# Brecca M. M. Gaffney

Department of Mechanical and Materials Engineering, Human Dynamics Laboratory, University of Denver, Denver, CO 80208 e-mail: brecca.gaffney@gmail.com

# Cory L. Christiansen

Department of Physical Medicine and Rehabilitation, University of Colorado, Denver, CO 80045; Denver Geriatric Research Education and Clinical Center, VA Eastern Colorado Health Care System, Denver, CO 80012 e-mail: cory.christiansen@ucdenver.edu

# Amanda M. Murray

Department of Physical Medicine and Rehabilitation, University of Colorado, Denver, CO 80045; Denver Geriatric Research Education and Clinical Center, VA Eastern Colorado Health Care System, Denver, CO 80012 e-mail: Amanda.Murray2@utoledo.edu

# Casey A. Myers

Department of Mechanical and Materials Engineering, Center for Orthopaedic Biomechanics, University of Denver, Denver, CO 80208 e-mail: casey.myers1@gmail.com

# Peter J. Laz

Department of Mechanical and Materials Engineering, Center for Orthopaedic Biomechanics, University of Denver, Denver, CO 80208 e-mail: peter.laz@du.edu

# Bradley S. Davidson<sup>1</sup>

Department of Mechanical and Materials Engineering, Human Dynamics Laboratory, University of Denver, 2155 E Wesley Ave. ECS 443, Denver, CO 80208 e-mail: Bradley.davidson@du.edu

# The Effects of Prosthesis Inertial Parameters on Inverse Dynamics: A Probabilistic Analysis

Joint kinetic measurement is a fundamental tool used to quantify compensatory movement patterns in participants with transtibial amputation (TTA). Joint kinetics are calculated through inverse dynamics (ID) and depend on segment kinematics, external forces, and both segment and prosthetic inertial parameters (PIPS); yet the individual influence of PIPs on ID is unknown. The objective of this investigation was to assess the importance of parameterizing PIPs when calculating ID using a probabilistic analysis. A series of Monte Carlo simulations were performed to assess the influence of uncertainty in PIPs on ID. Multivariate input distributions were generated from experimentally measured PIPs (foot/shank: mass, center of mass (COM), moment of inertia) of ten prostheses and output distributions were hip and knee joint kinetics. Confidence bounds (2.5–97.5%) and sensitivity of outputs to model input parameters were calculated throughout one gait cycle. Results demonstrated that PIPs had a larger influence on joint kinetics during the swing period than the stance period (e.g., maximum hip flexion/extension moment confidence bound size: stance =  $5.6$  N·m, swing:  $11.4$  N·m). Joint kinetics were most sensitive to shank mass during both the stance and swing periods. Accurate measurement of prosthesis shank mass is necessary to calculate joint kinetics with ID in participants with TTA with passive prostheses consisting of total contact carbon fiber sockets and dynamic elastic response feet during walking. [DOI: 10.1115/1.4038175]

Keywords: amputee, prosthesis inertial parameters, probabilistic analysis, inverse dynamics

## Introduction

The number of people living with lower limb amputation is expected to exceed  $3.6 \times 10^6$  $3.6 \times 10^6$  $3.6 \times 10^6$  by the year 2050 [1], which is

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resulting in an increased interest in movement science research aimed at the understanding of how this population physically compensates for the loss of the joint(s). From 1979 to 2009, 584 investigations reported the biomechanics of amputee locomotion [[2\]](#page-6-0), and 122 (20%) of these investigations used joint kinetics as the dependent variable to assess the effects of limb loss during locomotion on the musculoskeletal system.

Joint kinetics are fundamental to understanding amputee gait patterns and their effects on joint loading, because they represent the net effect of all agonist and antagonist muscles spanning a joint and provide insight into specific motor patterns [[3](#page-6-0)]. In

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participants with lower limb amputation, joint moments during gait have been used to quantify the compensatory movements required to achieve forward progression [[4,5\]](#page-7-0) and to determine potential consequential joint loading patterns at the hip and knee [\[6\]](#page-7-0). For example, participants with transtibial amputation (TTA) adopt reduced knee extensor moments during loading on the amputated limb compared to able-bodied individuals (0.05 versus 0.70 N-m/kg m) [\[7](#page-7-0)]. As a result, hip extensor kinetics during loading are higher compared to able-bodied individuals, which may be a strategy used to assist in forward progression during the stance period [[5,8](#page-7-0)–[11\]](#page-7-0).

