volatility modelling and forecasting is abundant, with a strand of literature adopting the classical time series models, particularly the generalized autoregressive conditional heteroscedasticity (GARCH) family of models. In this literature, some studies investigated the cryptocurrency volatility modelling based on the in-sample forecasting strategy, (Balcilar et al., 2017; Charles & Darné 2019; Cheikh et al., 2020; Chu et al., 2017; Conrad et al., 2018; Dyhrberg, 2016; Huynh et al., 2020; Katsiampa, 2017; Naimy & Hayek, 2018; Pichl & Kaizoji, 2017; Gyamerah, 2019; Tiwari et al., 2019, among others), and some assessed volatility forecasting based on out-of-sample strategy for a specific forecasting horizon (Bezerra & Albuquerque, 2017; Catani et al., 2019; Naimy & Hayek, 2018; Peng et al., 2018; Xiao & Sun, 2020, among others). This corpus uses the conventional time series models like the GARCH family models, which were extended recently in light of outliers that characterize cryptocurrency markets (Aslan & Sensoy, 2020; Charles & Darné, 2019; Catani et al., 2019; Trucíos, 2019, among others). A second subset of the literature involves approaches inspired by operations research, such as neural networks (Adcock & Gradojevic, 2019; Jay et al., 2020; among others), machine learning, and deep learning (Lahmiri & Bekiros, 2019; Patel et al., 2020; Akyildirim et al., 2020, 2021; Sensoy, 2019; among others).

A new strand of literature highlighted the role of high frequency data (HFD) in improving modelling and forecasting volatility (Bollerslev et al., 2020; Patton & Sheppard, 2015). Notably, using HFD has several advantages. First, with this kind of data, we might use an observed measure of the volatility and not a proxy, which might reduce measurement errors. Second, HFD offers more useful information for predicting financial asset volatility. Third, HFD provides the advantage of disentangling continuous and discontinuous components of volatility, which might improve forecasts. Fourth, distinguishing between positive and negative returns as well as using all signed information allows us to consider the leverage effect that may improve volatility forecasting. As far as we know, only a few studies have employed HFD to model and forecast cryptocurrency volatility. Peng et al. (2018) used both daily and hourly data to forecast the volatility of three cryptocurrencies (Bitcoin, Ethereum, Dash) and three currencies (Euro, British pound, and Japanese yen in US dollars) based on traditional GARCH family models and a combination of the traditional GARCH model and the machine learning approach to volatility estimation. Their results favour the support vector regression–GARCH model. The heterogenous autoregressive (HAR) model proposed by Corsi (2009) was employed by Hu et al. (2019) to model and forecast Bitcoin volatility using HFD. The authors assessed the power of prediction of the different components of realized volatility (RV) and showed that the future RV has a positive relationship with downside risk and a negative relationship with the positive jump. They also showed that jump and signed jumps improve volatility forecasting only in long horizons. Yu (2019) employed HFD to forecast the Bitcoin volatility by considering leverage effects and economic policy uncertainty (EPU). It was found that the leverage effect might impact future volatility significantly. However, jumps and EPU seem not to impact future volatility during the in-sample period. The out-of-sample strategy confirms the superiority of the leverage effect model compared to the model including jumps. Shen et al. (2020) investigated the Bitcoin volatility model using HFD and different HAR models. The results favour including jump components to forecast Bitcoin volatility.

Our study relates to this last subset of literature that uses HFD to model and forecast the volatility of cryptocurrency markets.[1] More interestingly, we aim to contribute to the literature on cryptocurrency volatility modelling and forecasting with HFD in at least three aspects. First, as far as we know, our study is the first to assess the effect of the COVID-19 crisis on the dynamic of volatility modelling and forecasting, comparing with the pre-COVID-19 period using HFD. The underlying issue dealt with studies on bad and good volatility by Patton and Sheppard (2015) and Bollerslev et al. (2020), as during periods of turmoil, the negative returns are relatively more frequent than positive ones. We assess whether the signed information (positive and negative semi-variances, and signed jumps) may be useful during crisis periods in improving modelling and forecasting the cryptocurrency market volatility. Second, contrary to the major cited works dealing with HFD, our study investigates the four main cryptocurrency markets—Bitcoin, Ethereum (ETH), Ethereum Classic (ETC), and Ripple (XRP)—having 79% of cryptocurrency capitalization. Third, our study generates insights for both modelling and forecasting cryptocurrency volatility by employing in-sample and out-of-sample forecasting strategies.

[1] One of the main stylized facts of the Cryptocurrency markets is the huge volatility, which may affect the hedging strategies of investors, or their portfolio allocation. Therefore, dealing with volatility in our study allows us to propose some useful risk management insights.

We use HFD sampled every five minutes[2] for four cryptocurrency markets ranged from April 2018 to June 2020. This period allows us to assess the effect of the first wave of COVID-19. We decompose RV into continuous and discontinuous parts, and subsequently, into positive and negative semi-variance, to assess the potential asymmetric dynamic in volatility modelling and forecasting. Furthermore, we compute the signed jump based on the signed information (positive and negative returns). These components are incorporated in the five candidate models inspired by Corsi's (2009) HAR model. The results of cryptocurrency volatility modelling show that the future RV for all markets being studied, whatever the period of analysis, is better explained with the HAR model, and extended with positive and negative jumps. Our results during the COVID-19 crisis period demonstrate that the studied markets' future RV is only sensitive to negative jumps. The out-of-sample forecasting strategy supports that the best fit model for cryptocurrency volatility is one that includes positive and negative semi-variances, highlighting the asymmetric dynamic of cryptocurrency markets during crisis periods.

Our study contributes to the literature in several ways. First, we propose new insights for cryptocurrency volatility modelling and forecasting by using high frequency data and by controlling for different volatility types (continuous, jump, positive jump, negative jumps, etc.). Second, we consider that during the crisis period only bad volatility will drive the dynamics of the future volatility. Third, our study pioneers an empirical evidence supporting the investors' sensitivity to bad news during a period of turmoil.

The remainder of this paper is organized as follows. Section 2 presents the empirical methodology. Section 3 presents the data and descriptive statistics. Section 4 describes the empirical framework and discusses the results. Section 5 concludes the paper.

2 Empirical methodology

In this section, we present the methodology used in our study to identify the best fit model for cryptocurrency market volatility forecasting. Specifically, before discussing the candidate models, we present their different components (the decomposition of the RV measure into its continuous and discontinuous components, the decomposition of the variance into positive and negative semi-variances, and the decomposition between positive and negative jumps). Subsequently, we present the competing models for cryptocurrency market volatility forecasting. Finally, we present the forecasting strategy used to disentangle these models.

2.1 The intraday volatility measure

The empirical literature has proposed various volatility measures (Ftiti & Jawadi, 2019). Contrary to the traditional measures defined as proxies of an unobserved measure of the variability of a time series, the realized volatility (RV) measure introduced by Andersen and Bollerslev (1998) is a measure of the observed variability, includes more information, and reduces measurement errors (Andersen & Bollerslev, 1998; Ftiti et al., 2016). RV is defined as the sum of intraday sampled Δ-period squared returns $r^2_{t+j\Delta,\Delta}$, as given below:

[2] The 5-min sampling frequency is considered the optimal frequency to compute realized bi-power and tri-power variation measures (Bandi and Russell, 2004a, 2004b; Hansen and Lunde, 2006; Zhang et al., 2005).

$$RV_{t+1}(\Delta) = \sum_{j=1}^{\frac{1}{\Delta}} r^2_{t+j\Delta,\Delta} \tag{1}$$

2.2 Decomposition of RV into continuous and discontinuous components

The decomposition of RV into continuous and jump components is based on jump detection, for which various tests have been proposed in the literature. In this study, we use jump identification methodology of Andersen et al. (2007).

