

University students' fully digital study of mathematics: an identification of student-groups via their resources usage and a characterization by personal and affective characteristics

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We investigated university students' study of mathematics in the digital setting context of the COVID-19 pandemic. We gathered data from a survey of 89 students enrolled in a 'Linear Algebra 1' course including affective variables, learning strategies, social relatedness and resources considered useful. The results indicate students' high effort and self-regulation and a high variation in affective characteristics and social relatedness. All the traditional aspects of mathematics teaching (lecture, tutorials and lecture notes) were rated as particularly useful. In addition, the videos from external resources were rated as equally useful as the teaching team's videos. In contrast, traditional literature such as textbooks was rarely considered useful. The most useful resource rated was communication with peers, underlining the important role of social learning despite fully digital learning environments. Finally, a cluster analysis based on students' rated usefulness of the resources led to three different user-types. Whereas the 'digitals' find the external digital resources very useful (videos, webpages, etc.), the 'traditionalists' rate the digitalized traditional resources best (lecture, tutorials, etc.). All resources receive uniformly good ratings from a third group ('all resource users'). We reflect on our findings in light of the pandemic and describe directions for future research.

I. Teaching and Learning in the Pandemic

From March 2020, the COVID-19 pandemic meant that university teaching had to be delivered entirely digitally. Teachers had to switch to a digital setting and students needed to find their way using digital resources. However, the pandemic did not only restrict the mode of teaching. Students, for example, had

fewer opportunities to collaborate because campuses were closed, and in many places, private meetings were restricted. The pandemic affected many further aspects of their life like their financing, as many lost their jobs, their management of studying at home and their work–life balance (Aristovnik *et al.*, 2020; Kindler *et al.*, 2020; Traus *et al.*, 2020).

This change to the digital environment seems to be particularly significant in mathematics. Here, the internationally predominant form of teaching is the so-called chalk talk, i.e., the lecture is delivered on the chalkboard (Artemeva & Fox, 2011). Therefore, it relies traditionally on a face-to-face setting. Similarly, small group tutorials and full-class tutorials have also been traditionally held in an in-person context. In contrast, the digital context emphasizes the inclusion of learning videos and, therefore, asynchronous teaching and learning.

Understanding the changes in students' ways of studying in the digital environment may help us improve teaching in both online and in-campus scenarios in the future. Thus, our aim was to explore what happened from the students' perspective when studying mathematics in the digital setting: how did students rate the usefulness of resources available to them in the online setting? Further, we explored whether different types of learners could be identified in the digital study of mathematics based on their rated usefulness of the resources. Answering these questions might help us align ourselves to the new demands and uncover new possibilities for the time after the pandemic. In particular, teachers have put considerable effort into developing new digital learning materials, and we would like to know if students find them useful.

For this purpose, we researched a first-year 'Linear Algebra 1' course at Paderborn University in Germany. This course was entirely held in a digital setting using online lectures, online tutorials and learning videos in the Winter term of 2020/2021. In our study, the students (mainly studying for a bachelors' degree in mathematics or a teaching diploma in secondary school) were asked to answer an online survey in the middle of the semester. During the pandemic, several unpredictable and fundamental changes occurred that contributed to a completely new situation. Therefore, our research is descriptive and exploratory. In the next section, we will identify some variables that might be particularly important in this new situation.

2. Theoretical Background

In this section, we first give some background on students' use of resources. Acknowledging the fundamental changes, we will not derive hypotheses but just underline the potential role of the variables and resources in the digital study. In the second part, we identify the background for variables that have shown to play an essential role when studying mathematics, especially in the transition to university (self-efficacy, identification with university mathematics, peer learning, self-regulated learning, resisting frustration and social relatedness) and thus seem worth investigating to support our research on the use of resources in the digital setting during the first semester.

2.1. Resources when studying mathematics

As mentioned above, the teaching of mathematics at university (at least in Germany) relies heavily on the traditional lecture being written on the chalkboard and the students taking notes (e.g., Artemeva & Fox, 2011). In addition to small group tutorials, students may discuss the content of the lecture and work particularly on respective tasks. Some universities also offer full-class tutorials, where the whole group of students may ask questions and the content from the lecture is repeated in a less rigorous way.

It is usually the lecturer's claim to present the mathematical content completely, so that in principle, no other literature is necessary in addition to the lecture notes. Yet, students may use additional literature and sometimes do so. In recent years, platforms like YouTube offer a wide diversity of instructional videos for higher mathematics. Thus, students might also make use of such videos when studying mathematics. Similarly, digital chat rooms can be used for communicating about mathematics.

There are several studies investigating students' use of or preferences for resources when studying mathematics or a related subject. Concerning the lecturer's resources (live lecture, online lecture and mathematics learning support centre), [Inglis et al. \(2011\)](#) found that none of the 534 engineering and mathematics undergraduates in their study used more than one resource in favour of another. Either the students relied heavily on one of these resources or they used no resource at all. Comparing the resources provided by the lecturer with those available externally, [Rønning \(2014\)](#) found that more than 80% of his engineering students attended the live lectures to a large or rather large extent and almost 90% made extensive use of the textbook the course was based on. In his study, digital resources were used less (video recorded lectures (46%), videos from external sources (21%) and other web resources (28%)). Rønning concluded that although students have a wide variety of resources available, they prefer to use the traditional ones. [Maclaren \(2018\)](#) investigated the views on resources available outside the classroom in the case of engineering students. Here, the students rated the resources from the lecturer as significantly more effective than published textbooks or social media. However, these students also considered screencast material, when available at an appropriate level, as being effective learning resources.

