JOURNAL OF APPLIED STATISTICS
2021, VOL. 48, NO. 7, 1243–1268
https://doi.org/10.1080/02664763.2020.1759033



# A combined mixed-*s*-skip sampling strategy to reduce the effect of autocorrelation on the $\bar{X}$ scheme with and without measurement errors

Sandile Charles Shongwe [a], Jean-Claude Malela-Majika [a] and Philippe Castagliola [b]

[a]Department of Statistics, College of Science, Engineering and Technology, University of South Africa, Pretoria, South Africa; [b]Département Qualité Logistique Industrielle et Organisation, Université de Nantes & LS2N UMR CNRS 6004, Nantes, France

**ABSTRACT**

In order to reduce the effect of autocorrelation on the $\bar{X}$ monitoring scheme, a new sampling strategy is proposed to form rational subgroup samples of size *n*. It requires sampling to be done such that: (i) observations from two consecutive samples are merged, and (ii) some consecutive observations are skipped before sampling. This technique which is a generalized version of the mixed samples strategy is shown to yield a better reduction of the negative effect of autocorrelation when monitoring the mean of processes with and without measurement errors. For processes subjected to a combined effect of autocorrelation and measurement errors, the proposed sampling technique, together with multiple measurement strategy, yields an uniformly better zero-state run-length performance than its two main existing competitors for any autocorrelation level. However, in steady-state mode, it yields the best performance only when the monitoring process is subject to a high level of autocorrelation, for any given level of measurement errors. A real life example is used to illustrate the implementation of the proposed sampling strategy.

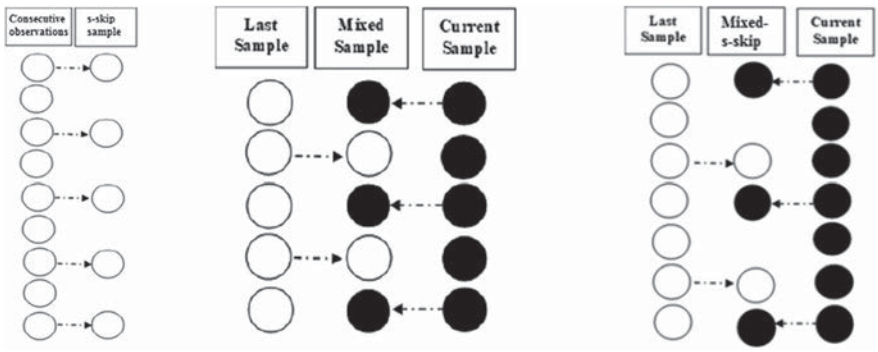
**ARTICLE HISTORY**
Received 12 October 2019
Accepted 17 April 2020

**KEYWORDS**
Autocorrelation; measurement errors; mixed samples strategy; multiple measurements; skipping sampling strategy; steady-state; zero-state

## 1. Introduction

The maximization of profit is the ultimate goal of any business-oriented organization. To meet this goal, the organization must present to its customers a high quality service and products, and avoid wasting time and products. This can only be possible if the production or any other similar process is thoroughly monitored. Statistical monitoring schemes are the most popular modern tools used to serve this broader purpose. The first modern monitoring scheme was proposed by W. A. Shewhart in the 1930s, see [21]. The original Shewhart monitoring scheme was designed under the assumptions of independent and identically distributed (i.i.d.) observations and perfect measurements. However, in

**CONTACT** Sandile Charles Shongwe sandile@tuks.co.za Department of Statistics, College of Science, Engineering and Technology, University of South Africa, Pretoria, South Africa
This article was originally published with errors, which have now been corrected in the online version. Please see Correction (http://dx.doi.org/10.1080/02664763.2020.1807783)




(a) $s$-skip (with $s$=1) (b) Mixed samples (c) Combined mixed-$s$-skip (with $s$=2)

**Figure 1.** Different sampling strategies to reduce the negative effect of autocorrelation.

practice, these assumptions are usually violated. The presence of autocorrelation and/or measurement errors is known to have a negative effect on the performance of monitoring scheme. A number of researchers have developed more interesting and advanced monitoring schemes under similar assumptions. There have been a great number of articles that investigated either the effect of autocorrelation or measurement errors; see for example [1,3,6,7,8,11,12,13,14,15,16,17,19,20,22,23,24,26,32,33,36]. For recent reviews on monitoring schemes under autocorrelation and measurements errors, readers are referred to [25] and [18], respectively.

The combination of the effect of autocorrelation and measurement errors has a profound negative effect on the efficiency of monitoring schemes. A combined effect of autocorrelation and measurement errors has been also investigated in the literature; see for example [5,27,28,34,35]. All the forgoing investigations have led to one conclusion, which is the deterioration of the performance of the monitoring schemes due by the negative effect of autocorrelation and measurement errors. Therefore, there is a growing need of reducing or eliminating this negative effect of autocorrelation and measurement errors.

In order to reduce the effect of autocorrelation, a number of authors have suggested the use of skipping and mixed samples strategies in order to construct rational subgroups. The skipping strategy (denoted as $s$-skip) is the combination of non-neighbouring observations by skipping $s$ observations before sampling to form a rational subgroup of size $n$, where $s$ is a positive integer (see Figure 1 (a)) – this was first proposed in [5] for the $\bar{X}$ scheme. Moreover, the s-skip strategy was used by the following: [13] for the basic Hotelling's $T^2$ chart, [6] for the synthetic $T^2$ chart, [10,29,30,31] for the synthetic and runs-rules $\bar{X}$ schemes. Next, the mixed sampling strategy is the mixture of the observations from two consecutive samples by skipping one observation (i.e. $s = 1$) in each sample (see Figure 1(b)) – this was first proposed in [7] for the $\bar{X}$ scheme. Moreover, the mixed samples strategy was used by the following: [14] for the basic Hotelling's $T^2$ chart and [6] for the synthetic $T^2$ chart. In general, from the abovementioned papers, it was observed that for small levels of autocorrelation, the $s$-skip strategy is more efficient than the mixed samples strategy; however, the converse is true for large levels of autocorrelation. Finally, the economic design of the $\bar{X}$ scheme with the skipping and mixed samples strategies is studied in [8].

Thus, in this paper, a new technique of reducing the effect of autocorrelation (with and without measurement errors) on the performance of a monitoring scheme is introduced and the Shewhart $\bar{X}$ scheme is used to demonstrate the new technique. In essence, the new sampling strategy is the combination of the $s$-skip and mixed samples strategies (see Figure 1(c)) – it is explained in detail in Section 2.3. The purpose of this paper is to improve on the research works by [5,7]; hence, the same models are used to account for autocorrelation and measurement errors. That is, a first order autoregressive model (i.e. AR(1)) accounts for the (within-sample) autocorrelation and the additive model with a constant standard deviation accounts for the measurement error.

The rest of the paper is organized as follows: In Section 2, the basic properties of autocorrelation, as well as the existing sampling strategies used to reduce the negative effect thereof are discussed along with the proposed technique. Moreover, their empirical run-length performance using the Shewhart $\bar{X}$ scheme is investigated. In Section 3, we present the run-length performance of the proposed sampling strategy under the presence of both autocorrelation and measurement errors, and compare its performance with other competing strategies. A real life example is used to demonstrate the proposed sampling strategies in Sections 2 and 3. Finally, the concluding remarks are given in Section 4.

## 2. Autocorrelated observations

Assume that, at time $t \geq 1$, the quality characteristic $\{Y_{t,i} : i = 1, 2, \ldots, n\}$ is a sequence of samples from an autocorrelated $N(\mu_0, \sigma_0)$ distribution that fits a stationary AR(1) model, given by

$$Y_{t,i} - \mu_0 = \phi(Y_{t,i-1} - \mu_0) + \varepsilon_i,\ t \geq 1,\ i = 1, 2, \ldots, n; \tag{1}$$

where $\phi$ is the level of serial dependence (or autocorrelation) assumed to satisfy $|\phi| < 1$ and $\varepsilon_i$ are i.i.d. normal $(0, \sigma_\varepsilon)$ random variables. Moreover, the nominal IC mean and standard deviation process parameters are denoted by $\mu_0$ and $\sigma_0$, respectively, where $\sigma_0 = \frac{\sigma_\varepsilon}{\sqrt{1-\phi^2}}$ and, without loss of generality, assume $\sigma_\varepsilon = 1$; see [2]. Note that after the occurrence of assignable causes, the process mean shifts from $\mu_0$ to $\mu_1 = \mu_0 + \delta\sigma_0$, so that $\delta = \frac{\mu_1 - \mu_0}{\sigma_0}$. A standard way to calculate the mean or the plotting statistic of the $\bar{X}$ scheme is

$$\bar{Y}_t = \frac{1}{n}\sum_{i=1}^{n} Y_{t,i} \tag{2}$$

While we assume dependence within the computation of $\bar{Y}_t$; however, between any $\bar{Y}_t$ and $\bar{Y}_r$ $(t \neq r)$ there is independence (i.e. no cross-correlation) – this is in line with the derivation in [2] for sub-grouped data. Hence, for the basic $\bar{X}$ scheme, the charting limits (i.e. the upper / lower control limit denoted by $(UCL/LCL)$) are given by:

$$UCL/LCL = \mu_{\bar{Y},0} \pm k\sigma_{\bar{Y},0} \tag{3}$$

where $\mu_{\bar{Y},0} = \mu_0$ and $\sigma_{\bar{Y},0} = \frac{\sigma_0}{\sqrt{n}}\rho$ are the mean and standard deviation of $\bar{Y}_t$, respectively; $k > 0$ is the design parameter that is related to the distance from the center line to the $UCL/LCL$ in terms of the standard deviation and finally, $\rho$ depends on which sampling

**Table 1.** $\rho$ terms for different sampling strategies when the process is i.i.d. and when it is under the effect of autocorrelation.

| Sampling strategy | $\rho$ |
|---|---|
| (i) i.i.d. | 1 |
| (ii) $s$-skip | $\sqrt{\frac{n+2\left(\frac{\phi^{(s+1)(n+1)}-n\phi^{2s+2}+(n-1)\phi^{s+1}}{(\phi^{s+1}-1)^2}\right)}{n}}$ |
| (iii) Mixed samples | $\sqrt{\frac{n_t+2\left(\frac{\phi^{2n_t+2}-n_t\phi^4+(n_t-1)\phi^2}{(\phi^2-1)^2}\right)}{n}+\frac{n_{t-1}+2\left(\frac{\phi^{2n_{t-1}+2}-n_{t-1}\phi^4+(n_{t-1}-1)\phi^2}{(\phi^2-1)^2}\right)}{n}}$ |
| (iv) Mixed-$s$-skip | $\sqrt{\frac{n_t+2\left(\frac{\phi^{(s+1)(n_t+1)}-n_t\phi^{2s+2}+(n_t-1)\phi^{s+1}}{(\phi^{s+1}-1)^2}\right)}{n}+\frac{n_{t-1}+2\left(\frac{\phi^{(s+1)(n_{t-1}+1)}-n_{t-1}\phi^{2s+2}+(n_{t-1}-1)\phi^{s+1}}{(\phi^{s+1}-1)^2}\right)}{n}}$ |

strategy is implemented as shown in Table 1. Note that for the i.i.d. case, $\rho$ is simply equal to 1.

There are two main sampling strategies that are used in the SPM literature to reduce the effect of autocorrelation and these are discussed below in subsections 2.1 and 2.2. Then, in Section 2.3, the combined mixed-$s$-skip strategy is introduced and the corresponding empirical analysis is done in Section 2.4.

### 2.1. *s*-skip sampling strategy

In the presence of autocorrelation, the $s$-skip strategy involves sampling of non-neighboring observations and is particularly used as a remedial approach to reduce autocorrelation. Costa and Castagliola [5] showed that the process in Equation (1) that incorporates the $s$-skip sampling strategy remains an AR(1) process; however, defined as $\{Y_{t,(s+1)i-s} : t \geq 1; i = 1, 2, 3, \ldots, n\}$ with parameter $\phi^{s+1}$ (instead of $\phi$):

$$Y_{t,i} - \mu_0 = \phi^{s+1}(Y_{t,i-s-1} - \mu_0) + \varepsilon'_i \tag{4}$$

with $\varepsilon'_i = \varepsilon_i + \phi\varepsilon_{i-1} + \phi^2\varepsilon_{i-2} + \ldots + \phi^s\varepsilon_{i-s}$. Let $\bar{Y}_t$ denote the plotting statistic at sampling point $t$, no longer calculated as in Equation (2), but using

$$\bar{Y}_t = \frac{1}{n}\sum_{i=1}^{n} Y_{t,(s+1)i-s} \tag{5}$$

Hence, for the $\bar{X}$ scheme with the $s$-skip strategy the charting limits are as given in Equation (3); however, with $\rho$ given in Table 1. Some other works in the literature that considered the $s$-skip sampling strategy are [6,7,8,10,13,29,30,31].

### 2.2. Mixed samples strategy

The mixed sampling strategy proposed by Franco et al. [7] implements the '1-skip' rule in two consecutive samples to merge the observations within the two samples, at times $t$-1 and $t$, into a single sample having size $n$. That is, $\bar{Y}_t$ is not calculated as in either Equation

(2) or (5) but, instead, it is computed as

$$\bar{Y}_t = \frac{n_{t-1}}{n}\left(\frac{1}{n_{t-1}}\sum_{i=1}^{n_{t-1}} Y_{t-1,2i}\right) + \frac{n_t}{n}\left(\frac{1}{n_t}\sum_{i=1}^{n_t} Y_{t,\,2i-1}\right) \tag{6}$$

where $n_{t-1}$ and $n_t$ are sizes of the subsamples taken at times $t-1$ and $t$, respectively, satisfying $n_{t-1} + n_t = n$. In their paper, Franco et al. [7] suggested the following combinations for $n_{t-1}$ and $n_t$: when $n$ is odd then $n_{t-1} = (n-1)/2$ and $n_t = (n+1)/2$; however, when $n$ is even then $n_{t-1} = n_t = n/2$. Hence, for the $\bar{X}$ scheme with mixed samples strategy, the charting limits are as given in Equation (3); however, with $\rho$ given in Table 1. Some other works in the literature that used mixed samples are [6,8,14].

