

Fermatean fuzzy set extensions of SAW, ARAS, and VIKOR with applications in COVID-19 testing laboratory selection problem

Sait Gül 

Faculty of Engineering and Natural Sciences,
Management Engineering Department,
Bahçeşehir University, Beşiktaş, İstanbul,
34353, Turkey

Correspondence

Sait Gül, Faculty of Engineering and Natural
Sciences, Management Engineering
Department, Bahçeşehir University, Beşiktaş,
İstanbul 34353, Turkey.
Email: sait.gul@eng.bau.edu.tr

Abstract

The multiple attribute decision-making models are empowered with the support of fuzzy sets such as intuitionistic, q -rung orthopair, Pythagorean, and picture fuzzy sets, and also neutrosophic sets, etc. These concepts generate varying representation opportunities for the decision-maker's preferences and expertise. Pythagorean and Fermatean fuzzy sets are special cases of q -rung orthopair fuzzy set when $q = 2$ and $q = 3$, respectively. From a geometric perspective, the latter provides a broader representation domain than the former does. In this study, the emerging concept of Fermatean fuzzy set is studied in detail and three well-known multi-attribute evaluation methods, namely SAW, ARAS, and VIKOR are extended under Fermatean fuzzy environment. In this manner, the decision-makers will have more freedom in specifying their preferences, thoughts, and expertise, and the abovementioned decision approaches will be able to handle this new type of data. The applicability of the propositions is shown in determining the best Covid-19 testing laboratory which is an important topic of the ongoing global health crisis. To validate the proposed methods, a benchmark analysis covering the results of the existing Fermatean fuzzy set-based decision methods, namely TOPSIS, WPM, and Yager aggregation operators is presented.

KEYWORDS

ARAS method, Fermatean fuzzy sets, multi-attribute evaluation, SAW method, VIKOR method

1 | INTRODUCTION

When the decision-makers do not have enough data such as cost, sales, volume, etc. at a certain time, linguistic evaluations are made while stating the preferences, opinions, feelings, and expertise in general. Various fuzzy environments supply tools for making these evaluations. Zadeh (1965) initiated the concept of fuzzy sets as a symbolization apparatus of human judgements. In the conventional definition of fuzzy sets, decision-makers can just determine membership degree (μ) between 0 and 1. The membership degree is accepted as a measure for the optimism or agreement level of judgement. Thus, it possesses a positive meaning.

In decades, several fuzzy sets for smoothing the representation of the vagueness and ambiguity which are hidden in the human subconscious are developed. Atanassov (1986) started the concept of intuitionistic fuzzy sets (IFS) by adding a new element into the fuzzy set: non-membership degree (ν). This element puts a level of resilience into the representation of judgements since the decision-maker can declare his/her pessimistic view or disagreement level. Thus, non-membership degree has a negative meaning. Accordingly, Atanassov (1986) also introduced a new measure

regarding hesitancy which has a neutral meaning: $\pi = 1 - \mu - \nu$. Therefore, IFS can cope with three dimensions of judgements (membership, non-membership, and hesitancy). In real life, we can represent these degrees with yes, no, and abstain.

After the development of IFS, other extensions such as neutrosophic sets (Smarandache, 2019), Pythagorean fuzzy sets (Yager, 2014), q-rung orthopair fuzzy sets (Yager, 2017), spherical fuzzy sets (Kutlu Gündoğdu & Kahraman, 2019), picture fuzzy sets (Cuong & Kreinovich, 2013), etc. have been introduced. The basic difference among these understandings is regarding the consideration of varying hesitancy degrees. While neutrosophic sets allow the decision-makers to express their hesitancies with an independent item, spherical and picture fuzzy sets include a limited level of independent hesitancy. These sets can be accepted as the generalization of intuitionistic and Pythagorean fuzzy sets, but the drawback related to them is the training requirement of decision-makers about this opportunity. In many MADM applications, there is usually a time limitation and budget to collect data from the decision-makers who must be informed about the details of assigning membership, non-membership, and hesitancy degrees. Therefore, three-dimensional fuzzy sets involving these three directly assignable degrees are not considered in this study. Rather than adding this complexity to the decision process, this study prefers broadening the decision domain of the decision-maker.

FFS is firstly proposed by Senapati and Yager (2020) as a special case of q-rung orthopair fuzzy sets (q-ROFS). The theory of q-ROFS which is developed by Yager (2017) requires the sum of the q^{th} power of membership (e.g., support for an idea) and non-membership (e.g., support against an idea) degrees should be equal to or smaller than 1. It is obvious that when q increases the space of acceptable orthopairs will increase and this geometric area supplies more independence to users or decision-makers while declaring their preferences, ideas, and claims. By setting $q = 2$, Yager (2014) rename the q-ROFS as Pythagorean fuzzy sets (PFS) and developed basic operations on them. Also, different researchers contributed to the literature of PFS, for example, Zhang and Xu (2014) introduced the PFS extension of TOPSIS, Garg (2016a) proposed a general PFS information aggregation operator, P. Liu and Wang (2018) developed different aggregation operators for PFSs, Wei and Wei (2018) gave similarity measures for PFSs, Xiao and Ding (2019) shared their propositions including divergence measures of PFSs, etc.

Senapati and Yager (2020) set $q = 3$ and this novel q-ROFS is called Fermatean fuzzy sets (FFS). Under this new concept, the decision-makers have more freedom since they can specify their ideas about agreeing (membership) and/or disagreeing (non-membership) regarding the state of a subject. For illustration, an example may be like that: in an election, a decision-maker may think that a candidate satisfies her expectations with a possibility of 0.80 where this candidate dissatisfies the expectations with a possibility of 0.75. It is obvious that the PFS concept does not handle this situation because $0.80^2 + 0.75^2 = 1.20 > 1$. However, the calculation of $0.80^3 + 0.75^3 = 0.93 < 1$ shows that this idea can be coped with FFS. An interesting point about the relations between fuzzy set definitions can be specified for q-ROFS, FFSs, and neutrosophic sets. Smarandache (2019) proved that neutrosophic set is a generalization of q-ROFS which is also a generalization of FFS by setting $q = 3$. Each element of a q-ROFS is defined in $[0,1]$ where the sum of their q^{th} power (cubes in FFS) is also defined in $[0,1]$. It is seen that any q-ROFS (and also FFS, accordingly) is a special case of neutrosophic set where the sum of each three-element is in $[0,3]$ since the definition range of neutrosophic set covers the definition range of q-ROFS.