Formally, the decomposition of the related volatility between continuous and jump components is based on the concept of bi-power variation (BV), introduced by Barndorff-Nielsen and Shephard (2004) and defined as follows:

$$BV_{t+1}(\Delta) = \mu_1^{-2} \sum_{j=2}^{\frac{1}{\Delta}} \left| r_{t+j\Delta,\Delta} \right| \left| r_{t+(j-1)\Delta,\Delta} \right| \tag{2}$$

where $\mu_1 = \sqrt{2/\pi}$.

Theoretically, a consistent estimator of the jump (J) contribution to the quadratic variation process is defined based on the difference between the RV and the BV. Formally, the jump is defined as follows:

$$J_{t+1}(\Delta) = RV_{t+1}(\Delta) - BV_{t+1}(\Delta) \tag{3}$$

To identify statically significant jumps, we use the Z statistic proposed by Huang and Tauchen (2005), using the jump-robust realized tri-power quarticity (TQ)[3]:

$$J_{t+1}(\Delta) = RV_{t+1}(\Delta) - BV_{t+1}(\Delta) \tag{4}$$

$$TQ_{t+1}(\Delta) \equiv \Delta^{-1}\mu_{\frac{4}{3}}^{-3}(1-4\Delta)^{-1} \sum_{j=5}^{1/\Delta} \left| r_{t+j\Delta,\Delta} \right|^{4/3} \left| r_{t+(j-2)\Delta,\Delta} \right|^{4/3} \left| r_{t+(j-4)\Delta,\Delta} \right|^{4/3}. \tag{5}$$

$$\mu_{\frac{4}{3}} \equiv 2^{\frac{2}{3}}\Gamma\left(\frac{7}{6}\right) \cdot \Gamma\left(\frac{1}{2}\right)^{-1}. \tag{6}$$

If there is no jump, $Z_{t+1}(\Delta)$ follows a standard normal distribution. Following Andersen et al. (2007) and Giot et al. (2010), we use a significance level (α = 0.01%) to compute the jump and the continuous components as follows:

$$J_{t+1,\alpha}(\Delta) = I\left[Z_{t+1}(\Delta) > \varphi_\alpha\right] \times \left[RV_{t+1}(\Delta) - BV_{t+1}(\Delta)\right]. \tag{7}$$

$$C_{t+1,\alpha}(\Delta) = I\left[Z_{t+1}(\Delta) \le \Phi_\alpha\right] \times RV_{t+1}(\Delta) + I\left[Z_{t+1}(\Delta) > \varphi_\alpha\right] \times BV_{t+1}(\Delta). \tag{8}$$

[3] To control for microstructure frictions, we used staggered measures of bi-power variation (BV) and the tri-power quarticity (TQ) proposed by Huang and Tauchen (2005) and Andersen et al. (2007).

2.3 Decomposition of RV into positive and negative semi-variance

Barndorff-Nielsen et al. (2010) were the first to introduce the concept of realized semi-variance, intuitively inspired from the semi-variance estimators of Markowitz (1952). Notably, Patton and Sheppard (2015) and Bollerslev et al. (2020) highlighted the importance of this estimator for volatility modelling. The positive and the negative realized semi-variance estimators are based on disentangling positive and negative intraday returns. Formally, the positive and the negative realized semi-variance estimators, as defined by Bollerslev et al. (2020) and Patton and Sheppard (2015) are computed as follows:

$$RS^{+} = \sum_{i=1}^{n} r_i^2 I\left\{r_i > 0\right\} \tag{9}$$

$$RS^{-} = \sum_{i=1}^{n} r_i^2 I\left\{r_i < 0\right\} \tag{10}$$

2.4 The signed jump

Like the decomposition of RV into continuous and discontinuous components, the realized semi-variance might be decomposed to its continuous and jump components following Barndorff-Nielsen et al. (2010), as follows:

$$RS^{+} \xrightarrow{p} 1/2 \int_0^t \delta^2(s)ds + \sum_{0 \le s \le t} \Delta p_s^2 I\{\Delta ps > 0\} \tag{11}$$

$$RS^{-} \xrightarrow{p} 1/2 \int_0^t \delta^2(s)ds + \sum_{0 \le s \le t} \Delta p_s^2 I\{\Delta ps < 0\} \tag{12}$$

The signed jump is defined, based on Barndorff-Nielsen et al. (2010), as the difference between the downside realized semi-variance (RS^{+}) and the upside realized semi-variance (RS^{-}). Formally, the signed jump is presented as follows:

$$\Delta J^2 = RS^{+} - RS^{-} \xrightarrow{p} \sum\nolimits_{0 \le s \le t} \Delta p_s^2 I\{\Delta ps > 0\} - \sum\nolimits_{0 \le s \le t} \Delta p_s^2 I\{\Delta ps < 0\} \tag{13}$$

Also, we compute the positive (ΔJ^{2+}) and negative jumps (ΔJ^{2+}) as follows:

$$\Delta J^{2+} \equiv (RS^{+} - RS^{-}) I\left\{\left(RS^{+} - RS^{-}\right) > 0\right\} \tag{14}$$

$$\Delta J^{2-} \equiv (RS^{+} - RS^{-}) I\left\{\left(RS^{+} - RS^{-}\right) < 0\right\} \tag{15}$$

2.5 The candidate models for cryptocurrency market volatility forecasting

The cryptocurrency markets have experienced high variability, with the prices moving upside and downside. More specifically, some extreme variabilities are observed across trading days. Therefore, we aim to test whether the different components of RV as well as those of the semi-realized volatility improve volatility forecasting of the cryptocurrency markets. Interestingly, our objective involves assessing whether the jump, positive and negative semi-variance components and signed jump perform the volatility modelling and forecasting of the cryptocurrency markets.

The empirical literature is rich in terms of volatility modelling and forecasting of time-series. Here, we do not follow the literature based on the conventional models such as the GARCH family models but is based on the strand of literature related to the HAR-RV model introduced by Corsi (2009). The HAR-RV model provides the advantage of considering the investors' heterogeneity, enabling us to consider high-frequency trading information. Formally, the HAR-RV model is defined as follows:

$$RV^{(d)}{}_{t+1d} = \varphi_0 + \varphi^{(d)} RV_t^{(d)} + \varphi^{(w)} RV_t^{(w)} + \varphi^{(m)} RV_t^{(m)} + \varepsilon_{t+1d} \tag{16}$$

where $RV^{(d)}{}_{t+1d}$ is the ex post volatility estimate. $\left(RV_t^{(d)}\right)$, $\left(RV_t^{(w)}\right)$, and $(RV_t^{(m)})$ denote daily, weekly, and monthly structure of the RV. ε_{t+1d} denotes the error term. This benchmark model can be represented as follows:

<u>Model 1:</u> ***HAR* – *RV*** (Benchmark model)

$$RV_{i,t+1,t+h} = \varphi_{i,0} + \varphi_{i,1} RV_{i,t} + \varphi_{i,5} RV_{i,t-1,t-4} + \varphi_{i,22} RV_{i,t-5,t-21} + \varepsilon_{i,t+1} \tag{17}$$

where i refers to the cryptocurrency markets in our analysis. As we investigate four markets, therefore, $i =$ Bitcoin, ETH, ETC, and XRP. $[t+1, t+h]$ denotes the period of analysis and h denotes the forecasting horizon; $\varepsilon_{i,t+1}$ denotes the forecasting error term for the market (i) at time $(t+1)$.