It is an open question to what extent these results can be transferred to the unique situation of the digital study of mathematics during the COVID-19 pandemic. In this time, all the traditional features of mathematics teaching at university either disappeared or changed significantly. Learning videos and digital chat can be considered to have become more significant. However, the usefulness of resources also has to be evaluated in relation to the aims and learning expectations within the context of the respective courses (e.g., [Maclaren, 2018](#)). When students draw on resources from outside, their quality and appropriateness to the course are not assured.

2.2. *Important variables in the secondary–tertiary transition*

Studying mathematics is known to be highly demanding. The transition to university bears many problems (e.g., [Gueudet, 2008](#)) and may lead to high dropout (e.g., [Geisler, 2020](#)). Research has identified many variables that contribute to explain these problems. However, the extent to which digital studying affects these variables is an open question.

2.2.1. Affective variables. In general, affective variables like self-efficacy play a central role in self-regulated mathematics learning ([Schukajlow et al., 2017](#)). Particularly in the transition to university, affective variables may explain students' dropout ([Rach & Heinze, 2017](#); [Geisler, 2020](#)). Unfortunately, the student affect seemed to be negatively impacted by the pandemic ([Händel et al., 2020](#); [Kindler et al., 2020](#)). It is, however, an open question if this is also true for mathematics.

In investigating online mathematics study, it seemed necessary to focus on students' self-efficacy and identification with university mathematics. Self-efficacy describes the personal belief of having the capability to effect changes by one's action ([Pajares, 2008](#)) or to attain designated types of performances ([Bandura, 1986](#)). Thus, self-efficacy connects to students' learning strategies, predicting students' effort and persistence, for example [Pajares \(2008\)](#), [Schunk & Mullen \(2012\)](#) and [Wolters & Pintrich \(1998\)](#), particularly when learning mathematics online ([Kim et al., 2014](#)). Students' identification with university

mathematics further highlights their identity—a research area within mathematics education growing in attention in recent years (Hannula *et al.*, 2016; Graven & Heyd-Metzuyanim, 2019). Students' identity may explain their learning behaviour during educational transitions (Black, 2010; Black & Hernandez-Martinez, 2016). As students' identity is strongly shaped or sometimes even defined by their social interactions (Darragh, 2016; Radovic *et al.*, 2018), studying under pandemic conditions may strongly impact students' identification with university mathematics.

2.2.2. Learning strategies. Learning strategies in general may also partly explain students' success in university mathematics (Rach & Heinze, 2017; Liebendörfer *et al.*, 2020). However, students' self-regulated learning changed during the pandemic: students reported problems focusing on their studies when they had to study at home (Kindler *et al.*, 2020; Traus *et al.*, 2020). While in the survey by Traus *et al.* (2020), having to manage their study much more was perceived as a disadvantage by almost half of the students; more than 60% of the students perceived the new flexibility in organizing their work as advantageous. Here, we investigate self-regulated learning with a specific focus on planning, as a new aspect unearthed by the pandemic is the greater need for planning.

Another focus is on peer learning as one learning strategy because students could not easily collaborate under the given conditions. Collaboration in one location was clearly limited and online collaboration had its own problems, e.g., using mathematical notation in digital tools was even challenging for instructors after switching to digital teaching (Cassibba *et al.*, 2020; Irfan *et al.*, 2020). Thus, peer learning might have been particularly affected by the pandemic. Finally, we also assessed 'resisting frustration', a strategy describing students' effort in terms of persisting with their work even if the work is frustrating. This strategy of persistence has most prominently predicted students' success in a recent study in engineering mathematics (Liebendörfer *et al.*, 2020).

2.2.3. Social relatedness. Social relatedness relates to feelings of being cared for and caring for others. This concept is central to the development of norms and values as well as to the development of long-lasting interest (Krapp, 2002). In university mathematics, many students described social relatedness as the key to getting through the difficult transition to tertiary level (Liebendörfer, 2018). Social relatedness has drastically changed during the pandemic due to the substantial contact restrictions. Early studies reported, for example, that students had fewer peer contacts (Busse & Zeeb, 2020; Traus *et al.*, 2020).

3. Research Questions

Given the fundamental changes in teaching and learning during the pandemic, we sought an exploratory analysis of student use of resources, also in comparison with students' affect, learning strategies and social relatedness. Whereas many studies rely on mean values of the whole population, we also wanted to analyse if there are subgroups showing different behaviours.

First, we wanted to investigate how students evaluate resources in the digital setting in terms of usefulness for their learning in the digital setting.

RQ 1: How do students evaluate the various resources available to them in the digital mathematics curriculum as useful to their learning process?

Here, the question arises, whether there may be different types or groups of students in terms of rated usefulness of resources.

RQ 2a: How can we cluster student groups according to the resources they found useful in a wholly online learning setting?

As we found different groups of students, it also seemed worth investigating to what extent these groups might be characterized by personal characteristics and further variables.

RQ 2b: To what extent can a connection be made between the clusters and the biographical and psychological variables?

4. Design and Methods

4.1. Design, context and participants

The study at hand was conducted by issuing an online survey in December 2020, in the middle of the semester (lasting from October to February), to students of a first-year 'Linear Algebra 1' course that was held entirely digitally. We chose the middle of the semester because the students should already have settled into their own learning process but should not yet be in concrete preparation for the exam. This course is half of the module 'Linear Algebra' taught during the first two semesters of a mathematics or physics bachelor's programme, computer science or the study programme for a teaching degree for secondary education. Students were informed that participation in the study was voluntary and were assured of anonymity and confidentiality. They were also guaranteed that their answers would not influence the course grades in any way. A total of 232 students were formally enrolled in the course that semester. However, based on the number of homework assignments handed in each week and the number of students registered for the examination, we estimate that some 130 students engaged in the course. The resources used in the context of the course are shown in Table 1.