## 2.3. Mixed-s-skip samples strategy

The proposed combined mixed-$s$-skip strategy is a generalization of the mixed samples one in the sense that it implements the '$s$-skip' rule in two consecutive samples to merge the observations within the two samples into one sample having size $n$. Hence, instead of having Equations (2) or (5) or (6) as a plotting statistic, the combined mixed-$s$-skip samples strategy uses

$$\bar{Y}_t = \frac{n_{t-1}}{n}\left(\frac{1}{n_{t-1}}\sum_{i=1}^{n_{t-1}} Y_{t-1,(s+1)i}\right) + \frac{n_t}{n}\left(\frac{1}{n_t}\sum_{i=1}^{n_t} Y_{t,\,(s+1)i-s}\right). \tag{7}$$

Hence, for the $\bar{X}$ scheme with mixed-$s$-skip samples strategy, the charting limits are as given in Equation (3); however, with $\rho$ given in Table 1. Note that when $s = 1$, the proposed strategy is the same as the mixed samples strategy in [7].

Next, the run-length properties of the Shewhart $\bar{X}$ scheme that incorporates the mixed-$s$-skip samples strategy depends on whether the process shift occurred: (i) at the beginning of the monitoring process; or (ii) it begins in IC and stays IC for a while and goes OOC just before sampling point $t$. These two latter modes of analysis are known as the (i) zero-state and, (ii) steady-state modes, respectively. Since the Shewhart $\bar{X}$ scheme is the simplest monitoring scheme that makes use of the information in the most recently inspected sample (and not in past samples), then its run-length distribution follows a geometric distribution with parameter $(1-\beta)$, where $\beta$ represents the Type II error . Hence, the zero-state average and standard deviation run-length (denoted by *ZSARL* and *ZSSDRL*) are given by

$$ZSARL = \frac{1}{1-\beta} \text{ and } ZSSDRL = \frac{\sqrt{\beta}}{1-\beta}, \tag{8}$$

respectively; where

$$\beta = \Phi\left(k - \delta\sqrt{n} \times \frac{1}{\rho}\right) - \Phi\left(-k - \delta\sqrt{n} \times \frac{1}{\rho}\right). \tag{9}$$

Following a similar procedure as in [7], it follows that the steady-state average and standard deviation run-length (denoted by *SSARL* and *SSSDRL*) of the $\bar{X}$ scheme using mixed-$s$-skip samples strategy are given by

$$SSARL = \frac{\beta_1}{1-\beta} + 1 \text{ and } SSSDRL = \frac{\sqrt{\beta_1(1+\beta-\beta_1)}}{1-\beta}, \quad (10)$$

respectively; where $\beta$ is the same as in Equation (9) and

$$\beta_1 = \Phi\left(k - \delta\sqrt{n} \times \frac{n_t}{n} \times \frac{1}{\rho}\right) - \Phi\left(-k - \delta\sqrt{n} \times \frac{n_t}{n} \times \frac{1}{\rho}\right). \quad (11)$$

To evaluate the performance of the proposed scheme from an overall performance perspective, the expected $ARL$ ($EARL$) and the expected $SDRL$ ($ESDRL$) are used because users tend not to know beforehand what exact shift value(s) is(are) targeted, see for instance [4]. The $EARL$ and $ESDRL$ measure the performance of a monitoring scheme over a range of shift values, i.e. $\delta_{min}$ to $\delta_{max}$ – which are the lower and the upper bound of $\delta$, respectively. The $EARL$ and $ESDRL$ are given by

$$EARL = \int_{\delta_{min}}^{\delta_{max}} ARL(\delta) \times f(\delta)\, d\delta \text{ and } ESDRL = \int_{\delta_{min}}^{\delta_{max}} SDRL(\delta) \times f(\delta)\, d\delta, \quad (12)$$

subject to $ARL(\delta = 0) = ARL_0$, with $\delta \in [\delta_{min}, \delta_{max}]$. Note that the shifts within the interval $[\delta_{min}, \delta_{max}]$ usually occur according to a probability distribution function (p.d.f.) equal to $f(\delta)$ which is usually unknown, where $ARL(\delta)$ and $SDRL(\delta)$ are the $ARL$ and $SDRL$ as a function of the shift $\delta$ in the parameter under surveillance. In the absence of any particular information, it is usually assumed that the shifts in the process mean happen with an equal probability, then $f(\delta) = 1/(\delta_{max} - \delta_{min})$ i.e. a Uniform $(\delta_{min}, \delta_{max})$ distribution. The proposed scheme is designed such that, we fix $k$ at a specific value, so that the attained IC $ARL$ is equal to the target nominal $ARL$ (denoted by $ARL_0$). Thus, we choose the scheme that yields the best overall performance for a range of specified shifts; i.e. the smallest $EARL$ or $ESDRL$. Equation (12) can equivalently be written as

$$EARL = \frac{1}{\Delta} \sum_{\delta=\delta_{min}}^{\delta_{max}} ARL(\delta) \text{ and } ESDRL = \frac{1}{\Delta} \sum_{\delta=\delta_{min}}^{\delta_{max}} SDRL(\delta), \quad (13)$$

where $\Delta$ is the number of increments from $\delta_{min}$ to $\delta_{max}$ of a Riemann sum. To preserve writing space, we use increments of size 0.25 in the summations in Equation (13), with $\delta_{min} = 0$ and $\delta_{max} = 3$. Based on the latter, it follows that $\Delta = 13$.

## 2.4. Empirical analysis for autocorrelated data

Table 2 illustrates the well-known negative effect of autocorrelation, that is, as the level of autocorrelation increases from 0 to some value between 0 and 1, the corresponding performance deteriorates, especially when $\phi$ is very high. In each panel of Table 2, it is observed that the $EARL$ and $ESDRL$ (computed using Equation (13)) deteriorate as $\phi$ increases. To investigate to what extent increasing the value of $\phi$ has deteriorated the scheme's performance as compared to the i.i.d. case (i.e. $\phi = 0$), we define the percentage difference (%Diff). More specifically, $\%\text{Diff}_A$ is defined as a percentage difference

**Table 2.** The *ARL*, *SDRL*, *EARL* and *ESDRL* of the $\bar{X}$ scheme when $\phi \in \{0, 0.3, 0.9\}$ when $\delta_{\min} = 0$, $\delta_{\max} = 3$ and $n = 4$ with no remedial approach.

| | *ARL* | | | | *SDRL* | | |
|---|---|---|---|---|---|---|---|
| Shift | i.i.d. | $\phi = 0.3$ | $\phi = 0.9$ | | i.i.d. | $\phi = 0.3$ | $\phi = 0.9$ |
| 0 | 370.4 | 370.4 | 370.4 | | 369.9 | 369.9 | 369.9 |
| 0.25 | 155.2 | 199.5 | 272.0 | | 154.7 | 199.0 | 271.5 |
| 0.5 | 43.9 | 71.1 | 142.6 | | 43.4 | 70.6 | 142.1 |
| 0.75 | 15.0 | 27.6 | 71.7 | | 14.5 | 27.1 | 71.2 |
| 1 | 6.3 | 12.3 | 37.7 | | 5.8 | 11.8 | 37.2 |
| 1.25 | 3.2 | 6.3 | 21.0 | | 2.7 | 5.7 | 20.5 |
| 1.5 | 2.0 | 3.6 | 12.4 | | 1.4 | 3.1 | 11.9 |
| 1.75 | 1.5 | 2.4 | 7.8 | | 0.8 | 1.8 | 7.3 |
| 2 | 1.2 | 1.7 | 5.2 | | 0.5 | 1.1 | 4.7 |
| 2.25 | 1.1 | 1.4 | 3.7 | | 0.3 | 0.7 | 3.1 |
| 2.5 | 1.0 | 1.2 | 2.7 | | 0.2 | 0.5 | 2.2 |
| 2.75 | 1.0 | 1.1 | 2.1 | | 0.1 | 0.3 | 1.5 |
| 3 | 1.0 | 1.0 | 1.7 | | 0.0 | 0.2 | 1.1 |
| *EARL* | 46.4 | 53.8 | 73.2 | *ESDRL* | 45.7 | 53.2 | 72.6 |
| $\%\text{Diff}_A$ | | 16.1% | 57.8% | $\%\text{Diff}_{SD}$ | | 16.4% | 58.9% |

of *EARL* at some specified value of $\phi$ from the corresponding i.i.d. case, i.e. $\%\text{Diff}_A = \frac{\text{EARL}_\phi - \text{EARL}_{\phi=0}}{\text{EARL}_{\phi=0}} \times 100\%$; where $EARL_\phi$ denote the *EARL* of the $\bar{X}$ scheme for some specified $\phi > 0$, whereas $EARL_{\phi=0}$ denote the *EARL* of the $\bar{X}$ scheme when $\phi = 0$. Similarly, let $\%\text{Diff}_{SD}$ denote the percentage difference of *ESDRL* at some specified value of $\phi$ from the corresponding i.i.d. case; hence, $\%\text{Diff}_{SD} = \frac{\text{ESDRL}_\phi - \text{ESDRL}_{\phi=0}}{\text{ESDRL}_{\phi=0}} \times 100\%$. Based on $\%\text{Diff}_A$ and $\%\text{Diff}_{SD}$, it is observed that the higher the value of $\phi$, the worse the deterioration as compared to the i.i.d. case. Finally, in Table 2, it is observed that at each $\delta$ value, the *SDRL* is slightly lower than the *ARL* and consequently the same holds for the *ESDRL* and *EARL* over the range of considered shift values.

Next, in Tables 3 and 4, the effect of using the mixed-$s$-skip samples strategy (in zero- and steady-state modes) when $s \in \{1, 2, \ldots, 10\}$ is investigated for the cases shown in Table 2. 'No remedy' denotes a scenario where there is no remedial approach used to offset the negative effect of autocorrelation. Note that when $s = 1$ for the mixed-$s$-skip strategy, the resulting performance corresponds to the one in [7], that is, $s = 1$ is a special case corresponding to mixed samples strategy. In essence, Tables 3 and 4 compare the performances of the $\bar{X}$ schemes using the $s$-skip, mixed samples and mixed-$s$-skip strategies.

From Table 3, it is observed that for relatively small values of $\phi$, then at each corresponding value of $s$, the mixed-$s$-skip strategy is either uniformly better or the same as the $s$-skip strategy for all possible values of $s$, in the zero-state mode. Also, it is seen that for small $\phi$, the negative effect of autocorrelation can be theoretically eradicated by skipping at least 5 observations before sampling to form a subgroup of size 4 when $\phi = 0.3$. While the mixed-$s$-skip strategy in zero-state does theoretically eradicate the effect of autocorrelation, this is not the case when the process is in the steady-state mode. For instance, while the $\%\text{Diff}_A$ converges to 0% when $\phi = 0.3$ in zero-state; however, in steady-state mode, it converges to 1.5%. This is visually illustrated in Figure 2(a) where the 'SS: Mixed-$s$-skip' (which denotes the mixed-$s$-skip in steady-state mode) line graph converges to a value of the *EARL* that is slightly higher than the i.i.d. case value; however, the line graphs of the 'ZS: Mixed-$s$-skip' (which denotes the mixed-$s$-skip in zero-state mode) and the $s$-skip are equal to the i.i.d.

**Table 3.** The *ARL* and *EARL* of the *s*-skip, mixed samples, mixed-*s*-skip (zero- and steady-state) $\bar{X}$ scheme when $\phi = 0.3$, $s \in \{1, 2, \ldots, 10\}$, $\delta_{\min} = 0$, $\delta_{\max} = 3$ and $n = 4$ (with $n_t = n_{t-1} = 2$).

| Type | Shift | i.i.d. | No remedy | s = 1 | s = 2 | s = 3 | s = 4 | s = 5 | s = 6 | s = 7 | s = 8 | s = 9 | s = 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| s-skip | 0 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 |
| | 0.25 | 155.2 | 199.5 | 168.7 | 159.3 | 156.4 | 155.6 | 155.3 | 155.3 | 155.2 | 155.2 | 155.2 | 155.2 |
| | 0.5 | 43.9 | 71.1 | 51.3 | 46.0 | 44.5 | 44.1 | 44.0 | 43.9 | 43.9 | 43.9 | 43.9 | 43.9 |
| | 0.75 | 15.0 | 27.6 | 18.2 | 15.9 | 15.2 | 15.1 | 15.0 | 15.0 | 15.0 | 15.0 | 15.0 | 15.0 |
| | 1 | 6.3 | 12.3 | 7.7 | 6.7 | 6.4 | 6.3 | 6.3 | 6.3 | 6.3 | 6.3 | 6.3 | 6.3 |
| | 1.25 | 3.2 | 6.3 | 3.9 | 3.4 | 3.3 | 3.3 | 3.3 | 3.2 | 3.2 | 3.2 | 3.2 | 3.2 |
| | 1.5 | 2.0 | 3.6 | 2.4 | 2.1 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| | 1.75 | 1.5 | 2.4 | 1.7 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 |
| | 2 | 1.2 | 1.7 | 1.3 | 1.2 | 1.2 | 1.2 | 1.2 | 1.2 | 1.2 | 1.2 | 1.2 | 1.2 |
| | 2.25 | 1.1 | 1.4 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 |
| | 2.5 | 1.0 | 1.2 | 1.1 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| | 2.75 | 1.0 | 1.1 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| | 3 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| | EARL | 46.4 | 53.8 | 48.4 | 47.0 | 46.5 | 46.4 | 46.4 | 46.4 | 46.4 | 46.4 | 46.4 | 46.4 |
| | $\%\text{Diff}_A$ | | 16.1% | 4.5% | 1.3% | 0.4% | 0.1% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| Steady-state: Mixed-s-skip | 0 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 |
| | 0.25 | 155.2 | 199.5 | 164.3 | 158.3 | 156.5 | 155.9 | 155.8 | 155.7 | 155.7 | 155.7 | 155.7 | 155.7 |
| | 0.5 | 43.9 | 71.1 | 49.2 | 46.0 | 45.0 | 44.7 | 44.7 | 44.6 | 44.6 | 44.6 | 44.6 | 44.6 |
| | 0.75 | 15.0 | 27.6 | 17.8 | 16.4 | 16.0 | 15.8 | 15.8 | 15.8 | 15.8 | 15.8 | 15.8 | 15.8 |
| | 1 | 6.3 | 12.3 | 8.0 | 7.4 | 7.2 | 7.2 | 7.2 | 7.2 | 7.2 | 7.2 | 7.2 | 7.2 |
| | 1.25 | 3.2 | 6.3 | 4.5 | 4.2 | 4.2 | 4.1 | 4.1 | 4.1 | 4.1 | 4.1 | 4.1 | 4.1 |
| | 1.5 | 2.0 | 3.6 | 3.1 | 2.9 | 2.9 | 2.9 | 2.9 | 2.9 | 2.9 | 2.9 | 2.9 | 2.9 |
| | 1.75 | 1.5 | 2.4 | 2.4 | 2.3 | 2.3 | 2.3 | 2.3 | 2.3 | 2.3 | 2.3 | 2.3 | 2.3 |
| | 2 | 1.2 | 1.7 | 2.1 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| | 2.25 | 1.1 | 1.4 | 1.9 | 1.9 | 1.8 | 1.8 | 1.8 | 1.8 | 1.8 | 1.8 | 1.8 | 1.8 |
| | 2.5 | 1.0 | 1.2 | 1.8 | 1.7 | 1.7 | 1.7 | 1.7 | 1.7 | 1.7 | 1.7 | 1.7 | 1.7 |
| | 2.75 | 1.0 | 1.1 | 1.7 | 1.6 | 1.6 | 1.6 | 1.6 | 1.6 | 1.6 | 1.6 | 1.6 | 1.6 |
| | 3 | 1.0 | 1.0 | 1.6 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 |
| | EARL | 46.4 | 53.8 | 48.4 | 47.4 | 47.2 | 47.1 | 47.1 | 47.0 | 47.0 | 47.0 | 47.0 | 47.0 |
| | $\%\text{Diff}_A$ | | 16.1% | 4.3% | 2.3% | 1.7% | 1.5% | 1.5% | 1.5% | 1.5% | 1.5% | 1.5% | 1.5% |

(*continued*).

