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How should local Brick-and-Mortar retailers offer delivery service in a pandemic World? Self-building Vs. O2O platform

Bo He [a,c], Prakash Mirchandani [b], Qichao Shen [a], Guang Yang [a,*]

[a] School of Economics and Business Administration, Chongqing University, Chongqing 400030, China
[b] Katz Graduate School of Business, University of Pittsburgh, Pittsburgh, PA 15260, USA
[c] Chongqing Key Laboratory of Logistics at Chongqing University, Chongqing 400030, China

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ABSTRACT

The Covid-19 pandemic has dramatically changed consumer purchase behavior, and the “stay-at-home order” policy has altered the operations of brick-and-mortar (B&M) retail stores. These changes have induced local B&M retailers to start online retailing with home delivery as an added option. B&M retailers can choose to offer online retailing on their own (referred to as self-building mode) or via a third-party online-to-offline (O2O) platform (referred to as platform mode). This paper investigates how the interplay between capacity, pricing, and online retailing mode is affected by the absence/presence of the pandemic. We characterize the equilibrium between the B&M retailer and the O2O platform provider. We find that the impact of the “stay-at-home orders” on B&M retailers differs by the online retailing mode. Interestingly, we find that the “stay-at-home orders” does not necessarily lower the B&M retailer’s profit if they engage in online retailing. Under self-building mode, the stay-at-home order leaves the B&M retailer with just the online channel. We identify the threshold delivery cost above (below) which the B&M retailer’s profit is lower (higher) than before. Under the platform mode, the “stay-at-home order” alters the retailer’s sales channel from dual channel to single channel, which mitigates the competition between the retailer and the O2O platform. The retailer’s profit increases if it has sufficiently high capacity. Finally, we extend the model to examine the effect of a “reopening policy” with a government subsidy. We find that although the subsidy improves the B&M retailer’s profitability, it may hurt the consumer surplus under some conditions. We suggest that governments take the B&M retailer’s capacity and operations mode into account when designing subsidy policies.

1. Introduction

As the COVID-19 pandemic has spread globally, more states, regions, and countries have issued “stay-at-home orders” to reduce the spread of the coronavirus. Therefore, many national retailers have shuttered all of their physical stores. In response, customers worldwide have radically changed their purchasing behavior, and are favoring locally-sourced products and neighborhood stores.[1] Moreover, consumers are embracing digital commerce and retailers are implementing innovative omnichannel operations.

Local Brick-and-Mortar (B&M) retailers have traditionally served a limited geographical area, but following the pandemic, have

* Corresponding author.
E-mail address: yang_guang@cqu.edu.cn (G. Yang).

[1] https://www.accenture.com/us-en/insights/retail/coronavirus-consumer-behavior-research

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started offering an online channel for their products. By engaging in online retail along with their existing physical store retail, local B&M retailers are enhancing customer convenience, improving market visibility, and enlarging catchment area. Local B&M retailers' online retailing format focuses on local consumers. They are offering quick home delivery service when consumers are not willing or not able to shop at the store in-person. For example, two stores in Pittsburgh with a local presence, East End Food Coop[2] that specializes in local and organic produce, and Labriola's,[3] that specializes in Italian foods, use Mercato's platform for online marketing and providing local delivery service. On the other hand, many B&M retailers, such as Katz's Deli,[4] that have launched their own online selling and local delivery services to dodge excessive fees charged by 3rd party platforms.

Based on these observations from practice, we note that local B&M retailers can implement online retailing and offer delivery service via two specific modes: *self-building mode* and *platform mode*. In the self-building mode, a B&M retailer develops an app and/or a website and delivers the products to local consumers on their own. With the platform mode, the B&M retailer sells on an independent Online-to-offline (O2O) platform, such as Instacart, Postmates, and DoorDash that also offers delivery service to local customers. The O2O platform charges the B&M retailer a commission for providing the marketplace and delivery services. In either case, due to limited budget, shelf-space, or product inventory, local B&M retailers face a capacity constraint. In this paper, we investigate how capacity influences a local B&M retailer's choice of the specific online retailing mode.

Despite the growth in local B&M retailers shifting to become "brick-and-click" retailers, some key characteristics such as offering delivery service, impact of capacity, and choice of specific mode of operations are underexplored in the existing academic literature on online retailing. Moreover, the outbreak of the COVID-19 pandemic has changed consumer purchase behavior and the B&M retailers' operations. Fig. 1 illustrates the impact of COVID-19 pandemic on the B&M retailer's operations.

The outbreak of COVID-19 has led to "stay-at-home orders", which in turn have prevented customers from shopping at the physical store in person. This is shown by the blue arrows in Fig. 1, and the "stay-at-home orders" due to COVID-19 prevent the customer from shopping in person at the B&M store (indicated by the red "X"). The mustard arrows depict the self-building mode, where the customer places an online order directly with the store (indicated by the blue box with an ordering device) and the store delivers the product to the customer (indicated with a mustard box with a truck). The green arrows depict the platform mode where the customer place the order though the platform, and the platform is also responsible for delivering the product.

With COVID-19 being brought under control and the increasing availability of vaccines, some countries and states have started lifting the "stay-at-home orders" and reopening their economies. Even so, customer behavior continues to be different from their pre-pandemic behavior. Therefore, for both with and without the lockdown orders, capturing the interaction between features such as delivery service, capacity, price, and specific mode of operations, and investigating their effect on online operations is of profound importance for B&M retailers. We explore several research issues in this context. First, how do the "stay-at-home orders" and "reopening policy" influence B&M retailers' operations in the presence of the pandemic? Do the "stay-at-home orders" and "reopening policy" decrease the B&M retailers' profit, and if not, under what conditions does it increase? Second, when the local B&M retailer's online retailing mode requires offering delivery service to customers, how does the delivery service affect consumer purchase behavior and the resulting demand structure? Third, how do the B&M retailer's capacity, pricing, and online retailing mode interact with each other in the absence/presence of lockdowns associated with the epidemic?

To address the above questions, we investigate the interactions between capacity, pricing, and the "self-building mode/platform mode" choice of a B&M retailer in the absence/presence of the pandemic. We identify capacity thresholds to characterize the B&M retailer's optimal decisions and profit. We describe the equilibrium between the B&M retailer and the independent O2O platform when the B&M retailer uses the platform mode. We also computationally compare the B&M retailer's profits under the self-building and platform modes. With the reopening of the economy, and the rescinding of the "stay-at-home orders" customers can patronize B&M store in person. However, because the epidemic may not be fully under control, customers still run a risk when shopping at the B&M store; this leads to disutility (Choi 2020) of in-person shopping. According to the recent Accenture's research report referenced previously, consumers are shopping with (i) greater awareness of the environment, health, and cost, and (ii) favoring locally-sourced products and neighborhood stores. Therefore, the "reopening policy" situation is different from the situation before the COVID-19 pandemic. We investigate how this difference affects a B&M store's policies and profits in Section 7. We also examine the case when governments offer subsidies, as many governments are doing, to B&M retailers.

We find that the "stay-at-home orders" do not necessarily lower the B&M retailer's profit if it engages in online retailing. Specifically, under the self-building mode, the B&M retailer builds its own online platform and delivery capability. The B&M retailer's profit is higher than before the outbreak of the pandemic if its delivery cost is low. With the platform mode, the "stay-at-home order" policy moves the retailer from dual-channel to single-channel sales, thus mitigating the competition between the retailer and the O2O platform. In this case, the retailer's profit will increase if it has sufficient capacity. Under the "reopening policy" situation, we find that the consumers' disutility from patronizing the physical store leads to a decrease of the retailer price and total demand; consequently, the B&M retailer's profit decreases with the self-building mode. With the platform mode, when the capacity is intermediate and the O2O platform reduces its commission, the B&M retailer obtains the same profit as before; otherwise, its profit decreases. Under "the "reopening policy" with government subsidies to B&M retailers, B&M's retailers' profits increase but the consumer surplus does not necessarily improve. The consumer surplus may decrease despite the government subsidy because a B&M store with constrained capacity may increase the delivery fee to induce consumers to purchase offline, thereby shifting demand to the offline channel.

[2] https://www.mercato.com/shop/east-end-food-coop

[3] https://www.labriolaitalianmarkets.com/onlineorders

[4] https://www.6sqft.com/katzs-deli-launches-its-own-local-delivery-service/

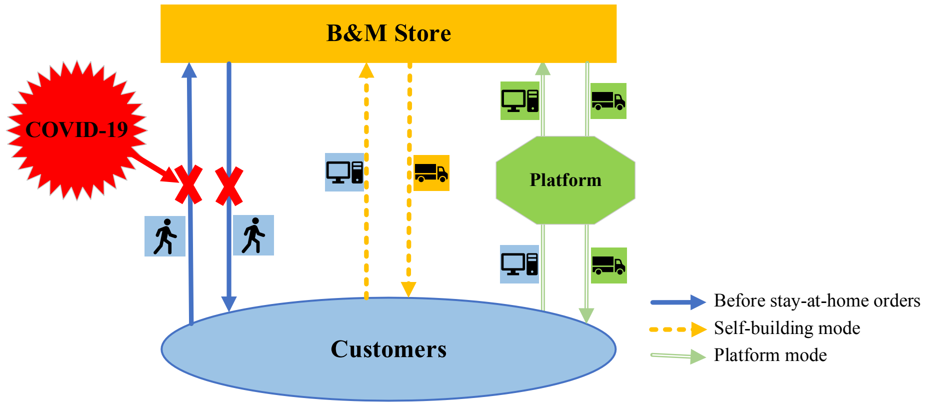

Fig. 1. B&M retailer's operations before and during the COVID-19 pandemic.