In multiple attribute decision making (MADM), there are common three consecutive steps. The multi-attribute evaluation process begins with structuring the decision model covering the basic definitions of alternatives, attributes and decision-makers, limitations, and data type. Then, the decision model represented by a decision matrix is fulfilled with the objective and subjective data, and a data collection process is performed for this purpose. Normalization is operated if needed. Finally, the decision model is analyzed by an appropriate MADM method. The second and third steps are constructed and performed considering the description given in the first step. From a MADM perspective, the motivations of the study are listed as follows:

1. The basic enhancement of FFS in terms of MADM is its extensive context easing the judgement representation issue. After the data type is described as FF number in the first step, all the consecutive operations are performed accordingly as FF numbers.
2. The data gathering process and normalization operation are fuzzified under FFS and the MADM method in the third step is modified for letting it be operated under FFS.
3. Thanks to its broader geometric area providing more opportunities for the decision-makers, FFS-based extensions of MADM applications are needed.
4. Also, the appropriately defined Covid-19 issues which require the imprecise evaluations of the experts should be handled in a broader context of FFS-based MADM to obtain a vigorous and more reliable solution.

In terms of the motivations defined above, this study attempts to contribute to the literature as follows:

1. The originality of the paper arises from the propositions of FFS integrated SAW (Simple Additive Weighting), ARAS (Additive Ratio Assessment), and VIKOR (Vise KriterijumsaOptimizacija I Kompromisno Resenje) MADM methods to analyze the decision model including FF numbers.
2. In these novel contexts, the decision-makers have more freedom in specifying their preferences, thoughts, and expertise.
3. The imprecision and vagueness hidden in human judgements are modelled in a broader context provided by the concept of FFS.

4. The method propositions are applied to an ongoing problem that aims to select the best testing laboratory for diagnosing Covid-19 infected patients. A real case from the literature is analyzed for this purpose.
5. The validity of the methods is tested via benchmarking their ranking results with the results of the applications found by previously developed FFS-based TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution), WPM (Weighted Product Method), and Yager aggregation operators.

The study has this organization: Section 2 gives the preliminaries of FFS and its operations as well as the literature survey results; Section 3 provides the introductions and algorithms of the novel propositions called FFS-SAW, FFS-ARAS, and FFS-VIKOR; Section 4 covers the application results of the propositions on the evaluation of the alternative authentic laboratories for Covid-19 diagnosis and the validity check which is based on the comparisons of the results obtained through different approaches; Section 5 concludes the study with the findings and further research agenda.

2 | FERMATEAN FUZZY SETS

To handle the uncertainties, first unresolved effort was made by Zadeh (1965). Since then, fuzzy sets have been applied in many directions such as decision making (Ekel, 2002), medical diagnosis (Yao & Yao, 2001), and pattern recognition (Pedrycz, 1990). Keeping in the importance of fuzzy sets, many extensions have been introduced like rough sets (Pawlak, 1982), soft sets (Molodtsov, 1999), intuitionistic fuzzy sets (Atanassov, 1986), linear diophantine fuzzy sets (Riaz & Hashmi, 2019), bipolar valued fuzzy sets (Lee, 2000) and bipolar soft sets (Mahmood, 2020; Shabir & Naz, 2013). Although all these extensions have their own advantages, the notion of IFS (Atanassov, 1986) has gained much more attention from the researchers as compared to the others, see Ejegwa et al., 2020; Rahman et al., 2020; Garg, 2016b.

In IFS, more uncertainty and imprecision can be modelled since both agreement (membership) and disagreement (non-membership) levels are provided. The limitation of IFS is that the sum of membership and non-membership degrees is restricted to a unit interval. To cope with this restriction, Yager (2014) extended the representation domain of IFS via relaxing the boundary and setting the squared sum of the degrees within the unit interval. This new type of fuzzy set was called PFS. A similar boundary is valid for PFS because some expert opinions cannot be modelled by both IFS and PFS, for example, (0.80, 0.75) since their sum is 1.55 (>1) and their square sum is 1.20 (>1). As a generalization of PFS, Yager (2017) established the theory of q-ROFS such that the sum of q^{th} power of degrees is bounded by 1. Recently, Senapati and Yager (2020) specialized q-ROFS by setting $q = 3$ and recalled it as FFS such that the sum of cubes is defined in a closed unit interval. FFS has a broader representation domain of human judgements because it covers the areas of IFS and PFS, see Figure 1. For instance, (0.80, 0.75) can be operated by FFS because the sum of the cubes is equal to 0.93 (<1).

The basics of FFS and operations defined on them are explained in this section.

Definition 1. Let X be a universal set. Then a FFS A in X is defined as follows:

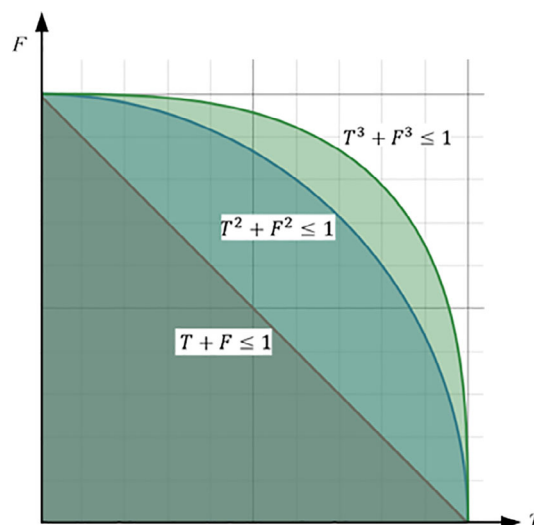


FIGURE 1 Benchmark of IFS, PFS, and FFS

$$A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\} \quad (1)$$

where μ_A, ν_A are mappings from X to $[0, 1]$. For all $x \in X$, $\mu_A(x)$ is called positive membership degree of $x \in A$ and $\nu_A(x)$ is negative membership degree of $x \in A$. The only condition of FFS that must be satisfied is given as:

$$0 \leq (\mu_A(x))^3 + (\nu_A(x))^3 \leq 1, \forall x \in X \quad (2)$$

and $\pi_A(x) = \sqrt[3]{1 - (\mu_A(x))^3 - (\nu_A(x))^3}$ is called the degree of indeterminacy (hesitancy) of x in A (Senapati & Yager, 2020).