Our second candidate model is an extension of our benchmark model in line with Andersen et al. (2007), which involves substituting the RV regressors through its continuous and discontinuous components. More formally, Model 2 is presented as follows:

<u>Model 2</u> ***HAR* – *CV* – *J***

$$RV_{i,t+1,t+h} = \varphi_{i,0} + \varphi_{i,1} CV_{i,t} + \varphi_{i,5} RV_{i,t-1,t-4} + \varphi_{i,22} RV_{i,t-5,t-21} + \varphi_{i,J} J_{i,t} + \varepsilon_{i,t+1} \tag{18}$$

where CV and J denotes the continuous part and the jump components, respectively.

The third candidate model extends the benchmark model by controlling for potential cryptocurrency volatility inertia, decomposing it into positive and negative parts. This model is written as follows:

<u>Model 3</u> ***HAR* – *SRV***

$$RV_{i,t+1,t+h} = \varphi_{i,0} + \varphi_{i,1}^{+} RV_{i,t}^{+} + \varphi_{i,1}^{-} RV_{i,t}^{-} + \varphi_{i,5} RV_{i,t-1,t-4} + \varphi_{i,22} RV_{i,t-5,t-21} + \varepsilon_{i,t+1} \tag{19}$$

The next model is an extension of Model 3. After assessing the role of signed information in RV, we focus on Model 4 on the role of signed information coming from the jump component. Therefore, Model 4 aims to assess the effect of a signed jump, which gives us the cryptocurrency volatility forecasting.

Model 4: $\boldsymbol{HAR - RV - \Delta J^2}$

$$RV_{i,t+1,t+h} = \varphi_{i,0} + \varphi_{i,1}BV_{i,t} + \varphi_{i,5}RV_{i,t-1,t-4} + \varphi_{i,22}RV_{i,t-5,t-21} + \varphi_{i,J^*}\Delta J^2_{i,t} + \varepsilon_{i,t+1} \tag{20}$$

Model 5 is intuitively inspired from Model 3 and is an extension of Model 4. The difference involves considering the potential cryptocurrency volatility inertia of signed information in the discontinuous part of the RV (in the jump). This model is defined as follows:

Model 5: $\boldsymbol{HAR - RV - \Delta J^{2+}\Delta J^{2-}}$

$$RV_{i,t+1,t+h} = \varphi_{i,0} + \varphi_{i,1}BV_{i,t} + \varphi_{i,5}RV_{i,t-1,t-4} + \varphi_{i,22}RV_{i,t-5,t-21} + \varphi_{i,J^+}J^{2+}_{i,t} + \varphi_{i,J^-}\Delta\left|J^{2-}_{i,t}\right| + \varepsilon_{i,t+1} \tag{21}$$

2.6 The modelling and volatility forecasting of cryptocurrency markets

To assess the predictive power of different components of RV as developed above, we propose an in-sample and an out-of-sample strategy.

2.6.1 In-sample strategy

The in-sample forecasting strategy aims to investigate the appropriate model for cryptocurrency volatility modelling. More specifically, we compare the power of prediction among the different candidate models. Our estimation employs weighted linear squares regression with a fitted value of an OLS regression, after correcting standard errors from further heteroscedasticity in the data using the Eicker–White approach.

2.6.2 Out-of-sample strategy

Our objective is to determine which kind of models/information drives better cryptocurrency volatility. Specifically, the out-of-sample strategy aims to assess which models outperform the cryptocurrency market volatility forecasting, based on the econometrics tests. Formally, the five candidate models are estimated over the period[4] $(\mathrm{N_i} - \mathrm{h})$, and then we forecast the volatility of each market for an horizon h. The h-step-ahead dynamic forecasts are calculated for $\mathrm{t} = \mathrm{k_i}, \ldots, \mathrm{T_i}$, where $\mathrm{k_i}$ is the forecasting starting date and $\mathrm{T_i}$ is the end date of the series for market i.

In line with Corsi's (2009) model and taking into account the cascade structure of RV (daily, weekly, and monthly), we adopt three forecasting horizons: daily (h = 1), weekly (5 days), and monthly (22 days) in the out-of-sample forecasting exercise. Then, the forecasted models are compared based on the forecast accuracy tests. The empirical literature may be categorized into two main strands. The first strand of literature is based on pairwise accuracy tests, such as those of Diebold and Mariano (2002) and Harvey et al. (1997). The second strand of literature compares between large set of models, using tests such as

[4] N_i is the number of observations for market i, while h denotes the forecasting horizon.

the Reality Check (RC) method developed by White (2000), the superior predictive ability (SPA) test developed by Hansen (2005), and the model confidence set (MCS) method of Hansen et al. (2011).[5] In our analysis, we adopt the MCS method for several reasons. The pairwise comparison tests suffer from a data snooping bias (White, 2000), and the MCS method has several advantages compared to the well-known tests of RC and SPA. Although the RC method considers data snooping, giving unbiased results, it has limitations in the presence of poor and irrelevant candidate models. This limitation has been addressed in the SPA test, allowing simultaneous comparison of a large set of models, regardless of whether irrelevant and poor candidate models have been included. However, the SPA test cannot discriminate between large competing models. MCS overcomes this limit, as it deals with a smaller set of models, called the *model confidence set*, containing the best models providing equal predictive ability at a given level of confidence. The MCS method contains the best forecasts, for a given level of confidence, that do not differ significantly in terms of their forecast performance.[6]

3 Data

Four cryptocurrency markets are investigated in this study—the Bitcoin, the Ethereum (ETH), the Ethereum Classic (ETC), and the Ripple (XRP)—representing 79% of the global cryptocurrency market capitalization, with a market capitalization over USD 1 billion each. The period of analysis was from April 2018 to June 2020. The data were collected from Bloomberg database.

To assess the effect of COVID-19, two subsamples are defined: the pre-COVID-19 sample from April 1, 2018 to December 31, 2019, and the COVID-19 sample from January 1, 2020 to June 30, 2020. We decided to close our COVID-19 sample period at the end of June 2020 to assess the first COVID-19 wave, given that the second wave is in progress.

Table 1 presents the descriptive statistics on the different components of RV for the four cryptocurrency markets, revealing some noteworthy aspects. First, we observe that, on average, the RV for all cryptocurrency markets rose during the COVID-19 period. The more pronounced increase was observed for Bitcoin, as it has approximately twice the RV during the COVID-19 period, compared to the pre-COVID-19 period. Interestingly, we observed the intensity of both positive RV and negative RV increased during the COVID-19 period, compared to the pre-COVID-19 period. Regarding the jump intensity, the results were inconclusive. ETH, ETC, and XRP exhibited a lower jump intensity during the COVID-19 period than the pre-COVID-19 period. However, for Bitcoin, the jump during the COVID-19 period was more significant than that during the pre-COVID-19 period. These findings might be more understandable when distinguishing between positive and negative jumps. We observed the intensity of upward or downward jumps were more significant during the COVID-19 period than during the pre-COVID-19 period, except in the case of XRP. These preliminary observations highlighted the importance of the different components of RV measures in explaining risk in cryptocurrency markets.

[5] For more information about these two subsets of the literature, please see Jawadi et al. (2019, pp. 132–133).

[6] The MCS procedure is a model selection algorithm, filtering a set of competitors from a given large set of models.