Joint kinetic calculations of amputee gait often do not accurately account for the variable segment inertial parameters of the individual prostheses. Inverse dynamics (ID), used to calculate joint kinetics during gait, depend on the accurate measurement of segment kinematics and external forces, as well as the accurate estimation of the segment inertial parameters (segment mass, center of mass (COM) location, and moments of inertia) [[12](#page-7-0)]. Uncertainty in segment inertial parameter estimation has been shown to impact hip and knee joint kinetics calculated using ID during walking in able-bodied individuals by upwards of 5 N-m during the stance period and 8 N·m during the swing period [[13\]](#page-7-0). When the variability of inertial parameter uncertainty is increased, as is the case for prosthetic limbs, the impact on joint kinetics is likely larger, yet this effect remains unknown.

Despite the large variability in below-knee prosthesis size and design, the most common methods for quantifying prosthesis inertial parameters (PIPs) do not account for the actual prosthesis worn by the participant, which affects the accuracy of ID solutions. The two most commonly used methods to quantify PIPs in link-segment models used for ID calculations are to: (1) make no adjustments by using the intact segment inertial parameters [\[6,7,9,14,15](#page-7-0)] or (2) make uniform adjustments by reducing the mass (2.325% body weight or 50% reduction of intact shank mass) and superiorly shifting the COM (25% of segment length) in the shank [\[16–18](#page-7-0)]. PIPs are known to be different than intact segment parameters, which provide input parameter uncertainty that will impact joint kinetics. For example, the mass of the residual limb combined with a common prosthetic limb for a participant with TTA is only 30–40% of the mass of the intact shank and foot [[19,20](#page-7-0)]. Therefore, making no adjustments to the PIPs when calculating joint kinetics likely results in inaccurate estimates. In addition, the residual limb gradually undergoes reduction in volume that is highly variable among individuals [\[21,22](#page-7-0)], resulting in refitting of the prosthesis over time. Both the alterations of residual leg size and prostheses will result in a change in PIPs within the same participant with amputation over time. As a result, a uniform scale factor is likely not accurate for all prosthetic limbs, particularly for mature prosthetic limbs  $(18+$  months postamputation) [\[23\]](#page-7-0).

To address the uncertainty in these estimation methods, PIPs for an individual's prosthesis can be experimentally measured for use in ID calculations. Prosthesis mass and COM location can be measured using a reaction board technique [\[24](#page-7-0),[25\]](#page-7-0) and moments of inertia can be measured with an oscillation rack [[26\]](#page-7-0); however, these processes increase experimental collection times by  $\sim$ 30 min per anatomical plane of movement [[26,27](#page-7-0)]. To address the burden on experimental collection time, while maintaining the accuracy these methods provide, a recent model was created to estimate PIPs based on subject-specific adjusted body mass (ABM) and residual limb shank length [\[27\]](#page-7-0). In the published study, the model used mean values of experimentally measured shank and foot PIPs of 11 below knee passive prostheses. The prosthetic foot and shank masses were expressed as a percentage of adjusted body mass. The prosthetic shank center of mass and radius of gyration lengths were expressed as a percentage of the prosthetic shank length (knee joint center to ankle joint center). The results demonstrate that peak hip and knee joint moments calculated from the estimation model compared to a model with experimentally measured PIPs were not statistically different during the stance period, but were smaller during the swing period. However, this model does not provide insight regarding which PIP has the most influence on joint kinetics calculated using ID.

An alternative solution to calculating joint kinetics that would allow researchers to maintain the accuracy of PIPs input parameters is needed. This could be accomplished by measuring a subset of PIPs to minimize the experimental collection time. For example, if uncertainty of a specific inertial parameter of a prosthesis had a substantially larger effect on joint kinetics than the remaining inertial parameters, researchers could measure that specific parameter and estimate the remaining parameters. To our knowledge, the influence of each individual PIP on joint kinetics is unknown, which would allow researchers to prioritize parameters most critical to the ID calculations.