FFS is a special and more concise case of q -ROFS and provides a broader representation opportunity for the decision-makers than IFS and PFS can. To clarify the differences among these three fuzzy set definitions, their geometry is studied in Figure 1. When we set $\mu_A(x) = T$ and $\nu_A(x) = F$, it is noticed that IFS represents all the points beneath the line of $T + F \leq 1$, PFS represents all the points beneath the curve of $T^2 + F^2 \leq 1$, and FFS represent all the points beneath the curve of $T^3 + F^3 \leq 1$. Thus, the area covered by FFS is broader than the area represented by IFS and PFS.

The basic mathematical operations are defined by Senapati and Yager (2019a, 2020) as listed above. These studies can be reviewed for some assertions and proofs.

Definition 2. Let $F_1 = (\mu_1, \nu_1)$, $F_2 = (\mu_2, \nu_2)$ and $F = (\mu, \nu)$ be FFSs, then their operations are as follows:

$$F_1 \oplus F_2 = \left(\sqrt[3]{\mu_1^3 + \mu_2^3 - \mu_1^3 \mu_2^3}, \nu_1 \nu_2 \right) \quad (3)$$

$$F_1 \otimes F_2 = \left(\mu_1 \mu_2, \sqrt[3]{\nu_1^3 + \nu_2^3 - \nu_1^3 \nu_2^3} \right) \quad (4)$$

$$F_1 \ominus F_2 = \left(\sqrt[3]{\frac{\mu_1^3 - \mu_2^3}{1 - \mu_2^3}}, \frac{\nu_1}{\nu_2} \right) \text{ if } \mu_1 \geq \mu_2 \text{ and } \nu_1 \leq \min \left\{ \nu_2, \frac{\nu_2 \pi_1}{\pi_2} \right\} \quad (5)$$

$$F_1 \oslash F_2 = \left(\frac{\mu_1}{\mu_2}, \sqrt[3]{\frac{\nu_1^3 - \nu_2^3}{1 - \nu_2^3}} \right) \text{ if } \nu_1 \geq \nu_2 \text{ and } \mu_1 \leq \min \left\{ \mu_2, \frac{\mu_2 \pi_1}{\pi_2} \right\} \quad (6)$$

$$\tau F = \left(\sqrt[3]{(1 - (1 - \mu^3)^\tau)}, \nu^\tau \right) \quad (7)$$

$$F^\tau = \left(\mu^\tau, \sqrt[3]{(1 - (1 - \nu^3)^\tau)} \right) \quad (8)$$

$$F^C = (\nu, \mu) \quad (9)$$

Definition 3. Let $F = (\mu, \nu)$ be a FFS, then the score and accuracy functions are defined as follows:

$$sc(F) = \mu^3 - \nu^3 \quad (10)$$

$$acc(F) = \mu^3 + \nu^3 \quad (11)$$

Definition 4. Let $F_1 = (\mu_1, \nu_1)$ and $F_2 = (\mu_2, \nu_2)$ be FFSs, then they are ranked according to the following rules:

1. If $sc(F_1) < sc(F_2)$, then $F_1 < F_2$
2. If $sc(F_1) > sc(F_2)$, then $F_1 > F_2$
3. If $sc(F_1) = sc(F_2)$, then

- a. If $acc(F_1) < acc(F_2)$, then $F_1 < F_2$
- b. If $acc(F_1) > acc(F_2)$, then $F_1 > F_2$
- c. If $acc(F_1) = acc(F_2)$, then $F_1 \approx F_2$.

Definition 5. Let $F_1 = (\mu_1, v_1)$ and $F_2 = (\mu_2, v_2)$ be FFSs, then the Euclidean distance between them is defined as follows:

$$d_{euc}(F_1, F_2) = \sqrt{\frac{1}{2} [(\mu_1^3 - \mu_2^3)^2 + (v_1^3 - v_2^3)^2 + (\pi_1^3 - \pi_2^3)^2]} \quad (12)$$

Definition 6. Let $F_1 = (\mu_1, v_1)$ and $F_2 = (\mu_2, v_2)$ be FFSs, then the average (arithmetic mean) operator is given as follows:

$$aver(F_1, F_2) = \left(\frac{\mu_1^3 + \mu_2^3}{2}, \frac{v_1^3 + v_2^3}{2} \right) \quad (13)$$

Since the first appearance of FFS in the literature, researchers have made contributions to this new concept. Senapati and Yager (2020) extended TOPSIS under FFS environment to solve a location selection problem for a house while Senapati and Yager (2019a) proposed a FFS-based extension of WPM (weighted product method) and applied it in a bridge construction method selection problem. Liu, Liu, and Chen (2019) developed Fermatean fuzzy linguistic set and showed its application with a new extension of TOPSIS. Similarly, Liu, Liu, and Wang (2019) proposed some novel distance measures and demonstrated their usability in novel extensions of TOPSIS and TODIM (Tomada de Decisão Interativa Multicritério). The last two studies applied their methodology in the same problem described by Senapati and Yager (2020).

In terms of averaging, aggregation, and geometric operators, few studies are found in the literature. Senapati and Yager (2019b) identified Fermatean fuzzy weighted average, geometric, power average, and power geometric operators and verified their applicability in a MADM problem that requires aggregation. Aydemir and Yilmaz Gunduz (2020) defined some operators, namely Fermatean fuzzy Dombi weighted average and geometric operators, ordered weighted average and geometric operators, hybrid weighted average, and geometric operators. They demonstrated the usability of the operators with TOPSIS. These two papers chose to apply their propositions to the problems defined by Senapati and Yager (2019a, 2020). Garg et al. (2020b) also proposed six new operators: Fermatean fuzzy Yager weighted average, ordered weighted average, hybrid weighted average, weighted geometric, ordered weighted geometric, and hybrid weighted geometric operators. They selected TOPSIS to check the validity of the proposed operators and researched selecting an authentic lab for the COVID-19 test. Akram et al. (2020) developed Fermatean fuzzy Einstein weighted averaging, ordered weighted averaging, generalized Fermatean fuzzy Einstein weighted averaging, and ordered weighted averaging operators. Then, the operators' applicability to the MADM field was shown on a problem of effective sanitizer selection for reducing Covid-19 impact. Similarly, Shahzadi and Akram (2021) proposed four new Fermatean fuzzy soft Yager operators of weighted average, ordered weighted average, weighted geometric, and ordered weighted geometric, and their applicability is introduced in an application of antivirus mask selection problem. Wang et al. (2019) proposed some mean operators of hesitant Fermatean 2-tuple linguistic terms and utilized them for solving an investment selection problem.