Table 1 Descriptive statistics on RV for the four cryptocurrency markets

	Mean			Min			Max			STD			OBS		
	Total period	Pre-COVID-19	COVID-19	Overall period	Pre-COVID-19	COVID-19	Overall period	Pre-COVID-19	COVID-19	Overall period	Pre-COVID-19	COVID-19	Overall period	Pre-COVID-19	COVID-19
Bitcoin															
RV	1.7280	1.4385	2.6008	0.0339	0.0339	0.1581	104.9415	21.0211	104.9415	5.2586	2.2351	9.7733	522	392	129
CV	1.5423	1.2512	2.4198	0.0239	0.0239	0.1581	104.9415	21.0211	104.9415	5.2032	2.1648	9.7008	522	392	129
Jump	0.5875	0.5520	0.7353	0.0109	0.0109	0.1064	5.7051	4.2330	5.7051	0.8140	0.7063	1.1632	165	133	32
RV+	0.8400	0.6941	1.2802	0.0180	0.0180	0.0747	59.3782	8.8280	59.3782	2.8280	1.0904	5.3325	522	392	129
RV-	0.8879	0.7444	1.3207	0.0140	0.0140	0.0722	45.5633	13.1563	45.5633	2.5456	1.2862	4.5724	522	392	129
ΔJ^2	−0.0479	−0.0504	−0.0405	−13.5110	−5.2914	−13.5110	13.8150	5.9847	13.8150	1.1414	0.8310	1.7800	522	392	129
$\Delta J2^+$	0.3897	0.3533	0.5064	0.0001	0.0001	0.0095	13.8150	5.9847	13.8150	1.1031	0.7284	1.8552	235	179	56
$\Delta J2^-$	−0.4063	−0.3896	−0.4543	−13.5110	−5.2914	−13.5110	−0.0003	−0.0003	−0.0004	1.0447	0.7584	1.6132	287	213	74
Ethereum															
RV	2.6852	2.3928	3.5670	0.0657	0.0657	0.3333	122.1360	17.5362	122.1360	6.2453	2.9332	11.4196	522	392	129
CV	2.4700	2.1484	3.4401	0.0657	0.0657	0.3333	122.1360	17.5362	122.1360	6.1662	2.7285	11.3902	522	392	129
Jump	0.8573	0.8790	0.7498	0.0259	0.0259	0.1075	6.0732	6.0732	3.9337	1.1703	1.1902	1.0855	131	109	22
RV+	1.2967	1.1325	1.7916	0.0339	0.0339	0.1672	70.3548	8.5599	70.3548	3.3793	1.4069	6.3082	522	392	129
RV−	1.3885	1.2602	1.7754	0.0318	0.0318	0.1529	51.7812	11.6718	51.7812	3.0195	1.7076	5.2711	522	392	129
ΔJ^2	−0.0918	−0.1277	0.0162	−14.2471	−7.7299	−14.2471	18.5736	5.5610	18.5736	1.4390	1.0894	2.1794	522	392	129
$\Delta J2^+$	0.5060	0.4589	0.6391	0.0003	0.0003	0.0012	18.5736	5.5610	18.5736	1.3424	0.7168	2.3418	241	178	63
$\Delta J2^-$	−0.6046	−0.6156	−0.5695	−14.2471	−7.7299	−14.2471	−0.0001	−0.0001	−0.0024	1.3179	1.1068	1.8475	281	214	67
Ethereum classic															
RV	4.0827	3.8647	4.7401	0.0854	0.0854	0.2334	134.3475	47.3073	134.3475	7.7585	5.2018	12.6695	522	392	129
CV	3.5155	3.1998	4.4673	0.0636	0.0636	0.1703	134.3475	45.7566	134.3475	7.4327	4.5016	12.6682	522	392	129
Jump	1.3278	1.4242	0.8865	0.0218	0.0218	0.0718	18.2822	18.2822	6.4971	2.2028	2.3542	1.2304	223	183	40
RV+	2.0119	1.8844	2.3967	0.0398	0.0398	0.0968	77.2078	23.9316	77.2078	4.2057	2.6016	7.1220	522	392	129
RV−	2.0707	1.9803	2.3434	0.0456	0.0456	0.0907	57.1397	23.3757	57.1397	3.7112	2.7499	5.7100	522	392	129

Table 1 (continued)

	Mean			Min			Max			STD			OBS		
	Total period	Pre-COVID-19	COVID-19	Overall period	Pre-COVID-19	COVID-19	Overall period	Pre-COVID-19	COVID-19	Overall period	Pre-COVID-19	COVID-19	Overall period	Pre-COVID-19	COVID-19
ΔJ^2	−0.0588	−0.0960	0.0532	−13.6459	−6.2445	−13.6459	20.0681	8.5113	20.0681	1.6513	1.2659	2.4776	522	392	129
$\Delta J2^+$	0.6158	0.5271	0.8659	0.0025	0.0025	0.0039	20.0681	8.5113	20.0681	1.5255	1.0238	2.4316	275	203	72
$\Delta J2^-$	−0.8099	−0.7652	−0.9556	−13.6459	−6.2445	−13.6459	−0.0005	−0.0005	−0.0065	1.4518	1.1564	2.1581	247	189	58
XRP															
RV	2.8364	2.8136	2.9052	0.1066	0.1066	0.1705	86.8585	79.5571	86.8585	6.1150	5.1493	8.4052	522	392	129
CV	2.6306	2.5830	2.7742	0.0831	0.0831	0.1167	86.8585	79.5571	86.8585	6.0208	5.0209	8.3648	522	392	129
Jump	0.8202	0.8530	0.6814	0.0369	0.0369	0.0538	7.3616	7.3616	4.1395	1.3344	1.4124	0.9450	131	106	25
RV+	1.3947	1.3928	1.4003	0.0536	0.0536	0.0738	46.1847	43.5755	46.1847	3.1688	2.7322	4.2341	522	392	129
RV−	1.4418	1.4208	1.5049	0.0472	0.0472	0.0939	40.6738	35.9816	40.6738	3.0724	2.5601	4.2725	522	392	129
ΔJ^2	−0.0471	−0.0280	−0.1047	−12.5339	−12.5339	−12.3928	7.5938	7.5938	5.5109	1.2524	1.2340	1.3098	522	392	129
$\Delta J2^+$	0.4779	0.5182	0.3480	0.0001	0.0001	0.0026	7.5938	7.5938	5.5109	0.8902	0.9275	0.7496	262	200	62
$\Delta J2^-$	−0.5762	−0.5971	−0.5173	−12.5339	−12.5339	−12.3928	−0.0012	−0.0012	−0.0103	1.3405	1.2581	1.5585	260	192	68

Max., Min., STD, and OBS denote the maximum, minimum, standard deviation, and the number of observations. Except OBS, all values are by 10^3