Probabilistic analyses provide the most comprehensive approach for understanding the impact that individual input parameter uncertainties and variability have on biomechanical calculations [[28](#page-7-0)]. The most common probabilistic method to assess the influence of input variability on output is Monte Carlo simulation, which is a method that repeatedly samples input probability distributions to create output distributions [[29\]](#page-7-0). The most common metrics used to quantify the impact of input variability on outputs are confidence bounds and sensitivity factors. Confidence bounds quantify the overall impact of input variability on outputs by providing the approximate output levels associated with a specific probability (e.g., 5–95%), and should be distinguished from confidence intervals (CIs) depending on the distribution type of the probabilistic simulations. Confidence intervals are a population-based measure typically used for approximating the mean of the entire population based on sample. However, when the output distribution of a probabilistic simulation is Gaussian, confidence bounds can be interpreted similarly to confidence intervals [\[30](#page-7-0)]. Sensitivity factors quantify the effect of changing an individual input parameter on an output [\[13,31](#page-7-0)]. Previous research from our group on the effects of uncertainty in segment inertial parameters on inverse dynamics in healthy participants concluded that the magnitude of joint moments was most sensitive to variability in segment mass rather than variability in segment COM location or segment moment of inertia [[13,32](#page-7-0),[33](#page-7-0)]. However, probabilistic methods have not been applied to examine the impact of correctly parameterizing PIPs on lower-extremity joint moments during gait for people with amputation. Therefore, the effect of variability in PIPs on joint kinetics during amputee walking remains unknown. Given the difference in joint kinetics  $[4,5,9]$  $[4,5,9]$  $[4,5,9]$  and segment inertial parameters [\[19,20](#page-7-0)] between patients with transtibial amputation and able-bodied individuals, it is unlikely that the results will be the same. If different, these results would allow researchers to prioritize which PIPs are important to experimentally measure without significantly increasing experimental collection times.

The objective of this investigation was to assess the importance of properly quantifying PIPs when calculating joint kinetics in a participant with TTA during gait.

To accomplish this, we used a series of Monte Carlo simulations to determine the range of possible outcomes in the presence of PIPs uncertainty, and the sensitivity of output variables (joint moments) to changes in input variables (PIPs).

## Methods

The influence of uncertainty in PIPs on ID calculations was assessed using a series of Monte Carlo simulations. Using the output distributions that were generated, confidence bounds and sensitivity factors were used to quantify the amount of output variability due to input variability and the individual influence of each PIP on ID solutions, respectively. Based on these findings, we provide an experimental application to demonstrate the effects of modeling the PIP that ID are most sensitive to by using four common methods in a cohort of ten patients with unilateral TTA.

Experimental Collection and Data Processing. One participant with a unilateral transtibial right limb amputation (age: 52 yr; amputation type: dysvascular; time since amputation: 17 months; height: 1.86 m; mass (with prosthesis): 100.45 kg) walked on a

10-m over-ground platform at 1.0 m/s. Whole body kinematics were collected from 63 reflective markers, sampled at 100 Hz (Vicon, Centennial, CO). Ground reaction forces (GRF) were collected from two embedded force platforms, sampled at 2000 Hz (Bertec, Columbus, OH). One gait cycle (residual limb heel strike to residual limb heel strike) was used for the probabilistic analysis. Prior to the experimental collection, the participant signed an informed consent from a protocol approved by the Colorado Multiple Institutional Review Board.

Kinematic and GRF data were low-pass filtered with a fourth order Butterworth filter (6 Hz and 20 Hz cutoff frequencies, respectively). A 15-segment model was created in VISUAL 3D (C-Motion, Inc., Germantown, MD) (head, upper arms, forearms, hands, trunk, pelvis, thighs, shanks, and feet). Intact segment masses were scaled as a percentage of total body mass, and intact segment inertias were based on segment geometry [[34\]](#page-7-0).

Prosthesis Inertial Parameters. The mass, center of mass location, and moment of inertia of the prosthesis (prosthesis  $shank +$  prosthesis foot  $+$  shoe) were experimentally measured. Prosthesis mass was measured using a standard scale. The center of mass of the prosthesis in the superior–inferior direction was determined using a reaction board technique [\[24,25](#page-7-0)]. The moment of inertia of the prosthesis was determined in each orthogonal anatomic planes (sagittal, frontal, transverse) using an oscillation technique [[26,35\]](#page-7-0). These methods were used to measure PIPs from ten total transtibial prostheses to provide input data for the probabilistic analysis (Table 1). All prostheses were passive consisting of total contact carbon fiber sockets and dynamic elastic response feet. All patients with amputation used sleeves and/or pin/liner suspensions at the interface between the residual limb and socket, which were included as part of the prosthesis.