As seen from the literature review, TOPSIS, WPM, and TODIM have been extended under FFS in the literature until now. Thus, different MADM methods are required to be modified for handling the decision problems which are defined under FFS environment. This study has attempted to make the appropriate modifications of SAW, ARAS, and VIKOR for fulfilling this gap.

3 | FFS-BASED MADM

In this section, we introduce the novel FFS extensions of three well-known MADM methods, namely, SAW, ARAS, and VIKOR in order to smooth the preference representation challenge of the decision-makers when they are consulted for their knowledge and expertise in solving a decision problem. Also, some contemporary fuzzy extensions of the mentioned methods are expressed for a better understanding of the state-of-the-art. As seen from the brief literature surveys about the methods, their current extensions under IFS and PFS have limited capabilities than FFS has as depicted in Figure 1.

3.1 | FFS-SAW

SAW which is known under different names such as factor rating or weighted sum method is a well-known and commonly used method in the MADM field. In the original methodology developed by Churchman and Ackoff (1954), the normalized performance scores of alternatives that are

obtained with respect to the attributes are weighted via multiplying with the weights of the attributes, and then the alternatives are ranked in descending order of these scores. The alternative having the highest score is accepted as the best one (Triantaphyllou & Mann, 1989).

In the literature, there are different fuzzy extensions of SAW. Kaur and Kumar (2013) proposed an IFS extension of SAW in handling the best vendor selection problem while Büyüközkan and Güler (2020) developed a hesitant fuzzy linguistic SAW method and applied it to the evaluation of the smartwatches. Also, Boltürk and Kahraman (2020) showed the SAW version under the PFS environment for an Automated Storage and Retrieval Systems selection problem. To the best of our knowledge, the literature does not have any FFS based SAW extension. The algorithm of the proposed FFS-SAW method is as follows:

Step 1. Concerning the preferences and ideas of decision-makers, decision matrix (D) is constructed. If there is a group decision-making problem in hand, k decision-makers might give their matrices such as $D^e = [x_{ij}^e]$ where $x_{ij}^e = (\mu_{ij}^e, \nu_{ij}^e)$ and $e = 1, \dots, k; i = 1, \dots, m; j = 1, \dots, n$. Here, x_{ij}^e represents the e^{th} decision-maker's preference for alternative i with respect to attribute j . The aggregated decision matrix of $D = [x_{ij}]$ where $x_{ij} = (\mu_{ij}, \nu_{ij})$ is obtained via the Fermatean fuzzy weighted averaging (FFWA) operator which is developed by Senapati and Yager (2019b). w_e shows the weight of the e^{th} decision-maker and represents his/her expertise level for the problem at hand.

$$\text{FFWA} : x_{ij} = \left(\sum_{e=1}^k w_e \mu_{ij}^e, \sum_{e=1}^k w_e \nu_{ij}^e \right) \quad (14)$$

Step 2. The performance scores will be ready for operations after a normalization process since they require to be converted to a comparable and unitless type. x_{ij} values are normalized via Equation (15) as Aydemir and Yılmaz Gunduz (2020) stated.

$$x_{ij} = \begin{cases} x_{ij} = (\mu_{ij}, \nu_{ij}) & \text{if } j \text{ is a benefit attribute,} \\ (x_{ij})^c = (\nu_{ij}, \mu_{ij}) & \text{if } j \text{ is a cost attribute.} \end{cases} \quad (15)$$

Step 3. For performing aggregation of normalized performance scores in SAW, w_j representing the weight of the j^{th} attribute is needed. The weight set can be obtained objectively or subjectively by applying any methodology defined in the literature. Equation (16) is performed in calculating the aggregated performance scores.

$$\text{SAW}_i = \sum_{j=1}^n w_j x_{ij} = \sum_{j=1}^n w_j (\mu_{ij}, \nu_{ij}) = (\mu_i, \nu_i) \quad (16)$$

This operation is made iteratively via multiplications and then additions. First, the scores are weighted with the multiplication operation given in Equation (17). Then, these weighted scores per attribute are summed by performing an addition operation where Equation (18) gives an example including the summation of the first and second performance values.

$$w_j (\mu_{ij}, \nu_{ij}) = \left(\sqrt[3]{(1 - (1 - (\mu_{ij})^3)^{w_j}), (\nu_{ij})^{w_j}} \right) = (\mu_{ij}^w, \nu_{ij}^w) \quad (17)$$

$$(\mu_{i1}^w, \nu_{i1}^w) \oplus (\mu_{i2}^w, \nu_{i2}^w) = \left(\sqrt[3]{((\mu_{i1}^w)^3 + (\mu_{i2}^w)^3 - (\mu_{i1}^w)^3 (\mu_{i2}^w)^3)}, \nu_{i1}^w \nu_{i2}^w \right) \quad (18)$$

Step 4. In the final step, the aggregated $\text{SAW}_i = (\mu_i, \nu_i)$ FF numbers are defuzzified via score and accuracy functions and the ruleset which is given in Definition 4 is utilized for ranking alternatives. The related functions are rewritten with the appropriate notions in the current step and given in Equations (19) and (20). In general, only descending order of $sc(\text{SAW}_i)$ crisp values are found sufficient for ranking the alternatives. The best alternative will have the highest defuzzified value.

$$sc(\text{SAW}_i) = (\mu_i)^3 - (\nu_i)^3 \quad (19)$$

$$acc(\text{SAW}_i) = (\mu_i)^3 + (\nu_i)^3 \quad (20)$$

3.2 | FFS-ARAS

ARAS which is developed by Zavadskas and Turskis (2010) aims to prioritize the alternatives which are assessed with respect to significant attributes with the aim of reaching a more extensive decision. ARAS method is based on the utility function value impelling the complex relative

efficiency of alternatives which is proportional to the relative impact of weighted scores. Each alternative and an optimal one derived from the decision matrix are relatively compared.

Some papers which modified ARAS with contemporary fuzzy concepts are found in the literature. Büyüközkan and Göçer (2018) utilized an interval-valued IFS version of ARAS for the supplier selection problem of digital supply chains. Mishra et al. (2020) proposed an IFS version of ARAS in a real-life application of information technologies personnel selection. Mohagheghi and Mousavi (2019) developed interval-valued PFS-based ARAS and demonstrated the validity of the method in a sustainable project portfolio evaluation problem. Çalış Boyacı (2020) developed the hesitant fuzzy linguistic ARAS version and performed it in the selection of the most eco-friendly city of Turkey. The first attempt integrating ARAS and FF numbers belongs to this study for the first time in the literature and the algorithm of the proposition is given below.