**Table 3.** Continued.

| Type | Shift | i.i.d. | No remedy | s = 1 | s = 2 | s = 3 | s = 4 | s = 5 | s = 6 | s = 7 | s = 8 | s = 9 | s = 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Zero-state: Mixed-s-skip | 0 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 |
| | 0.25 | 155.2 | 199.5 | 163.9 | 157.9 | 156.0 | 155.5 | 155.3 | 155.3 | 155.2 | 155.2 | 155.2 | 155.2 |
| | 0.5 | 43.9 | 71.1 | 48.5 | 45.3 | 44.3 | 44.0 | 43.9 | 43.9 | 43.9 | 43.9 | 43.9 | 43.9 |
| | 0.75 | 15.0 | 27.6 | 17.0 | 15.6 | 15.1 | 15.0 | 15.0 | 15.0 | 15.0 | 15.0 | 15.0 | 15.0 |
| | 1 | 6.3 | 12.3 | 7.2 | 6.6 | 6.4 | 6.3 | 6.3 | 6.3 | 6.3 | 6.3 | 6.3 | 6.3 |
| | 1.25 | 3.2 | 6.3 | 3.7 | 3.4 | 3.3 | 3.3 | 3.2 | 3.2 | 3.2 | 3.2 | 3.2 | 3.2 |
| | 1.5 | 2.0 | 3.6 | 2.2 | 2.1 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| | 1.75 | 1.5 | 2.4 | 1.6 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 |
| | 2 | 1.2 | 1.7 | 1.3 | 1.2 | 1.2 | 1.2 | 1.2 | 1.2 | 1.2 | 1.2 | 1.2 | 1.2 |
| | 2.25 | 1.1 | 1.4 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 |
| | 2.5 | 1.0 | 1.2 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| | 2.75 | 1.0 | 1.1 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| | 3 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| | EARL | 46.4 | 53.8 | 47.7 | 46.8 | 46.5 | 46.4 | 46.4 | 46.4 | 46.4 | 46.4 | 46.4 | 46.4 |
| | $\%\mathrm{Diff}_A$ | | 16.1% | 2.8% | 0.9% | 0.3% | 0.1% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |

**Table 4.** The *ARL* and *EARL* of the *s*-skip, mixed samples, mixed-*s*-skip (zero- and steady-state) $\bar{X}$ scheme when $\phi = 0.9$, $s \in \{1, 2, \ldots, 10\}$, $\delta_{\min} = 0$, $\delta_{\max} = 3$ and $n = 4$ (with $n_t = n_{t-1} = 2$).

| Type | Shift | i.i.d. | No remedy | s = 1 | s = 2 | s = 3 | s = 4 | s = 5 | s = 6 | s = 7 | s = 8 | s = 9 | s = 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| s-skip | 0 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 |
| | 0.25 | 155.2 | 272.0 | 263.0 | 254.4 | 246.1 | 238.2 | 230.8 | 224.0 | 217.6 | 211.7 | 206.4 | 201.5 |
| | 0.5 | 43.9 | 142.6 | 131.2 | 121.0 | 112.0 | 104.0 | 96.9 | 90.7 | 85.2 | 80.5 | 76.3 | 72.6 |
| | 0.75 | 15.0 | 71.7 | 63.6 | 56.7 | 50.9 | 45.9 | 41.7 | 38.1 | 35.1 | 32.5 | 30.3 | 28.3 |
| | 1 | 6.3 | 37.7 | 32.6 | 28.4 | 24.9 | 22.1 | 19.7 | 17.7 | 16.1 | 14.8 | 13.6 | 12.6 |
| | 1.25 | 3.2 | 21.0 | 17.8 | 15.3 | 13.3 | 11.6 | 10.3 | 9.2 | 8.3 | 7.6 | 7.0 | 6.4 |
| | 1.5 | 2.0 | 12.4 | 10.4 | 8.9 | 7.7 | 6.7 | 5.9 | 5.3 | 4.8 | 4.4 | 4.0 | 3.7 |
| | 1.75 | 1.5 | 7.8 | 6.5 | 5.6 | 4.8 | 4.2 | 3.7 | 3.3 | 3.0 | 2.8 | 2.6 | 2.4 |
| | 2 | 1.2 | 5.2 | 4.4 | 3.7 | 3.2 | 2.9 | 2.6 | 2.3 | 2.1 | 2.0 | 1.9 | 1.8 |
| | 2.25 | 1.1 | 3.7 | 3.1 | 2.7 | 2.3 | 2.1 | 1.9 | 1.8 | 1.6 | 1.5 | 1.5 | 1.4 |
| | 2.5 | 1.0 | 2.7 | 2.3 | 2.0 | 1.8 | 1.7 | 1.5 | 1.4 | 1.3 | 1.3 | 1.2 | 1.2 |
| | 2.75 | 1.0 | 2.1 | 1.9 | 1.6 | 1.5 | 1.4 | 1.3 | 1.2 | 1.2 | 1.1 | 1.1 | 1.1 |
| | 3 | 1.0 | 1.7 | 1.5 | 1.4 | 1.3 | 1.2 | 1.2 | 1.1 | 1.1 | 1.1 | 1.1 | 1.0 |
| | *EARL* | 46.4 | 73.2 | 69.9 | 67.1 | 64.6 | 62.5 | 60.6 | 59.0 | 57.5 | 56.3 | 55.2 | 54.2 |
| | $\%\text{Diff}_A$ | | 57.8% | 50.8% | 44.7% | 39.4% | 34.7% | 30.7% | 27.2% | 24.1% | 21.4% | 19.0% | 16.9% |
| Steady-state: Mixed-s-skip | 0 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 |
| | 0.25 | 155.2 | 272.0 | 214.8 | 210.3 | 206.1 | 202.2 | 198.4 | 194.9 | 191.7 | 188.6 | 185.8 | 183.1 |
| | 0.5 | 43.9 | 142.6 | 83.2 | 79.7 | 76.5 | 73.5 | 70.8 | 68.3 | 66.0 | 64.0 | 62.1 | 60.4 |
| | 0.75 | 15.0 | 71.7 | 34.4 | 32.5 | 30.8 | 29.2 | 27.9 | 26.6 | 25.5 | 24.5 | 23.6 | 22.8 |
| | 1 | 6.3 | 37.7 | 16.2 | 15.2 | 14.3 | 13.5 | 12.9 | 12.2 | 11.7 | 11.2 | 10.8 | 10.4 |
| | 1.25 | 3.2 | 21.0 | 8.7 | 8.2 | 7.7 | 7.3 | 7.0 | 6.7 | 6.4 | 6.1 | 5.9 | 5.7 |
| | 1.5 | 2.0 | 12.4 | 5.4 | 5.1 | 4.8 | 4.6 | 4.4 | 4.2 | 4.1 | 4.0 | 3.8 | 3.7 |
| | 1.75 | 1.5 | 7.8 | 3.8 | 3.6 | 3.4 | 3.3 | 3.2 | 3.1 | 3.0 | 2.9 | 2.8 | 2.8 |
| | 2 | 1.2 | 5.2 | 2.9 | 2.8 | 2.7 | 2.6 | 2.6 | 2.5 | 2.4 | 2.4 | 2.3 | 2.3 |
| | 2.25 | 1.1 | 3.7 | 2.4 | 2.4 | 2.3 | 2.2 | 2.2 | 2.2 | 2.1 | 2.1 | 2.1 | 2.0 |
| | 2.5 | 1.0 | 2.7 | 2.1 | 2.1 | 2.1 | 2.0 | 2.0 | 2.0 | 1.9 | 1.9 | 1.9 | 1.9 |
| | 2.75 | 1.0 | 2.1 | 2.0 | 1.9 | 1.9 | 1.9 | 1.8 | 1.8 | 1.8 | 1.8 | 1.8 | 1.8 |
| | 3 | 1.0 | 1.7 | 1.8 | 1.8 | 1.8 | 1.8 | 1.7 | 1.7 | 1.7 | 1.7 | 1.7 | 1.7 |
| | *EARL* | 46.4 | 73.2 | 57.5 | 56.6 | 55.8 | 55.0 | 54.2 | 53.6 | 53.0 | 52.4 | 51.9 | 51.5 |
| | $\%\text{Diff}_A$ | | 57.8% | 24.1% | 22.1% | 20.3% | 18.6% | 17.0% | 15.6% | 14.3% | 13.1% | 12.0% | 11.0% |

(*continued*).

**Table 4.** Continued.

| Type | Shift | i.i.d. | No remedy | s = 1 | s = 2 | s = 3 | s = 4 | s = 5 | s = 6 | s = 7 | s = 8 | s = 9 | s = 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Zero-state: Mixed-s-skip | 0 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 |
| | 0.25 | 155.2 | 272.0 | 214.4 | 210.0 | 205.8 | 201.8 | 198.1 | 194.6 | 191.3 | 188.2 | 185.4 | 182.8 |
| | 0.5 | 43.9 | 142.6 | 82.6 | 79.1 | 75.8 | 72.8 | 70.1 | 67.6 | 65.4 | 63.3 | 61.4 | 59.7 |
| | 0.75 | 15.0 | 71.7 | 33.6 | 31.7 | 30.0 | 28.5 | 27.1 | 25.9 | 24.7 | 23.7 | 22.8 | 22.0 |
| | 1 | 6.3 | 37.7 | 15.4 | 14.4 | 13.5 | 12.7 | 12.0 | 11.4 | 10.9 | 10.4 | 9.9 | 9.6 |
| | 1.25 | 3.2 | 21.0 | 7.9 | 7.4 | 6.9 | 6.5 | 6.1 | 5.8 | 5.5 | 5.3 | 5.0 | 4.8 |
| | 1.5 | 2.0 | 12.4 | 4.5 | 4.2 | 4.0 | 3.7 | 3.5 | 3.4 | 3.2 | 3.1 | 3.0 | 2.9 |
| | 1.75 | 1.5 | 7.8 | 2.9 | 2.7 | 2.6 | 2.4 | 2.3 | 2.2 | 2.1 | 2.1 | 2.0 | 1.9 |
| | 2 | 1.2 | 5.2 | 2.0 | 1.9 | 1.8 | 1.8 | 1.7 | 1.6 | 1.6 | 1.5 | 1.5 | 1.5 |
| | 2.25 | 1.1 | 3.7 | 1.6 | 1.5 | 1.5 | 1.4 | 1.4 | 1.3 | 1.3 | 1.3 | 1.2 | 1.2 |
| | 2.5 | 1.0 | 2.7 | 1.3 | 1.3 | 1.2 | 1.2 | 1.2 | 1.2 | 1.1 | 1.1 | 1.1 | 1.1 |
| | 2.75 | 1.0 | 2.1 | 1.2 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.0 | 1.0 |
| | 3 | 1.0 | 1.7 | 1.1 | 1.1 | 1.1 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| | *EARL* | 46.4 | 73.2 | 56.8 | 55.9 | 55.0 | 54.3 | 53.5 | 52.9 | 52.3 | 51.7 | 51.2 | 50.8 |
| | $\%Diff_A$ | | 57.8% | 22.6% | 20.6% | 18.7% | 17.0% | 15.5% | 14.0% | 12.7% | 11.6% | 10.5% | 9.5% |