The course consisted of two lectures (1.5 h each) per week held synchronously using video communication software. A set of lecture notes was provided to the students, which was gradually expanded as the course progressed. In addition to the lectures, a full-class tutorial (1.5 h per week) was offered via

TABLE 1. Resources used in the course 'Linear Algebra 1'

	Frequency and duration	Held/produced by whom
Synchronous offers		
Lecture	2x/week; 1.5 h each	Lecturer
Full-class tutorial	1x/week; 1.5 h	Lecturer
Small group tutorial	1x/week; 1.5 h	Experienced student tutors
Learning support centre	2x/week; 3 and 1 h ¹	Experienced student tutors
Asynchronous offers		
Videos from the teaching team		
Explanation on the notes from the full-class tutorial	Every week	Lecturer
Explanations on the solutions to the weekly homework	Every week	Lecturer
Supplementary resources		
Standard textbooks of linear algebra, videos from external resources, learning management system (forum and a tool for anonymous questions)		

¹ In the learning support centre, special office hours are offered for the various courses in mathematics (Linear Algebra, Calculus, Geometry, etc.). In addition to these office hours, the learning centre is also open as a place of learning for the students.

TABLE 2. *Participants in the online survey: gender and study programme (n = 81, 8 students did not answer these questions)*

	Gender		Total
	Men	Women	
Teaching degree for secondary education	20	36	56
Computer science	7	2	9
Mathematics bachelor's programme	9	5	14
Physics bachelor's programme	1	1	2
Total	37	44	81

video communication software, where students could ask questions to the lecturer. The notes from this tutorial (examples, explanations, etc.) were subsequently recorded and made available to the students as videos. Small group tutorials were conducted every week (with about 15 to 20 students each) where students worked on small tasks under a tutor's supervision. For some of these tasks, a staff member produced videos with the corresponding solutions. Finally, a digital learning support centre was offered to the students (two times a week; 3 and 1 h). Here, the students could work in breakout rooms and ask a tutor when they needed help.

Students had to successfully complete the weekly homework in order to receive permission to take the final exam of the course.² The lecturer produced weekly videos in which he explained the solutions to these exercises. Standard textbooks of linear algebra were recommended to students, and videos from external providers were only in exceptional cases. The whole organization of the course was based on a learning management system, where a public forum and an anonymous tool for posing questions were provided.

In total, 89 students (37 male, 44 female, 8 'no answer'; 60 students aged 20 years or younger) participated in the study; 58 being in their first semester at university. We assessed students' final secondary school graduation mark ($M = 2.15$, $SD = 0.49$)³ as well as the study programme students were enrolled in (Table 2).

4.2. Instruments⁴

4.2.1. *Resources.* Based on the list of all resources from the course (see Section 4.1), we formulated the following items (authors' translation):

For my study in the course "Linear Algebra 1" ... is very useful: Attending the lecture; attending full-class tutorial; attending the small group tutorial; the use of the lecture notes; the videos of the professor and his assistants; communication with other students; the videos recommended by the professor and/or his assistants; the use of webpages; the use of videos from external resources; the use of supplementary literature; digital chat.

² The students had to achieve a total of at least 30% of the achievable score in the homework. There was also the requirement that at least 25% of the possible points had to be achieved within each of the first and second halves of the semester.

³ Marks are scaled from 1 to 4; 1 being the best mark.

⁴ All items presented in this section are also listed in the appendix.

We did not incorporate questions concerning the learning support centre in this context, as this 'resource' did not seem to be equally appropriate for a one item quantitative investigation at this time. Students' use of the learning support centre initially exceeded what was previously expected by the teaching team. For this reason, not all students were able to participate according to their wishes. This reason seemed to contradict a single quantitative measurement of perceived usefulness.

4.2.2. Psychological variables. We assessed *self-efficacy* in mathematics using items from PISA 2003 (Ramm *et al.*, 2006) that had been adjusted to tertiary mathematics in the context of the WiGeMath project (Hochmuth *et al.*, 2018). One example of the four items used was the following statement: 'I am confident that I can perform well on homework and examinations.' (authors' translation).

We assessed students' *identification with university mathematics* with a scale that was constructed by the authors. The items had been formulated to cover different aspects of positive identification with university mathematics and being a mathematician. Finally, after piloting the scale in the former course (see below), we came up with the following five items (authors' translation):

- I would describe myself as a mathematician.
- I am proud to (also) study mathematics.
- I feel comfortable when other people call me a mathematician.
- I feel connected to the mathematical community.
- I am happy when others see me as a mathematician.

To assess *peer learning*, we used a three-item scale from the LimSt-questionnaire (learning strategies in mathematical studies; Liebendörfer *et al.*, 2021) developed specifically for university mathematics. One example item is the following statement: 'I meet with other students to develop ideas for solutions together.' (authors' translation).

Existing scales for self-regulated learning are reported to have questionable reliability in the literature. Thus, we constructed a new scale in the context of a pilot study (see below). The items were formulated to reflect the planning phase particularly as planning seemed to have gained much more importance in the studying at home context. Finally, we came up with the following four items (authors' translation):

- I plan my learning and work in the context of the course.
- My learning behaviour follows a conscious pattern.
- I divide my learning and working time sensibly.
- I set myself goals that I want to achieve in my learning.

