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The Effects of Language Instruction on Math Development

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Abstract

How does language shape mathematical development? In this article, we consider this question by reviewing findings from cross-sectional and longitudinal research. In this literature, we find that differences in the structures of languages and individual variation in language ability are associated with mathematical performance in both obvious and unexpected ways. We then consider the causal nature of these relations, with a focus on experimental studies that have tested the effects of language instruction on mathematical outcomes. Findings from this work show that certain forms of language instruction meaningfully improve performance in several mathematical domains, providing strong evidence of a linguistic pathway in mathematical development. However, much additional research is needed to understand how language instruction may be integrated optimally into math education. We conclude with recommendations for research.

Keywords

mathematics; language; intervention

Many life experiences rely heavily on various forms of mathematical knowledge (National Research Council, 2012). Therefore, a key goal of formal schooling is to ensure that students develop competence within a broad range of mathematical domains. Language has long been hypothesized to play a pivotal role in mathematical development (Carey, 2004). Accordingly, the education literature is replete with recommendations for incorporating language as a major feature of mathematics instruction (Schleppegrell, 2007). In this article, we consider the basis for such recommendations. We begin by mapping out hypothesized associations between language and mathematical performance. We then turn to experimental studies that have rigorously tested the causal effects of language instruction on mathematical outcomes. Findings from this literature indicate that certain forms of language instruction enhance mathematics performance, providing strong evidence for a linguistic pathway in mathematical development. However, as we discuss, many important theoretical and practical questions remain unanswered.

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Language and Mathematical Development

As part of a broader communicative system, language functions to represent and convey various forms of knowledge. Its multiple modes (e.g., comprehension, production) and dimensions (e.g., vocabulary, syntax, phonology) are tightly interwoven (Frank et al., 2021), but gradually dissociate throughout development (Lonigan & Milburn, 2017). In this article, we discuss how mathematical development is shaped in obvious and unexpected ways by differences in the structures and contents of children's languages, as well as by individual differences in their vocabulary knowledge, syntactic knowledge, and phonological processing.

Vocabulary Knowledge

Many languages express quantification with a variety of specific (e.g., *1, 2, 3*), approximate (e.g., *several*), universal (e.g., *every*), and comparative (e.g., *more*) terms (Lidz, 2016). An understanding of this core vocabulary, as well as a command of technical terms (e.g., *hypotenuse*) and specialized uses of common words (e.g., *regroup, simplify, significance*), is important for representing and conveying mathematical knowledge (Schleppegrell, 2007). Indeed, in two recent meta-analytic reviews (Lin et al., 2021; Peng et al., 2020), researchers found an overall significant relation between vocabulary knowledge and mathematical performance. The significance of these relations held after controlling for other linguistic (e.g., listening comprehension) and cognitive (e.g., working memory) factors, suggesting that beyond serving as a retrieval aide, vocabulary also provides a medium for mathematical reasoning.

The association between vocabulary knowledge and mathematical performance is moderated by task demands (Lin et al., 2021; Peng et al., 2020) and individual variation in mathematical and vocabulary knowledge (Peng et al., 2020; Ünal et al., 2021). For example, whereas some tasks can be performed with minimal or even no vocabulary knowledge (e.g., selecting the larger of two approximate magnitudes; Spelke & Tsivkin, 2001), others draw heavily on vocabulary (e.g., word problems; Lin et al., 2021). Accordingly, performance on language-demanding math tasks suffers when a person's relevant vocabulary knowledge is limited, as occurs when attempting to solve a word problem in an unfamiliar language (Xu et al., 2021). Furthermore, as mathematical skill increases, the relative importance of vocabulary diminishes (Ünal et al., 2021). This suggests that vocabulary instruction may be particularly important in the initial phases of learning and with new mathematical content.

Syntactic Knowledge

Syntax refers to how units of linguistic meaning (e.g., morphemes, words) are joined to represent and convey combinations of thought. Languages differ in their syntactic structures. For example, whereas English and Russian reliably mark singular (*1*) and plural (≥*2*) quantities (e.g., *cat* [singular], *cats* [plural]), Japanese makes no such obligatory distinction (Sarnecka et al., 2007).

How might children use these grammatical number markers in their languages to learn about number concepts? Researchers investigated this question by examining whether exposure to singular/plural markings affected children's development of number knowledge (Sarnecka et al., 2007). They began by assessing the frequency with which singular/plural markings appeared in several corpora of linguistic exchanges with preschool-aged children raised learning English, Russian, or Japanese. As expected, singular/plural markings were common in English and Russian but absent in Japanese. They then compared the counting skills and cardinal knowledge of preschool-aged monolingual children raised learning English, Russian, or Japanese. In each group, children's counting skills exceeded their cardinal knowledge. Moreover, children learned the meanings of the number words *one, two*, and *three* in that order. Yet compared to children learning Japanese, significantly higher proportions of English and Russian children understood the meanings of the words *one*, *two*, and *three*. This suggests that the structure of children's languages may affect the rate, but not the pattern, of number word learning.