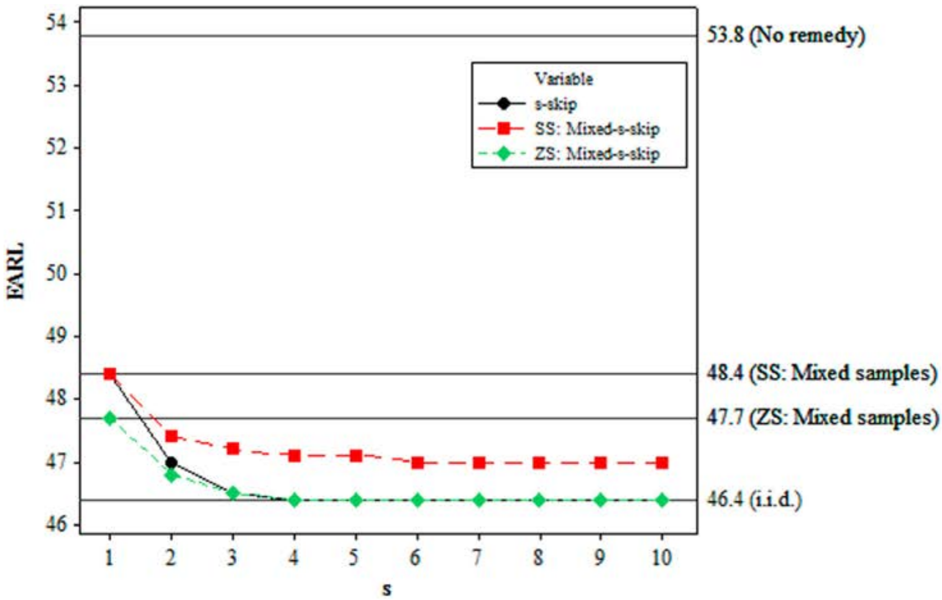
**Table 5.** The *ESDRL* (and $\%\text{Diff}_{\text{SD}}$ – in brackets) of the *s*-skip, mixed samples, mixed-*s*-skip (zero- and steady-state) $\bar{X}$ scheme when $\phi \in \{0, 0.3, 0.9\}$, $s \in \{1, 2, \ldots, 10\}$, $\delta_{\min} = 0$, $\delta_{\max} = 3$ and $n = 4$ (with $n_t = n_{t-1} = 2$).

| | $\phi = 0.3$ | | | $\phi = 0.9$ | | |
|---|---|---|---|---|---|---|
| i.i.d. | 45.7 | 45.7 | 45.7 | 45.7 | 45.7 | 45.7 |
| No remedy | 53.2 (16.4%) | 53.2 (16.4%) | 53.2 (16.4%) | 72.6 (58.9%) | 72.6 (58.9%) | 72.6 (58.9%) |
| s = 1 | 47.8 (4.6%) | 47.2 (3.2%) | 47.0 (2.9%) | 69.4 (51.8%) | 56.3 (23.2%) | 56.3 (23.1%) |
| s = 2 | 46.3 (1.3%) | 46.2 (1.2%) | 46.1 (0.9%) | 66.5 (45.6%) | 55.4 (21.2%) | 55.3 (21.0%) |
| s = 3 | 45.9 (0.4%) | 46.0 (0.6%) | 45.8 (0.3%) | 64.1 (40.2%) | 54.5 (19.3%) | 54.5 (19.2%) |
| s = 4 | 45.8 (0.1%) | 45.9 (0.4%) | 45.7 (0.1%) | 61.9 (35.5%) | 53.7 (17.6%) | 53.7 (17.4%) |
| s = 5 | 45.7 (0.0%) | 45.9 (0.3%) | 45.7 (0.0%) | 60.0 (31.4%) | 53.0 (16.0%) | 52.9 (15.8%) |
| s = 6 | 45.7 (0.0%) | 45.8 (0.3%) | 45.7 (0.0%) | 58.4 (27.8%) | 52.4 (14.5%) | 52.3 (14.4%) |
| s = 7 | 45.7 (0.0%) | 45.8 (0.3%) | 45.7 (0.0%) | 57.0 (24.6%) | 51.7 (13.2%) | 51.7 (13.0%) |
| s = 8 | 45.7 (0.0%) | 45.8 (0.3%) | 45.7 (0.0%) | 55.7 (21.8%) | 51.2 (12.0%) | 51.1 (11.8%) |
| s = 9 | 45.7 (0.0%) | 45.8 (0.3%) | 45.7 (0.0%) | 54.6 (19.4%) | 50.7 (10.9%) | 50.6 (10.7%) |
| s = 10 | 45.7 (0.0%) | 45.8 (0.3%) | 45.7 (0.0%) | 53.6 (17.3%) | 50.2 (9.9%) | 50.1 (9.7%) |
| | *s*-skip | Steady-state: Mixed-*s*-skip | Zero-state: Mixed-*s*-skip | *s*-skip | Steady-state: Mixed-*s*-skip | Zero-state: Mixed-*s*-skip |

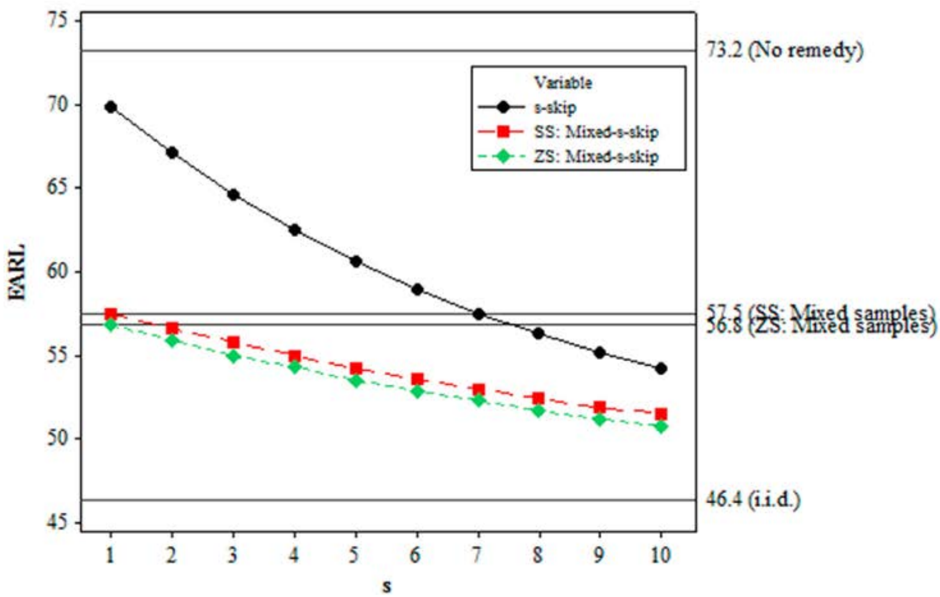
case when $s \geq 5$. Also, while the mixed-*s*-skip and *s*-skip strategies are flexible as *s* increase, the mixed samples strategy is static. More importantly, Figure 2(a) illustrate that the no remedial approach has the worst performance and adds the motivation for the use of any remedial approach to counteract the negative effect of autocorrelation.

Next, in Table 4, it is observed that when $\phi$ is very large (i.e. close to 1), then at each corresponding value of *s*, the mixed-*s*-skip strategy is uniformly better than the *s*-skip strategy for all possible values of *s*, in both the zero- and steady-state modes. While for small $\phi$ values, it is possible to skip few observations and consequently eradicate the negative effect of autocorrelation, this is not the case with large $\phi$ values. Moreover, it is observed that the mixed-*s*-skip strategy in zero-state has slightly smaller values of the *EARL* than those in steady-state mode; hence, the $\%\text{Diff}_{\text{A}}$ is slightly lower in zero-state mode. This is illustrated in Figure 2(b), where it is observed that the mixed samples strategy is not flexible, and thus, even the *s*-skip strategy outperforms the mixed samples for large $\phi$ values as the value of *s* increases. Thus, it is observed that the new mixed-*s*-skip strategy introduces the flexibility such that even for high values of *s*, it maintains its competitiveness when compared to the *s*-skip strategy. Note that [7] only used the steady-state mode to evaluate the performance of the mixed samples $\bar{X}$ scheme and concluded that it has a better performance than the *s*-skip strategy for large $\phi$ values; however, Figure 2 shows that this is not the case for large *s* values. Note though, the new generalized version of mixed samples rather yields a better performance than the *s*-skip for all possible values of *s*. Even as $s \rightarrow \infty$, the *EARL* or *ESDRL* of the mixed-*s*-skip and *s*-skip strategies converge to an approximately equal values with those of the mixed-*s*-skip strategy slightly smaller, e.g. for $s = 30$ with $\phi = 0.95$ and $n = 10$, the resulting *EARL*s, with $\delta_{min} = 0$ and $\delta_{max} = 3$, are equal to 39.51 and 39.05 for the *s*-skip and (steady-state) mixed-*s*-skip strategies, respectively.

A similar pattern as that of the *ARL*s (i.e. Tables 3 and 4) is observed for the *SDRL*s for each of the considered strategies at different values of $\delta$ and *s*; hence, in Table 5 we only show the *ESDRL* and $\%\text{Diff}_{\text{SD}}$, which have a similar pattern as the *EARL* and $\%\text{Diff}_{\text{A}}$ shown in Tables 3 and 4, as the value of *s* increases.




(a) $\phi$=0.3




(b) $\phi$=0.9

**Figure 2.** The *EARL* of the *s*-skip, mixed samples and mixed-*s*-skip $\bar{X}$ scheme in zero-state (ZS) and steady-state (SS) when $\phi \in \{0, 0.3, 0.9\}$ and $n = 4$.

The effect of the sample size on the performance of the mixed-$s$-skip $\bar{X}$ scheme in zero-state mode is illustrated in Figure 3 when $n \in \{3, 4, 7, 10\}$ which implies that $(n_{t-1}, n_t) \in \{(1,2), (2,2), (3,4), (5,5)\}$, respectively. It is evident from Figure 3 that increasing the sample size leads to an improved performance of the mixed-$s$-skip $\bar{X}$ scheme. While Figure 3 is specifically done for zero-state and $s = 3$, a similar pattern is observed in steady-state as well as for other values of $s$.

### 2.5. Implementation example for monitoring autocorrelated data

The yogurt cup filling process dataset taken from page 1418 of Franco et al. [7] is displayed on Table 6, which shows the weights of different yogurt cups taken at different sampling points. The dataset has 24 samples (each of size 5 yogurt cups taken every hour). The Phase I analysis of this process indicated that the weight of a yogurt cup, $Y_{t,i}$, fits an AR(1) model with parameter $\phi = 0.7$, an IC mean estimate, $\mu_0 = 125g$ and an IC standard deviation, $\sigma_0 = 1g$. For illustration purpose, assume that the data in Table 6 is a full dataset and here we show how to implement the mixed-$s$-skip sampling strategy to form rational subgroups of size $n = 3$ (i.e. with $n_{t-1} = 1$ and $n_t = 2$). In the last two columns, the corresponding plotting statistics at each sampling point are shown when $s \in \{1,2\}$. For instance, for $s = 2$ and $t \in \{2,3\}$, then using Equation (7), these are calculated as follows:

$$\bar{Y}_2 = \frac{1}{3}\left(\frac{1}{1}Y_{1,3}\right) + \frac{2}{3}\left(\frac{1}{2}(Y_{2,1} + Y_{2,4})\right) = \frac{1}{3}(126.45 + 125.56 + 123.77) = 125.26,$$

$$\bar{Y}_3 = \frac{1}{3}\left(\frac{1}{1}Y_{2,3}\right) + \frac{2}{3}\left(\frac{1}{2}(Y_{3,1} + Y_{3,4})\right) = \frac{1}{3}(123.60 + 127.18 + 126.32) = 125.70.$$

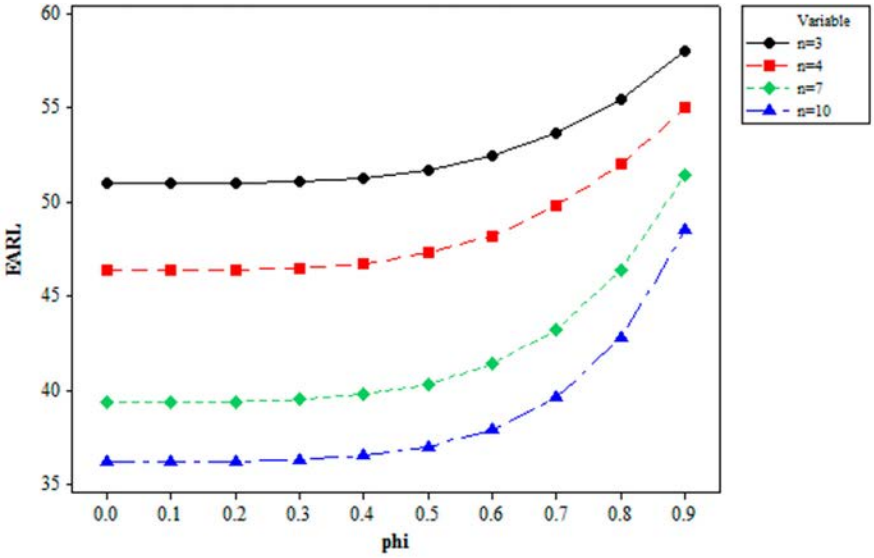
For $n = 3$, $\phi = 0.7$ and assuming that the process is in a steady-state mode, we obtain $\rho$ (see Equation (3) and Table 1) equal to 1.1518 and 1.1085 for $s$ equal to 1 and 2, respectively. That is, as $s$ increases, the value of $\rho$ converges towards the value of 1. Hence, the *UCL* / *LCL* for the $\bar{X}$ scheme are given by 126.99 / 123.01 and 126.92 / 123.08 for $s$ equal to 1 and 2, respectively. That is, as $s$ increases, the control limits become narrow. As it can be seen in Figure 4(a) and (b), for each mixed-$s$-skip sampling strategy with $s \in \{1,2\}$, the $\bar{X}$ scheme does not yield an OOC signal when $s = 1$; however, it issues the first OOC signal at sampling point $t = 16$ when $s = 2$. In summary, this example shows that the $\bar{X}$ scheme's control limits become narrow as $s$ increases (i.e. $\rho$ decreases towards 1 as $s$ increase); hence, there is an improvement in the OOC detection rate especially as $\phi$ is relatively large in the steady-state mode.