Table 2 Estimation of candidates models

Models	Bitcoin					Ethereum				
	Model 1	Model 2	Model 3	Model 4	Model 5	Model 1	Model 2	Model 3	Model 4	Model 5
φ_0	0.0009*** (0.0002)	0.0010*** (0.0002)	0.0004* (0.0003)	0.0005** (0.0002)	0.00038 (0.0003)	0.0014*** (0.0003)	0.0014*** (0.0003)	0.0008*** (0.0002)	0.0009 *(0.0005)	0.0008*** (0.0005)
φ_1	0.4634*** (0.0407)	0.4748 (0.3038)		0.6188*** (0.1933)	0.2882*** (0.0862)	0.4456*** (0.0412)	0.4496*** (0.0416)		0.5819*** (0.2236)	0.5253*** (0.1780)
φ_5	0.0333 (0.0408)	0.0324 (0.0392)	0.0618 (0.0192)	0.0601*** (0.0199)	0.0605*** (0.0125)	0.0414 (0.0412)	0.0411 (0.0412)	0.0727** (0.0350)	0.0724** (0.0320)	0.0720** (0.0313)
φ_{22}	−0.0146 (0.0406)	−0.0087 (0.0071)	−0.0250* (0.0117)	−0.0204** (0.0106)	−0.0260** (0.0120)	−0.0039 (0.0410)	−0.0029 (0.0410)	−0.0125 (0.0347)	−0.0106 (0.0161)	−0.0110 (0.0162)
β_J		−0.2735 (0.2855)					0.1646 (0.4070)			
φ_1^+			−1.9477* (1.0600)					−1.5872*** (0.1528)		
φ_1^-			3.1475*** (1.4122)					2.7353*** (0.1711)		
φ_{j^*}				−2.5874** (1.2197)					−2.1725** (1.1456)	
φ_{j^+}					−0.2456 (0.4159)					−1.7991** (0.8549)
φ_{j^-}					3.9328** (1.7391)					2.3864 (1.5997)
Adj.R^2	0.2142	0.2182	0.4963	0.5086	0.5473	0.1990	0.1981	0.4245	0.4269	0.4266
	Ethereum classic					XRP				
φ_0	0.0023*** (0.0004)	0.0024*** (0.0006)	0.0018*** (0.0004)	0.0020*** (0.0004)	0.0019*** (0.0004)	0.0018*** (0.00034)	0.0019*** (0.0003)	0.0017*** (0.00033)	0.0018*** (0.0003)	0.0013*** (0.0003)
φ_1	0.4300*** (0.0416)	0.4385* (0.2392)		0.5737*** (0.0421)	0.5094*** (0.0610)	0.3830*** (0.0425)	0.3915* (0.2059)		0.4108*** (0.0426)	0.0706 (0.0617)
φ_5	0.0237 (0.0415)	0.0244 (0.0239)	0.0291 (0.0382)	0.0330 (0.0382)	0.0364 (0.0382)	0.0006(.0425)	0.0007 (0.0269)	0.0073 (0.0420)	0.0076 (0.0418)	−0.0004 (0.0397)

Table 2 (continued)

Models	Bitcoin					Ethereum				
	Model 1	Model 2	Model 3	Model 4	Model 5	Model 1	Model 2	Model 3	Model 4	Model 5
φ_{22}	0.0001 (0.0414	0.0003 (0.0262)	−0.0149 (0.0381)	−0.0139 (0.0381)	−0.0135 (0.0381)	−0.0016 (0.0425)	0.0055 (0.0218)	−0.0061 (0.0420)	0.0004 (0.0418)	0.0128 (0.03971)
β_J		0.2985 (0.2828)					−0.0104 (0.30823)			
φ_1^+			−1.2115*** (0.1806)					−0.3729* (0.2065)		
φ_1^-			2.3061*** (0.2053)					1.1641*** (0.2131)		
φ_{j^*}				−1.7942*** (0.1897)					−0.8348*** (0.2054)	
φ_{j^+}					−1.3105*** (0.3825)					2.1546*** (0.5291)
φ_{j^-}					2.1518*** (0.3103)					2.3578*** (0.2856)
Adj.R^2	0.1822	0.1812	0.3071	0.3082	0.3099	0.1412	0.1417	0.1641	0.1707	0.2532

The coefficients φ_0, φ_1, φ_5, φ_{22}, φ_J, φ_1^+, φ_1^-, φ_{j^*}, φ_{j^+}, and φ_{j^-} are defined in the models 1–5. Adj.R^2 denotes the adjusted R squared. *,**, *** denote the significance level of 10%, 5%, and 1%

4 Empirical analysis

4.1 The dynamic of cryptocurrency market volatility over the entire sample

4.1.1 In-sample forecasting results

The first step of our econometric analysis is to assess the best fit model in cryptocurrency volatility modelling based on the estimation of the original HAR-RV and the proposed four extended models. Table 2 presents the results of the in-sample forecasting strategy for the four markets during the complete period of study, ranging from April 2018 to June 2020. Two interesting findings are shown in this table. First, we note the decomposition of volatility into continuous and discontinuous components improves the cryptocurrency volatility modelling. Moreover, Model 2 has a higher adjusted R-squared than the original HAR-RV. Second, taking into account the potential asymmetric dynamic of the volatility is an important issue, as we show that the models including the decomposition of RV into positive and negative volatility (Models 3 and 4) as well as the model decomposing the volatility into positive and negative jumps (Model 5) perform better the other models, in term of the adjusted R-squared.

Although Model 5 seems to be the best fit model in modelling the cryptocurrency market volatility, two notable facts need to be highlighted. First, the positive and negative jumps included in Model 5 are both significant in the case of ETC and XRP. However, in the case of Bitcoin, only the negative jumps may affect the realized volatility. Second, we observed some heterogeneous results in terms of the sensitivity of cryptocurrency market volatility to positive and negative jumps across the four markets. For example, the volatility sensitivity of the Bitcoin and the ETC to negative jumps is substantially more important compared to positive jumps. In the case of ETH, Model 4 exhibits the highest R-squared followed by Model 5. This finding also highlights the asymmetric dynamics of the ETH volatility, as the signed jump distinguishes between positive and negative returns.

Overall, our findings highlighted negative jumps (abrupt price decrease) during the previous trading day raised the volatility of the cryptocurrency market; however, past positive jumps (abrupt price increase) reduced cryptocurrency market volatility. These heterogeneities might be explained based on the crisis period included in our sample, i.e., the COVID-19 pandemic, which might substantially affect the dynamic of cryptocurrency volatility. Therefore, dividing our sample into pre-COVID-19 and COVID-19 periods is crucial to assess cryptocurrency volatility.

4.1.2 The out-of-sample results

The second step involves assessing the best fit accuracy of the candidate models based on the out-of-sample forecasting strategy for the three horizons: 1 month, 1 week, and 1 day. The comparison between the forecasting performance of the candidate models is based on the MCS methods, presented in Table 3. The results indicate some noteworthy findings. For the horizons 1 week and 1 day, the best fit model for Bitcoin and XRP is Model 5. This finding highlights that the dynamic of Bitcoin and XRP volatility has an asymmetric dynamic based on the discontinuous components (jumps). However, for the ETC and ETH volatilities, the best fit model is Model 3. This result shows that ETH and ETC volatility is more pronounced based on the asymmetric dynamic, which in turn depends on

Table 3 Results of forecasting accuracy tests

Candidate models	Bitcoin			Ethereum		
	1 month	1 week	1 day	1 month	1 week	1 day
Model 1	0.00242	0.00123	0.00128	0.00419	0.00263	0.00219
	(0.0024)	(0.0164)	(0.0000)	(0.0023)	(0.0088)	(0.0000)
Model 2	**0.00157**	0.00058	0.00091	**0.00241**	0. 00,127	0.00156
	(1.0000)	(0.1582)	(0.0000)	**(1.0000)**	(0.3821)	(0.0000)
Model 3	0.00757	0.00022	0.00014	0.01135	**0.00084**	**0.00037**
	(0.3157)	(0.6631)	(0.0000)	(0.3992)	**(1.0000)**	**(1.0000)**
Model 4	0.00586	0.00023	0.00016	0.00924	0.00098	0.00048
	(0.3175)	(0.6631)	(0.0000)	(0.3992)	(0.0088)	(0.0000)
Model 5	0.00943	**0.00019**	**0.00007**	0.00998	0.0098	0.00047
	(0.1903)	**(1.0000)**	**(1.0000)**	(0.3992)	(0.00088)	(0.0000)
	Ethereum classic			XRP		
Model 1	0.01391	0.00895	0.00494	0.00559	0.00409	0.00409
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Model 2	**0.00546**	0.00423	0.00341	**0.00253**	0.00216	0.00297
	(1.0000)	(0.0000)	(0.0000)	**(1.0000)**	(0.0000)	(0.0000)
Model 3	0.00952	**0.00252**	**0.00159**	0.00320	0.000190	0.00236
	(0.6226)	**(1.0000)**	**(1.0000)**	(0.6211)	(0.0000)	(0.0000)
Model 4	0.00847	0.00307	0.00197	0.00303	0.00196	0.00245
	(0.6226)	(0.0000)	(0.0000)	(0.6211)	(0.0000)	(0.0000)
Model 5	0.00959	0.00284	0.00178	0.00357	**0.00116**	**0.00154**
	(0.6226)	(0.0000)	(0.0000)	(0.6225)	**(1.0000)**	**(1.0000)**