To assess the strategy of resisting frustration, we again used a three-item scale from the LimSt questionnaire (Liebendörfer *et al.*, 2021). One example item is the following statement: 'I don't give up, even if the content is very difficult or complicated.' (authors' translation).

Finally, we measured *social relatedness* with three items from the relatedness satisfaction scale and one item from the relatedness frustration scale (reversed scored) by Longo *et al.* (2014). Some items have been adjusted for the tertiary level. One original example is the following statement: 'I feel I'm perfectly integrated into a group.' (ibid., p. 314).

All scales had been tested for reliability in the previous iteration of the course 'Linear Algebra 1' in Summer term 2020, which was also held as an online course. In total, 42 students participated in this pilot study. Here, the scales for self-efficacy, identification with university mathematics, peer learning, resisting frustration and social relatedness all proved reliable in the digital setting. We also used this pilot study to sharpen our instrument for assessing self-regulated learning. We conducted a factor analysis on eight items. Finally, considering item discrimination, we came up with a reliable scale consisting

of the four items named above. All scales used in this study showed acceptable internal consistencies ($0.75 < \alpha < 0.92$). All items were to be rated on a six-level Likert scale (with [1] denoting ‘totally disagree’ . . . up to [6] ‘totally agree’, with intermediate values not explicitly labelled following a predesigned questionnaire model in our online tool).

4.3. Statistical analysis

We have generally treated the Likert scale data as metric data, even though strictly speaking they are ordinal data. We followed this widespread practice because parametric methods are robust with respect to violations of assumptions that occur when Likert scales are used (Norman, 2010; Harpe, 2015). Thus, the differences in the results are usually marginal.

In relation to Research Question 2, a cluster analysis was conducted. Considering the different resources from the course ‘Linear Algebra 1’, we selected those items not relying on communicational facets, for we did not want to incorporate aspects of social relatedness here. The cluster analysis was based on cases with complete information only ($n=48$). We followed the following procedure. First, we looked for outliers (single linkage; squared Euclidean variance) and excluded one case. Ward’s method (Eszergár-Kiss & Caesar, 2017) was used to determine the number of clusters. At this stage, solutions of two or three clusters were possible. We used k -means clustering (e.g., Lust *et al.*, 2011) to decide the number of clusters, also based on content aspects: following our exploratory approach, we opted for the solution giving more meaningful clusters. When making use of two clusters, the group of the ‘all resource users’ would not have become visible. Accordingly, we finally came up with the solution of three clusters.

5. Results

We start this section with a preliminary analysis of students’ psychological and biographical variables for the whole group as these variables will later be used to characterize the clusters.

5.1. Preliminary analysis of students’ psychological and biographical variables

Selected descriptive results concerning the affective characteristics, learning strategies and social relatedness are shown in Fig. 1 and Table 3. It becomes evident that means and medians of self-efficacy and identification with university mathematics as well as the social relatedness were located around the centre of the scale. However, the results show a high variation as the maximum and the minimum of the scale are reached. Students further reported high values for all three learning strategies (peer learning, self-regulated learning and resisting frustration).

Most variables had positive correlations according to the theoretical assumptions that affective variables interact with student behaviour. Peer learning had positive correlations to social relatedness but no other variables, according to the observation in earlier studies that peer learning seems equally important for all students.

We found a small correlation between graduation mark and self-efficacy ($r_p = -.23$; $p < 0.05$). Note that lower marks are better, so the better students were at school, the higher their self-efficacy was. We found no other remarkable differences between the scales and neither the mark in graduation nor the subgroup of students regarding the semester. However, both gender and study programme had correlations with the affective variables. Women reported lower self-efficacy than men did with a high effect size ($m_{women} = 2.77$ ($n = 44$), $m_{men} = 3.89$ ($n = 37$); $p < 0.001$ with Cohen’s $d = 0.97$; t -test).

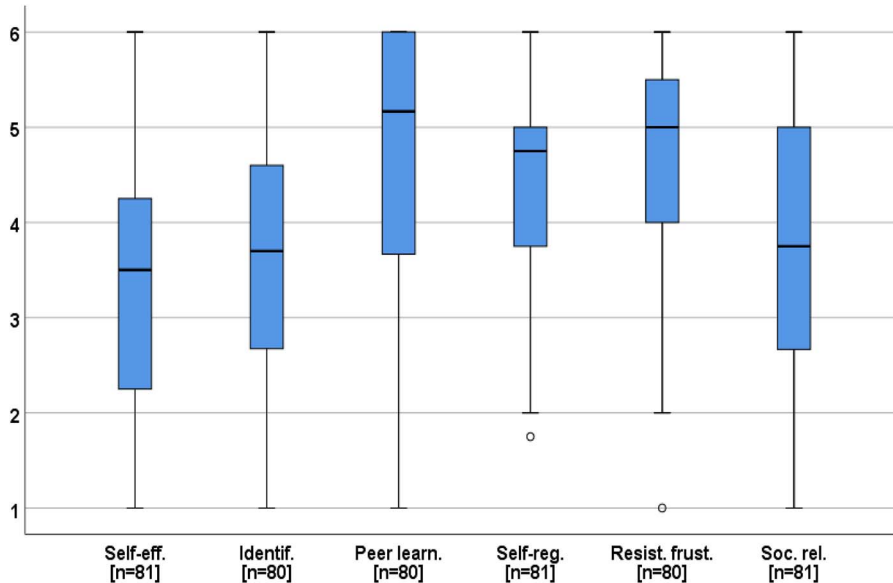


FIG. 1. Boxplots concerning the scales 'self-efficacy', 'identification with university mathematics', 'peer learning', 'self-regulated learning', 'resisting frustration' and 'social relatedness'.