Retailers then receive the subsidy at the cost of the customer. Therefore, governments should take a B&M retailer's capacity and operational mode into account when subsidizing retailers.

This paper focuses on the retailer's delivery service, which belongs to the retail logistics area. The contributions of this paper are as follows. First, to complement existing literature that investigates the impact of COVID-19 on the airlines industry (Amankwah 2020), food industry (Singh et al. 2021), healthcare industry (Govindan et al. 2020), and service industry (Choi 2020), this paper investigates how the pandemic will affect the retail industry with a focus on local B&M retailer's operations. Second, the existing literature related to the online retailing mode doesn't consider a retailer's capacity constraint, which is critical for local retailers to engage simultaneously in online and offline operations. By investigating the interactions between pricing, capacity, and modal choice for a B&M retailer, our research fills this gap. Third, this paper proposes managerial recommendations for B&M retailers for effectively facing the pandemic and insights for governments that offer subsidies to the retail industry.

The remainder of this paper is organized as follows. Section 2 provides the literature review. In Section 3, we describe our notation and present our model. Section 4 investigates the self-building mode and platform mode in the absence of the pandemic, while Section 5 explores the self-building mode and platform mode under "stay-at-home orders" after a pandemic outbreak. In Section 6, we examine the impact of "stay-at-home orders" on the B&M retailer's profitability. Section 7 extends the models to consider the case where B&M retailers reopen and analyze the effect of the "reopening policy" and government subsidies. Finally, we present the concluding remarks and propose directions for future research in Section 8.

2. Literature review

There are four streams of literature closely related to our work: (i) online order fulfillment policy, (ii) O2O delivery platform, (iii) "product + delivery service", (iv) operations management under COVID-19.

When engaging in online retailing, how to fulfill the online order is a critical problem (Ishfaq and Bajwa 2019). There are different means to implement online order fulfillment, such as distribution centers or dedicated order fulfillment facilities (Acimovic and Graves 2014; Jasin and Sinha 2015), drop-shipping (Gan et al. 2010), Buy-Online-and-Pick-up-in-Store (BOPIS) (Gao and Su 2017; Jin et al. 2018; McCarthy et al. 2019), ship-to-store (Cao et al. 2016; Gallino et al. 2017), ship-from-store (Bayram and Cesaret 2021; Difrancesco et al. 2021; He et al. 2021). Acimovic and Graves (2014) formulate the order-fulfillment problem as a continuous-time dynamic program that minimizes shipping costs. Gan et al. (2010) study a drop-shipping supply chain in which the retailer receives a customer's order and the supplier fills it. Gao and Su (2017) investigate how BOPIS affects the store's operations. Cao et al. (2016) study how the addition of ship-to-store channel will impact a retailer's operations. Bayram and Cesaret (2021) formulate a dynamic program to examine the ship-from-store mode which allows a retailer to fulfill online orders by using inventory from a nearby store. Lim et al. (2020) investigate how to jointly optimize replenishment, allocation, and fulfillment decisions in an anticipative manner. Unlike the use of these fulfillment modes, we investigate the implications of a local B&M retailer filling online orders by offering local delivery via a 3rd party O2O platform or by itself.

Emerging O2O delivery platforms (e.g., Instacart, Postmates, and DoorDash) that can provide online selling and local delivery service is one type of online marketplace innovation (Chen et al. 2020). Some papers study the delivery platform pricing scheme (Kung and Zhong 2017; Liu et al. 2019; Tong et al. 2020), competition between logistics companies with/without a common platform (Wang and Huang, 2020), O2O platform with/without self-scheduling delivery capacity (He et al. 2020) and different contractual forms between a delivery platform and a restaurant (Feldman et al. 2019). Kung and Zhong (2017) investigate a two-sided platform's optimal pricing strategy and compare three delivery pricing decisions. Tong et al. (2020) evaluates the impact of dynamic and static pricing strategies on platform performance. Feldman et al. (2019) consider a platform that provides delivery service for the restaurant, investigate relationships between couriers, and compare the performance of different contractual forms. He et al. (2021) investigate when a local B&M store should adopt an O2O strategy with three options under exogenously and endogenously determined retail prices. The above literature does not consider the value of delivery service to consumers, which plays a critical role in O2O retailing. To complement this stream of literature, this paper focuses on a B&M retailer's operations including pricing of product and delivery service as well as modal choice in the presence/absence of the COVID-19 epidemic.

Logistics can be a competitive lever in the online retailing sector (Tang and Veelenturf 2019). Large online retailers such as Amazon, Alibaba, and JD invest heavily in logistics and may use in-house capacity delivery service to customers. Although online consumers pay for the product and the delivery fee when purchasing online, there is scarce literature that considers the product and delivery service (product + logistics service) simultaneously. Guo et al. (2017) investigate the situation when a retailer optimizes the retail price to maximize the expected profit in different types of delivery service scenarios. Niu et al. (2019) study retailers who sell "product + logistics" to customers in the context of cross-border e-commence. Adding to this line of literature, this paper models how product and delivery service together influence consumers' decisions during "stay-at-home orders" due to a pandemic and the subsequent reopening policy. Specifically, we study how a B&M retailer can implement online selling and offer offline delivery service, and analyze the B&M retailer's pricing decision and operational mode choice with a capacity constraint.

The outbreak of COVID-19 has greatly influenced our society and economy. Governments have implemented social distancing, lockdown, and other counter measures to mitigate the spread of the disease, which has changed a variety of industries such as airlines (Amankwah 2020), food (Singh et al. 2021), healthcare (Govindan et al. 2020), and services (Choi 2020). Using a unified conceptual frame, Amankwah (2020) studies the global airlines' responses to COVID-19. Singh et al. (2021) investigate disruptions in the food supply chain and develop a public distribution system network and test its efficacy using a simulation model. Using physicians' knowledge and a fuzzy inference system, Govindan et al. (2020) develop a practical decision support system to help with demand management in the healthcare supply chain. Choi (2020) studies an innovative business model that "Bring-Service-Near-Your-Home" operations under the pandemic and examines how the government should use a subsidy scheme to help this business model. Besides, Ivanov (2020) and Ivanov and Dolgui (2021) study the ripple effect and viability issues in supply chains under the COVID-19 pandemic. In contrast to the above-mentioned papers, our paper focuses on online retailing and investigates consumer purchase behavior and B&M retailer's operations under the "stay-at-home orders" and "reopening policy" brought by the pandemic. To the best of our knowledge, this is the first paper that studies the impact of "stay-at-home orders" and "reopening policy" on B&M retailers' operations. We summarize the major difference of our paper and the most relevant papers in Table 1.

3. Model description

Let v_1 denote the maximum value that consumers assign to the product. Consumers are heterogeneous in the value they assign to the product; we model the heterogeneity by θ_1 which is uniformly distributed over [0, 1]. Thus, the net surplus for a consumer located at θ_1 from purchasing the product from the B&M retailer at unit price p is $u_1 = \theta_1 v_1 - p$. If a B&M retailer or delivery platform provides delivery service to consumers, we denote the maximum delivery service value to consumers as v_2 and normalize v_2 to 1. To exclude trivial cases and focus on general cases, we assume that the value of the product v_1 is greater than $3/2$. This assumption ensures that the difference between the product value and delivery service cost is sufficiently high, and reflects the situation where customers value the product more than the delivery service when purchasing online. Let θ_2 represent the consumer's willingness to pay for the delivery service, which is uniformly distributed over [0, 1]. Thus, if a consumer purchases the product online, his/her surplus is $u_2 = \theta_1 v_1 + \theta_2 v_2 - p - k$, where k is the delivery fee. For example, consumers can go to a restaurant to enjoy the meal; or they can order a meal online and pay a delivery fee to enjoy a meal at home. Therefore, the consumers are two-dimensionally heterogeneous (Tyagi 2004; Shi et al. 2013; Cao et al. 2016; Guo et al. 2017). A population of size J consumers is spread uniformly over the unit square. All parameters in this paper are non-negative.

If a B&M retailer engages in local online retailing, it charges the same retail price to both online and in-store consumers, which is consistent with current practice. Gallino and Moreno (2019) point out that it is becoming rare to find retailers that charge different prices across channels. The B&M retailer holds a quantity Q of capacity and can implement the local online retailing on their own, which we refer to as *self-building mode*. With self-building mode, if a consumer purchases a product online, the retailer will deliver the product to the consumer with a delivery fee k per unit and incur a delivery cost c per unit.

On the other hand, a B&M retailer can engage in online retailing via a third-party O2O platform. The O2O platform gives visibility to the retailer, helps consumers discover the product, facilitates informed purchasing decisions, and permits customers to enjoy on-demand delivery through the platform. We call this mode of operations the *platform mode*. With this mode, the platform will charge the B&M retailer a fixed commission τ (Jiang et al. 2011; Mantin et al. 2014; Hagiu and Wright 2014) per unit sale and a delivery fee k for each order to the online consumers. It costs the platform c_l to deliver each order. The delivery cost of the platform

Table 1
Comparison of this paper with the most relevant literature.