Step 1. The procedure for forming the decision matrix is the same as FFS-SAW.

Step 2. The normalization of the decision matrix is applied as given in FFS-SAW.

Step 3. The first distinctive feature of ARAS is the addition of the optimal solution into the decision matrix. For extracting this optimal alternative from the current decision matrix of $D = [x_{ij}]$ where $x_{ij} = (\mu_{ij}, \nu_{ij})$, the score function comparisons of the performance values are conducted as proposed by Senapati and Yager (2020). Equation (21) is performed for this purpose.

$$x_{0j} = (\mu_{0j}, \nu_{0j}) = \{x_{ij} \text{ where } \max_i < sc(x_{ij}) | i = 1, \dots, m > \} \quad (21)$$

The final decision matrix includes the corresponding FF number-based evaluation in the first row: $D = [x_{ij}]$ where $i = 0, \dots, m; j = 1, \dots, n$. The new index $i = 0$ represents the optimal alternative.

Step 4. The attributes' priorities are reflected in the performance values via the multiplication of the weights and the performance scores presented under each attribute. Equation (16) is applied for this task. The output of this step is the FF optimality values: $X_i = (\mu_i, \nu_i)$. It is clear that $X_i = SAW_i$.

Step 5. The utility value which is based on the comparison of X_i values of optimal alternative (X_0) and each alternative (X_i) is computed ($i = 1, \dots, m$). This utility value is $K_i = sc(X_i) / sc(X_0)$. After the alternatives are ranked according to their decreasing values of K_i , the most desirable alternative will have the highest utility value.

3.3 | FFS-VIKOR

The VIKOR method was developed by Opricovic (1998) as a MADM approach with the purpose of handling discrete decision problems having incommensurable and conflicting attributes. It can determine compromise solutions for a decision-making problem with conflicting attributes and support the decision-makers to reach a final decision. The compromise solution is a feasible solution that is the closest to the ideal one (Opricovic, 2011).

Extending VIKOR under various fuzzy environments forms a fruitful area in the literature of MADM. An extensive literature review focusing on fuzzy VIKOR versions may be seen in Kutlu Gündoğdu and Kahraman (2019). For instance, while Devi (2011) extended VIKOR under the IFS environment for robot selection problem, Wu et al. (2019) used interval-valued IF numbers in the application of VIKOR and demonstrated the usage of the method in financial risk assessment of rural tourism projects. PFS-based VIKOR propositions are identified by many researchers such as T. Y. Chen (2018) for 5 different MADM problems including service quality assessment of domestic airlines, investment decisions regarding Internet stocks, etc., Rani et al. (2019) for the evaluation of renewable energy technologies in India, and Gul et al. (2019) in a problem regarding the safety risk assessment of mines. Krishankumar et al. (2020) proposed a q-ROFS version of VIKOR with unknown weight information and presented its application in solving the green supplier selection problem. As seen from the literature, there is a gap concerning the FFS-based extension of VIKOR in the literature, and the detailed algorithm below aims at fulfilling this gap.

Step 1. The procedure for forming the decision matrix is the same as FFS-SAW and FFS-ARAS.

Step 2. The normalization of the decision matrix is applied as given in FFS-SAW and FFS-ARAS.

Step 3. VIKOR is based on the positive and negative ideal solutions which should be derived from the decision matrix of $D = [x_{ij}]$. Equations (22) and (23) can be used for these aims, respectively.

$$A^* = \{x_{ij} \text{ where } \max_i < sc(x_{ij}) | i = 1, \dots, m > \} \quad (22)$$

$$A^- = \{x_{ij} \text{ where } \min_i < sc(x_{ij}) | i = 1, \dots, m > \} \quad (23)$$

While the elements of A^* are depicted as $x_j^+ = (\mu_j^+, \nu_j^+)$, the elements of A^- is shown as $x_j^- = (\mu_j^-, \nu_j^-)$.

Step 4. VIKOR computes two initial measures: S_i represents the average gap between the alternative and the ideal solutions while R_i shows the maximal gap for improvement priority (Tzeng & Huang, 2011). FFS-based measures are calculated as given in Equations (24) and (25), respectively.

$$S_i = \sum_{j=1}^n w_j \frac{d_{euc}(x_{ij}, x_j^+)}{d_{euc}(x_j^-, x_j^+)} \quad (24)$$

$$R_i = \max_j \left\{ w_j \frac{d_{euc}(x_{ij}, x_j^+)}{d_{euc}(x_j^-, x_j^+)} \right\} \quad (25)$$

where d_{euc} depicts the Euclidean distance between FFSs [Equation (12)]. As seen from the equations, VIKOR method focuses on the distances between alternatives and ideal solutions.

Step 5. Q_i values representing the aggregation of average and maximal gaps (weighted distances) are calculated via Equation (26) where $S^* = \min_i S_i$, $S^- = \max_i S_i$, $R^* = \min_i R_i$, $R^- = \max_i R_i$.

$$Q_i = v \frac{S_i - S^*}{S^- - S^*} + (1 - v) \frac{R_i - R^*}{R^- - R^*} \quad (26)$$

v is identified as a weight for the strategy of the maximum group utility whereas $(1 - v)$ is the weight of the individual regret. These two strategies might be compromised by $v = 0.5$. Opricovic (2011) modified v as $v = (n+1)/2n$ from $(v + 0.5 * (n-1)/n = 1)$ because the attribute (1 of n) related to R is included in S , too.

Step 6. The alternatives are ranked in increasing order of S_i , R_i , and Q_i . The compromise solution will have the minimum Q_i if the following two conditions are satisfied.

C1. *Acceptable Advantage*: $[Q_i(2nd - ranked alt.) - Q_i(1st - ranked alt.)] / [Q_i(La - ranked alt.) - Q_i(1st - ranked alt.)]$ is called advantage rate and bounded with a threshold of $1/(m-1)$.

C2. *Acceptable Stability in decision-making*: The compromise alternative must also be ranked as the best one in the lists of S_i and/or R_i .

If one of the two conditions is not satisfied, a compromise solution sets can be generated.

1. If only C2 is not satisfied, the first and second-ranked alternatives are proposed.
2. If only C1 is not satisfied, the first Ω alternatives form the compromise solution set. The last Ω alternative is obtained by $[Q_i(\Omega th - ranked alt.) - Q_i(1st - ranked alt.)] < 1/(m-1)$.