TABLE 3. *Cronbach's alpha, medians, means, standard deviations and correlations of individual variables*

	α	Median	Mean	SD	(2)	(3)	(4)	(5)	(6)
(1) Self-efficacy	0.92	3.50	3.31	1.32	0.49**	0.01	0.19	0.34**	0.22*
(2) Identification with university mathematics	0.86	3.60	3.64	1.27	—	0.13	0.27*	0.38**	0.33**
(3) Peer learning	0.81	5.33	4.72	1.36	—	—	0.21	-0.01	0.63**
(4) Self-regulated learning	0.75	4.75	4.41	1.00	—	—	—	0.50**	0.29**
(5) Resisting frustration	0.86	5.00	4.58	1.14	—	—	—	—	0.10
(6) Social relatedness	0.81	3.89	3.80	1.34	—	—	—	—	—

* $p < 0.05$,

** $p < 0.01$.

This difference is also mirrored in programme of study ($m_{Teach} = 3.01$ ($n = 56$), $m_{Math} = 4.14$ ($n = 14$); $p = 0.004$ with Cohen's $d = 0.90$; t -test). We further found that students studying for a bachelor's degree in mathematics had a substantially higher identification with university mathematics than preservice teachers did ($m_{Teach} = 3.50$ ($n = 56$), $m_{Math} = 4.52$ ($n = 14$); $p = 0.007$ with Cohen's $d = 0.84$; t -test).

5.2. The usefulness of resources (RQ 1)

Research Question 1 focused on students' rated usefulness of the different resources used in the course. Respective results are shown in Fig. 2. All the (digitalized) traditional parts of mathematics teaching (lecture, full-class tutorial, small-group tutorial and lecture notes) were considered highly useful. The full-class tutorial was rated the highest, with 87% of the students giving positive ratings (ratings of 4 or higher). The digital resources (videos, webpages and chat) were also generally rated as useful, yet they

USEFULNES OF RESOURCES IN THE CONTEXT OF THE COURSE

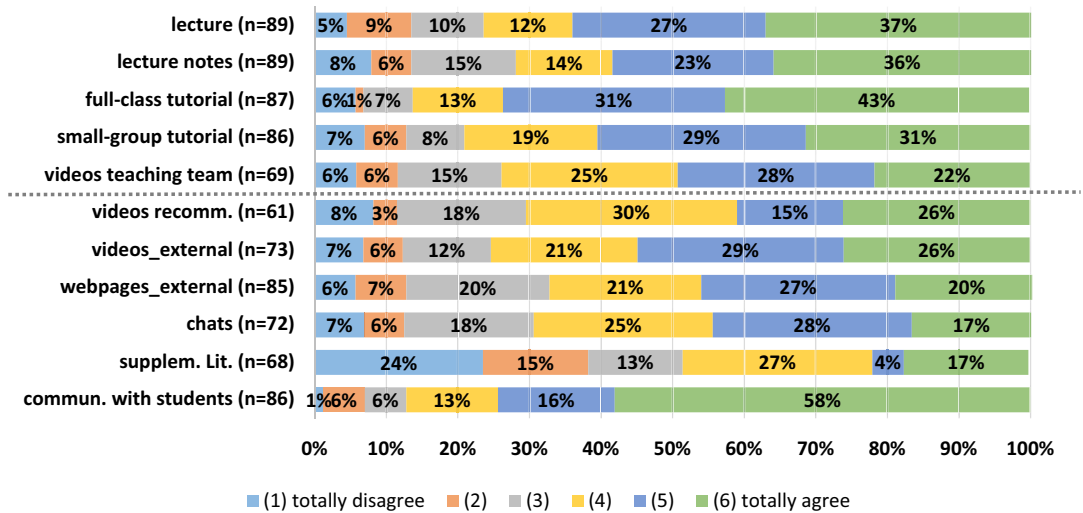


FIG. 2. Results concerning the usefulness of the resources in the context of the course.

achieved fewer ratings of 6 ('totally agree') in comparison with the traditional ones. Similarly, videos from external resources ('videos_external') had also been evaluated positively, achieving approximately equal positive ratings (76%) as the videos from the teaching team (76% ratings of 4 or higher). Videos recommended by the teaching team were evaluated only slightly lower. While the supplementary literature achieved the lowest ratings, communication with other students is rated overall as the most useful resource for learning in the course (58% 'totally agree').

We further note that almost every student answered the items concerning the traditional lecture components as well as webpages and communication with students, whereas videos, chats and literature were not answered by several students. This finding might indicate that some students never made use of these resources.

5.3. Identifying subgroups (RQ 2)

The cluster analysis (see Section 4.3) yielded three clusters that answer Research Question 2a. For interpreting these clusters, we look at the respective profiles concerning the resources considered to be useful shown in Fig. 3.

Cluster profile 1 is characterized by a minor rated usefulness of the traditional teaching material of mathematics (lecture, full-class tutorial and lecture notes), and a very positive evaluation of the digital (asynchronous, mostly external) resources. Students in this cluster mostly found webpages and videos from external resources useful. We thus called this group the 'digitals'. Profile 2 is characterized by positive evaluations of the traditional components of mathematics teaching (lecture, full-class tutorial and lecture notes), whereas the supplementary digital resources were rated less useful. Hence, we called this cluster the 'traditionalists'. Profile 3 is characterized by a (high) appreciation of all resources available. As there were no clear preferences for resources, we named this profile the 'all resource users'.