## 3. Autocorrelated observations with measurement errors

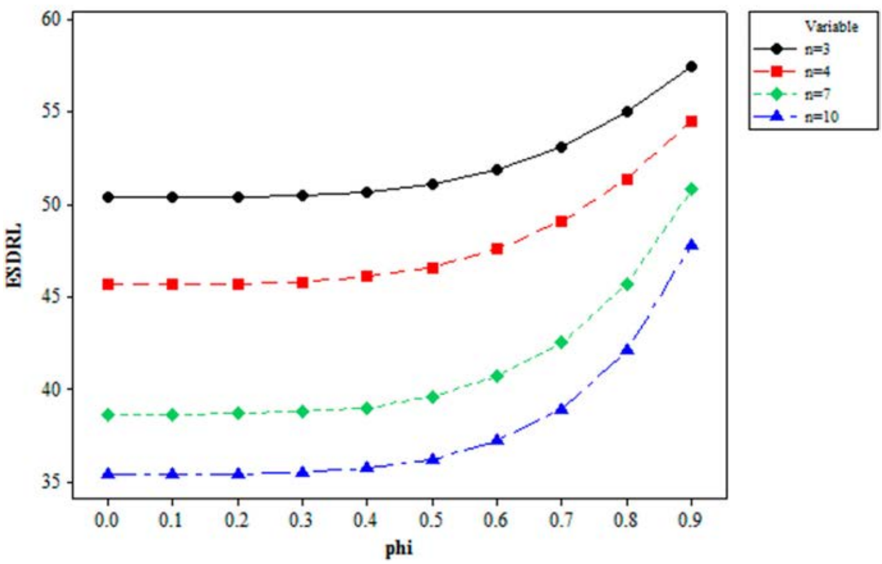
### 3.1. The proposed strategy

Assume that the $\{Y_{t,(s+1)i-s}\}$ observations from Equation (1) are not directly observable, but can only be assessed from the results $\{X_{t,(s+1)i-s,j} : t \geq 1;\ i = 1,2, \ldots, n;\ j = 1,2, \ldots, m\}$, where each element of the sequence can be expressed in terms of the additive model with a constant standard deviation, see [16], i.e.

$$X_{t,(s+1)i-s,j} = A + BY_{t,(s+1)i-s} + e_{t,(s+1)i-s,j}; \quad (14)$$




(a) *EARL*




(b) *ESDRL*

**Figure 3.** The *EARL* and *ESDRL* of the mixed-*s*-skip $\bar{X}$ scheme (with $s = 3$) in zero-state when $\phi \in \{0, 0.1, 0.2, \ldots, 0.9\}$, $n \in \{3, 4, 7, 10\}$ and $\delta_{\max} = 3$.

where $e_{t,(s+1)i-s,j} \sim N(0, \sigma_M)$ is a random error term due to measurement inaccuracy and $\sigma_M$ is the standard deviation of the measurement system, where $A$ and $B$ are two constants depending on the measurement system location error (for a sake of simplicity, in this paper, we assume that $A = 0$ and $B = 1$). Since we assume that $\{X_{t,(s+1)i-s,j}\}$ are from

**Table 6.** The yogurt filling cup process dataset from Franco et al. [7].

| $t$ | $Y_{t,1}$ | $Y_{t,2}$ | $Y_{t,3}$ | $Y_{t,4}$ | $Y_{t,5}$ | Mixed-1-skip | Mixed-2-skip |
|---|---|---|---|---|---|---|---|
| 1 | 124.74 | 126.12 | 126.45 | 124.66 | 125.11 | | |
| 2 | 125.56 | 123.24 | 123.60 | 123.77 | 123.54 | 125.09 | 125.26 |
| 3 | 127.18 | 127.38 | 127.18 | 126.32 | 126.55 | 125.87 | 125.70 |
| 4 | 124.41 | 124.22 | 124.29 | 126.10 | 124.60 | 125.36 | 125.90 |
| 5 | 125.37 | 124.87 | 123.65 | 123.16 | 122.29 | 124.41 | 124.27 |
| 6 | 124.83 | 126.62 | 126.24 | 125.86 | 127.53 | 125.31 | 124.78 |
| 7 | 124.22 | 124.15 | 124.14 | 123.82 | 124.18 | 124.99 | 124.76 |
| 8 | 123.91 | 124.28 | 126.31 | 126.06 | 127.08 | 124.79 | 124.70 |
| 9 | 125.40 | 125.14 | 125.60 | 123.90 | 124.92 | 125.09 | 125.20 |
| 10 | 125.53 | 125.36 | 124.24 | 123.71 | 123.64 | 124.97 | 124.95 |
| 11 | 125.79 | 123.91 | 124.28 | 125.19 | 125.98 | 125.14 | 125.07 |
| 12 | 124.55 | 126.61 | 126.98 | 126.84 | 127.60 | 125.15 | 125.22 |
| 13 | 126.15 | 125.60 | 124.26 | 126.17 | 126.65 | 125.67 | 126.43 |
| 14 | 123.54 | 124.42 | 123.52 | 123.53 | 122.95 | 124.22 | 123.78 |
| 15 | 124.02 | 123.78 | 122.60 | 122.42 | 123.26 | 123.68 | 123.32 |
| 16 | 124.02 | 123.78 | 122.60 | 122.42 | 123.26 | 123.47 | 123.01 |
| 17 | 125.03 | 124.72 | 123.62 | 124.99 | 124.37 | 124.14 | 124.21 |
| 18 | 125.17 | 125.10 | 124.45 | 124.03 | 125.11 | 124.78 | 124.27 |
| 19 | 124.22 | 125.64 | 125.19 | 124.39 | 125.40 | 124.84 | 124.35 |
| 20 | 123.35 | 122.90 | 122.31 | 122.42 | 120.09 | 123.77 | 123.65 |
| 21 | 124.65 | 125.45 | 124.43 | 124.83 | 124.34 | 123.99 | 123.93 |
| 22 | 124.88 | 125.27 | 124.73 | 123.09 | 123.14 | 125.02 | 124.13 |
| 23 | 123.59 | 124.27 | 123.70 | 124.62 | 123.41 | 124.19 | 124.31 |
| 24 | 124.24 | 125.87 | 124.62 | 125.99 | 124.19 | 124.38 | 124.64 |

an imperfect measurement system (i.e. $\gamma = \frac{\sigma_M}{\sigma_0} > 0$, which denotes the ratio of the measurement system variability to the process variability) then it is standard practice to take multiple measurements (of size $m$, with $m > 1$) as a remedial approach in reducing the effect of measurement errors, see [16]. Hence, instead of Equations (9) as a plotting statistic, the combined mixed-$s$-skip samples strategy and $m$-multiple measurements strategy (denoted as mix-$s$&$m$) uses $m$ separate measurements, each of size $n$ (i.e. a total of $m \times n$ observations), so that the plotting statistic is given by

$$\bar{X}_t = \frac{1}{mn}\sum_{i=1}^{n}\sum_{j=1}^{m} X_{t,(s+1)i-s,j}$$
$$= \frac{1}{n}\left(\sum_{i=1}^{n_{t-1}} Y_{t-1,(s+1)i} + \sum_{i=1}^{n_t} Y_{t,\,(s+1)i-s} + \frac{1}{m}\sum_{i=1}^{n}\sum_{j=1}^{m} e_{t,(s+1)i-s,j}\right). \qquad (15)$$

The $\bar{X}$ scheme's $UCL/LCL$ are given by:

$$UCL/LCL = \mu_{\bar{X},0} \pm k\sigma_{\bar{X},0} \qquad (16)$$

with $\mu_{\bar{X},0} = \mu_0$ and $\sigma_{\bar{X},0} = \frac{\sigma_0}{\sqrt{n}}\varphi$ are the mean and standard deviation of $\bar{X}_t$, respectively; $k$ is as defined in Equation (3) and $\varphi$ depends on which sampling strategy is implemented to account for both autocorrelation and measurement errors. Note that for the i.i.d. case, with perfect measurement, $\varphi$ is equal to 1. The expressions for $\varphi$ when the sampling strategy implemented is the mix-$s$&$m$ strategy, the $s$-skip strategy with $m$-multiple measurements strategy (denoted by $s$&$m$) proposed in [5] and the mixed samples strategy with $m$-multiple measurements strategy (denoted by mix&$m$) are each shown in Table 7.

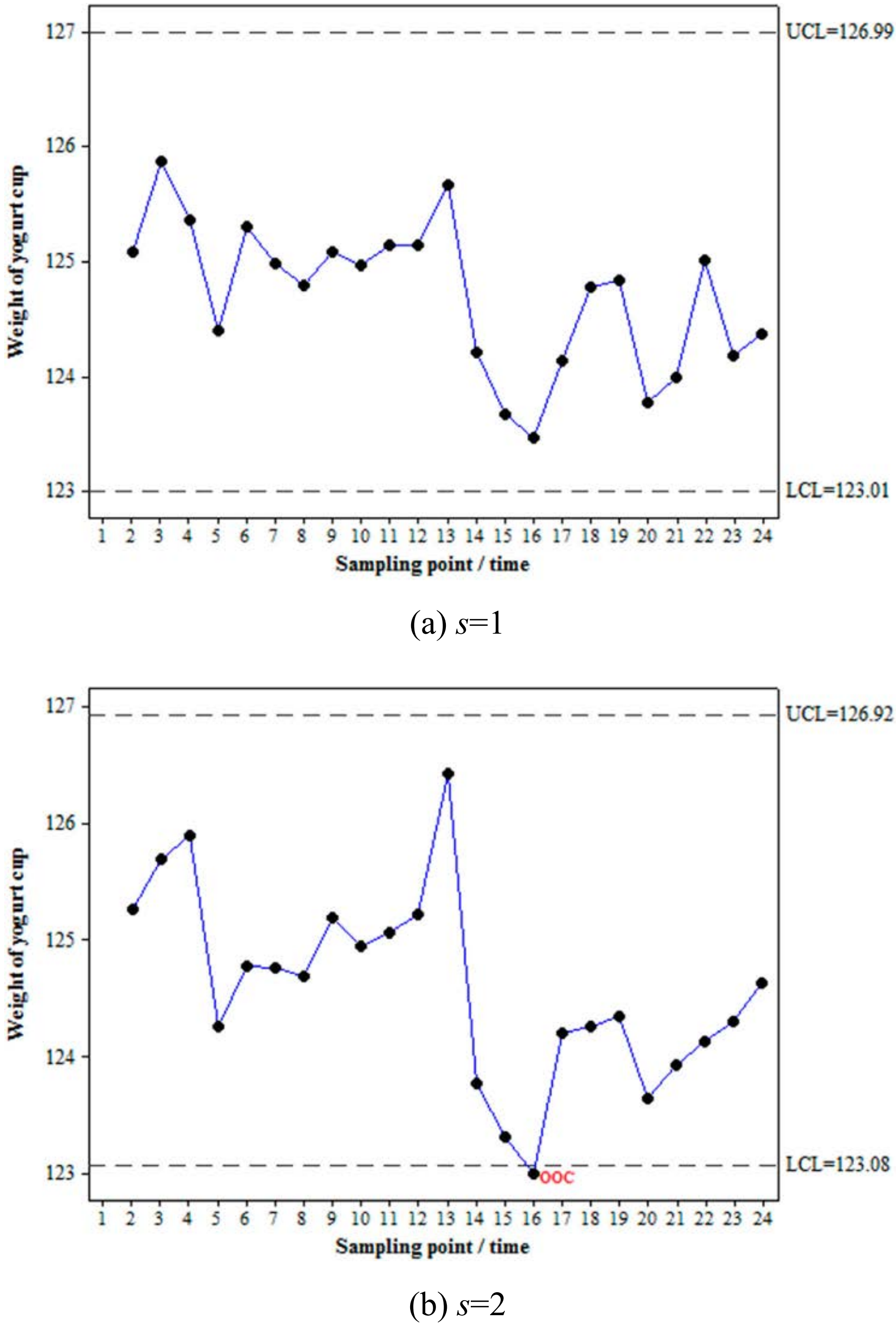



**Figure 4.** The weight of yogurt cups example for the $\bar{X}$ scheme with the mixed-*s*-skip strategy.

**Table 7.** $\varphi$ terms for different sampling strategies when the process is i.i.d. and when it is under the effect of autocorrelation and measurement errors.

| Sampling strategy | $\varphi$ |
|---|---|
| (i) i.i.d. | 1 |
| (ii) *s*&*m* | $\sqrt{\left(\frac{m+\gamma^2}{m}\right)+\left(\frac{n+2\left(\frac{\phi^{(s+1)(n+1)}-n\phi^{2s+2}+(n-1)\phi^{s+1}}{(\phi^{s+1}-1)^2}\right)}{n}\right)-1}$ |
| (iii) Mix&*m* | $\sqrt{\left(\frac{m+\gamma^2}{m}\right)+\left(\frac{n_t+2\left(\frac{\phi^{2n_t+2}-n_t\phi^4+(n_t-1)\phi^2}{(\phi^2-1)^2}\right)}{n}+\frac{n_{t-1}+2\left(\frac{\phi^{2n_{t-1}+2}-n_{t-1}\phi^4+(n_{t-1}-1)\phi^2}{(\phi^2-1)^2}\right)}{n}\right)-1}$ |
| (iv) Mix-*s*&*m* | $\sqrt{\left(\frac{m+\gamma^2}{m}\right)+\left(\frac{n_t+2\left(\frac{\phi^{(s+1)(n_t+1)}-n_t\phi^{2s+2}+(n_t-1)\phi^{s+1}}{(\phi^{s+1}-1)^2}\right)}{n}+\frac{n_{t-1}+2\left(\frac{\phi^{(s+1)(n_{t-1}+1)}-n_{t-1}\phi^{2s+2}+(n_{t-1}-1)\phi^{s+1}}{(\phi^{s+1}-1)^2}\right)}{n}\right)-1}$ |

Note that the *ZSARL*, *ZSSDRL*, *SSARL* and *SSSDRL* of the $\bar{X}$ scheme with mix-*s*&*m* strategy are also given by Equations (8) and (10); however, with

$$\beta = \Phi\left(k-\delta\sqrt{n}\times\frac{1}{\varphi}\right)-\Phi\left(-k-\delta\sqrt{n}\times\frac{1}{\varphi}\right) \tag{17}$$

and

$$\beta_1 = \Phi\left(k-\delta\sqrt{n}\times\frac{n_t}{n}\times\frac{1}{\varphi}\right)-\Phi\left(-k-\delta\sqrt{n}\times\frac{n_t}{n}\times\frac{1}{\varphi}\right). \tag{18}$$

Note that [7] proposed the mixed samples strategy for autocorrelated data but without taking measurement errors into account. Thus, in an effort to extend on their work so that it accounts for measurement errors, the mix&*m* is introduced here by taking $s = 1$ in Equations (15), (17) and (18). In the next subsection, the run-length performance of the *s*&*m* strategy is compared with the two new strategies (i.e. mix&*m* and mix-*s*&*m*) in both zero- and steady-state mode of analysis.