Numbers reported in this table denote the values of the mean squared error (MSE) loss function. Values in (.) denote the (10^{-3}) value of the MCS procedure. The MCS p-values are computed based on 10,000 bootstrap samples with MSE as a loss function (The results of the loss functions based on MSE are not presented to save space but are available upon request). The confidence level for MCS is 90%. Models 1 through 5 denote $HAR - RV$, $HAR - CV - J$, $HAR - SRV$, $HAR - RV - \Delta J^2$, and $HAR - RV - \Delta J^{2+} \Delta J^{2-}$, respectively

distinguishing between positive and negative RV. For the relatively long-term forecasting horizon (1 month), the results of the best fit model were homogenous across all the markets, as MCS showed the superiority of Model 2. This finding highlights the importance of considering the discontinuous and continuous components of volatility in the interest of the long-term investors.

4.2 The dynamic of cryptocurrency market volatility during the pre-COVID crisis

4.2.1 In sample forecasting results

Table 4 presents the results of the estimation of the five candidate models for volatility modelling for the four cryptocurrency markets, during the pre-COVID-19 period. Two main differences were observed compared to the results of the whole sample. First, Model 5 becomes the best fit model for cryptocurrency volatility modelling for all the

Table 4 Estimation of candidate models

Models	Bitcoin					Ethereum				
	Model 1	Model 2	Model 3	Model 4	Model 5	Model 1	Model 2	Model 3	Model 4	Model 5
φ_0	0.0007*** (0.0001)	0.0007*** (0.0001)	0.0007*** (0.0001)	0.0007*** (0.0001)	0.0007*** (0.0001)	0.00116*** (0.0002)	0.00114*** (0.0002)	0.0011*** (0.0002)	0.0011*** (0.0002)	0.00117*** (0.0002)
φ_1	0.5326*** (0.0462)	0.5695*** (0.0935)		0.5620*** (0.0905)	0.6781*** (.1121)	0.5452*** (0.0444)	0.2167 (0.1762)		0.6060*** (0.0498)	0.6943*** (0.0605)
φ_5	−0.0085 (0.0463)	−0.0105 (0.0389)	−0.0045 (0.0370)	−0.0087 (0.0389)	−0.0262 (0.0407)	−0.0415 (0.0444)	−0.0387 (0.0443)	−0.0479 (0.0447)	−0.0409 (0.0445)	−0.0380 (0.0442)
φ_{22}	−0.0038 (0.0446)	0.0139 (0.0291)	−0.0033 (0.0319)	0.0114 (0.0294)	0.0176 (0.0285)	0.0238 (0.0440)	0.0321 (0.0441)	0.0227 (0.0440)	0.0374 (0.0438)	0.0454 (0.0436)
β_J		−0.0987 (0.1173)					0.5804*** (0.0479)			
φ_1^+			0.4397** (0.1793)					0.7083*** (0.1457)		
φ_1^-			0.60723*** (0.1903)					0.4160*** (0.118)		
φ_{j^*}				−0.0598 (0.1630)					−0.1436 (0.1239)	
φ_{j^+}					−0.4908* (0.2675)					−0.4569* (0.26757)
φ_{j^-}					0.4105* (0.2298)					0.4874*** (0.1834)
Adj.R^2	0.2751	0.2926	0.2740	0.3003	0.3013	0.2881	0.2933	0.2888	0.2930	0.3127
	Ethereum classic					XRP				
φ_0	0.0012*** (0.0003)	0.0011*** (0.0003)	0.0011*** (0.0003)	0.0012*** (0.0003)	0.0015*** (0.0003)	0.0019*** (0.0003)	0.0020*** (0.0003)	0.0020*** (0.0003)	0.0020*** (0.0003)	0.0017*** (0.0003)
φ_1	0.6172*** (0.0413)	0.7030**** (0.0481)		0.7458*** (9.0474)	0.8148*** (0.0503)	0.2944*** (0.0500)	0.3040* (0.1835)		0.2766*** (0.0518)	−0.0481 (0.0726)
φ_5	0.00517 (0.0412)	0.0079 (0.0406)	0.3180** (0.1678)	0.0095 (0.0403)	−0.0089 (0.0400)	−0.01312 (0.0502)	−0.0127 (0.0243)	−0.0161 (0.0496)	−0.0148 (0.0497)	−0.0043 (0.0474)

Table 4 (continued)

Models	Bitcoin					Ethereum				
	Model 1	Model 2	Model 3	Model 4	Model 5	Model 1	Model 2	Model 3	Model 4	Model 5
φ_{22}	0.0765* (0.040)	0.0708 (0.0403)	0.07955** (0.0407)	0.0737 (0.0401)	0.0742 (0.0394)	0.0356 (0.0501)	0.0441 (0.0317)	0.0413 (0.0495)	0.0498 (0.0496)	0.0572 (0.0473)
β_J		0.2330** (0.1217)					0.0367 (0.3348)			
φ_1^+			0.9343*** (0.1772)					0.9076*** (0.2070)		
φ_1^-			0.3180*** (0.1678)					−0.3628* (0.2211)		
φ_{j^*}				−0.4610*** (0.1662)					−0.5798*** (0.2104)	
φ_{j^+}					−0.3131 (0.2684)					−3.3748*** (0.5003)
φ_{j^-}					1.1615*** (0.2526)					0.7271*** (0.2936)
Adj.R^2	0.3918	0.3957	0.3957	0.4147	0.4337	0.0807	0.0798	0.1010	0.0985	0.1800

The coefficients φ_0, φ_1, φ_5, φ_{22}, φ_J, φ_1^+, φ_1^-, φ_{j^*}, φ_{j^+}, and φ_{j^-} are defined in the models 1–5. Adj.R^2 denotes the adjusted R squared. *,**, *** denote the significance level of 10%, 5%, and 1%

Table 5 Results of forecasting accuracy tests

Candidate models	Bitcoin			Ethereum		
	1 month	1 week	1 day	1 month	1 week	1 day
Model 1	0.00185	0.00106	0.00062	0.00229	0.00200	0.00152
	(0.8350)	(0.0000)	(0.0000)	(0.0110)	(0.0000)	(0.0000)
Model 2	0.00169	0.00040	0.00046	0.00103	0. 00,072	0.00092
	(0.8350)	(0.0635)	(0.0000)	(0.9928)	(0.0000)	(0.0000)
Model 3	0.00202	**0.00035**	**0.00037**	**0.00102**	**0.00066**	**0.00087**
	(0.7547)	**(1.0000)**	**(1.0000)**	**(1.0000)**	**(1.0000)**	**(1.0000)**
Model 4	0.00169	0.00039	0.00044	0.00113	0.00075	0.00096
	(0.8350)	(0.0635)	(0.0000)	(0.1708)	(0.0000)	(0.0000)
Model 5	**0.00167**	0.00047	0.00051	0.00124	0.00089	0.00111
	(1.0000)	(0.0000)	(0.0000)	(0.0853)	(0.0000)	(0.0000)
	Ethereum classic			**XRP**		
Model 1	0.00631	**0.01280**	0.001494	0.00291	0.00106	0.00165
	(0.1700)	**(1.0000)**	(0.0000)	(0.1381)	(0.0000)	(0.0000)
Model 2	0.00467	0.01612	**0.00090**	**0.00257**	0.00058	0.00135
	(0.7483)	(0.4763)	**(1.0000)**	**(1.0000)**	(0.4272)	(0.0000)
Model 3	0.00766	0.02820	0.00097	0.00344	0.00077	0.00156
	(0.2776)	(0.3526)	(0.0000)	(0.0930)	(0.0079)	(0.0000)
Model 4	0.00572	0.01990	0.00123	0.00330	0.00064	0.00146
	(0.2426)	(0.5375)	(0.0000)	(0.0930)	(0.0216)	(0.0000)
Model 5	**0.00449**	0.01388	0.00188	0.00527	**0.00024**	**0.00051**
	(1.0000)	(0.5375)	(0.0000)	(0.1589)	**(1.0000)**	**(1.0000)**