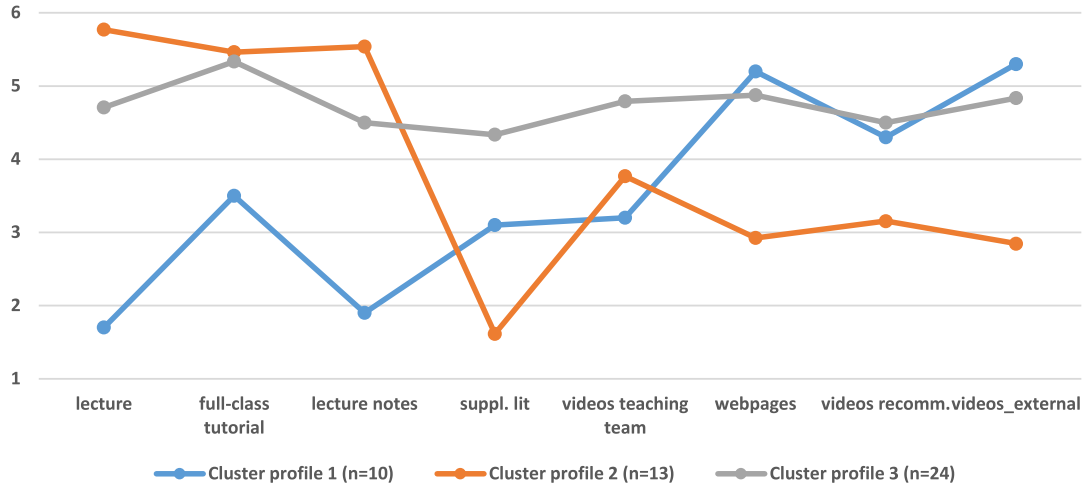


FIG. 3. Students' use of resources in the three clusters.

TABLE 4. Biographical data and personal variables by cluster profiles

		Total	Digitals (n = 10)		Traditionalists (n = 13)		All resource users (n = 24)	
Age	≤20 years	36	9		9		18	
	21–25 years	10	1		4		5	
First semester? ⁵	Yes	35	10		7		18	
	No	11	0		6		5	
Gender	Male	18	2		7		9	
	Female	28	8		6		14	
Study programme	Teaching degree	35	10		9		16	
	Comp. Science	3	0		0		3	
	Math Bch.	7	0		4		3	
	Physics Bch.	1	0		0		1	
			Mean	SD	Mean	SD	Mean	SD
Secondary school graduation mark ⁶			2.29	0.62	2.19	0.35	2.27	0.56
Self-efficacy			2.28	1.18	3.92	1.19	3.32	1.22
Identification			3.00	0.93	4.07	1.20	3.88	1.32
Peer learning			4.90	1.36	4.90	1.17	4.69	1.35
Self-regulation			4.00	1.31	4.54	0.96	4.64	0.91
Resisting frustration			4.60	1.24	4.74	0.75	4.88	1.22
Social relatedness			3.50	1.70	4.15	1.31	3.91	1.48

These clusters indicate that different personal characteristics matter in answering Research Question 2b (Table 4). All 'digitals' were in their first semester at university and studying for a teaching degree;

⁵ Although the course is intended to be offered to first-year students, some students are not in their first semester at the university. This may be because they have changed their study programme or have to repeat the course because they have not passed the module exam.

⁶ Marks are scaled from 1 to 4; 1 being the best mark.

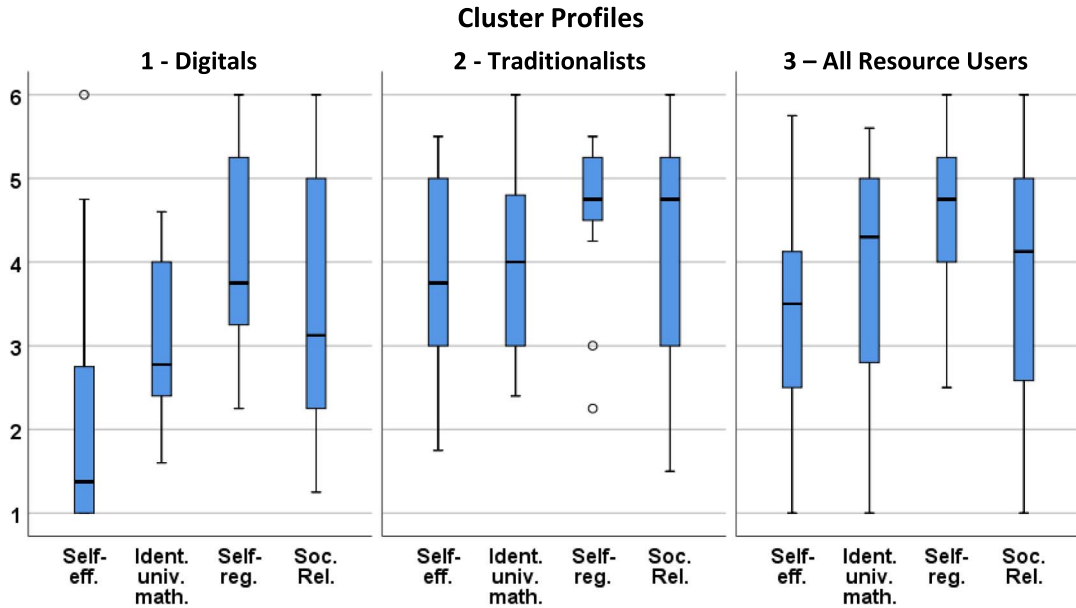


FIG. 4. Boxplots of self-efficacy, identification with university mathematics, self-regulated learning and social relatedness regarding the three cluster profiles.

most of them were women. They reported substantially less self-efficacy and identification with the study programme, less self-regulated learning and less social relatedness than the other groups. These results become even more apparent when looking at the corresponding boxplots (Fig. 4). Even the upper quartile of the ‘digitals’ self-efficacy was located in the lower part of the scale; its median was 1.38. The medians of self-regulated learning and social relatedness were also below the value of 4 in contrast to the other groups’ respective values.