Related literature	Online order fulfillment policy	Delivery platform	Product + delivery service	General services	Impact of external event (COVID-19)
Gao and Su 2017; Jin et al. 2019; Bayram and Cesaret 2020	✓				
Tong et al. 2020; He et al. 2020	✓	✓			
Guo et al. 2017			✓		
Niu et al. 2019	✓		✓		
Choi 2020				✓	✓
This paper	✓	✓	✓		✓

mode is lower than that of the self-building mode (i.e., $c_l < c$), because O2O platforms usually utilize on-demand gig-worker capacity and sophisticated scheduling algorithms to achieve better economies of scale. We normalize c_l to zero. Furthermore, O2O platforms can have more significant advantages in online marketing such as name recognition and high visibility (e.g., Kitchens et al. 2018; Ryan, et al. 2012) than does a local B&M retailer. Therefore, B&M retailers who want to expand their market size often sell through an O2O platform. To model this advantage of the O2O platform mode over the self-building mode, we let the number of customers for the self-building mode, J, equal 1, and the number of customers for the O2O platform mode equal to γ, where $\gamma > 1$. This parameter also reflects the spillover effects of the platform (online channel) on the offline channel (Abhishek et al. 2016). The retail price should satisfy $p \leqslant v_1$ to ensure that consumers obtain a positive surplus. The delivery fee k charged to consumers should satisfy $k \leqslant v_2$ to ensure that consumers are willing to use the delivery service. The retailer's delivery cost c should satisfy $c < v_2$ to ensure that the feasibility of the self-building mode. For notational convenience, we use superscripts '1′, '2′ to denote the self-building mode and platform mode, respectively, and use subscripts 's', 'o' to denote the expressions for the retailer and the platform, respectively. 'Π' and 'π' represent the profit functions in the absence of the pandemic and the presence of the pandemic, respectively.

4. Without COVID-19 pandemic

In this section, we investigate the retailer's online retailing format in the absence of the COVID-19 pandemic. We first examine the self-building and platform modes separately, and then compare these two modes to explore the conditions under which the B&M retailer prefers one mode over the other.

4.1. Demand structure

When a B&M retailer engages in local online retailing, customers have two options for purchasing the products. If they visit the B&M store in person to buy the product, the net surplus is $u_1 = \theta_1 v_1 - p$. If they make the purchase online, they pay p for the product, and k for the delivery. Thus, the net surplus for a customer at (θ_1, θ_2) is $u_2 = \theta_1 v_1 + \theta_2 - p - k$. Consumers will choose between in-store pickup and home delivery depending on which option yields a greater surplus. In this paper, we use lexicographic ordering to model the consumer decision process. With a lexicographic order, a consumer evaluates alternatives first on the most important attribute, and if there are ties, on the second most important attribute, and so forth (Kohli and Jedidi 2007). In a similar vein, we assume that consumers evaluate the product value first and require $u_1 > 0$ holds before making the purchase. Customers then purchase in the B&M retailer's store if $u_1 > u_2$, and purchase online with home delivery if $u_2 > u_1$. We depict the demand structure for offline and online channels in Fig. 2 and divide the demand structure into two cases according to conditions $p + k < v_1$ and $v_1 < p + k < v_1 + k$. In either case, the demand allocation for offline channel equals $(1 - \frac{p}{v_1})k$ and the demand allocation for the online channel equals $(1 - k)(1 - \frac{p}{v_1})$.

4.2. Self-building mode

With this mode, the demand for the B&M retailer's offline channel is $d_s^1 = (1 - \frac{p}{v_1})k$, and the demand for the online channel is $d_o^1 = (1 - k)(1 - \frac{p}{v_1})$. The total demand is $D_s^1 = d_s^1 + d_o^1$. The B&M retailer determines the retail price p and the delivery fee k charged to online consumers to maximize its profit function as follows:

$$\max_{p,k} \Pi_s^1 = p d_s^1 + (p + k - c) d_o^1 \quad \text{s.t. } d_s^1 + d_o^1 \leqslant Q \tag{1}$$

where the first term in the objective function denotes the profit from the offline channel and the second term denotes the profit from the online channel. The constraint ensures that the total demand cannot exceed the capacity. Proposition 1 characterizes the B&M retailer's optimal decisions.

Proposition 1. *Let $Q_0 \triangleq \frac{1}{2} + \frac{(1-c)^2}{8v_1}$. With self-building mode, the B&M retailer's optimal decisions are as follows:*

(1) *If $Q < Q_0$, the optimal delivery fee is $k^* = \frac{1+c}{2}$, the optimal retail price is $p^* = v_1(1 - Q)$, and the optimal profit is $\Pi_s^{1*} = \frac{(1-c)^2 Q}{4} + Q(1 - Q)v_1$; and*

(2) *If $Q \geqslant Q_0$, the optimal delivery fee is $k^* = \frac{1+c}{2}$, the optimal retail price is $p^* = \frac{v_1}{2} - \frac{(1-c)^2}{8}$, and the optimal profit is $\Pi_s^{1*} = \frac{(4v_1 + (1-c)^2)^2}{64 v_1}$.*

From Proposition 1, we can see that the capacity threshold Q_0 plays a critical role in the B&M retailer's decisions. When the capacity is below the threshold Q_0, the optimal retail price decreases with capacity Q, while the optimal profit increases with the capacity Q. In addition, the total demand $d_s^1 + d_o^1$ equals to the capacity Q, where the offline demand is $d_s^1 = kQ$, and the online demand is $d_o^1 = (1 - k)Q$. When the capacity is above the threshold Q_0, the optimal retail price and optimal profit are independent of Q. The total demand equals Q_0, where the offline demand is $d_s^1 = kQ_0$, and the online demand is $d_o^1 = (1 - k)Q_0$. The delivery fee is independent of capacity Q, which is consistent with practice. Fig. 3 demonstrates the impact of Q on the B&M retailer's optimal price and optimal profit.

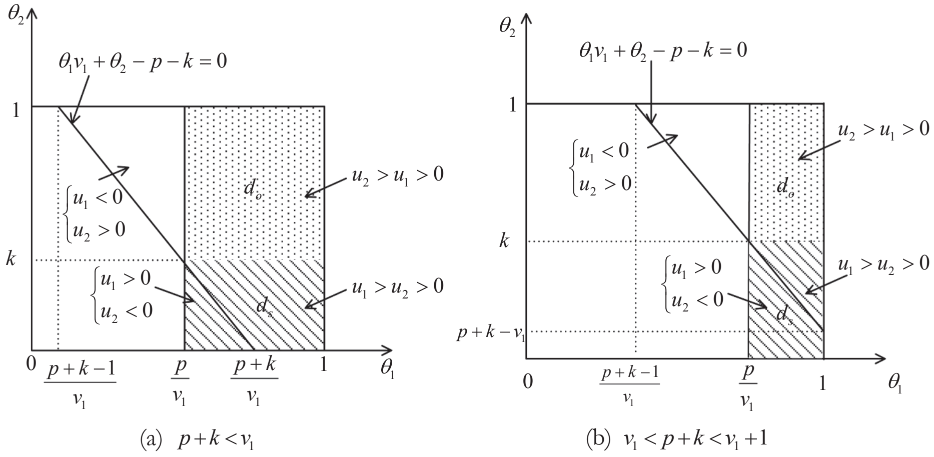

Fig. 2. Demand structure in the absence of the pandemic.

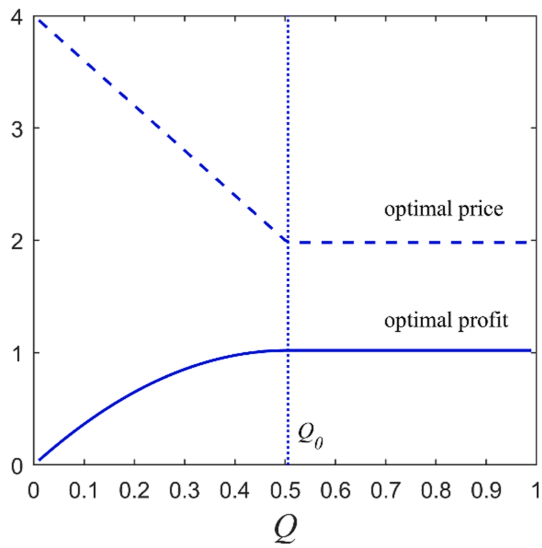

Fig. 3. Effect of capacity under self-building mode ($v_1 = 4, c = 0.6$).

4.3. Platform mode

With this mode, following industrial practice, we assume that the O2O platform charges a B&M retailer-specific commission τ and a uniform delivery fee k to all online customers. That is, the O2O platform determines the commission based on the B&M retailer, and charges the same delivery fee to all the B&M store's online customers. As a result, the delivery fee charged by the platform is exogenously determined and the commission is endogenously determined from the perspective of the B&M retailer. The decision sequence is as follows: first, the O2O platform determines the commission, then the B&M retailer decides on the retail price. Finally, consumers make purchase decisions. The demand for the B&M retailer is $d_s^2 = \gamma(1 - \frac{p}{v_1})k$, and the demand for the online channel is $d_o^2 = \gamma(1 - k)(1 - \frac{p}{v_1})$. The total demand is $D_s^2 = \gamma(d_s^2 + d_o^2)$. The B&M retailer sets the retail price to maximize its profit, which is given by

$$\max_p \Pi_s^2 = pd_s + (p - \tau)d_o \quad s.t.\ d_s^2 + d_o^2 \leqslant Q \tag{2}$$

The O2O platform sets the commission to charge the B&M retailer to maximize its profit, that is,

$$\max_\tau \Pi_o^2 = (\tau + k)d_o^2 \tag{3}$$

There exists a sequential game between the B&M retailer and the O2O platform. To obtain the subgame perfect Nash equilibrium, we first solve the B&M retailer's decision problem using backward induction.

Lemma 1. *With platform mode, if* $\tau < \frac{(\gamma-2Q)v_1}{(1-k)\gamma}$*, the B&M retailer's optimal retail price is* $p^* = v_1(1-\frac{Q}{\gamma})$*; otherwise, the B&M optimal retailer's retail price is* $p^* = \frac{1}{2}(v_1 + \tau - \tau k)$*.*

Lemma 1 indicates that if the commission τ is low, the B&M retailer will set the retail price such that the demand matches the capacity, and the retail price is independent of the commission fee. If the commission τ is high, the retail price is an increasing function of commission fee. Next, we analyze the O2O platform's decision problem. Define $f(k) = k - k^3 + v_1(1-3k)$ and let $\overline{k}$ be a unique solution to $f(k) = 0$.[5] Proposition 2 characterizes the O2O platform's optimal decision.