4 | AN APPLICATION

The FFS-based extensions of SAW, ARAS, and VIKOR are applied to the problem described by Garg et al. (2020b). The proposed approaches are tried to be validated on a decision problem focusing on COVID-19 because this pandemic period prevails at present and this application keeps its actuality today.

Huang et al. (2020) stated that coronaviruses are enveloped non-segmented positive-sense RNA viruses belonging to the family Coronaviridae and the order Nidovirales. They are broadly distributed in all mammals. In December 2019, a series of pneumonia cases of unknown cause emerged in Wuhan, the capital city of Hubei province of China, with clinical presentations greatly resembling viral pneumonia. Deep sequencing analysis from lower respiratory tract samples indicated a novel coronavirus, which was named 2019 novel coronavirus (2019-nCoV). Guan et al. (2020) mentioned that the World Health Organization declared this coronavirus disease 2019 (Covid-19) as a public health emergency of international concern: CO is COrona, VI stands for Virus, and D represents Disease.

In order to eliminate or minimize at least the spread of Covid-19, countries are trying some manners such as quarantining citizens, limiting travel, testing and treating patients, carrying out contact tracing, and cancelling large gatherings, etc. Covid-19 is more than a health crisis since the speed of the virus spread is noncomparable with other well-known viruses. This pandemic is currently impacting negatively the countries' economic conditions, quality of social life, and political circumstances all over the world.

In the field of MADM, studies are focusing on some specific needing of the Covid-19 pandemic. Ashraf and Abdullah (2020) developed spherical fuzzy information-based TOPSIS method for evaluating the emergency alternatives caused by the Covid-19 pandemic in China. Alqahtani and Rajkhan (2020) integrated AHP (Analytic Hierarchy Process) and TOPSIS in evaluating e-learning critical success factors during the Covid-19

pandemic. De Nardo et al. (2020) utilized PAPRIKA (Potentially All Pairwise Rankings of All Possible Alternatives) for prioritizing hospital admissions of patients affected by Covid-19. Ren et al. (2020) developed a hesitant fuzzy MADM approach for medicine selection problem for patients with mild symptoms of Covid-19. Zolfani et al. (2020) applied a novel Grey-based decision framework to select the best location for a temporary Covid-19 hospital. Sayan et al. (2020) compared the results of fuzzy-based TOPSIS and PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluation) methods which are applied in the evaluations of Covid-19 diagnostic tests. Yang et al. (2020) proposed spherical fuzzy aggregation operators and their applications are shown on the antivirus mask selection problem. Mohammed et al. (2020) combined entropy measure and TOPSIS to benchmark the Covid-19 diagnostic models. Ashraf et al. (2020) proposed spherical fuzzy-based TOPSIS and COPRAS (Complex Proportional Assessment) methods with the aim of assessing the emergency alternatives during the Covid-19 period. Samanlıoğlu and Kaya (2020) applied the hesitant fuzzy AHP method in the problem of evaluation of the COVID-19 pandemic intervention strategies.

During this emergency period, the people who have symptoms of the Covid-19 virus must take a medical test for a diagnose. Garg et al. (2020b) described an authentic laboratory selection problem for suggesting a solution methodology for this need. The alternative laboratories are represented by $Y_i, i = 1, \dots, 5$. The decision analyst considered 3 attributes affecting the most effective laboratory selection decision: time limit (C_1), accurate result (C_2), and location flexibility for the client (C_3). The weights are determined as $w_1 = 0.3, w_2 = 0.4, w_3 = 0.3$. The decision matrix is directly stated in the study as depicted in Table 1. All the attributes are evaluated as benefit types and the problem is not described as a group decision-making problem. It might be but the study does not include any detail about the data collection stage. So, we assume that there was a decision committee evaluating alternatives with respect to attributes collaboratively. The decision matrix of $D = [x_{ij}], i = 1, \dots, 5; j = 1, 2, 3$ is the consensus result that was reached at the end of their discussions within the committee.

4.1 | Application of FFS-SAW

This proposition is based on the weighting of performance scores of alternatives. Steps 1 and 2 are directly skipped because of the nature of the data set in hand. For illustration, the weighting of the first alternative's performance scores is given as follows:

$$0.3 * (0.700, 0.400) = \left(\sqrt[3]{(1 - (1 - 0.700^3)^{0.3}), 0.400^{0.3}} \right) = (0.491, 0.760)$$

$$0.4 * (0.600, 0.300) = \left(\sqrt[3]{(1 - (1 - 0.600^3)^{0.4}), 0.300^{0.4}} \right) = (0.453, 0.618)$$

$$0.3 * (0.800, 0.300) = \left(\sqrt[3]{(1 - (1 - 0.800^3)^{0.3}), 0.300^{0.3}} \right) = (0.579, 0.697)$$

After the multiplications, the SAW_i values are found via Equation (16) by performing iterative calculations shown in Equation (18). The calculations of the first alternative are shown below.

$$(0.491, 0.760) \oplus (0.453, 0.618) = \left(\sqrt[3]{(0.491^3 + 0.453^3 - 0.491^3 * 0.453^3), 0.760 * 0.618} \right) = (0.585, 0.469)$$

$$(0.585, 0.469) \oplus (0.579, 0.697) = \left(\sqrt[3]{(0.585^3 + 0.579^3 - 0.585^3 * 0.579^3), 0.469 * 0.697} \right) = (0.708, 0.327) = SAW_1$$

TABLE 1 Fermatean fuzzy decision matrix of D

	C_1		C_2		C_3	
Y_1	0.700	0.400	0.600	0.300	0.800	0.300
Y_2	0.800	0.600	0.700	0.500	0.500	0.200
Y_3	0.500	0.300	0.600	0.800	0.600	0.400
Y_4	0.700	0.500	0.900	0.300	0.900	0.400
Y_5	0.600	0.100	0.400	0.100	0.300	0.400

After aggregating performance scores, the score function values are required in order to rank the alternatives in descending order: $sc(SAW_1) = 0.708^3 - 0.327^3 = 0.320$.

All the weighted values of the alternatives, aggregated performance scores of SAW_i , and the related score function values are given in Table 2. The last column shows the rank of alternatives: $Y_4 > Y_1 > Y_2 > Y_5 > Y_3$. So, FFS-SAW finds laboratory number 4 is the best testing facility.

4.2 | Application of FFS-ARAS

The same initiating conditions are valid for FFS-ARAS application, too. As specified in Step 4 of FFS-ARAS, $SAW_i = X_i$. Thus, Table 2 will be the output table of the application of FFS-ARAS. The distinctive feature of ARAS includes the determination of the optimal alternative. For this purpose, all the values in Table 1 are defuzzified via performing Equation (21). The crisp values, the positive and negative ideals are depicted in Table 3.