The ‘traditionalists’ had the highest proportion of males, non-first semesters, older students and students in the mathematics bachelor’s programme. They showed a particularly high self-efficacy and social relatedness. The ‘all resource users’ formed the largest group of about half of the students in our study. They showed no prominent characteristics.

6. Discussion

6.1. Students’ evaluation of the resources (RQ 1)

RQ 1: How do students evaluate the various resources available to them in the digital mathematics curriculum as useful to their learning process?

Except for the supplementary literature, most students rated all resources available in the course as useful. According to the percentages of students rating the maximum point of the scale, the traditional aspects of mathematical teaching at university (lecture, lecture notes, full-class tutorial and small-group tutorial) are considered more useful than the digital ones. Our results confirm earlier research showing that students prefer resources that are closely connected to the lecture (Inglis *et al.*, 2011; Rønning, 2014; Maclaren, 2018) even in the digital environment. It would be worth investigating reasons why students find the

traditional elements particularly helpful. Students might value the coherence in style and mathematical details (like definitions and notation), since their learning is mainly driven by the compulsory exercises (Liebendörfer, 2014; Göller, 2016, 2020). Accordingly, students might appreciate knowing that these resources focus on content that is closely related to the tasks. Similarly, they might explicitly prefer resources pertaining to content that will be highly relevant to the examination.

However, the importance of external digital resources should not be underestimated. Despite the large number of videos produced by the teaching team focused on answering students' questions from the full-class tutorial and explaining the weekly homework assignments, the videos from external resources achieve comparable ratings in terms of usefulness. It becomes apparent that videos from external resources play a prominent part in students' way of studying mathematics. It seems remarkable that webpages and chat were also rated as useful by the majority of students. While we should generally assume that students can learn a lot with these resources, we do not know and cannot control the quality of such external digital resources, which would be worth investigation. Furthermore, students' check of the reliability of external digital resources could play a crucial role in this context. In a small study, Mac an Bhaird *et al.* (2020) found that only a minority of their students who use online resources 'always' consider the reliability of these resources.

The resource that was rated most useful in our study was communication with other students (58% 'totally agree'). This result demonstrates that despite all the technical problems, effective communication between students can also take place in the digital setting. One could further assume that communication with other students gains particular subjective significance in a setting that relies on individuals working at home.

6.2. *Different clusters using different resources (RQ 2)*

RQ 2a: How can we cluster student groups according to the resources they found useful in a wholly online learning setting?

The cluster analysis revealed three different profiles: the 'digitals' making extensive use of external and digital resources ($n = 10$), the 'traditionalists' focussing on the traditional aspects of mathematics teaching (lecture, full-class tutorial and lecture notes) ($n = 13$) and the 'all resource users' making use of all resources available.

RQ 2b: To what extent can a connection be made between the clusters and the biographical and psychological variables?

The profile of the 'digitals' could be sharpened by taking into account personal data. These persons were likely to be younger students (aged 20 years or younger, first-year students), women and all studying for a teaching degree. The 'digitals' further displayed lower self-efficacy, identification with university mathematics, self-regulated learning and social relatedness. As these variables may explain students' dropout (Rach & Heinze, 2017; Liebendörfer, 2018; Geisler, 2020), the 'digitals' could be considered students at risk. Finding the core recommended materials unhelpful could indicate that they have substantial trouble understanding the content. We should note, however, that the 'digitals' had similar graduation marks and resisted frustration similarly to their peers indicating similar starting ability and engagement.

The interpretation of this group is thus not entirely straightforward. The 'digitals' could be students who never wanted to engage fully and took the course because digital teaching easily affords students to also take other courses. But this interpretation seems to contradict with students' strongly resisting frustration. They could also be students who want to become teachers without specifically liking

mathematics as a subject in all its different aspects. However, even that would not explain why they find external videos and websites helpful but not the lecture notes. Therefore, it seems most likely to us that these are students who are overwhelmed by the subject at this moment. Possibly, these students may have trouble following the regular instruction and therefore try finding additional resources to keep on track.

The fact that the ‘digitals’ all studied for a teaching degree could be explained by two reasons. First, student teachers can be particularly burdened at the beginning of their studies because they have to learn two subjects with different ways of thinking. Second, if mathematics majors could hardly cope with the requirements they might have dropped out of their studies already, as we know they sometimes drop out within the first weeks (Liebendörfer, 2018). The high proportion of women would then be indirectly explained by the fact that women are generally more strongly represented in pre-service teachers. In addition, specific commitments like family care have appeared or increased during the pandemic, which may have affected students’ learning and by which women were more affected (Aristovnik *et al.*, 2020).