Prosthetic inertial parameters were experimentally measured and separated into a two-segment model: (1) prosthesis shank (residual  $limb +$  prosthesis socket  $+$  prosthesis pylon (with connectors)) and (2) prosthesis foot. The prosthesis shank mass was defined as 66% of total prosthesis mass, and the prosthesis foot mass was defined as 34% of total prosthesis mass [[26,36\]](#page-7-0). The center of mass and moment of inertia of the prosthesis foot were calculated using regression equations for an intact foot [[37](#page-7-0)], with the mass adjusted [[26\]](#page-7-0). The center of mass and moment of inertia of the prosthesis socket were calculated as the difference between total prosthesis (prosthesis socket + prosthesis foot) and the prosthesis foot  $[26]$ . The moment of inertia for the prosthesis socket and foot in all three planes was defined as:  $I_{\text{sagittal}}$  (sagittal plane),  $I_{\text{frontal}}$  (frontal plane), and  $I_{transverse}$  (transverse plane). The residual limb was modeled as a right frustum cone based using the residual limb length and the distal and proximal limb radii. The residual limb segment properties were determined with an estimated uniform tissue density of 1.1 g/cm3 [\[38,39\]](#page-7-0). Segment properties of the prosthesis shank were determined by combining the mass, center of mass, and moment of inertia (using the parallel axis theorem) of the residual limb and the prosthesis socket. PIPs and anthropometric data of the ten individual prostheses are supplied in Supplementary Tables 3–6 which are available under ["Supplemental Data](http://dx.doi.org/10.1115/1.4038175)" tab for this paper on the ASME Digital Collection.

Table 1 Mean (1 SD) of experimentally measured PIPs of ten transtibial prostheses were used as input parameters for the probabilistic analysis

<b>PIP</b>	Foot	<b>Shank</b>	
Mass $(kg)$	0.97(0.13)	3.28(0.45)	
COM(m)	0.06(0.01)	0.15(0.02)	
$I_{\text{sagittal}}\left(\text{kg}\cdot\text{m}^2\right)$	$1.20 \times 10^{-3}$ (3.71 $\times 10^{-4}$ )	0.07(0.03)	
$I_{\text{frontal}}\left(\text{kg}\cdot\text{m}^2\right)$	$1.30 \times 10^{-3}$ (4.09 $\times 10^{-4}$ )	0.06(0.02)	
$I_{\text{transverse}}$ (kg·m <sup>2</sup> )	$3.08 \times 10^{-4} (9.50 \times 10^{-5})$	0.03(0.01)	

Probabilistic Analysis. Monte Carlo simulations, which repeatedly samples multiple model input distributions to determine an output distribution [\[29\]](#page-7-0), were used to assess the sensitivity and bound sizes between changes in PIPs and joint moments in one patient with TTA. The input distributions were defined by the experimentally measured PIPs from ten transtibial prostheses (described in the Prosthesis Inertial Parameters section). A bootstrap analysis on the means of the PIPs (Table 1) with 10,000 replications was used to determine input distributions for the probabilistic analysis, which was used to estimate input distributions for all conceivable PIPs [[40\]](#page-7-0). To account for the relationships between segment input parameters, multivariate distributions were created for the prosthesis shank and foot. Each multivariate distribution was defined by the population mean, and the covariance matrix of the PIPs was generated by randomly selected input vectors [\[29\]](#page-7-0). To account for the range of input data provided from the ten transtibial prostheses, and include only physiologically feasible input parameters, PIPs were only included within the 2.5–97.5% of the multivariate distributions.

In each trial of the Monte Carlo simulation, a randomly generated segment parameter from the multivariate distributions for the prosthesis shank and foot were included into link-segment model to calculate ID using the experimentally collected kinematic and GRF data. A custom interface between VISUAL 3D and MATLAB (The MathWorks, Inc., Natick, MA) was developed to perturb the prosthesis shank and foot PIPs by altering the model file within a Monte Carlo simulation. For every simulation, hip (flexion/extension, adduction/abduction, internal/external rotation) and knee (flexion/ extension) moments were calculated across one gait cycle (residual limb heel strike to residual limb heel strike). A series of Monte Carlo simulations of 2000 trials were performed to achieve convergence (differences in mean confidence bounds of less than 0.1 N-m [[13,41\]](#page-7-0)).