Numbers reported in this table denote the values MSE loss function. Values in (.) denote the (10^{-3}) value of the MCS procedure. The MCS p-value are computed based on 10,000 bootstrap samples with mean squared errors as a loss function (The results of the loss functions based on MAE are not presented to save place but are available upon request). The confidence level for MCS is 90%. Models 1 through 5 represent $HAR-RV$, $HAR-CV-J$, $HAR-SRV$, $HAR-RV-\Delta J^2$, and $HAR-RV-\Delta J^{2+}\Delta J^{2-}$, respectively

markets. Second, we did not observe a divergence in terms of cryptocurrency volatility sensitivity across positive and negative jumps in the case of Bitcoin and ETH. The sensitivity of their volatility to positive and negative jumps were similar in the absolute value. However, there is an asymmetric sensitivity of XRP volatility in favour of negative jumps. Finally, ETC was only sensitive to negative jumps.

4.2.2 The out-of-sample results

In terms of forecasting performance, Model 3 is the best fit model for Bitcoin and ETH for the 1-week and 1-day forecasting horizons (Table 5). However, for XRP and ETC, the best fit model varies across the forecasting horizons. Notably, the best fit model of ETC volatility forecasting is Model 5 and Model 3 for the 1-month and 1-day forecasting horizons, respectively. For the XRP volatility forecasting, the best fit model is Model 5 for the 1-week and 1-day forecasting horizons.

Table 6 Estimation of candidates models

Models	Bitcoin					Ethereum				
	Model 1	Model 2	Model 3	Model 4	Model 5	Model 1	Model 2	Model 3	Model 4	Model 5
φ_0	0.0019 (0.0013)	0.0018* (0.0010)	0.0001 (0.0004)	0.0001 (0.0004)	0.0002 (0.0004)	0.0026* (0.0015)	0.0027*** (0.0010)	0.0007 (0.0008)	0.0008 (0.0007)	0.00134* (0.00077)
φ_1	0.4409*** (0.0990)	0.4558*** (0.0888)		0.8442*** (0.0771)	0.4507* (0.2556)	0.4150*** (0.1004)	0.4184 (0.3660)		0.8709*** (0.0947)	−0.0268 (0.2506)
φ_5	0.0241 (0.0990)	0.0263 (0.0422)	0.0877*** (0.0184)	0.0854*** (0.0193)	0.0746*** (0.0155)	0.0453 (0.1004)	0.0435 (0.0558)	0.1002** (0.0518)	0.0989*** (0.0317)	0.0890* (0.0476)
φ_{22}	0.0295 (0.0987)	0.0203 (0.0142)	0.0442*** (0.0104)	0.0405*** (0.0093)	0.0438*** (0.0108)	0.0233 (0.1001)	0.0246 (0.0115)	0.0453 (0.051)	0.0465*** (0.0128)	0.0443 (0.0473)
β_J		−0.6079 (0.7475)					−0.7279 (1.0864)			
φ_1^+			−3.8745*** (0.6038)					−3.9088*** (0.2908)		
φ_1^-			5.5070*** (0.7868)					5.6349*** (0.3493)		
φ_{j^*}				−4.7956*** (0.5780)					−4.7989*** (0.6075)	
φ_{j^+}					−1.7727 (1.8879)					1.0724 (1.6255)
φ_{j^-}					5.8964*** (1.0900)					7.9927*** (0.9172)
Adj.R^2	0.1682	0.2069	0.7825	0.8055	0.8211	0.1471	0.1384	0.7738	0.7800	0.8100
	Ethereum classic					XRP				
φ_0	0.00287** (0.0014)	0.0031** (0.0014)	0.0003 (0.0009)	0.0003 (0.0009)	0.0001 (0.0009)	0.0020* (0.0011)	0.0021** (0.0008)	0.0011* (0.0007)	0.0012*** (0.0007)	0.0011 (0.0006)
φ_1	0.3267*** (0.0929)	0.3298*** (0.0932)	0.0691 (0.0629)	0.8368*** (0.0736)	1.2340*** (0.3404)	0.4856*** (0.0970)	0.4915 (0.3841)		0.4724*** (0.0594)	−0.0507 (0.1814)
φ_5	0.0310 (0.0927)	0.0299 (0.0930)	−0.0542 (0.0607)	0.0688 (0.0602)	0.0751 (0.0603)	0.0147 (0.0967)	0.0112 (0.0411)	0.0762 (0.0599)	0.0750 (0.0592)	−0.0008 (0.0617)

Table 6 (continued)

Models	Bitcoin					Ethereum				
	Model 1	Model 2	Model 3	Model 4	Model 5	Model 1	Model 2	Model 3	Model 4	Model 5
φ_{22}	−0.0355 (0.0895)	−0.0290 (0.0902)		−0.0509 (0.0580)	−0.0524 (0.0579)	−0.0544 (0.0972)	−0.0491 (0.0420)	−0.0497 (0.05999)	−0.0484(0.0593)	−0.0460 (0.0561)
β_J		−0.7175 (1.4678)					−0.8277 (0.8723)			
φ_1^+			−3.4868*** (0.3490)					−4.0621*** (0.3967)		
φ_1^-			5.06896*** (0.4315)					4.9903*** (0.3930)		
φ_{j^*}				−4.4928*** (0.3747)					−4.5446*** (0.3859)	
φ_{j^+}					−7.1921*** (0.5522)					3.6779 (2.7334)
φ_{j^-}					3.5632*** (0.8631)					6.5071*** (0.7437)
Adj.R^2	0.0836	0.0793	0.5790	0.6147	0.6163	0.2063	0.2005	0.6979	0.7046	0.7318

The coefficients φ_0, φ_1, φ_5, φ_{22}, φ_J, φ_1^+, φ_1^-, φ_{j^*}, φ_{j^+}, and φ_{j^-} are defined in the models 1–5. Adj.R^2 denotes the adjusted R squared. *,**, *** denote the significance level of 10%, 5%, and 1%