This all suggests that students’ mastery of the content provides the central explanation for the clusters. The resources provided directly by the teaching team may align more with the high demands of the course and thus be less useful for students with serious problems understanding the content. In line with this interpretation, self-efficacy is the variable which distinguishes the ‘digitals’ from the ‘traditionalists’ most clearly and with a high value (Cohen’s $d = 1.24$ SD based on the variance of the entire sample). This perspective also fits the observation that the ‘all resource users’ have lower self-efficacy than the ‘traditionalists’. ‘All resource users’ might seek out other resources when they get stuck using the traditional resources. This could also explain why all but one student in a later semester belong to the ‘traditionalists’ or the ‘all resource users’ and consider the traditional aspects of mathematics teaching as useful: students who could not master the content might have dropped out.

This perspective corresponds to findings from more experienced mathematics students. Reinhold *et al.* (2021) found during the pandemic that ‘particularly interested, motivated, and mathematically talented students need face-to-face communication and discussions’ (p. 8). In our interpretation, they would particularly benefit from traditional face-to-face situations because they can participate in the discourse (Solomon, 2007). However, it seems surprising that no correlation between final school marks and cluster profile could be identified although school marks generally predict success in university mathematics (Halverscheid & Pustelnik, 2013; Rach & Ufer, 2020). Acknowledging the difference between mastery during semester and performance in final examinations, we believe that our interpretation is the most plausible.

In any case, this issue deserves further exploration. In particular, we should clarify reasons for why students become ‘digitals’ by being overburdened. Prior research found students’ adoption of their learning strategies to the new kind of mathematics crucial for keeping track (Rach & Heinze, 2017). In the digital learning environment, the ‘digitals’ may have taken a strategic approach that allows content to be roughly understood in the short term but gives too little sense of achievement in the long term. In addition, specifics of the pandemic like caring commitments should be investigated.

6.3. Limitations

The results presented here are based on only one study with one specific implementation. Due to different ways universities have offered digital teaching, results would need to be cross-checked with further studies before they can be generalized. Also, our results on the study programmes and clusters should be considered with caution because of the small sample sizes in the subgroups.

Some of our instruments were newly developed. We measured students' *identification with university mathematics* in a specific way that may not focus on aspects that are important for identification in other researchers' perspectives. Similarly, self-regulated learning was assessed with a rather narrow focus on planning but less on regulation. In addition, labelling intermediate values of all Likert scales might have improved measurement accuracy.

Another limitation is that we could not evaluate the use of the learning support centre. The centre has been set up with restricted capacity but seemed to be very useful for the students. Unfortunately, the demand was unexpectedly high, so that the centre could not serve everyone equally. We could thus not validly evaluate its usefulness. However, the importance of the learning support centre becomes evident in the positive student responses to open questions in the evaluation of the course. Research on learning support centres might be especially important in the digital setting and should be included in further studies.

6.4. Conclusion

Our study shows that the self-developed materials of teachers have very high usefulness for students in the digital setting. This may be due to the teaching team's remarkable commitment in this case, but generally, it shows that the high effort required to produce such resources can be worthwhile. On the other hand, it is evident that resources from external sources also play an essential role in students' way of learning. In particular, the use of videos from the internet cannot be neglected. This also seems to be significant because the quality of these resources and their alignment with the course are beyond the teaching team's control.

Our study further shows that social contact is very significant, even under pandemic conditions. A constructive exchange between students is possible and should be supported by the universities as much as they can. Providing a digital learning support centre where students can meet online seems to be a good starting point.

Finally, we were able to identify a student cluster that has not been described in previous research. That is, students who do not consider the classical materials as being useful and therefore probably rely more on external resources when learning mathematics. We assume that these students have problems mastering the content and may thus search for alternative (probably more basic) explanations elsewhere.

It seems worth noting that the 'digitals' group in particular relies heavily on external resources, whose quality and alignment with the course is not supervised or guaranteed. This result raises the question of the extent to which we can help such students and how far we want to. Currently, more than half of the mathematics major students in Germany drop out of their studies (Heublein & Schmelzer, 2018; see also Heublein, 2014). One could argue that these students lack support resources. However, it seems questionable whether such resources would enable most of them to achieve their study goals in the long term. Dealing with heterogeneity became visible as an important issue in the context of this study. Discussing it, however, is a fundamental question that goes beyond the focus of this paper.

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Appendix: Items

'Mathematical self-efficacy' (author's translation):

I am confident that I can master the mathematical skills that are taught.

I am confident that I can perform well on homework and exams.

I am confident that I can understand even the most complicated material presented by the lecturer.

I am confident that I can understand even the most challenging material in the literature.

'Identification with university mathematics' (authors' translation):

I would describe myself as a mathematician.

I am proud to (also) study mathematics.

I feel comfortable when other people call me a mathematician.

I feel connected to the mathematical community.

I am happy when others see me as a mathematician.

'Peer learning' (authors' translation):

- I enlist the help of other students when I have serious comprehension problems.
- I meet with other students to develop ideas for solutions together.
- When I have an approach to a solution, I discuss it with other students.

'Self-regulated learning' (authors' translation):

- I plan my learning and work in the context of the course.
- My learning behaviour follows a conscious pattern.
- I divide my learning and working time sensibly.
- I set myself goals that I want to achieve in my learning.

'Resisting frustration' (authors' translation):

- I don't give up, even if the content is very difficult or complicated.
- Even when I get frustrated, I still keep studying.
- Even if I don't make any progress at all, I keep trying until I get it right.

'Social relatedness' (authors' translation and adjustment for the tertiary level; compare [Longo *et al.*, 2014](#)):

- I feel perfectly integrated into a group.
- I feel very close and connected with my fellow students.
- I feel the fellow students I interact with really care about me.
- Sometime, I feel a bit rejected by my fellow students.