Proposition 2. *Suppose* $k \in [0, \overline{k}]$, $Q_1 \triangleq \frac{\gamma k}{1+k}$*, and* $Q_2 \triangleq \frac{1}{4}\gamma + \frac{\gamma(1-k)k}{4v_1}$*. Then, the O2O platform's optimal commission is as follows:*

(1) *if* $0 < Q < Q_1, \tau^* = v_1(1-\frac{Q}{\gamma})$*;*

(2) *if* $Q_1 \leqslant Q \leqslant Q_2, \tau^* = \frac{(\gamma-2Q)v_1}{(1-k)\gamma}$; and

(3) *if* $Q > Q_2$, $\tau^* = \frac{v_1 - k(1-k)}{2(1-k)}$.

Proposition 2 indicates that two capacity thresholds affect the O2O platform's commission. Observe that when the capacity is less than the threshold Q_2, the commission τ^* decreases with Q; otherwise, the commission does not depend on Q. Following Lemma 1 and Proposition 2, we can derive the B&M retailer's optimal retail price and the optimal profit, which are summarized in Table 2. We can see that when the capacity is less than the threshold Q_2, the optimal retail price decreases and the B&M retailer's optimal profit increases as the capacity Q increases; otherwise, the optimal retail price and B&M retailer's optimal profit are constants because the B&M retailer's total amount sold will not increase as capacity increases. Fig. 4 illustrates Proposition 2 and demonstrates the impact of capacity Q on the B&M retailer's optimal retail price and optimal profit as well as the O2O platform's commission. We can observe that when the capacity is less than the threshold Q_1, the commission curve overlaps with the retail price curve. This means that the O2O platform will charge the commission as high as the B&M retailer's retail price, and the B&M retailer cannot obtain positive profit from the online channel when its capacity is low.

4.4. Retailing mode comparison

In this section, we compare the self-building mode with the platform mode to examine under what conditions the B&M retailer should prefer one mode over the other. Table 2 summarizes the optimal decisions and profits under both modes. For tractability, we resort to numerical computations to compare the B&M retailer's profits under the self-building mode with that under the platform mode. Fig. 5 demonstrates the relationship of B&M retailer's profits under the two modes with respect to delivery cost (c) and the delivery fee (k).

From Fig. 5, we observe that when the capacity is low (Fig. 5(a) and (d)), the B&M retailer prefers the self-building mode over the platform mode. Although the platform mode can provide a larger market size than the self-building mode for the B&M retailer, the low capacity restricts the B&M retailer from utilizing this advantage. When the capacity is intermediate (Fig. 5(b) and (e)), if the delivery cost c of self-building mode is high and the delivery fee k charged by the O2O platform is high, then the platform mode will yield greater profit than the self-building mode for the B&M retailer. When the capacity is high (Fig. 5(c) and (f)), for a given delivery fee k, the platform mode will yield a greater profit than the self-building mode for the B&M retailer as the delivery cost c of the self-building mode increase. The self-building mode will yield a greater profit than the platform mode for the B&M retailer as the delivery fee decreases. This conclusion seems counterintuitive. The reason is that with the platform mode, more customers will buy from the platform as the delivery fee k decreases. Because the marginal profit of the online channel is less than that of the offline channel, the increase in online demand leads to a decrease in offline demand, which is to the detrimental of the B&M retailer. Therefore, the B&M retailer is more likely to choose the self-building mode as the delivery fee k decreases. From Fig. 5, we have the following observation.

Observation 1. *If the B&M retailer's capacity is no less than its market potential, the B&M retailer will be more likely to choose the self-building mode as the delivery cost of the self-building decreases or the delivery fee charged by the platform decreases.*

5. With COVID-19 pandemic "stay-at-home orders"

After the outbreak of COVID-19, countries around the world have enforced a lockdown or "stay-at-home orders" to slow the spread of COVID-19. Only essential businesses were allowed to remain open, and many retailers closed their B&M stores. Under this scenario, customers could make online purchases only. In this section, we investigate the B&M retailer operations under the "stay-at-home orders" in the presence of COVID-19 pandemic. We examine the self-building mode and the platform mode, then compare these two modes to examine the conditions under which one of the modes dominates the other.

[5] If $k > \overline{k}$, then $Q_1 > Q_2$. Therefore, the three intervals degenerate into the two intervals: $Q \leq Q_2$ (i.e., case (2) of Proposition 2) and $Q > Q_2$ (i.e., case (3) of Proposition 2). In other words, the case of three capacity intervals when $k \in [0, \overline{k}]$ implicitly contains the case of two capacity intervals.

Table 2
Results without the pandemic.

Mode	Condition	Optimal decisions	Optimal profits
Self-building mode	if$Q < Q_0$	$p^* = v_1(1-Q),\ k^* = \frac{1+c}{2}$	$\Pi_s^{1*} = \frac{(1-c)^2 Q}{4} + Q(1-Q)v_1$
	if$Q \geqslant Q_0$	$p^* = \frac{v_1}{2} - \frac{(1-c)^2}{8},\ k^* = \frac{1+c}{2}$	$\Pi_s^{1*} = \frac{(4v_1 + (1-c)^2)^2}{64v_1}$
Platform mode	if$0 < Q < Q_1$	$\tau^* = v_1(1-\frac{Q}{\gamma}), p^* = v_1(1-\frac{Q}{\gamma})$	$\Pi_s^{2*} = v_1 Q(1-\frac{Q}{\gamma})k, \Pi_o^{2*} = (\tau^* + k)(1-k)Q$
	if$Q_1 \leqslant Q \leqslant Q_2$	$\tau^* = \frac{(2Q-\gamma)v_1}{(1-k)\gamma}, p^* = v_1(1-\frac{Q}{\gamma})$	$\Pi_s^{2*} = \frac{v_1 Q^2}{\gamma}, \Pi_o^{2*} = (\tau^* + k)(1-k)Q$
	if$Q > Q_2$	$\tau^* = \frac{v_1 - k(1-k)}{2(1-k)}, p^* = \frac{k^2 + (3v_1 - k)}{4}$	$\Pi_s^{2*} = \frac{\gamma(k^2 - k - v_1)^2}{16v_1}, \Pi_o^{2*} = \frac{\gamma(k^2 - k - v_1)^2}{8v_1}$

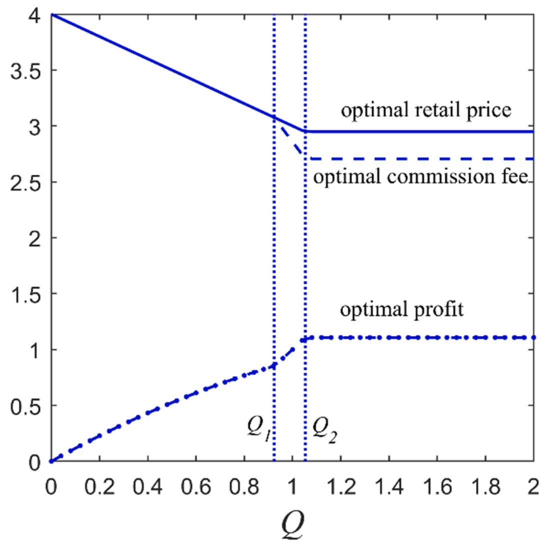

Fig. 4. Effects of capacity under platform mode ($v_1 = 4, \gamma = 4, k = 0.3$).

5.1. Demand structure

Under this scenario, if customers purchase online, the net surplus is $u_2 = \theta_1 v_1 + \theta_2 - p - k$. Consumers will purchase online if and only if $u_2 > 0$. There are two subcases according to whether the inequality $p + k \leqslant v_1$ holds or the inequality $v_1 < p + k < v_1 + 1$ holds. Fig. 6 demonstrates the demand structure. Specifically, if $p + k \leqslant v_1$, the demand allocation for online channel equals the shaded area (Fig. 6(a)), which is $\frac{1}{2}(1 - \frac{p+k}{v_1} + 1 - \frac{p+k-1}{v_1}) = 1 - \frac{p+k}{v_1} + \frac{1}{2v_1}$. If $v_1 < p + k < v_1 + 1$, the demand allocation for online channel equals the shaded area (Fig. 6(b)), which is $\frac{1}{2}(1 - \frac{p+k-1}{v_1})(1 - p - k + v_1)$.