### 3.2. Empirical analysis of autocorrelated data with measurement errors

In Tables 8 and 9, the negative effect of autocorrelation and measurement errors is illustrated for both $\phi$ and $\gamma$ equal to {0.3, 0.9}, the i.i.d. case (i.e. both $\phi$ and $\gamma$ equal to 0) and the no remedial approach case to offset the combined negative effect of autocorrelation and measurement error which is denoted as 'No remedy'. Note that in Tables 8 and 9, the aim is to illustrate the degree of the negative effect of a small / large level of autocorrelation and measurement errors in the short- and long-run scenarios; and more importantly, to illustrate how the magnitude of the zero- and steady-state *ARL*s are affected as $s$ and $m$ increase for different values of $\gamma$ and $\phi$. Firstly, in both tables, it is observed that when $s = 1$, the mix-*s*&*m* and mix&*m* have exactly the same OOC performance; however, when $s > 1$, then the mix-*s*&*m* strategy uniformly outperforms the mix&*m* strategy. Secondly, in Table 8, with $\phi = 0.3$, as $s$ increases (for any value of $m$) the *s*&*m* strategy becomes more competitive than the mix&*m* strategy – this is because the mix&*m* strategy is not flexible with respect to the $s$ variable. That is, for small $\phi$ values, the *s*&*m* strategy tends to outperform the mix-*s*&*m* strategy in steady-state; however, in zero-state, the mix-*s*&*m* strategy is uniformly

**Table 8.** The *ARL* and *SDRL* of the *s*-skip, mixed samples, mixed-*s*-skip (zero- and steady-state) $\bar{X}$ scheme when $\phi = 0.3$, $\gamma = 0.3$, $s \in \{1, 3, 5\}$, $m \in \{2, 4, 6\}$, $\delta_{\min} = 0$, $\delta_{\max} = 3$ and $n = 5$ (with $n_t = 3$ and $n_{t-1} = 2$).

| | | | | s = 1,m = 2 | | | | | s = 3,m = 4 | | | | | s = 5,m = 6 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Shift | i.i.d. | No remedy | *s&m* | SS: Mix&*m* | ZS: Mix&*m* | SS: Mix-s&m | ZS: Mix-s&m | *s&m* | SS: Mix&*m* | ZS: Mix&*m* | SS: Mix-s&m | ZS: Mix-s&m | *s&m* | SS: Mix&*m* | ZS: Mix&*m* | SS: Mix-s&m | ZS: Mix-s&m |
| *ARL* | 0 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 |
| | 0.25 | 133.2 | 186.4 | 151.1 | 147.8 | 147.4 | 147.8 | 147.4 | 136.6 | 145.9 | 145.5 | 136.7 | 136.3 | 134.7 | 145.2 | 144.8 | 135.1 | 134.7 |
| | 0.5 | 33.4 | 62.1 | 41.8 | 40.6 | 40.0 | 40.6 | 40.0 | 34.9 | 39.7 | 39.0 | 35.4 | 34.8 | 34.1 | 39.4 | 38.7 | 34.7 | 34.1 |
| | 0.75 | 10.8 | 23.2 | 14.1 | 14.1 | 13.4 | 14.1 | 13.4 | 11.3 | 13.7 | 13.0 | 12.0 | 11.3 | 11.0 | 13.6 | 12.9 | 11.8 | 11.0 |
| | 1 | 4.5 | 10.1 | 5.9 | 6.4 | 5.6 | 6.4 | 5.6 | 4.7 | 6.2 | 5.4 | 5.5 | 4.7 | 4.6 | 6.2 | 5.4 | 5.4 | 4.6 |
| | 1.25 | 2.4 | 5.1 | 3.1 | 3.7 | 2.9 | 3.7 | 2.9 | 2.5 | 3.6 | 2.8 | 3.3 | 2.5 | 2.4 | 3.6 | 2.8 | 3.2 | 2.4 |
| | 1.5 | 1.6 | 3.0 | 1.9 | 2.6 | 1.8 | 2.6 | 1.8 | 1.6 | 2.6 | 1.8 | 2.4 | 1.6 | 1.6 | 2.5 | 1.8 | 2.3 | 1.6 |
| | 1.75 | 1.2 | 2.0 | 1.4 | 2.1 | 1.4 | 2.1 | 1.4 | 1.3 | 2.1 | 1.3 | 1.9 | 1.3 | 1.2 | 2.0 | 1.3 | 1.9 | 1.2 |
| | 2 | 1.1 | 1.5 | 1.2 | 1.8 | 1.1 | 1.8 | 1.1 | 1.1 | 1.8 | 1.1 | 1.7 | 1.1 | 1.1 | 1.8 | 1.1 | 1.7 | 1.1 |
| | 2.25 | 1.0 | 1.2 | 1.1 | 1.6 | 1.1 | 1.6 | 1.1 | 1.0 | 1.6 | 1.0 | 1.5 | 1.0 | 1.0 | 1.6 | 1.0 | 1.5 | 1.0 |
| | 2.5 | 1.0 | 1.1 | 1.0 | 1.5 | 1.0 | 1.5 | 1.0 | 1.0 | 1.5 | 1.0 | 1.4 | 1.0 | 1.0 | 1.4 | 1.0 | 1.4 | 1.0 |
| | 2.75 | 1.0 | 1.1 | 1.0 | 1.3 | 1.0 | 1.3 | 1.0 | 1.0 | 1.3 | 1.0 | 1.3 | 1.0 | 1.0 | 1.3 | 1.0 | 1.3 | 1.0 |
| | 3 | 1.0 | 1.0 | 1.0 | 1.2 | 1.0 | 1.2 | 1.0 | 1.0 | 1.2 | 1.0 | 1.2 | 1.0 | 1.0 | 1.2 | 1.0 | 1.2 | 1.0 |
| | *EARL* | 43.3 | 51.4 | 45.8 | 45.8 | 45.2 | 45.8 | 45.2 | 43.7 | 45.5 | 44.9 | 44.2 | 43.7 | 43.5 | 45.4 | 44.9 | 44.0 | 43.5 |
| | $\%\text{Diff}_{\text{A}}$ | | 18.8% | 5.8% | 5.8% | 4.5% | 5.8% | 4.5% | 1.1% | 5.2% | 3.9% | 2.1% | 1.0% | 0.5% | 4.9% | 3.7% | 1.7% | 0.5% |
| *SDRL* | 0 | 369.9 | 369.9 | 369.9 | 369.9 | 369.9 | 369.9 | 369.9 | 369.9 | 369.9 | 369.9 | 369.9 | 369.9 | 369.9 | 369.9 | 369.9 | 369.9 | 369.9 |
| | 0.25 | 132.7 | 185.9 | 150.6 | 146.9 | 146.9 | 146.9 | 146.9 | 136.1 | 145.0 | 145.0 | 135.8 | 135.8 | 134.2 | 144.3 | 144.3 | 134.2 | 134.2 |
| | 0.5 | 32.9 | 61.6 | 41.3 | 39.5 | 39.5 | 39.5 | 39.5 | 34.4 | 38.5 | 38.5 | 34.3 | 34.3 | 33.6 | 38.2 | 38.2 | 33.6 | 33.6 |
| | 0.75 | 10.3 | 22.6 | 13.6 | 12.9 | 12.8 | 12.9 | 12.8 | 10.8 | 12.5 | 12.5 | 10.8 | 10.8 | 10.5 | 12.3 | 12.3 | 10.5 | 10.5 |
| | 1 | 4.0 | 9.6 | 5.4 | 5.1 | 5.1 | 5.1 | 5.1 | 4.2 | 4.9 | 4.9 | 4.2 | 4.2 | 4.1 | 4.9 | 4.9 | 4.1 | 4.1 |
| | 1.25 | 1.8 | 4.6 | 2.5 | 2.4 | 2.4 | 2.4 | 2.4 | 1.9 | 2.3 | 2.3 | 2.0 | 1.9 | 1.9 | 2.3 | 2.3 | 1.9 | 1.9 |
| | 1.5 | 0.9 | 2.5 | 1.3 | 1.3 | 1.2 | 1.3 | 1.2 | 1.0 | 1.3 | 1.2 | 1.1 | 1.0 | 1.0 | 1.3 | 1.2 | 1.1 | 1.0 |
| | 1.75 | 0.5 | 1.4 | 0.7 | 0.8 | 0.7 | 0.8 | 0.7 | 0.6 | 0.8 | 0.7 | 0.7 | 0.6 | 0.5 | 0.8 | 0.7 | 0.7 | 0.5 |
| | 2 | 0.3 | 0.9 | 0.4 | 0.6 | 0.4 | 0.6 | 0.4 | 0.3 | 0.6 | 0.4 | 0.6 | 0.3 | 0.3 | 0.6 | 0.4 | 0.6 | 0.3 |
| | 2.25 | 0.2 | 0.6 | 0.3 | 0.6 | 0.2 | 0.6 | 0.2 | 0.2 | 0.5 | 0.2 | 0.5 | 0.2 | 0.2 | 0.5 | 0.2 | 0.5 | 0.2 |
| | 2.5 | 0.1 | 0.4 | 0.1 | 0.5 | 0.1 | 0.5 | 0.1 | 0.1 | 0.5 | 0.1 | 0.5 | 0.1 | 0.1 | 0.5 | 0.1 | 0.5 | 0.1 |
| | 2.75 | 0.0 | 0.2 | 0.1 | 0.5 | 0.1 | 0.5 | 0.1 | 0.0 | 0.5 | 0.1 | 0.4 | 0.0 | 0.0 | 0.5 | 0.1 | 0.4 | 0.0 |
| | 3 | 0.0 | 0.1 | 0.0 | 0.4 | 0.0 | 0.4 | 0.0 | 0.0 | 0.4 | 0.0 | 0.4 | 0.0 | 0.0 | 0.4 | 0.0 | 0.4 | 0.0 |
| | *ESDRL* | 42.6 | 50.8 | 45.1 | 44.7 | 44.6 | 44.7 | 44.6 | 43.0 | 44.4 | 44.3 | 43.2 | 43.0 | 42.8 | 44.3 | 44.2 | 43.0 | 42.8 |
| | $\%\text{Diff}_{\text{SD}}$ | | 19.3% | 5.9% | 5.0% | 4.7% | 5.0% | 4.7% | 1.1% | 4.4% | 4.0% | 1.4% | 1.0% | 0.5% | 4.2% | 3.8% | 0.9% | 0.5% |

**Table 9.** The *ARL* and *SDRL* of the *s*-skip, mixed samples, mixed-*s*-skip (zero- and steady-state) $\bar{X}$ scheme when $\phi = 0.9$, $\gamma = 0.9$, $s \in \{1, 3, 5\}$, $m \in \{2, 4, 6\}$, $\delta_{\min} = 0$, $\delta_{\max} = 3$ and $n = 5$ (with $n_t = 3$ and $n_{t-1} = 2$).