The optimal alternative of ARAS is equal to the VIKOR's positive ideal alternative. To avoid repetition, both ideals are depicted in Table 3. Per each attribute, the positive ideal values are shown in italic, and negative ideal solutions are shown as underlined. Thus, the optimal alternative for FFS-ARAS is obtained as given in the vector below:

$$x_{0j} = [(0.800, 0.600), (0.900, 0.300), (0.900, 0.400)]$$

Table 4 includes Table 2 and also the related optimal alternative in the first row. K_i values are demonstrated to rank the alternatives. FFS-ARAS gives the same ranking as FFS-SAW: $Y_4 > Y_1 > Y_2 > Y_5 > Y_3$.

4.3 | Application of FFS-VIKOR

The decision matrix of the laboratory selection problem is known, and the ideal and negative ideal solutions are depicted in Table 3. The vectors respectively representing them are as follows:

$$x_j^+ = [(0.800, 0.600), (0.900, 0.300), (0.900, 0.400)]$$

$$x_j^- = [(0.500, 0.300), (0.600, 0.800), (0.300, 0.400)]$$

TABLE 2 Results of FFS-SAW

	C ₁		C ₂		C ₃		SAW _i	sc(SAW _i)	Rank	
Y ₁	0.491	0.760	0.453	0.618	0.579	0.697	0.708	0.327	0.320	2
Y ₂	0.579	0.858	0.537	0.758	0.340	0.617	0.701	0.401	0.281	3
Y ₃	0.340	0.697	0.453	0.915	0.413	0.760	0.575	0.484	0.076	5
Y ₄	0.491	0.812	0.741	0.618	0.687	0.760	0.865	0.381	0.591	1
Y ₅	0.413	0.501	0.297	0.398	0.201	0.760	0.467	0.152	0.099	4

TABLE 3 Defuzzified decision matrix

	C ₁	C ₂	C ₃
Y ₁	0.279	0.189	0.485
Y ₂	0.296	0.218	0.117
Y ₃	<u>0.098</u>	<u>-0.296</u>	0.152
Y ₄	0.218	0.702	0.665
Y ₅	0.215	0.063	<u>-0.037</u>
x_j^+ (x_{0j})	0.296	0.702	0.665
x_j^-	0.098	<u>-0.296</u>	<u>-0.037</u>

TABLE 4 Results of FFS-ARAS

x_{0j}	C_1		C_2		C_3		X_i		$sc(X_i)$	K_i	Rank
	0.579	0.858	0.741	0.618	0.687	0.760	0.878	0.403	0.611		
Y_1	0.491	0.760	0.453	0.618	0.579	0.697	0.708	0.327	0.320	0.523	2
Y_2	0.579	0.858	0.537	0.758	0.340	0.617	0.701	0.401	0.281	0.459	3
Y_3	0.340	0.697	0.453	0.915	0.413	0.760	0.575	0.484	0.076	0.125	5
Y_4	0.491	0.812	0.741	0.618	0.687	0.760	0.865	0.381	0.591	0.967	1
Y_5	0.413	0.501	0.297	0.398	0.201	0.760	0.467	0.152	0.099	0.161	4

TABLE 5 Results of FFS-VIKOR

	d_{euc}			S_i	R_i	Q_i	Ranks		
	C_1	C_2	C_3				S_i	R_i	Q_i
Y_1	0.278	0.513	0.238	0.676	0.411	0.597	3	4	3
Y_2	0.000	0.348	0.634	0.549	0.278	0.402	2	2	2
Y_3	0.509	0.500	0.513	0.919	0.400	0.755	4	3	4
Y_4	0.229	0.000	0.000	0.135	0.135	0.000	1	1	1
Y_5	0.444	0.678	0.702	1.105	0.543	1.000	5	5	5

S and R values which are distance-based measures are the basic elements of the VIKOR method. For illustration, the distance between the first alternative and positive ideal solution for C_1 is shown below:

$$d_{euc}(x_{11}, x_1^+) = \sqrt{\frac{1}{2}[(0.700^3 - 0.800^3)^2 + (0.400^3 - 0.600^3)^2 + (0.840^3 - 0.648^3)^2]} = 0.278$$

The distance values for the remaining two attributes are found as $d_{euc}(x_{12}, x_2^+) = 0.513$ and $d_{euc}(x_{13}, x_3^+) = 0.238$. These three values can be shown as a vector: $[0.278, 0.513, 0.238]$. The vector representing the distances between positive and negative ideal solutions is formed as follows: $d_{euc}(x_j^-, x_j^+) = [0.509, 0.500, 0.702]$.

The next step involves the calculation of S_i and R_i values. For illustration,

$$S_1 = 0.3 * \frac{0.278}{0.509} + 0.4 * \frac{0.513}{0.500} + 0.3 * \frac{0.238}{0.702} = 0.676$$

$$R_1 = \max_j \left\{ 0.3 * \frac{0.278}{0.509}; 0.4 * \frac{0.513}{0.500}; 0.3 * \frac{0.238}{0.702} \right\} = 0.411$$

Table 5 shows all the required information gathered for FFS-VIKOR. For calculation of Q_i , the required parameters are found as follows: $S^* = \min_i S_i = 0.135$, $S^- = \max_i S_i = 1.105$, $R^* = \min_i R_i = 0.135$, $R^- = \max_i R_i = 0.543$. The first alternative's Q value is found as given:

$$Q_1 = 0.667 * \frac{0.676 - 0.135}{1.105 - 0.135} + (1 - 0.667) * \frac{0.411 - 0.135}{0.543 - 0.135} = 0.597$$

where $v = \frac{(n+1)}{2n} = \frac{(3+1)}{2*3} = 0.667$ as specified in Step 5 of FFS-VIKOR.

To reach a compromise solution, the two conditions should be checked.

C1. Acceptable Advantage: $\frac{Q_i(Y_2) - Q_i(Y_4)}{Q_i(Y_5) - Q_i(Y_4)} = \frac{0.402 - 0}{1 - 0} = 0.402 \geq \frac{1}{5-1} = 0.25$. So, the condition is satisfied.

C2. Acceptable Stability in decision-making: The compromise alternative (Y_4) is ranked as the best one in the lists of S_i and R_i . So, the second condition is satisfied.

It is concluded that the compromise solution of Y_4 is the best alternative that is proposed to the decision-makers. Considering Q values, the alternatives are ranked: $Y_4 > Y_2 > Y_1 > Y_3 > Y_5$.