Data Analysis. Confidence bounds and sensitivity factors were used to quantify the impact of variability of PIPs on the hip and knee joint moments (output distributions). Confidence bounds provide the associated probability that an output parameter lies within a chosen range between the upper and lower bounds. For this analysis, the 2.5% and 97.5% confidence bounds were calculated for each joint kinetic value to match the input distribution bounds, which provide a 95% probability that the joint kinetic value lies between the upper and lower bounds. The mean and standard deviation (SD) of the confidence bounds were calculated for three separate analyses: the entire gait cycle, the stance period, and the swing period [[13\]](#page-7-0). Sensitivity factors quantify the amount of change of an output parameter (hip and knee joint kinetics) due to a change in input parameter (PIPs). The sensitivity of joint kinetics to variations in PIPs were determined with Pearson product-moment correlation coefficients (r) between each PIP and the peak moment during the stance and swing periods, which allow the individual importance of PIPs on joint kinetics to be quantified based on the magnitude of correlation coefficient. Sensitivities were categorized as weakly sensitive  $(r = 0.2-0.4)$ , moderately sensitive  $(r = 0.4{\text -}0.6)$ , or highly sensitive  $(r = 0.6{\text -}1.0)$ , based on correlation coefficients [[13\]](#page-7-0). To assess the statistical significance of the correlation coefficients, 95% CIs were calculated for the correlation coefficients. CIs that did not cross zero were considered to be statistically significant ( $\alpha$  = 0.05) [[30\]](#page-7-0). To place the relation between PIPs and output kinetic into clinically meaningful units, the slope of each relation was calculated and scaled by the SD of the input PIP (Table [2](#page-3-0)). This scaling step indicates the amount of expected variance of the output variable given  $a \pm 1$ SD change in input parameter [[13\]](#page-7-0).

Experimental Application on Amputee Cohort. To demonstrate the influence of how a specific PIP (prosthesis shank mass) is modeled when calculating ID, a post hoc experimental analysis was performed on a cohort of ten participants with unilateral TTA (Table [3](#page-3-0)). Each participant performed three over-ground walking trials at a

<span id="page-3-0"></span>Table 2 The slope of the sensitivity relationships between peak (minimum and maximum) joint moments during the stance and swing periods and shank PIPs. Slopes were scaled by the SD of the input parameter, indicating the expected change of the output following  $a + 1$  SD change in input.

	Hip flex/ext		Hip abd/add		Hip int/ext		Knee flex/ext	
	Min (ext)	Max (flex)	Min (abd)	Max (add)	Min (ext)	Max(int)	Min(flex)	Max (ext)
Stance								
<b>Mass</b>	0.03	$1.12^{a}$	$0.45^{\rm a}$	$0.35^{\rm a}$	$0.14^{a}$	$-0.14^{a}$	0.11	$-0.32^{\rm a}$
<b>COM</b>	0.05	$-0.10$	0.17	$0.31^{a}$	$-0.05$	0.00	$0.12^a$	$-0.15$
I <sub>sagittal</sub>	$0.21^{\rm a}$	$-0.13$	0.00	0.08	$-0.03$	$-0.04$	0.08 <sup>b</sup>	$0.29^{\rm a}$
$I_{frontal}$	$-0.05$	$-0.03$	0.05	0.10	0.02	0.07	0.03	0.02
$I_{\text{transverse}}$	0.01	$-0.05$	$-0.01$	$-0.11$	0.05	$-0.09$	$-0.01$	0.03
Swing								
<b>Mass</b>	$-0.95^{\rm b}$	0.32 <sup>b</sup>	$-0.54^{\rm a}$	0.20 <sup>b</sup>	$-0.22^{\rm a}$	0.07	$-0.53^{\rm a}$	$-0.44^{\rm a}$
<b>COM</b>	$-1.08^{a}$	0.20	0.19	$0.23^{\rm a}$	0.08	0.15	$-1.00b$	$-0.21$
$I_{\text{sagittal}}$	$-0.45$	$0.42^{\rm a}$	0.17	0.09	0.01	0.37 <sup>a</sup>	$-1.82^{\rm a}$	$0.39^{\rm a}$
$I_{\text{frontal}}$	$-0.09$	0.05	0.09	0.04	0.03	$-0.04$	0.15	0.03
$I_{\text{transverse}}$	$-0.03$	$-0.01$	0.02	$-0.05$	0.02	0.21 <sup>b</sup>	$-0.07$	0.04

<sup>a</sup>Highly sensitive  $(r = 0.6 - 1.0)$ .

<sup>a</sup>Highly sensitive (r = 0.6–1.0).<br><sup>b</sup>Moderately sensitive (r = 0.4–0.6), both of which are statistically significant (95% confidence interval did not cross zero). The level of significance was set at  $\alpha = 0.05$ .