| | | | | $s=1, m=2$ | | | | | $s=3, m=4$ | | | | | $s=5, m=6$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Shift | i.i.d. | No remedy | *s&m* | SS: Mix&*m* | ZS: Mix&*m* | SS: Mix-*s&m* | ZS: Mix-*s&m* | *s&m* | SS: Mix&*m* | ZS: Mix&*m* | SS: Mix-*s&m* | ZS: Mix-*s&m* | *s&m* | SS: Mix&*m* | ZS: Mix&*m* | SS: Mix-*s&m* | ZS: Mix-*s&m* |
| *ARL* | 0 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 | 370.4 |
| | 0.25 | 133.2 | 282.1 | 266.3 | 229.2 | 229.0 | 229.2 | 229.0 | 242.6 | 221.8 | 221.5 | 210.3 | 210.0 | 223.0 | 219.1 | 218.9 | 196.9 | 196.6 |
| | 0.5 | 33.4 | 156.6 | 135.2 | 95.7 | 95.2 | 95.7 | 95.2 | 108.3 | 89.1 | 88.6 | 79.6 | 79.1 | 89.9 | 86.8 | 86.3 | 69.6 | 69.1 |
| | 0.75 | 10.8 | 82.3 | 66.4 | 41.3 | 40.7 | 41.3 | 40.7 | 48.6 | 37.6 | 36.9 | 32.4 | 31.8 | 37.6 | 36.3 | 35.6 | 27.2 | 26.6 |
| | 1 | 4.5 | 44.6 | 34.3 | 19.9 | 19.2 | 19.9 | 19.2 | 23.6 | 17.8 | 17.1 | 15.1 | 14.4 | 17.5 | 17.2 | 16.4 | 12.5 | 11.8 |
| | 1.25 | 2.4 | 25.4 | 18.9 | 10.7 | 10.0 | 10.7 | 10.0 | 12.5 | 9.6 | 8.8 | 8.1 | 7.4 | 9.1 | 9.2 | 8.5 | 6.8 | 6.0 |
| | 1.5 | 1.6 | 15.3 | 11.1 | 6.5 | 5.7 | 6.5 | 5.7 | 7.2 | 5.9 | 5.1 | 5.0 | 4.2 | 5.2 | 5.6 | 4.9 | 4.3 | 3.5 |
| | 1.75 | 1.2 | 9.7 | 7.0 | 4.4 | 3.6 | 4.4 | 3.6 | 4.5 | 4.0 | 3.2 | 3.5 | 2.7 | 3.3 | 3.9 | 3.1 | 3.1 | 2.3 |
| | 2 | 1.1 | 6.4 | 4.6 | 3.3 | 2.5 | 3.3 | 2.5 | 3.1 | 3.0 | 2.2 | 2.7 | 1.9 | 2.3 | 2.9 | 2.2 | 2.4 | 1.7 |
| | 2.25 | 1.0 | 4.5 | 3.3 | 2.6 | 1.9 | 2.6 | 1.9 | 2.2 | 2.5 | 1.7 | 2.3 | 1.5 | 1.7 | 2.4 | 1.7 | 2.1 | 1.3 |
| | 2.5 | 1.0 | 3.3 | 2.5 | 2.2 | 1.5 | 2.2 | 1.5 | 1.7 | 2.1 | 1.4 | 2.0 | 1.3 | 1.4 | 2.1 | 1.4 | 1.8 | 1.2 |
| | 2.75 | 1.0 | 2.5 | 1.9 | 2.0 | 1.3 | 2.0 | 1.3 | 1.4 | 1.9 | 1.2 | 1.8 | 1.1 | 1.2 | 1.9 | 1.2 | 1.7 | 1.1 |
| | 3 | 1.0 | 2.0 | 1.6 | 1.8 | 1.2 | 1.8 | 1.2 | 1.3 | 1.7 | 1.1 | 1.6 | 1.1 | 1.1 | 1.7 | 1.1 | 1.5 | 1.0 |
| | *EARL* | 43.3 | 77.3 | 71.0 | 60.8 | 60.2 | 60.8 | 60.2 | 63.6 | 59.0 | 58.4 | 56.5 | 55.9 | 58.7 | 58.4 | 57.8 | 53.9 | 53.3 |
| | $\%\text{Diff}_\text{A}$ | | 78.7% | 64.2% | 40.4% | 39.0% | 40.4% | 39.0% | 47.1% | 36.4% | 35.0% | 30.7% | 29.2% | 35.8% | 35.0% | 33.6% | 24.5% | 23.1% |
| *SDRL* | 0 | 369.9 | 369.9 | 369.9 | 369.9 | 369.9 | 369.9 | 369.9 | 369.9 | 369.9 | 369.9 | 369.9 | 369.9 | 369.9 | 369.9 | 369.9 | 369.9 | 369.9 |
| | 0.25 | 132.7 | 281.6 | 265.8 | 228.5 | 228.5 | 228.5 | 228.5 | 242.1 | 221.0 | 221.0 | 209.5 | 209.5 | 222.5 | 218.4 | 218.4 | 196.1 | 196.1 |
| | 0.5 | 32.9 | 156.1 | 134.7 | 94.7 | 94.7 | 94.7 | 94.7 | 107.8 | 88.1 | 88.1 | 78.6 | 78.6 | 89.4 | 85.8 | 85.8 | 68.6 | 68.6 |
| | 0.75 | 10.3 | 81.8 | 65.9 | 40.2 | 40.2 | 40.2 | 40.2 | 48.1 | 36.4 | 36.4 | 31.3 | 31.2 | 37.1 | 35.1 | 35.1 | 26.1 | 26.1 |
| | 1 | 4.0 | 44.1 | 33.8 | 18.6 | 18.6 | 18.6 | 18.6 | 23.1 | 16.6 | 16.6 | 13.9 | 13.9 | 17.0 | 15.9 | 15.9 | 11.3 | 11.2 |
| | 1.25 | 1.8 | 24.9 | 18.4 | 9.5 | 9.5 | 9.5 | 9.5 | 12.0 | 8.3 | 8.3 | 6.9 | 6.8 | 8.5 | 8.0 | 8.0 | 5.5 | 5.5 |
| | 1.5 | 0.9 | 14.8 | 10.6 | 5.2 | 5.2 | 5.2 | 5.2 | 6.7 | 4.6 | 4.5 | 3.7 | 3.7 | 4.7 | 4.4 | 4.3 | 3.0 | 2.9 |
| | 1.75 | 0.5 | 9.2 | 6.4 | 3.1 | 3.1 | 3.1 | 3.1 | 4.0 | 2.7 | 2.7 | 2.2 | 2.2 | 2.8 | 2.6 | 2.5 | 1.8 | 1.7 |
| | 2 | 0.3 | 5.9 | 4.1 | 2.0 | 1.9 | 2.0 | 1.9 | 2.5 | 1.7 | 1.7 | 1.4 | 1.4 | 1.7 | 1.6 | 1.6 | 1.1 | 1.1 |
| | 2.25 | 0.2 | 4.0 | 2.7 | 1.3 | 1.3 | 1.3 | 1.3 | 1.7 | 1.2 | 1.1 | 1.0 | 0.9 | 1.1 | 1.1 | 1.0 | 0.8 | 0.7 |
| | 2.5 | 0.1 | 2.8 | 1.9 | 1.0 | 0.9 | 1.0 | 0.9 | 1.1 | 0.9 | 0.7 | 0.7 | 0.6 | 0.8 | 0.8 | 0.7 | 0.7 | 0.4 |
| | 2.75 | 0.0 | 2.0 | 1.3 | 0.8 | 0.6 | 0.8 | 0.6 | 0.8 | 0.7 | 0.5 | 0.6 | 0.4 | 0.5 | 0.7 | 0.5 | 0.6 | 0.3 |
| | 3 | 0.0 | 1.5 | 1.0 | 0.6 | 0.4 | 0.6 | 0.4 | 0.6 | 0.6 | 0.4 | 0.6 | 0.3 | 0.4 | 0.6 | 0.3 | 0.5 | 0.2 |
| | *ESDRL* | 42.6 | 76.8 | 70.5 | 59.6 | 59.6 | 59.6 | 59.6 | 63.1 | 57.9 | 57.8 | 55.4 | 55.3 | 58.2 | 57.3 | 57.2 | 52.8 | 52.7 |
| | $\%\text{Diff}_\text{SD}$ | | 80.4% | 65.6% | 40.1% | 39.9% | 40.1% | 39.9% | 48.2% | 36.0% | 35.8% | 30.1% | 30.0% | 36.6% | 34.6% | 34.4% | 23.9% | 23.7% |

better than all the competing strategies. Note though, for large $\phi$ values (see Table 9 with $\phi = 0.9$), the *s*&*m* strategy is outperformed by the mix-*s*&*m* strategy in both the zero- and steady-state modes. Thirdly, increasing $s$ and $m$ improves the performance of each of the monitoring schemes. Fourthly, while for the process under the effect of low level autocorrelation only (say, $\phi = 0.3$ in Table 3 or Figure 2(a)), skipping at least 5 observations ensures that we can theoretically get rid of all autocorrelation and the scheme yields the same OOC performance as the i.i.d. version, but for the combined negative effect of autocorrelation and measurement errors (e.g. $\phi = 0.3$ and $\gamma = 0.3$), it is not possible to theoretically get rid of all the negative effect of autocorrelation and measurement errors for moderate values of $s$ and $m$ so that $\%\text{Diff}_{\text{A}}$ or $\%\text{Diff}_{\text{SD}}$ converges to 0.0% - this only happens for unreasonably large values of $s$ and $m$. That is, a combined effect of both autocorrelation and measurement errors introduces more variability in the process than autocorrelation only. Lastly, similar to Figure 3, increasing the sample size yields an improved performance for the mix-*s*&*m* strategy as well as the other strategies.

In essence, we observed that at any possible corresponding values of $s$ and $m$:

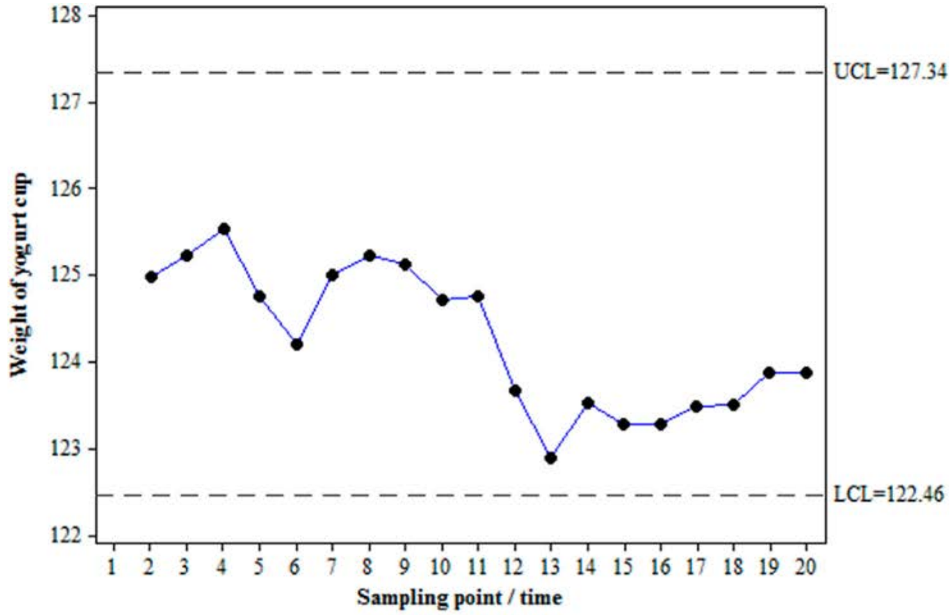
- In zero-state mode, the mix-*s*&*m* strategy has a better OOC performance than the mix&*m* and *s*&*m* strategies.
- In steady-state mode, the *s*&*m* strategy tends to outperform the mix&*m* and mix-*s*&*m* strategies for large values of $s$ when $\phi$ is small (for any value of $\gamma$). However, when $\phi$ is large (for any value of $\gamma$), the mix-*s*&*m* strategies uniformly outperform the *s*&*m* and mix&*m* strategies, whereas the mix&*m* strategy only outperforms the *s*&*m* strategy for relatively small to moderate values of $s$.

### 3.3. Implementation example for monitoring autocorrelated data with measurement errors

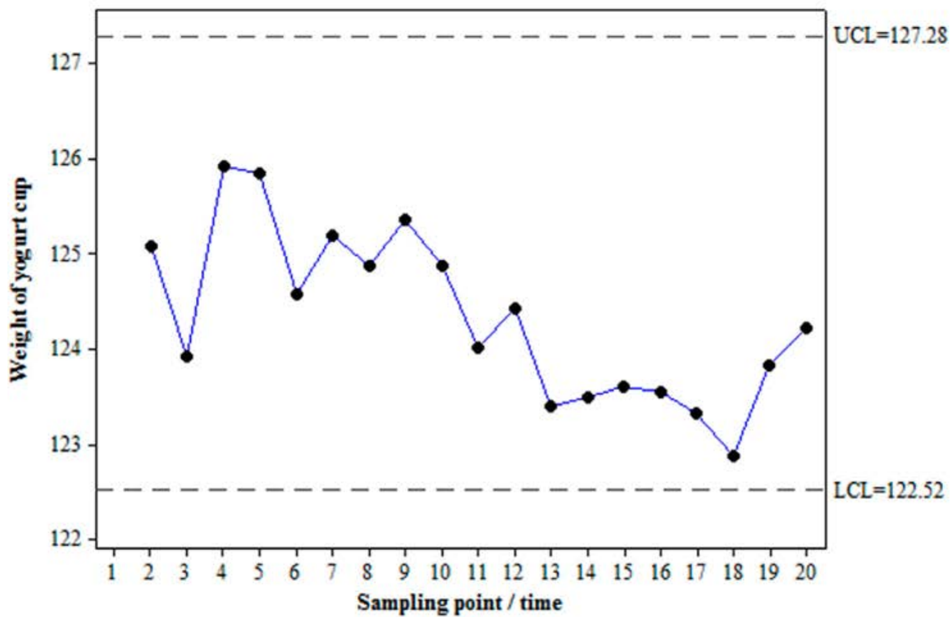
The yogurt cup filling process dataset taken from page 670 of Costa and Castagliola [5] is displayed on Table 10, which shows the weights of different yogurt cups taken at different sampling points. The dataset has 20 samples (each of size 5 yogurt cups taken every hour and each of them weighted $m = 2$ times) corresponding to a 20-hours sequence of production and for the sake of illustration, assume this is a full dataset. The Phase I analysis of this process indicated that the weight of a yogurt cup, $X_{t,i,j}$, fits an AR(1) model with parameter $\phi = 0.38$, an IC mean estimate, $\mu_0 = 124.9g$ and an IC standard deviation, $\sigma_0 = 0.76g$. An R&R study indicates that the measurement system standard deviation, $\sigma_M = 0.24g$, so that $\gamma = 0.316$. The aim of this example is to show how to implement the mix-*s*&*m* sampling strategy to form rational subgroups of size $n = 3$ (i.e. with $n_{t-1} = 1$ and $n_t = 2$) when $s \in \{1,2\}$, $m = 2$ when the process is in a steady-state mode. In the last two columns, the corresponding plotting statistics at each sampling point are shown. For instance, for $s = m = 2$ and $t \in \{2,3\}$, these are calculated as follows:

$$\bar{X}_2 = \frac{1}{2 \times 3}((X_{1,3,1} + X_{1,3,2}) + (X_{2,1,1} + X_{2,1,2}) + (X_{2,4,1} + X_{2,4,2})) = 125.08,$$

$$\bar{X}_3 = \frac{1}{2 \times 3}((X_{2,3,1} + X_{2,3,2}) + (X_{3,1,1} + X_{3,1,2}) + (X_{3,4,1} + X_{3,4,2})) = 123.92.$$