Table 7 Results of the forecasting accuracy tests

Candidate models	Bitcoin			Ethereum		
	1 month	1 week	1 day	1 month	1 week	1 day
Model 1	0.00919	0.00508	0.00410	0.01405	0.00788	0.00561
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Model 2	**0.00276**	0.00243	0.00299	**0.00514**	0. 00,420	0.00434
	(1.0000)	(0.0000)	(0.0000)	**(1.0000)**	(0.0326)	(0.0000)
Model 3	0.01784	0.00208	**0.00001**	0.04135	**0.00167**	**0.00018**
	(0.5278)	(0.4512)	**(1.0000)**	(0.2981)	**(1.0000)**	**(1.0000)**
Model 4	0.01427	**0.00027**	0.00002	0.03410	0.00184	0.00023
	(0.5278)	**(1.0000)**	(0.0000)	(0.3823)	(0.1111)	(0.0000)
Model 5	0.01825	0.00039	0.00001	0.07389	0.00199	0.00019
	(0.1025)	(0.4512)	(0.0000)	(0.1492)	(0.1496)	(0.0000)
	Ethereum classic			**XRP**		
Model 1	0.01330	0.01256	0.00779	0.01037	0.00504	0.00436
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Model 2	**0.00690**	0.00814	0.00636	**0.00317**	0.00229	0.00314
	(1.0000)	(0.0000)	(0.0000)	**(1.0000)**	(0.0622)	(0.0000)
Model 3	0.02437	**0.00047**	**0.00006**	0.1067	**0.00076**	**0.00031**
	(0.5377)	**(1.0000)**	**(1.0000)**	(0.6201)	**(1.0000)**	**(1.0000)**
Model 4	0.02050	0.00050	0.00015	0.01967	0.00079	0.00033
	(0.5159)	(0.3734)	(0.0000)	(0.6201)	(0.1878)	(0.0000)
Model 5	0.01438	0.00063	0.00033	0.01443	0.00085	0.00042
	(0.6225)	(0.1530)	(0.0000)	(0.6233)	(0.4504)	(0.0000)

Numbers reported in this table denote the values of MSE loss function. Values in (.) denote the (10^{-3}) value of the MCS procedure. The MCS p-value are computed based on 10,000 bootstrap samples with mean squared errors as a loss function (The results of the loss functions based on MSE are not presented to save space but are available upon request). The confidence level for MCS is 90%. Models 1 through 5 represent $HAR - RV$, $HAR - CV - J$, $HAR - SRV$, $HAR - RV - \Delta J^2$, and $HAR - RV - \Delta J^{2+}\Delta J^{2-}$, respectively

4.3 The dynamic of cryptocurrency market volatility during the COVID-19 crisis period

4.3.1 In-sample forecasting results

The COVID-19 period volatility modelling results are presented in Table 6. The results present some specificities compared to the other samples. Although Model 5 exhibits the highest adjusted R-squared, the dynamic of the volatility modelling of the four markets has changed. For the four markets, only the volatility is statistically significant for negative jumps. In other words, during periods of turmoil, only bad jumps impacted the cryptocurrency market volatility, which were insensitive to good jumps. This result showed that during crisis periods, the investors in the cryptocurrency market were very stressed which led them to over-react to negative news.

More interestingly, we observed a high adjusted R-squared for all models and specifically for Model 5, compared to previous samples. Our findings show that the power of prediction of the best fit model during the crisis period was around 80% for Bitcoin, ETH, and XRP volatility modelling, and around 62% for XRP volatility modelling. This result was important for investors and hedgers in terms of portfolio diversification and developing hedging strategies during crisis periods.

Overall, our findings during the crisis period showed some notable aspects, leading to explain the divergence on the dynamic of cryptocurrency volatility observed during the whole period sample. Indeed, we showed only the occurrence of negative jumps (abrupt price decrease) during the previous trading day raised the cryptocurrency market volatility; however, past positive jumps (abrupt price increase) did not impact the cryptocurrency market volatility.

4.3.2 The out-of-sample results

In terms of the forecast accuracy performance, the results were mostly in favour of Model 3 across all forecasting horizons, thereby revealing the importance of considering the asymmetric dynamic in cryptocurrency market forecasting (Table 7). More interestingly, Model 5 is usually retained, across all horizons, in the set of models performing the cryptocurrency market forecasting. However, Model 3 is ranked the best model in the set of models.

From an economic point of view, our findings seemed to provide useful insights in line with a previous study (Huynh et al., 2020). Huynh et al. (2020) shows that alternatives coins send market signals faster than the largest market capitalization coin (Bitcoin). Our findings highlighted that the responses of altcoins were likely the reverse of Bitcoin (during the whole period of analysis), especially the negative jumps. This result shows that the altcoins were both sensitive to bad and good signals (signals) from the market and they were repercussed on their volatility. However, Bitcoin was more sensitive to bad signals (news). Our findings added to the literature showing that during period of turmoil, the volatility sensitivity of main coin and altcoin converge were only affected by bad signals (news).

Overall, our results contributed to the literature in two aspects. First, we showed that the predictability of cryptocurrency volatility was performed based on the HAR model, which was extended by including positive and negative jumps. More specifically, we showed that during the crisis period, only negative jumps had a predictive power concerning future realized volatility. This finding was important as it might explain the results of previous findings (Shen et al., 2020; Yu, 2019) that show the jumps do not have a predictive power for future volatility. In other words, these studies, by omitting the decomposition of jumps into positive and negative jumps, might have diluted the effect on jumps. Second, our results proposed new insights regarding cryptocurrency volatility forecasting, as we show that the best fit model was the model that takes into account the inertia effect of the volatility in the semi-variance components. It is important to consider the asymmetric behaviour while investigating the future dynamic of volatility.

For robustness check, have run our empirical analysis based on the upper and lower sampling frequency of 10-min and 1-min respectively. The results were still globally unchanged.[7]

[7] Tables results are not reported to save space. Results are available upon authors request.

5 Conclusion

The stylized facts of the cryptocurrency market show it was characterized by abrupt price movements and high uncertainty, raising the question of volatility forecasting during crisis periods. This study investigates the volatility dynamics of four major cryptocurrency markets: Bitcoin, ETC, ETH, and XRP. It is based on the recent literature on use of HFD in modelling and forecasting the volatility of financial assets. Particularly, our study contributes to the literature on cryptocurrency by investigating volatility forecasting in the context of the current COVID-19 crisis. After computing the different components of the RV—CV, jumps, positive and negative semi-variances, and signed jumps—we developed various candidate models, extending the original HAR model of Corsi (2009). Subsequently, both in-sample and out-of-sample strategies were performed to propose further insights in terms of modelling and forecasting cryptocurrency volatility during non-crisis and crisis periods.

Our findings showed the difference in the dynamic of cryptocurrency volatility across the study samples (non-crisis and crisis periods). The extended HAR model that includes the positive and negative jumps appears to be the best model for predicting future volatility in both periods. However, the difference was that during the crisis period, only the negative jump component was statistically significant. This result implied future volatility was explained by bad volatility during crisis periods. Turmoil periods led cryptocurrency market investors to be very stressed and over-react to negative news. In terms of volatility forecasting, the results showed that globally, the extended HAR model, which includes positive and negative semi-variances, performed better than other models. These findings might aid investors and hedgers in refining their forecasting to execute their portfolio diversification strategy and perform better at hedging strategies, respectively.

It should be noted that in this study, we focused on whether the signed jump had predictive power in modelling and forecasting cryptocurrency volatility, during a turmoil period like the COVID-19 outbreak. We did not look for the predictive power of COVID-19 indicators such infected patients or deaths. Ftiti et al. (2021) have proposed useful insights showing the role of non-fundamental news in predicting the stock market returns volatility. Future research should clarify our results that showed the volatility was sensitive only to bad jumps during the health crisis through the medium of fundamental and non-fundamental news. More interestingly, we did not develop a risk management strategy based on distinguishing between bad and good volatility. Therefore, future studies should focus on portfolio optimization by considering our results. It merits further investigation into whether the optimization portfolio is based only on bad risk, and this may improve portfolio allocation and hedging strategy.

Authors' contributions All authors contributed to the study's conception and design. Material preparation and data collection were performed by [Wael Louhichi] and [Hachmi Ben Ameur]. The econometric methodology was performed by [Zied Ftiti]. All drafts have been written by all authors. All authors commented on previous versions of the manuscript.

Data availability Upon request to the corresponding author.

Code availability Upon request to the corresponding author.

Declaration

Conflict of interest None.

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