5.2. Self-building mode

With this mode, the demand equals $d_{o1} = 1 - \frac{p+k}{v_1} + \frac{1}{2v_1}$ if $p + k < v_1$ and $d_{o2} = \frac{1}{2}(1 - \frac{p+k-1}{v_1})(1 - p - k + v_1)$ if $v_1 < p + k < v_1 + 1$. Correspondingly, the total demand d_s^1 is d_{o1} or d_{o2}. Therefore, the B&M retailer's optimization problems are given by:

$$\max_{p,k} \pi_s^1 = (p + k - c)d_{o1} \quad s.t.\ d_{o1} \leqslant Q \tag{4}$$

and

$$\max_{p,k} \pi_s^1 = (p + k - c)d_{o2} \quad s.t.\ d_{o2} \leqslant Q \tag{5}$$

respectively. Defining $T = p + k$, the optimization problems (4) and (5) can be rewritten as follows:

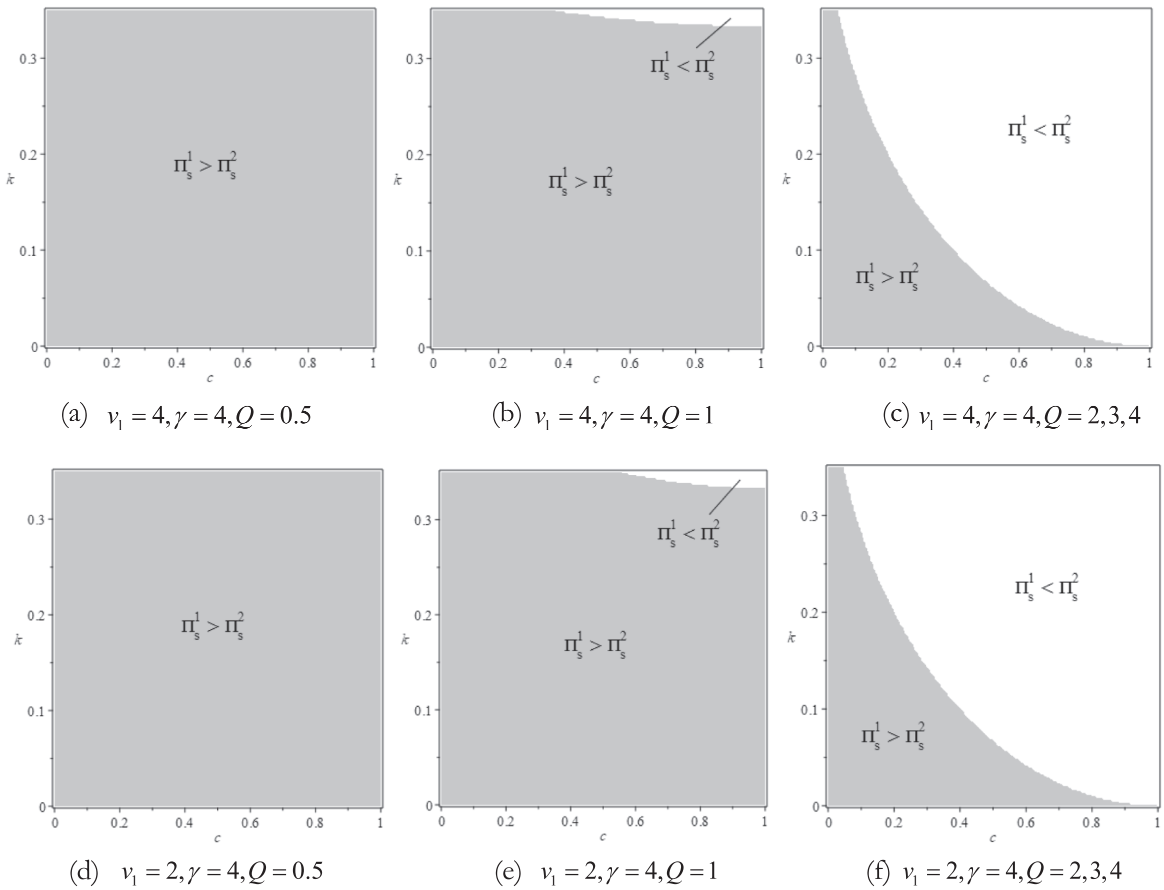

Fig. 5. Comparison of self-building mode with platform mode.

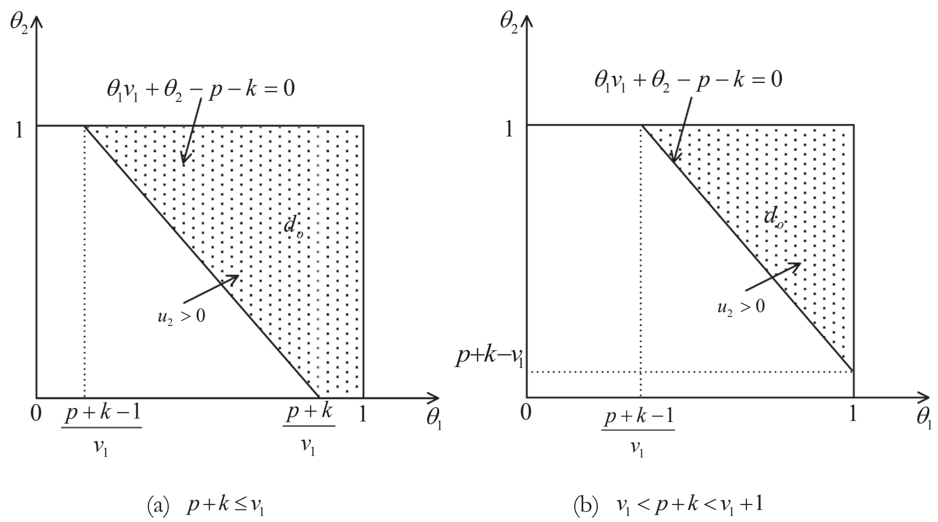

Fig. 6. Demand structure under “stay-at-home orders”

$$\begin{aligned} & \max_T \pi_s^1 = (T-c)\left(1-\frac{T}{v_1}+\frac{1}{2v_1}\right) \\ & \quad s.t.\ 1-\frac{T}{v_1}+\frac{1}{2v_1} \leqslant Q \end{aligned} \tag{6}$$

and

$$\begin{aligned} & \max_T \pi_s^1 = \frac{1}{2}(T-c)\left(1-\frac{T-1}{v_1}\right)(1-T+v_1) \\ & \quad s.t.\ \frac{1}{2}\left(1-\frac{T-1}{v_1}\right)(1-T+v_1) \leqslant Q \end{aligned} \tag{7}$$

respectively. Solving the above two problems, we obtain Proposition 3.

Proposition 3. *Let* $\overline{Q}_0 \triangleq \frac{2v_1-2c+1}{4v_1}$. *With self-building mode, the B&M retailer's optimal decisions and profits are as follows:*

(1) *If* $0 < Q \leqslant \overline{Q}_0$, *the optimal retail price and delivery fee satisfy* $p^* + k^* = (1-Q)v_1 + \frac{1}{2}$, *and the optimal profit is* $\pi_s^{1*} = (v_1(1-Q) - c + \frac{1}{2})Q$; *and*

(2) *If* $\overline{Q}_0 < Q$, *the optimal retail price and delivery fee satisfy* $p^* + k^* = \frac{2v_1+2c+1}{4}$, *and the optimal profit is* $\pi_s^{1*} = \frac{(2v_1-2c+1)^2}{16v_1}$.

Proposition 3 indicates the capacity threshold $\overline{Q}_0$ plays a critical role in the B&M retailer's decisions. When the capacity is below the threshold $\overline{Q}_0$, the total price that comprises the retail price and delivery fee decreases while the optimal profit increases with the capacity Q. In addition, the price is set such that the total demand d_{o1} equals the capacity Q. When the capacity is above the threshold $\overline{Q}_0$, the optimal retail price and optimal profit are independent of Q. The total demand equals $\overline{Q}_0$. Fig. 7 demonstrates the impact of Q on the B&M retailer's optimal total price and optimal profit.

5.3. Platform mode

With this mode, the online demand is $d_{o1} = \gamma(1 - \frac{p+k}{v_1} + \frac{1}{2v_1})$ if $p+k < v_1$ and $d_{o2} = \frac{\gamma}{2}(1 - \frac{p+k-1}{v_1})(1-p-k+v_1)$ if $v_1 < p+k < v_1 + 1$. Correspondingly, the total demand d_s^2 is d_{o1} or d_{o2}. To study the interaction between the B&M retailer and the O2O platform, we use backward induction to solve this sequential game. We first solve the B&M retailer's optimization problems, which are given by:

$$\begin{aligned} & \max_p \pi_s^2 = (p-\tau)d_{o1} \\ & \quad s.t.\ d_{o1} \leqslant Q, \end{aligned} \tag{8}$$

and

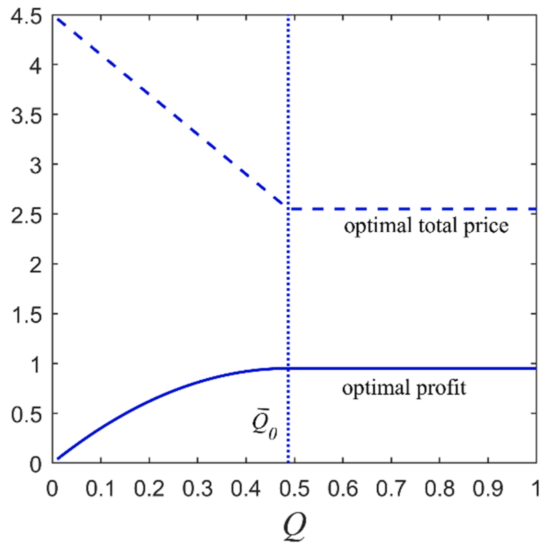

Fig. 7. Effect of capacity under self-building mode ($v_1 = 4, c = 0.6$).

$$\max_{p} \pi_s^2 = (p-\tau)d_{o2}$$
$$s.t.\ d_{o2} \leqslant Q \tag{9}$$

respectively. After solving the above problems, we characterize the B&M's retail price response function in Lemma 2.

Lemma 2. *With platform mode, the B&M retailer's optimal retail price is given as follows:*

(1) If $Q \leqslant \frac{\gamma}{2v_1}$, then $p^* = v_1 - k$;
(2) If $Q > \frac{\gamma}{2v_1}$ and $\tau \leqslant \frac{(2v_1+1-2k)\gamma - 4Qv_1}{2\gamma}$, then $p^* = v_1 - k + \frac{1}{2} - \frac{Qv_1}{\gamma}$;
(3) If $Q > \frac{\gamma}{2v_1}$ and $\frac{(2v_1+1-2k)\gamma - 4Qv_1}{2\gamma} < \tau \leqslant v_1 - k - \frac{1}{2}$, then $p^* = \frac{1}{2}(v_1 + \tau - k + \frac{1}{2})$;
(4) If $Q > \frac{\gamma}{2v_1}$ and $v_1 - k - \frac{1}{2} < \tau$, then $p^* = v_1 - k$.