TABLE 6 Comparison results

	WPM	TOPSIS-1	TOPSIS-2	SAW	VIKOR	ARAS	Original
Y_1	2	2	2	2	3	2	2
Y_2	3	3	3	3	2	3	3
Y_3	5	4	5	5	4	5	5
Y_4	1	1	1	1	1	1	1
Y_5	4	5	4	4	5	4	4

4.4 | Comparison of MADM methods

In order to check the validity of the proposed methods, their results are compared with other methods. Firstly, the problem aiming at selecting the best testing facility for Covid-19 diagnosis is respectively analyzed by the FFS-TOPSIS method developed by Senapati and Yager (2020) and FFS-WPM proposed by Senapati and Yager (2019a). The algorithms are not repeated here, so the interested readers can study the two specified articles. But in short, TOPSIS is a method focusing on finding the best alternative which is relatively the closest one to the positive ideal solution. This relative measurement requires a combination of the highest separation from the negative ideal and the lowest separation from the positive ideal alternatives. WPM uses an aggregation process that is based on the multiplications of the weighted performance scores where the weights are considered as powers of the scores.

The comparison results are demonstrated in Table 6. TOPSIS-I represents the solution based on the closeness index while TOPSIS-II states the alternative ranks obtained via considering the relative closeness measurement proposed by Hadi-Vencheh and Mirjaberi (2014) as a modification of TOPSIS. The columns of SAW, ARAS, and VIKOR show the novel methods' ranking results while the original solution developed by Garg et al. (2020b) is given in the last column.

It is seen that alternative Y_4 is best ranked for all the methods applied. The rankings of the other alternatives are slightly changing.

1. Y_1 is mostly the second-ranked alternative; only FFS-VIKOR listed it in the third order.
2. Y_2 takes third place in the list; only FFS-VIKOR considered it as the second-best.
3. Y_3 is mostly the worst alternative, but FFS-TOPSIS-I and FFS-VIKOR saw it as the fourth-ranked alternative.
4. Y_5 is seen in fourth place; FFS-TOPSIS-I and FFS-VIKOR considered it as the worst alternative.

So, it is concluded that the newly proposed methods are validated since there are no significant differences among the ranking lists. The differences summarized above can be neglected because there is only 1 difference in terms of the related alternative's rank. For example, in terms of FFS-VIKOR, the alternative pairs of Y_1 - Y_2 and Y_3 - Y_5 have exchanged their ranks when the rankings are benchmarked with other methods.

As an advantage of the proposed methods, it is seen that the FFS-based MADM application is superior to the IFS and PFS-based MADM since the representation domain of FFS is broader than IFS and PFS methods. It is obvious that the proposed FFS-based models cover the IFS and PFS-based applications. Besides, as seen from the comparisons of different FFS-based MADM methods, the alternative rankings are slightly changing. Thus, the methods proposed were validated in this manner. The limitations and the future research possibilities are given in the next section.

5 | CONCLUSION AND FUTURE STUDIES

In MADM, the decision-makers are consulted about their preferences, knowledge, and expertise while studying the decision problem in hand so that various assessment tools are provided to ease the data gathering process for them. IFS and PFS are the alternative tools to realize this purpose, but they have a limited capability than FFS. Their mathematical boundaries are extended in FFS by considering the sum of the cubes of the membership and non-membership degrees rather than directly taking their sum (like IFS) or their squared sum (like PFS). This extension presents a broader preference domain to the decision-makers.

As the first contribution of the study to the literature, it respectively integrates this novel type of q-ROFS, namely, Fermatean fuzzy set with three well-known MADM methods for the first time in the literature: SAW, ARAS, and VIKOR. FFS is a very early concept under the family of fuzzy sets. Thus, its capability and validity should be checked via making such kind of applications as this study has attempted. After giving the developed algorithms called FFS-SAW, FFS-ARAS, and FFS-VIKOR, their applications are shown in a very up-to-date problem covering the testing facility (laboratory) selection for the diagnosis of Covid-19. The results show that there are no significant differences between the proposed

methods and the previous methods of TOPSIS, WPM, and Yager aggregation operators. This finding supports the robustness of the methods proposed newly.

The study also has some boundaries. First, aggregation operations were not shown in real-life applications because the decision-makers of the laboratory selection problem produced a team decision matrix. Future studies can work on the aggregation of the decision matrices formed by different decision-makers. Second, rather than enforcing the decision-makers to allocate directly positive and negative membership degrees, a future study may work on providing appropriate linguistic terms that have FF number correspondences so that the data collection process is eased and becomes more practical. Third, there is no independent consideration of hesitancy (indeterminacy) degree in FFS but it is measured indirectly from the known parameters (membership and non-membership degrees). In FFS, it is claimed that the independent allocation of a hesitancy degree by the decision-maker creates additional complexity and a longer time is required to inform them about the origins of this idea. So, FFS neglects it. If there is enough time to collect data and the decision-makers are trained about the allocation of their hesitancies, further study can consider this opportunity. Spherical (Akram et al., 2021; Ashraf & Abdullah, 2019; Gül, 2021; Kutlu Gündoğdu & Kahraman, 2019; Mahmood et al., 2019) and t-spherical fuzzy sets (Y. Chen et al., 2021; Ullah et al., 2020), and also neutrosophic sets (Abdel-Baset et al., 2019; P. Liu & Cheng, 2019) are good tools for capturing the independent hesitancy data. Also, rather than considering an independent hesitancy value, q-ROFS versions of the models can be developed in order to expand the representation domain (Garg et al., 2020a; Jin et al., 2021). Finally, novel aggregation operators, entropy, similarity, and distance measures as well as inclusion (subsethood degree) measures can also be defined for FFS.

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CONFLICT OF INTEREST

The authors declare no conflicts of interest.

DATA AVAILABILITY STATEMENT

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

ORCID

Sait Gül  <https://orcid.org/0000-0002-6011-0848>

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AUTHOR BIOGRAPHY

Sait Gül is an assistant professor at Management Engineering Department of Bahçeşehir University Faculty of Engineering and Natural Sciences. He received his B.Sc. degrees in Maritime Transport and Management Engineering (2007) and Industrial Engineering (2008) from Istanbul University and holds his M.Sc. in Engineering Management (2012) and Ph.D. in Industrial Engineering (2017), both from Istanbul Technical University. His research interests contain operations research, multiple criteria decision analysis, fuzzy theory applications, knowledge management and big data analysis.

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