Table 3 Participant characteristics of the ten participants with unilateral TTA used for the experimental distributions

Age (years)	BMI (kg/m <sup>2</sup> )	Time since amputation (months)	Residual limb length (cm)	Socket type	Prosthetic foot
$56.8 \pm 4.3$	$30.6 \pm 4.6$	$17.4 \pm 5.1$	$14.8 \pm 4.3$	Total contact carbon fiber	Dynamic elastic response

fixed speed of 1.0 m/s ( $\pm$ 5% with gait timers) in which whole-body kinematics and ground reaction forces were collected. The same processing techniques described in the Experimental Collection and Data Processing section were used to calculate joint kinetics from four link-segment models. Four different methods of modeling prosthesis mass within each link-segment model were implemented: (1) experimental measurement, (2) intact segment parameters based on Dempster's regressions [\[34\]](#page-7-0), (3) 50% reduction of intact shank mass (2.35% body mass [[8,11,17,18](#page-7-0)]), and( 4) the general model (GM) (3.3% of the adjusted body mass) [\[27\]](#page-7-0). Within each of these models, the prosthesis shank and foot were separated into two separate segments (as described in Prosthesis Inertial Parameters section).

For each ID solution calculated from each link-segment model, the peak hip moment on the amputated limb in the sagittal plane during the stance and swing period was used for comparison. The error in magnitudes of peak moments during both the stance and swing period for each participant was calculated across the experimentally measured model and three estimation models. In addition, the percentage of peak hip moments from the amputee cohort that were outside of the 95% confidence bound from the probabilistic analysis was determined, to indicate the percentage of hip joint moments outside of the expected range given the input uncertainty of shank mass.

## Results

Confidence Bounds. The 2.5–97.5% confidence bounds represent the possible range of the combined effect of variability in PIPs of the prosthesis shank and foot (Fig. 1). During the entire



Fig. 1 Mean (1 SD) 2.5–97.5% confidence bounds for hip and knee joint moments during the stance and swing period



Fig. 2 Mean (1 SD) 2.5–97.5% confidence bounds for hip and knee joint moments (flexion/ extension (F/E), abduction/adduction (A/A), internal/external rotation (I/E) during the stance and swing period

gait cycle, the mean bound sizes for the hip flexion/extension moments  $(2.6 \pm 2.3 \text{ N}\cdot\text{m})$  and knee flexion/extension moments  $(1.8 \pm 1.4 \text{ N}\cdot\text{m})$  were substantially larger than other joints and anatomical planes (hip abduction/adduction:  $1.3 \pm 0.7$  N·m; hip internal/external rotation:  $0.6 \pm 0.4$  N·m) (Fig. 2). During the swing period, mean bound sizes were 47.7% and 93.9% larger in the sagittal plane hip and knee moments, respectively, compared to the stance period (Fig. 2). See Supplementary Fig. 1, which available under the ["Supplemental Data"](http://dx.doi.org/10.1115/1.4038175) tab for this paper on the ASME Digital Collection, for hip and knee joint power confidence bounds.

Sensitivity Factors. Statistically significant sensitivity factors (correlations) for hip and knee kinetics and three PIPs (shank mass, shank COM, and shank  $I_{sagital}$ ) were identified in both the stance and swing periods. During the stance period, all hip moments were highly sensitive to shank mass, and knee moments were moderately sensitive to shank mass; hip adduction and knee flexion moments were moderately sensitive to shank COM; and hip flexion and knee extension moments were highly and moderately sensitive to  $I_{\text{sagittal}}$ , respectively (Fig. 3(*a*)). No statistically significant sensitivity factors existed between foot PIPs and joint moments (Supplementary Fig. 2 is available under the ["Supplemental Data](http://dx.doi.org/10.1115/1.4038175)" tab for this paper on the ASME Digital Collection). See Supplementary Figs. 3 and 4, which are available under ["Supplemental Data](http://dx.doi.org/10.1115/1.4038175)" tab for this paper on the ASME Digital Collection, for the hip and knee joint power sensitivity factors, respectively.

Experimental Application on Amputee Cohort. The error in prosthesis shank mass of models 2–4 compared to model 1 are shown in Fig. [4.](#page-5-0)



Fig. 3 Sensitivity factors calculated between peak hip and knee joint moments and shank PIPs during the stance (top row) and swing (bottom row) periods during one gait cycle. \* Indicates highly sensitive  $(r = 0.6 - 1.0)$  and + indicates moderately sensitive  $(r = 0.4 - 0.6)$ , both of which are statistically significant (95%) confidence interval did not cross zero). The level of significance was set at  $\alpha = 0.05$ .

<span id="page-5-0"></span>

Fig. 4 Mean (1 SD) error of prosthesis shank mass between experimentally measured shank mass and shank mass estimated using: Intact: intact parameters based on Dempster's regressions (black), Scaled: 50% reduction of intact shank mass (red), and GM: 3.3% of the adjusted body mass

During both the stance and swing periods, the largest error occurred when modeling the prosthesis shank mass using intact segment parameters when compared to experimental measurements. During the stance period, the largest error occurred at the peak hip flexion moment during terminal stance  $(5.9 \pm 3.5 \text{ N} \cdot \text{m})$ (Fig. 5); in which 80% of values in the amputee cohort were larger than the 2.5% confidence bound (Fig.  $5(b)$ ). During the swing period, the largest error occurred at the peak hip extension moment during terminal swing  $(6.6 \pm 4.6 \text{ N} \cdot \text{m})$  (Fig. 5); in which 60% of values in the amputee cohort were larger than the 2.5% confidence bound (Fig.  $5(b)$ ).