Lemma 2 indicates that the B&M retailer's retail price depends on the capacity threshold $\frac{\gamma}{2v_1}$ as well as the commission thresholds $\frac{(2v_1+1-2k)\gamma - 4Qv_1}{2\gamma}$ and $v_1 - k - \frac{1}{2}$. The optimal solution to the problem in equation (9) is dominated by the solution to the problem (8), which is revealed in Lemma 3 (see Appendix). Thus, we do not need to consider the optimization problem (9). Next, we investigate the O2O platform's decision problem. The O2O platform's objective function is given by

$$\max_{\tau} \pi_o^2 = (\tau + k)d_{o1} \tag{10}$$

Anticipating the B&M retailer's retail price response function, the O2O platform's optimal commission is characterized by Proposition 4.

Proposition 4. *Let $\overline{Q}_1 \triangleq \frac{\gamma}{2v_1}$ and $\overline{Q}_2 \triangleq \frac{\gamma(2v_1+1)}{8v_1}$. Then, the O2O platform's optimal commission is as follows:*

(1) *if $0 < Q \leqslant \overline{Q}_1, \tau^* = v_1 - k$;*
(2) *if $\overline{Q}_1 < Q \leqslant \overline{Q}_2$, $\tau^* = v_1 - k + \frac{1}{2} - \frac{2Qv_1}{\gamma}$; and*
(3) *if $\overline{Q}_2 < Q$, $\tau^* = \frac{v_1}{2} - k + \frac{1}{4}$.*

Proposition 4 indicates that there are two capacity thresholds $\overline{Q}_1$ and $\overline{Q}_2$ that affect the O2O platform's optimal commission fee. Observe that the commission τ^* decreases in the capacity Q when the capacity is intermediate; otherwise, the commission τ^* doesn't vary with the capacity Q. Following Proposition 3 and Proposition 4, we can derive the B&M retailer's optimal retail price and the optimal profit as well as the O2O platform's optimal profit, which are summarized in Table 3. The B&M retailer's optimal retail price p^* and the O2O platform's optimal commission τ^* constitute an equilibrium strategy under different conditions, which are presented in Table 3.

Fig. 8 demonstrates the impact of capacity Q on the B&M retailer's optimal retail price and optimal profit, as well as the O2O platform's commission fee. As in Fig. 3, when the capacity is less than the threshold $\overline{Q}_1$, the commission curve overlaps with the retail price curve, which indicates that the O2O platform will charge a commission as high as the B&M retailer's retail price. Thus, the B&M retailer cannot obtain a positive profit from the online channel. Because the online channel is the unique sales channel for the B&M retailer, the B&M retailer will earn zero profit when its capacity is low. When the capacity falls between thresholds $\overline{Q}_1$ and $\overline{Q}_2$, the retail price decreases with capacity; however, the commission also decreases more sharply with the capacity, resulting in an increase in the B&M retailer's profit. When the capacity exceeds the threshold $\overline{Q}_2$, the retail price, the commission, and the B&M retailer's profit do not vary with the capacity.

5.4. Retailing mode comparison

In this section, we compare the self-building mode with the platform mode to examine under what conditions the B&M retailer will prefer one mode over the other. We do this comparison numerically. Fig. 9 demonstrates the B&M retailer's profit with self-building

Table 3
Results in presence of the pandemic.

Mode	Condition	Optimal decisions	Optimal profits
Self-building mode	if$Q \leqslant \overline{Q}_0$	$p^* + k^* = (1-Q)v_1 + \frac{1}{2}$	$\pi_s^{1*} = (v_1(1-Q) - c + \frac{1}{2})Q$
	if$\overline{Q}_0 < Q$	$p^* + k^* = \frac{2v_1 + 2c + 1}{4}$	$\pi_s^{1*} = \frac{(2v_1 - 2c + 1)^2}{16v_1}$
Platform mode	if$0 < Q \leqslant \overline{Q}_1$	$\tau^* = v_1 - k, p^* = v_1 - k$	$\pi_s^{2*} = 0, \pi_o^{2*} = \frac{\gamma}{2}$
	if$\overline{Q}_1 < Q \leqslant \overline{Q}_2$	$\tau^* = v_1 - k + \frac{1}{2} - \frac{2Qv_1}{\gamma}, p^* = v_1 - k + \frac{1}{2} - \frac{Qv_1}{\gamma}$	$\pi_s^{2*} = \frac{v_1 Q^2}{\gamma}, \pi_o^{2*} = (v_1 + \frac{1}{2} - \frac{2Qv_1}{\gamma})Q$
	if$\overline{Q}_2 < Q$	$\tau^* = \frac{v_1}{2} - k + \frac{1}{4}, p^* = \frac{3v_1}{4} - k + \frac{3}{8}$	$\pi_s^{2*} = \frac{\gamma(2v_1+1)^2}{64v_1}, \pi_o^{2*} = \frac{\gamma(2v_1+1)^2}{32v_1}$

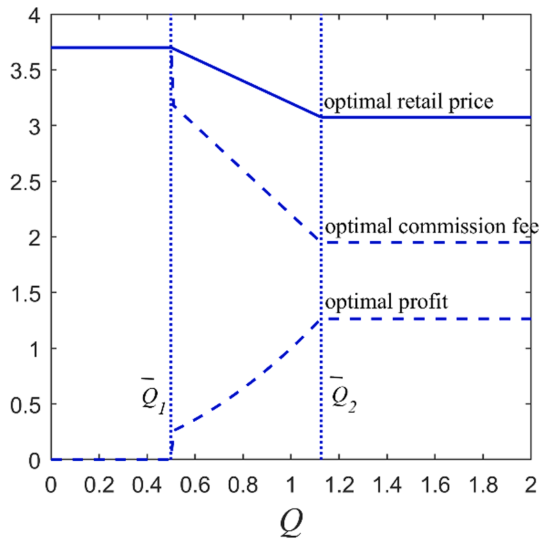

Fig. 8. Effects of capacity under platform mode ($v_1 = 4, \gamma = 4, k = 0.3$).

mode and platform mode as a function of delivery cost (c). We have the following observations:

Observation 2. When the capacity is small relative to the market potential of self-building mode, the self-building mode yields higher profit than the platform mode (Fig. 9 (a), (c), (e), and (h)).

Observation 3. When the capacity is intermediate relative to the market potential of self-building mode, if the product value v_1 and the market potential of the platform mode γ are both low or high, then the platform mode dominates the self-building mode when the delivery cost of self-building mode is high (Fig. 9 (b) and (i)); otherwise, the self-building mode dominates the platform mode (Fig. 9 (d) and (f)).

Observation 4. When the capacity is high relative to the market potential of self-building mode, if the market potential of the platform mode is more significant, then the platform mode yields greater profit than the self-building mode (Fig. 9 (g) and (j)).

6. The impact of the "stay-at-home orders"

In this section, we investigate the impact of the "stay-at-home orders" on the B&M retailer's profitability under the self-building and platform modes. The outbreak of the pandemic has changed customer purchase behavior. Comparing Fig. 2(a) with Fig. 6(a), we can see that a proportion of customers who did not purchase in the offline store earlier will buy products from the online channel; these additional customers are denoted by the triangle ($u_1 < 0,\ u_2 > 0$) in Fig. 2(a). Also, a portion of customers who purchased in the offline store before will not buy products in the online channel, which is denoted by the triangle ($u_1 > 0,\ u_2 < 0$) in Fig. 2(a). The changes in demand structure will lead to changes in the B&M retailer's profit, which is specified in Proposition 5.

Proposition 5. *With self-building mode, the optimal profits of a B&M retailer in the presence/absence of the pandemic have the following relationships: if $c < \sqrt{2}-1$, $\pi_s^{1^*} > \Pi_s^{1^*}$; otherwise, $\pi_s^{1^*} \leqslant \Pi_s^{1^*}$.*

Proposition 5 indicates that if the delivery cost with respect to service value v_2 is low, the "stay-at-home orders" will increase the B&M retailer's optimal profit. If the delivery cost is high, the reverse holds. The reason is that consumers can only purchase online under the "stay-at-home orders"; thus, there only exists the online channel. With the self-building mode, the B&M retailer provides the delivery on its own. If the B&M retailer has a cost advantage in delivery, customer migration to the online channel will increase the B&M retailer's profit. On the contrary, if the B&M retailer does not have a cost advantage in delivery, pure online selling will hurt the retailer's profit. Note that we have defined $Q_2 \triangleq \frac{1}{4}\gamma + \frac{\gamma(1-k)k}{4v_1}$ in Proposition 2 and $\overline{Q}_1 \triangleq \frac{\gamma}{2v_1}$ in Proposition 4.