(a) $s$=1 & $m$=2




(b) $s$=2 & $m$=2

**Figure 5.** The weight of yogurt cups example for the $\bar{X}$ scheme with the mix-*s*&*m* strategy.

**Table 10.** The yogurt filling cup process dataset from Costa and Castagliola [5].

| $t$ | $X_{t11}$ | $X_{t12}$ | $X_{t21}$ | $X_{t22}$ | $X_{t31}$ | $X_{t32}$ | $X_{t41}$ | $X_{t42}$ | $X_{t51}$ | $X_{t52}$ | Mix-1&2 | Mix-2&2 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 124.9 | 124.8 | 125.9 | 125.9 | 125.2 | 124.8 | 124.6 | 124.1 | 124.8 | 124.4 | | |
| 2 | 124.9 | 125.2 | 125.5 | 125.0 | 124.1 | 123.9 | 125.2 | 125.2 | 125.0 | 125.6 | 124.98 | 125.08 |
| 3 | 125.1 | 125.1 | 125.2 | 124.8 | 125.4 | 125.3 | 122.9 | 122.4 | 125.4 | 125.4 | 125.23 | 123.92 |
| 4 | 126.1 | 125.9 | 124.6 | 124.8 | 125.7 | 125.5 | 126.4 | 126.5 | 124.9 | 125.7 | 125.53 | 125.93 |
| 5 | 125.8 | 125.7 | 122.6 | 122.6 | 124.1 | 123.5 | 126.1 | 126.3 | 124.9 | 125.0 | 124.75 | 125.85 |
| 6 | 125.0 | 125.2 | 125.5 | 124.8 | 124.8 | 125.0 | 124.9 | 124.8 | 124.8 | 124.2 | 124.20 | 124.58 |
| 7 | 124.2 | 124.6 | 125.8 | 125.3 | 125.4 | 125.5 | 126.4 | 126.2 | 125.1 | 125.2 | 125.00 | 125.20 |
| 8 | 124.9 | 124.9 | 123.8 | 123.2 | 125.1 | 125.3 | 124.0 | 124.5 | 124.4 | 124.2 | 125.22 | 124.87 |
| 9 | 125.9 | 125.8 | 124.4 | 124.8 | 126.3 | 125.7 | 124.9 | 125.2 | 125.2 | 125.1 | 125.12 | 125.37 |
| 10 | 124.2 | 124.3 | 126.2 | 125.5 | 125.6 | 125.0 | 124.4 | 124.4 | 124.1 | 124.3 | 124.72 | 124.88 |
| 11 | 123.7 | 123.6 | 123.4 | 123.3 | 124.7 | 124.8 | 123.1 | 123.1 | 123.1 | 122.8 | 124.75 | 124.02 |
| 12 | 124.0 | 124.1 | 122.6 | 122.4 | 123.6 | 123.6 | 124.4 | 124.5 | 123.6 | 123.1 | 123.67 | 124.42 |
| 13 | 122.0 | 122.5 | 123.9 | 124.0 | 123.7 | 124.1 | 124.3 | 124.4 | 121.9 | 122.9 | 122.88 | 123.40 |
| 14 | 122.4 | 123.0 | 122.8 | 123.1 | 123.7 | 124.2 | 123.7 | 124.1 | 122.8 | 123.1 | 123.53 | 123.50 |
| 15 | 123.9 | 123.6 | 124.1 | 124.5 | 123.4 | 122.9 | 123.1 | 123.1 | 124.5 | 125.1 | 123.28 | 123.60 |
| 16 | 121.9 | 122.3 | 123.4 | 123.3 | 123.5 | 123.3 | 125.3 | 125.5 | 123.3 | 123.6 | 123.27 | 123.55 |
| 17 | 123.3 | 122.9 | 123.6 | 123.5 | 124.2 | 123.8 | 123.4 | 123.6 | 123.5 | 123.4 | 123.48 | 123.33 |
| 18 | 122.0 | 122.2 | 123.6 | 123.4 | 124.7 | 125.0 | 122.6 | 122.5 | 124.5 | 123.9 | 123.50 | 122.88 |
| 19 | 124.0 | 123.9 | 123.1 | 123.4 | 123.9 | 124.5 | 122.6 | 122.8 | 124.2 | 123.5 | 123.88 | 123.83 |
| 20 | 125.5 | 124.9 | 122.2 | 122.3 | 123.2 | 123.2 | 123.2 | 123.3 | 123.2 | 123.2 | 123.88 | 124.22 |

When $n = 3$, $\phi = 0.380$ and $\gamma = 0.316$, then using Equation (16) and Table 7, the resulting $\varphi$ are equal to 1.0706 and 1.0423 for the mix-1&2 and mix-2&2 strategies, respectively. That is, as $s$ increases, the value of $\varphi$ converges towards the value of 1 (given that $m$ is constant) and thus the control limits become narrow. That is, the $UCL$ / $LCL$ for the $\bar{X}$ scheme are given by 127.34 / 122.46 and 127.28 / 122.52 for the mix-1&2 and mix-2&2 strategies, respectively. Note though, as can be seen from Figure 5, the mix-$s$&$m$ strategy do not signal any assignable causes in the process being monitored. In essence, this example illustrates the weakness of the mix-$s$&$m$ strategy when the level of autocorrelation is small and the process is in a steady-state mode, see the last paragraph of Section 3.2.

## 4. Conclusion

In an effort to reduce the negative effect of autocorrelation, the mixed-$s$-skip sampling strategy which combines the $s$-skip and mixed samples methodology is proposed. Using the average and standard deviation of the run-length distribution, it is shown that, in the zero-state mode, the mixed-$s$-skip strategy yields the best OOC performance than the $s$-skip and mixed samples strategies for any level of autocorrelation when evaluating the run-length processes with and without measurement errors. However, in steady-state mode, it only yields the best OOC performance when the level of autocorrelation is very high for processes with and without measurement errors.

This new remedial approach sampling strategy is recommended instead of the mixed samples strategy (for any autocorrelation level value in both zero- and steady-state modes) and $s$-skip strategy (for any autocorrelation level value in zero-state; however, only for large autocorrelation values in steady-state) when the process is under the negative effect of autocorrelation with and without measurement errors. Finally, although it yields better performance, the drawback of the proposed strategy is that it requires way more observations, time and effort to implement as compared to the standard no remedy approach.

For future research purpose, we intend to investigate the performance of this sampling strategy when the autocorrelation parameter, $\phi$, as well as the distributional parameters, are estimated from some historical Phase I data; that is, in part, in a similar manner as done in Garza-Venegas et al. [9]. Moreover, the new mixed-*s*-skip strategy can easily be applied for a variety of other monitoring schemes (i.e. the basic $T^2$ scheme, synthetic $\bar{X}$ or $T^2$ schemes, exponentially weighted moving average or Cumulative Sum, etc., with key basic concepts discussed in [5,6,13,14,24]) and different quality characteristics (i.e. median, standard deviation, etc.).

## Data availability statement

The data used in the application example is available from the papers by Costa and Castagliola [5] and Franco et al. [7].

## Acknowledgements

The authors would like to gratefully acknowledge the three anonymous referees and the Associate Editor for taking their valuable time to thoroughly read and give suggestions to improve the paper.

## Disclosure statement

No potential conflict of interest was reported by the authors.

## ORCID

*Sandile Charles Shongwe* http://orcid.org/0000-0003-2243-8196
*Jean-Claude Malela-Majika* http://orcid.org/0000-0001-7236-7678
*Philippe Castagliola* http://orcid.org/0000-0002-9532-4029

## References

[1] S. Ahmad, M. Riaz, S. Hussain, and S.A. Abbasi, *On auxiliary information-based control charts for autocorrelated processes with application in manufacturing industry*. Int. J. Adv. Manuf. Technol. 100 (2019), pp. 1965–1980.

[2] L.C. Alwan, and D. Radson, *Time-series investigation of subsample mean chart*. IIE Trans. 24 (1992), pp. 66–80.

[3] X.-B. Cheng, and F.-K. Wang, *VSSI median control chart with estimated parameters and measurement errors*. Qual. Reliab. Eng. Int. 34 (2018), pp. 867–881.

[4] X.Y. Chew, K.W. Khaw, and W.C. Yeong, *The efficiency of run rules schemes for the multivariate coefficient of variation: A Markov chain approach*. J. Appl. Stat. (2019). doi:10.1080/02664763.2019.1643296.

[5] A.F.B. Costa, and P. Castagliola, *Effect of measurement error and autocorrelation on the* $\bar{X}$*chart*. J. Appl. Stat. 38 (2011), pp. 661–673.

[6] P. Dargopatil, and V. Ghute, *New sampling strategies to reduce the effect of autocorrelation on the synthetic* $T^2$ *chart to monitor bivariate process*. Qual. Reliab. Eng. Int. 35 (2019), pp. 30–46.

[7] B.C. Franco, P. Castagliola, G. Celano, and A.F.B. Costa, *A new sampling strategy to reduce the effect of autocorrelation on a control chart*. J. Appl. Stat. 41 (2014), pp. 1408–1421.

[8] B.C. Franco, G. Celano, P. Castagliola, and A.F.B. Costa, *Economic design of Shewhart control charts for monitoring autocorrelated data with skip sampling strategies*. Int. J. Prod. Econ. 151 (2014), pp. 121–130.

[9] J.A. Garza-Venegas, V.G. Tercero-Gómez, L. Lee-Ho, P. Castagliola, and G. Celano, *Effect of autocorrelation estimators on the performance of the $\bar{X}$control chart*. J. Stat. Comput. Simul. 88 (2018), pp. 2612–2630.

[10] X.L. Hu, and J. Sun, *Synthetic $\bar{X}$chart for AR(1) autocorrelated processes*. Proceedings of the 27th Chinese Control and Decision Conference (2015), pp. 7–12. doi:10.1109/CCDC.201527161658.

[11] X. Huang, S. Bisgaard, and N. Xu, *Model-based multivariate monitoring charts for autocorrelated processes*. Qual. Reliab. Eng. Int. 30 (2014), pp. 527–543.

[12] A.A. Kalgonda, and S.R. Kulkarni, *Multivariate quality control chart for autocorrelated processes*. J. Appl. Stat. 31 (2004), pp. 317–327.

[13] R.C. Leoni, A.F.B. Costa, B.C. Franco, and M.A.G. Machado, *The skipping strategy to reduce the effect of the autocorrelation on the $T^2$ chart's performance*. Int. J. Adv. Manuf. Technol. 80 (2015), pp. 1547–1559.

[14] R.C. Leoni, M.A.G. Machado, and A.F.B. Costa, *The $T^2$ chart with mixed samples to control bivariate autocorrelated processes*. Int. J. Prod. Res. 54 (2016), pp. 3294–3310.

[15] S.-N. Lin, C.-Y. Chou, S.-L. Wang, and H.-R. Liu, *Economic design of autoregressive moving average control chart using genetic algorithms*. Expert. Syst. Appl. 39 (2012), pp. 1793–1798.

[16] K.W. Linna, and W.H. Woodall, *Effect of measurement error on Shewhart control charts*. J. Qual. Technol. 33 (2001), pp. 213–222.

[17] K.W. Linna, W.H. Woodall, and K.L. Busby, *The performance of multivariate control charts in the presence of measurement error*. J. Qual. Technol. 33 (2001), pp. 349–355.

[18] M.R. Maleki, A. Amiri, and P. Castagliola, *Measurement errors in statistical process monitoring: A literature review*. Comput. Ind. Eng. 103 (2017), pp. 316–329.

[19] P.E. Maravelakis, *EWMA chart and measurement error*. J. Appl. Stat. 31 (2004), pp. 445–455.

[20] P.E. Maravelakis, *Measurement error effect on the CUSUM control chart*. J. Appl. Stat. 39 (2012), pp. 323–336.

[21] D.C. Montgomery, *Statistical Quality Control: A Modern Introduction*, 7th ed., John Wiley & Sons, Singapore Pte. Ltd, Hoboken, NJ, 2013.

[22] H.D. Nguyen, Q.T. Nguyen, K.P. Tran, and D.P. Ho, *On the performance of VSI Shewhart control chart for monitoring the coefficient of variation in the presence of measurement errors*. Int. J. Adv. Manuf. Technol. 104 (2019), pp. 211–243.

[23] R. Noorossana, and Y. Zerehsaz, *Effect of measurement error on phase II monitoring of simple linear profiles*. Int. J. Adv. Manuf. Technol. 79 (2015), pp. 2031–2040.

[24] R. Osei-Aning, S.A. Abbasi, and M. Riaz, *Mixed EWMA-CUSUM and mixed CUSUM-EWMA for monitoring first order autoregressive processes*. Qual. Technol. Quant. Manag. 14 (2017), pp. 429–453.

[25] D.R. Prajapati, and S. Singh, *Control charts for monitoring the autocorrelated process parameters: A literature review*. Int. J. Prod. Qual. Manage. 10 (2012), pp. 207–249.

[26] H. Sabahno, and A. Amiri, *The effect of measurement errors on the performance of variable sample size and sampling interval $\bar{X}$control chart*. Int. J. Eng. Trans. A 30 (2017), pp. 995–1004.

[27] M. Scagliarini, *Estimation of $C_p$ for autocorrelated data and measurement errors*. Communications in Statistics-Theory and Methods 31 (2002), pp. 1647–1664.

[28] M. Scagliarini, *Inference on $C_{p^k}$for autocorrelated data in the presence of random measurement errors*. J. Appl. Stat. 37 (2010), pp. 147–158.

[29] S.C. Shongwe, and J.-C. Malela-Majika, *Shewhart-type monitoring schemes with supplementary w-of-w runs-rules to monitor the mean of autocorrelated samples*. Communications in Statistics – Simulation and Computation (2019). doi:10.1080/03610918.2019.1650180

[30] S.C. Shongwe, J.-C. Malela-Majika, P. Castagliola, and T. Molahloe, *Side-sensitive synthetic and runs-rules charts for monitoring AR(1) processes with skipping sampling strategies*. Communications in Statistics – Theory and Methods (2019). doi:10.1080/03610926.2019.1596284

[31] S.C. Shongwe, J.-C. Malela-Majika, and T. Molahloe, *One-sided runs-rules schemes to monitor autocorrelated time series data using a first-order autoregressive model with skipping sampling strategies*. Qual. Reliab. Eng. Int. 35 (2019), pp. 1973–1997.
[32] K.P. Tran, C. Heuchenne, and N. Balakrishnan, *On the performance of coefficient of variation charts in the presence of measurement errors*. Qual. Reliab. Eng. Int. 35 (2019), pp. 329–350.
[33] P.H. Tran, K.P. Tran, and A.C. Rakitzis, *A synthetic median control chart for monitoring the process mean with measurement errors*. Qual. Reliab. Eng. Int. 35 (2019), pp. 1100–1116.
[34] L. Xiaohong, and W. Zhaojun, *The CUSUM control chart for the autocorrelated data with measurement error*. Chinese Journal of Applied Probability 25 (2009), pp. 461–474.
[35] S.F. Yang, and C.M. Yang, *Effects of imprecise measurement on the two dependent processes control for the autocorrelated observations*. The International Journal of Advanced Manufacturing Technology 26 (2005), pp. 623–630.
[36] F.S. Zaidi, P. Castagliola, K.P. Tran, and M.B.C. Khoo, *Performance of the Hotelling $T^2$ control chart for Compositional data in the presence of measurement errors*. J. Appl. Stat. 46 (2019), pp. 2583–2602.