## **Discussion**

The objective of this investigation was to assess the importance of properly quantifying PIPs when calculating joint kinetics for a participant with TTA during gait. Using a probabilistic analysis, we determined if variability in PIPs influenced the ID calculation of joint moments by assessing the overall influence using confidence bounds and to identify the individual influence of specific PIPs on ID using sensitivity factors. Our results demonstrate that hip and knee joint moment calculations were most sensitive to prosthesis shank mass, COM location, and moment of inertia in the sagittal plane, based on sensitivity factors, which had influence during both the stance and swing periods. Because substantial differences in mass and COM location typically exist between intact and prosthetic limbs, but only small differences in moment of inertia, accurately measuring the mass and COM location is important. Using a reaction board technique [[24,25\]](#page-7-0), prosthesis mass and COM can be simultaneously measured with approximately a 5-min increase in experimental data collection time.

The estimated bounds indicated that differences in peak moments must exceed 5.6 N·m during the stance period and 11.4 N-m during the swing period to exceed the uncertainty introduced by inaccurate estimations of PIPs. Changes in PIPs were most influential during the swing period, shown by larger confidence bounds in the swing period than the stance period, which is con-sistent with previous results [\[13,26](#page-7-0)]. For example, the maximum range (bound size) of the sagittal plane hip moment during the stance period during terminal stance was 5.6 N-m and during the swing period during terminal swing was 11.4 N-m. Differences in moments of similar magnitudes have been used to identify between limb differences during amputee gait [[5](#page-7-0)]; however, the current analysis indicates that this difference is within the possible range that can be attributed to variance in PIPs. The difference of influence of PIPs on ID between the stance and swing periods is likely explained by the lack of ground reaction force contribution to the kinetics during the swing period, which shifts the influence



Fig. 5 (a) Ensemble averages of the hip flexion/extension moment during walking in ten patients with unilateral TTA that was calculated using inverse dynamics from a link-segment model with prosthesis mass that was modeled using: experimental measurements (blue), intact segment parameters based on Dempster's regressions (black) [\[34](#page-7-0)], mass reduced to 50% of intact shank mass (red) (2.35% body mass [\[8,11,17,18\]](#page-7-0)), and modeled with the GM developed by Ferris et al. (green) (3.3% of the ABM [[27](#page-7-0)]). (b) the error between peak flexion/extension moments calculated with shank mass experimentally measured and by modeling it using intact segment parameters (black), 50% scaled reduction (red), or the general model (3.3% ABM) (green). The gray circles indicate individual participant values, and the ratios indicate the number of participants whose peak moment error was larger than that of the peak 2.5% confidence bound determined from the probabilistic analysis. For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.

<span id="page-6-0"></span>onto segment parameters [[42\]](#page-7-0). In addition, these differences may also be explained by the larger moment arm of the shank and foot segment COM relative to the hip joint center throughout the swing period compared to the stance period. Furthermore, within both the stance and swing periods,  $I_{Sagittal}$  had a larger influence than  $I_{\text{Frontal}}$  and  $I_{\text{Transverse}}$ , because over-ground walking is primarily constrained to the sagittal plane. It is expected that  $I_{\text{Frontal}}$  and  $I<sub>Transverse</sub>$  would have a larger influence in multiplanar tasks (e.g., pivoting).

Changes in foot PIPs did not influence joint moment calculations during gait, which is likely explained by the similarity across the prostheses foot types (dynamic elastic response) used to develop the input distributions. This small variability of foot PIPs minimizes the overall influence of prostheses foot PIPs on joint kinetics. In addition, passive dynamic elastic response prosthetic feet do not generate an active plantar flexion moment [[43,44\]](#page-7-0), which contributes little torques that affect superior joints in the kinetic chain. We expect that powered prostheses or prostheses with an actively compliant ankle joint, which are larger in mass than dynamic elastic response feet and inertial properties that are closer to an intact foot, would have a similar impact on joint kinetics. This is supported by the difference in the current results to that of our previous work [\[13](#page-7-0)], which found that foot segment mass has a larger influence than all other segment inertial parameters on the hip flexion moment during the swing phase of the gait cycle during healthy walking. Therefore, we recommend that researchers do not use segment parameters based on intact segment estimates when calculating inverse dynamics during walking in patients with TTA using passive prostheses.