Proposition 6. *With platform mode, the optimal profits of a B&M retailer in the presence/absence of the pandemic have the following relationships:*

(1) *if $k^2 - k - v_1 + 2 < 0$ and $Q \leqslant Q_2$, then $\pi_s^{2^*} \leqslant \Pi_s^{2^*}$; if $k^2 - k - v_1 + 2 < 0$ and $Q > Q_2$, then $\pi_s^{2^*} > \Pi_s^{2^*}$;*

(2) *if $k^2 - k - v_1 + 2 \geqslant 0$ and $Q \leqslant \overline{Q}_1$, then $\pi_s^{2^*} \leqslant \Pi_s^{2^*}$; if $k^2 - k - v_1 + 2 \geqslant 0$ and $Q > \overline{Q}_1$, then $\pi_s^{2^*} > \Pi_s^{2^*}$.*

Proposition 6 indicates that if the capacity is below a threshold Q_2 or $\overline{Q}_1$, the "stay-at-home orders" will decrease the B&M retailer's optimal profit; otherwise, the reverse holds. Whether the capacity threshold is Q_2 or $\overline{Q}_1$ depends on whether the inequality $k^2 - k - v_1 + 2 < 0$ holds. The reason is that under the platform mode with the pandemic, the platform becomes the unique sales channel for the B&M retailer. Under this scenario, if the B&M retailer's capacity is relatively low, the platform's optimal commission equals the

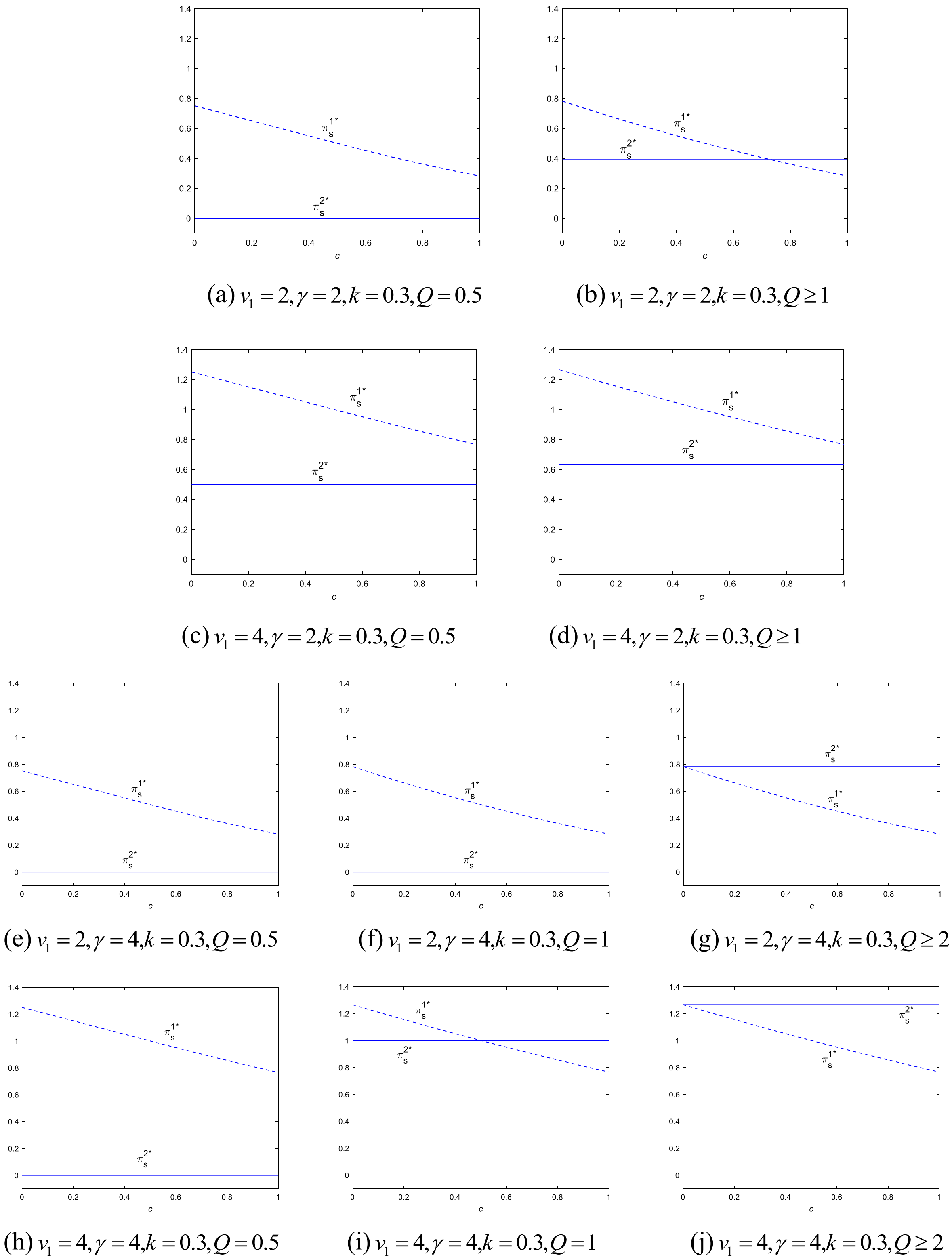

Fig. 9. Comparison of self-building mode with platform mode.

retail price, which leads to the zero profit for the B&M retailer. In contrast, without the pandemic, the B&M retailer can sell through both the offline channel and platform channel. Under this scenario, the B&M retailer can still obtain a positive profit even when the platform charges the commission equal to the retail price. Therefore, the B&M retailer's profit in the presence of the pandemic will be no larger than that in the absence of the epidemic. On the other hand, the occurring of the pandemic changes customer purchase behavior and demand structure. The channel relationship between the B&M retailer and the O2O platform changes from the dual sales channel to a single sales channel, which mitigates the competition between the B&M retailer and the O2O platform. When the B&M retailer has sufficient capacity, the mitigation of competition will increase the B&M retailer's profit.

Corollary 1. *The O2O platform's profit in the presence of the pandemic is greater than in the absence of the pandemic.*

Corollary 2. *In the presence of the pandemic, as delivery cost cincreases, the B&M retailer is more inclined to choose the platform mode.*

Corollary 1 indicates that with "stay-at-home orders", customers do not have any option other than purchasing online, which increases the O2O platform's profit. Furthermore, under the platform mode, the O2O platform's and B&M retailer's profits no longer rely on the delivery fee since all demand is satisfied through the online channel. Therefore, as the delivery cost of self-building mode increases, B&M retailer's profit with self-building mode decreases, which increases its likelihood to choose the platform mode.

7. Extension: "Reopening policy" with government subsidy under the pandemic

As the COVID-19 pandemic is slowly being brought under control, governments are beginning to reopen the economy. Therefore, B&M retailers are reopening their stores to the public. However, because they are establishing safety and hygienic protocols, B&M retailer costs have increased. To help retailers recover, governments have implemented a series of subsidy schemes.[6] In this section, we first analyze the case without government subsidies and then examine the case when governments offer subsidies to B&M retailers. By comparing these two cases, we investigate the effect of "reopening policy" on B&M retailers, and the effect of offering a subsidy scheme on B&M retailer's profit, consumer surplus, and social welfare. We use subscripts 'a' and 'b' to denote respectively the case without and with government subsidy.

According to Accenture, although fears about health are gradually subsiding, consumers remain uncomfortable about visiting public places. If customers visit the B&M retail store in person, they are at risk of being infected by the virus. Therefore, we assume that buying a product in person reduces the consumers' perceived value by Δ, and the net surplus from purchasing from the B&M retailer becomes $u_1 = \theta_1 v_1 - p - \Delta$. Specifically if $\Delta < k$, the demand allocation for the offline channel is $d_s^1 = (1 - \frac{p+\Delta}{v_1})(k - \Delta)$ with the self-building mode and $d_s^2 = \gamma(1 - \frac{p+\Delta}{v_1})(k - \Delta)$ with the platform mode; the demand allocation for the online channel is $d_o^1 = (1 - \frac{p+\Delta}{v_1})(1 - k + \Delta)$ with the self-building mode and $d_o^2 = \gamma(1 - \frac{p+\Delta}{v_1})(1 - k + \Delta)$ with the platform mode; if $\Delta \geqslant k$, customers will not purchase in the offline store. Hence, the demand for the online channel is $d_o^1 = 1 - \frac{p+\Delta}{v_1}$ with the self-building mode and $d_o^2 = \gamma(1 - \frac{p+\Delta}{v_1})$ with the platform mode. Although $\Delta \geqslant k$ implies that the B&M retailer sales occur only in the online channel, just as in the "stay-at-home orders" case, the underlying reasons are different. The former is due to the consumers' active decision, which is influenced by the delivery fee and the disutility of visiting a retail store due to the fear of catching the coronavirus. The latter is due to the consumers' passive decision imposed by the "stay-at-home orders". To avoid the trivial case, we assume that $\Delta \leqslant 1 + c$ to ensure positive demand in offline channel under the reopening policy. Following the same approach as in Proposition 1 and 2, we can derive the B&M retailer's optimal decisions and profits, which are summarized in Table 4 (see Appendix). Comparing the B&M retailer's profits in Tables 2 and 4, we obtain the results in Proposition 7.

Proposition 7. *Under the reopening policy, compared with the case without the pandemic,*

1. *the B&M retailer's profit with the self-building mode will decrease, that is,* $\Pi_s^{1^*} > \pi_{as}^{1^*}$;
2. *the B&M retailer's profit with the platform mode will (weakly) decrease, that is* $\Pi_s^{2^*} \geqslant \pi_{as}^{2^*}$, *furthermore, when* $Q_1 \leqslant Q \leqslant Q_{a2}$, $\Pi_s^{2^*} = \pi_{as}^{2^*}$, *where* $Q_1 = \frac{\gamma k}{1+k}$ *and* $Q_{a2} = \frac{\gamma\left(\Delta \cdot k - k^2 - \Delta + k + v_1\right)}{4 v_1}$.

Intuitively, Proposition 7 reveals that in the presence of the pandemic, the B&M store's profit with the self-building mode under the reopening policy will reduce. With the platform mode, as more customers who purchase in the physical channel now turn to buy from the online channel, if the online demand exceeds a certain threshold, the O2O platform will reduce the commission. The former condition implies that the B&M store's capacity is intermediate and can allocate more capacity to satisfy online demand. As a result, the increase of the B&M store's profit from the online channel will offset the decrease of the B&M store's profit from the offline channel, which leads to the B&M store obtaining the same profit as before the pandemic. However, when the capacity is sufficient, as customers who purchase from the online channel will not continue to increase, the O2O platform will not continue to reduce its commission, thereby the B&M store cannot obtain the same profit as before the outbreak of the pandemic. We can see that the "reopening policy" and the "stay-at-home orders" have a different impact on the B&M retailer's profitability. Although both have changed customer purchase behavior and demand structure, the "stay-at-home orders" eliminates the risk of infection and the corresponding disutility of patronizing the offline store while the "reopening policy" doesn't. As a result, the presence of the disutility will

[6] https://www.retailcouncil.org/coronavirus-info-for-retailers/covid-19-relief-measures-by-region/

weakly decrease the B&M retailer's profit.

Suppose the government employs a subsidy scheme that offers an amount of b per unit sold by B&M retailers. The B&M retailer's objective function with self-building mode is given by $\max_{p,k}\pi_{bs}^1 = (p+b)d_s^1 + (p+k-c)d_o^1$. The B&M retailer's and the platform' objective functions with platform mode are given by $\max_{p}\pi_{bs}^2 = (p+b)d_s^2 + (p-\tau)d_o^2$ and $\max_{\tau}\pi_{bo}^2 = (\tau+k)d_o$, respectively. The consumer surplus under the self-building mode is given by

$$CS^1 = \int_0^{k-\Delta}\int_{\frac{p+\Delta}{v_1}}^1(\theta_1 v_1 - p - \Delta)d\theta_1 d\theta_2 + \int_{k-\Delta}^1\int_{\frac{p+\Delta}{v_1}}^1(\theta_1 v_1 + \theta_2 - p - k)d\theta_1 d\theta_2 \tag{11}$$

The consumer surplus under the platform mode is given by

$$CS^2 = \gamma\left(\int_0^{k-\Delta}\int_{\frac{p+\Delta}{v_1}}^1(\theta_1 v_1 - p - \Delta)d\theta_1 d\theta_2 + \int_{k-\Delta}^1\int_{\frac{p+\Delta}{v_1}}^1(\theta_1 v_1 + \theta_2 - p - k)d\theta_1 d\theta_2\right) \tag{12}$$

Following the same approach as in Propositions 1 and 2, we can derive the B&M retailer's optimal decisions and profits, which are summarized in Table 5 (see Appendix). Proposition 8 summarizes the effect of subsidy scheme on retail price, the B&M retailer's profit, consumers surplus, and social welfare.

Proposition 8. *Under reopening policy, if the government offers subsidies to B&M retailers, we have the following:*

(1) *With the self-building and platform modes, if the B&M retailer has constrained capacity, the retail price does not change with the subsidy b, that is, $p_a^{i*} = p_b^{i*}$; otherwise, retail price decreases with the subsidy b, that is, $p_a^{i*} > p_b^{i*}$;*
(2) *With the self-building and platform modes, the B&M retailer's profit will increase compared to the case of without subsidy, that is, $\pi_{bs}^i > \pi_{as}^i, i \in \{1,2\}$;*
(3) *With the self-building mode, when the B&M retailer has constrained capacity, consumer surplus decreases with the subsidy b if $b + c + \Delta < 1$; otherwise, consumer surplus remains unchanged. When the B&M retailer does not have constrained capacity, the change in consumer surplus is ambiguous if $b + c + \Delta < 1$; otherwise, consumer surplus increases with the subsidy b. Social welfare may increase or decrease with the subsidy b.*
(4) *With the platform mode, if B&M retailer has constrained capacity, consumer surplus will remain unchanged as the subsidy b increases; when B&M retailer does not have constrained capacity, consumer surplus increases with the subsidy b. The change in social welfare increases with the subsidy b.*

Proposition 8(2) indicates the government subsidy scheme will increase the B&M retailer's profit under both operational modes. However, the government subsidy scheme does not necessarily lower the retail price (Proposition 8(1)). This is because when the B&M retailer's demand exceeds capacity, decreasing the retail price will not increase the B&M retailer's profit. Instead, the B&M retailer can increase its profit by maintaining its retail price despite receiving the subsidy. In this case, the consumer surplus is unchanged. By contrast, when the B&M retailer has unconstrained capacity, it can further increase its demand by decreasing the retail price, thereby increasing its profit. Under this scenario, the B&M retailer with the platform mode will pass through a portion of the subsidy to its consumers, thereby improving the consumer surplus (Proposition 8(4)). However, the effect of the subsidy on consumer surplus with the self-building mode is subtle. Specifically, when the B&M retailer has constrained capacity and the subsidy is small ($b < 1 - c - \Delta$), the B&M retailer will raise delivery fee k_b^* as the subsidy b increases, which leads to increased demand in the offline channel. Thus, the B&M retailer can obtain a greater subsidy from offline channel sales and improve its profitability. Under this scenario, since the retail price is not lowered and delivery fee is increased, the consumer surplus decreases (Proposition 8(3)). If the subsidy is large ($b \geqslant 1 - c - \Delta$), the B&M retailer cannot raise delivery fee anymore because the optimal delivery fee k_b^* is a constant. Thus, the consumer surplus remains unchanged even when the subsidy increases. When the B&M retailer does not have constrained capacity and the subsidy is small, it will decrease the retail price to increase the total demand and raise the delivery fee to increase the offline demand. So, the consumer surplus in the offline channel will increase, but the effect on the consumer surplus in the online channel is ambiguous. To summarize, the government subsidy scheme will help the B&M retailers but not necessarily improve the consumer surplus. Indeed, it may decrease the consumer surplus under certain condition. As the government subsidy b increases, social welfare with the platform mode will increase while the change in social welfare with the self-building mode is not clear. Thus, if the government aims to improve consumer surplus by subsidizing the B&M retailer, it should take the B&M retailer's capacity and operational mode into account.

8. Conclusion

The COVID-19 outbreak has dramatically changed consumer purchase behavior, and the stay-at-home and social-distancing measures have shut down hundreds of thousands of U.S. retailers. Local B&M retailers are facing critical challenges and adapting their operations with a shift to online retailing. Implementing such online retailing requires providing delivery service to customers within an acceptable time. There are two options for the local B&M retailers: self-building mode and platform mode. With the self-building mode, the B&M retailer's delivery cost plays a vital role since the B&M retailer provides the delivery service. With the platform mode, the O2O platform charges a commission to the retailer and offers the delivery service to online customers. Because these scenarios are new and evolving, it is imperative to investigate how the "stay-at-home orders" and "reopening policy" caused by the pandemic will affect the B&M retailer's operations.

We investigate the interplay between the B&M retailer's retailing price, capacity, and specific selling mode in the absence/presence of the COVID-19 pandemic. We also identify the equilibrium between the B&M retailer and the O2O platform when the B&M store adopts the platform mode. In the absence of the pandemic, we observe that if the B&M retailer's capacity is large relative to its market potential, the B&M retailer is more likely to choose the self-building mode as the delivery cost of self-building decreases or the delivery fee charged by the platform decreases. In the presence of the pandemic, we demonstrate the impact of relevant parameters on the retailing modes. Since when the epidemic will end is still not clear, and the consumer purchase behavior has been fundamentally changed, we have the following managerial insights and suggestions for the B&M retailers.

(1) Local B&M retailers who have not engaged in online retailing may be hit hardest by the pandemic, due to their reliance on only physical retail. This paper suggests that these local B&M retailers accelerate their transition to online retailing. As for the local B&M retailers that have engaged in online retailing, the "stay-at-home orders" doesn't necessarily lower their profit.
(2) Under the "stay-at-home orders", retailers with the self-building mode will obtain larger profits than before if delivery cost is low. And the retailer's capacity does not affect this profitability relationship. Thus, a B&M retailer with self-building mode should try to reduce its delivery cost. And it may not be necessary for them to expand capacity.
(3) Under "stay-at-home orders", with the platform mode, the B&M retailer's sales channel changes from dual-channel before the outbreak of the pandemic to a single-channel during the pandemic, which reduces competition between the retailer and the O2O platform. If the capacity exceeds a particular threshold value, the B&M retailer's profit will increase. Moreover, the capacity threshold relies on product value. When the product value is relatively low, the capacity threshold is relatively high. Hence, this paper suggests that B&M retailers who use the platform mode increase their capacity to above the capacity threshold.
(4) Under "reopening policy", the B&M store's profit will be lower than before due to the risk of the pandemic. If the government offers subsidies to B&M retailers, the B&M retailer's profitability will improve. However, it may hurt consumer surplus under some conditions. This may happen when B&M stores with constrained capacity want to obtain higher subsidy. They will increase the delivery fee such that more online consumers purchase offline, thereby increasing demand in offline channel. Therefore, we suggest that governments take a B&M retailer's capacity and operational mode into account while developing subsidy schemes.

There are several potential directions worthy of future research and may generate more insights for B&M retailers. First, this paper focuses on analytically deriving the research findings, empirical studies on investigating the impact on local B&M retailers' operations can be conducted to corroborate our analytical results. Second, the B&M retailer can improve its safety protocols to attract customers to purchase from the retailer by exerting effort and following the Centers for Disease Control and Prevention guidance. Incorporating this factor is worthy of future research. Third, our study focuses on B&M retailers that sell products with a capacity constraint, and future research can consider the B&M retailers that provide services with a capacity constraint.

CRediT authorship contribution statement

Bo He: Conceptualization, Methodology, Formal analysis, Writing - original draft, Writing - review & editing, Funding acquisition. **Prakash Mirchandani:** Writing - review & editing. **Qichao Shen:** Software, Visualization. **Guang Yang:** Validation, Formal analysis, Writing - review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supplementary material

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