Based on the current findings, and those of Smith et al., [[26\]](#page-7-0), we recommend accurate measurement of prosthesis mass and COM location when calculating ID in participants with lowerlimb amputation. Our results, and those of Smith et al. [[26\]](#page-7-0), partially disagree with two early investigations [[14,15\]](#page-7-0), that are commonly used to justify reasoning for using intact segment inertial parameters as PIPs. The first study often cited, Boccardi et al. [[14](#page-7-0)] demonstrated that less than 10% of the knee extensor moment is attributed to inertial parameters of intact shank and foot segments. However, because the results of this investigation were based on intact segment inertial parameters, it may not be valid to use these findings as justification for not altering PIPs when calculating joint kinetics in participants with lower-limb amputation. Second, Miller [\[15](#page-7-0)] noted negligible differences when substituting intact limb inertial properties for those of the prosthetic limb during running based on a sensitivity analysis between net knee moment and segmental inertial properties. However, PIPs of running prostheses are likely different than that of passive walking prostheses, and therefore, inferences of these findings should be limited to amputee running.

Our results demonstrate the large effect of shank mass on hip flexion/extension moment calculations using three common methods of estimating the shank mass (intact segment parameters based on Dempster's regression, uniform scaled reduction of intact shank mass by 50%, and the GM developed by Ferris et al., [[27](#page-7-0)] (3.3% of adjusted body mass)) compared to experimental measurement. Based on our results, modeling the prosthesis shank mass using intact parameters produced the largest error, in which the majority of results exceed the uncertainty introduced by inaccurate estimations of PIPs. The findings of Dempster and Aitkens [[34](#page-7-0)] were based on anthropometric measurements of eight cadavers, which are different than prosthetic limbs [[26,36](#page-7-0)]. In addition, due to the wide variability among the PIPs across various types of prostheses, it will rarely be the case that regression equations [\[34](#page-7-0)] of shank mass accurately represent the prosthetic segment parameters within the link-segment model. Therefore, we recommend experimental measurements of shank mass. If direct measures of shank mass are unavailable, a scaled reduction in prosthesis mass (either 50% reduction of intact mass or 3.3% of the adjusted body mass) will produce results within the range of expected uncertainty that we calculated.

There are several limitations of this investigation that should be considered. First, the input distributions for the probabilistic analysis were developed from a small sample size (ten prostheses), and may not represent the full range of prosthetic designs. However, we expect our results to generalize to participants with commonly used transtibial passive prostheses for two primary reasons: (1) the input distributions match what has previously been reported [\[26](#page-7-0)] and (2) we chose inclusion bounds on input distributions for the PIPs that are consistent with general population demographics [\[26,36](#page-7-0)]. Second, these conclusions are only applicable for prostheses that are similar to those included in this investigation (below knee passive prostheses with dynamic elastic response feet). We expect prostheses with drastically different PIPs (e.g., powered, blade) would have different effects on joint kinetics. Third, the mass of the prosthesis shank and foot was not directly measured; but rather the whole prosthesis mass was measured and then was distributed to the shank and foot segments based off of previous findings of similar prostheses [\[36](#page-7-0)]. Measuring each prosthesis segment independently is not common practice in an experimental collection involving a participant with amputation; and therefore, although not prosthesis specific, this approach still yields an improvement in ID analyses compared to overall uniform scaled adjustments or no adjustments. Fourth, there is an assumption in modeling the shank as a rigid segment due to axial pistoning at the interface between the residual limb and prosthesis socket. However, regardless of the precision of fit, there remains a potential for motion between the residual limb and socket. Such potential motion was not accounted for in our analyses. Finally, these results are only valid during walking. Other tasks, which are required for this population to achieve functional community ambulation (pivoting, stair climbing), require different joint kinetics than walking for completion [\[45](#page-7-0)].

## **Conclusion**

This investigation demonstrated the individual effects of PIPs on ID during walking in a participant with unilateral TTA. The results indicated that changes in the prosthesis shank mass had the largest influence on joint moment calculations. Because the mass of the prosthesis can be easily measured, we recommend that researchers interested in calculating joint kinetics in participants with amputation include an accurate measure of prosthesis shank mass in the link-segment model when calculating ID. Including this measurement will positively affect the remaining inertial parameter estimates in most situations because these calculations rely on segment mass [\[37](#page-7-0)].

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