In similar studies, preschool-aged children raised learning Slovenian or Saudi Arabic, languages that grammatically mark singular (*1*), dual (*2*), and plural (≥*3*) quantities, understood the number word *two* earlier than children of the same age raised learning English, which only distinguishes between singular (*1*) and plural (≥*2*) quantities (Almoammer et al., 2013). Across these and other studies (see Barner, 2017), evidence suggests that children draw insights about the meanings of number words from the syntactic structures of their languages.

Information about number concepts can also be found in the structure of the verbal count sequence. For example, in Turkish, the logic of the base-10 counting system is transparently represented in the number word sequence from 11-20 (*on* [10], *on bir* [11], *on iki* [12], … *on dokuz* [19]). In contrast, the 11-20 sequence of English number words provides no obvious cues to the system's logic (*ten* [10], *eleven* [11], *twelve* [12], … *nineteen* [19]). Researchers investigated whether these differences affected the rate at which preschool-aged children raised learning Turkish (in Turkey) or English (in Canada) learned to count within this range (Cankaya et al., 2014). Across four sessions, children played an 11-20 counting game in their first language. Following these sessions, the Turkish-speaking children significantly outperformed their English-speaking counterparts in counting accurately from 11-20. This suggests that the relative transparency of Turkish may have eased the task. Other cross-linguistic comparisons provide additional evidence that children induce information about the structure of the count sequence from syntactic features of their languages (LeFevre et al., 2018).

How might variation in syntactic knowledge affect mathematical development? Although the literature on this topic is sparse, several researchers have found significant relations. A cross-sectional study found significant associations between U.S. first- and second-grade children's syntactic knowledge and performance in arithmetic and word- problem solving (Chow & Ekholm, 2019). Indeed, compared to other language variables (i.e., morphology, general vocabulary), syntactic knowledge was the strongest predictor of children's mathematical performance. Another study compared the language profiles of first- and second-grade U.S. children with and without math difficulties (Chow et al., 2021). Children without math difficulties significantly outperformed those with difficulties on a measure of syntax but not on general vocabulary or morphology. This suggests that individual differences in syntactic knowledge may play a particularly important role in shaping early mathematical development. Yet another study examined the association between syntactic knowledge and mathematical development in an older, cross-sectional sample of children in grades three to eight (Truckenmiller et al., 2016). The study similarly found a significant association between syntactic knowledge and mathematical performance. These findings suggest that across the elementary and middle school grades, children's mathematical development varies, in part, as a function of their syntactic knowledge.

Phonological Processing

Phonological processing involves an awareness of the sound structure of language (*phonological awareness*), the ability to mentally store information as phonological representations (*phonological memory*), and the capacity to rapidly access these representations (*rapid automatized naming*; Yang et al., 2021). A meta-analysis found an overall significant relation between phonological processing and mathematical performance (Yang et al., 2021). This association was moderated by phonological component, mathematical domain, task demand (accuracy, fluency), and participants' age. For example, whereas accuracy-based tasks were closely associated with phonological awareness, fluency-based tasks were associated more closely with rapid automatized naming.

Across development, associations between components of phonological processing and performance within a mathematical domain can also shift. For example, when children first encounter an arithmetic problem (e.g., 2 + 2 = x), they rely on phonological awareness and phonological memory to recognize the symbols and execute the counting strategies required to solve it. However, through repeated encounters with that problem, children gradually commit its solution to memory (Hecht et al., 2001). Once this representation is established in long-term memory, children shift to an automatic retrieval strategy, relying then on rapid automatized naming (Yang et al., 2021).

How might individual differences in phonological processing affect mathematical development? Researchers in the United Kingdom examined this question by longitudinally tracking the mathematical development of a sample of children at three time points between the ages of 5 and 7 (Jordan et al., 2010). Participants included 1) those with phonological difficulties, 2) those with phonological and mathematical difficulties, and 3) those without either type of difficulty (i.e., typically developing). Although the three groups initially fared similarly, by the third time point, the typically developing children significantly outperformed those with phonological difficulties on number facts, arithmetic, and mathematical concepts. Other longitudinal and cross-sectional studies have also demonstrated that difficulties in phonological processing can prevent children from establishing stable foundations in number knowledge, arithmetic, and word-problem solving, resulting in ongoing math difficulties (see Peng et al., 2018).

Language and Math Instruction

In the preceding sections, we discussed examples of developmental associations between language and mathematical performance. Such observations have long motivated recommendations to incorporate language instruction into math education (Rothman &

Cohen, 1989). However, observing correlational associations between language and math, even longitudinally, cannot tell us how instruction in language affects mathematics performance (Bailey et al., 2018). Therefore, we turn our attention in this section to experimental studies that have tested the effects of language instruction on math outcomes. Although limited in size and scope, this nascent literature provides important theoretical and practical insights into the causal effects of various forms of language instruction on children's number knowledge, arithmetic, and word-problem solving performance.

Number Knowledge

Number knowledge involves an understanding of counting and cardinality. Findings from observational studies suggest that variations in the amount and type of early language input affect children's development in these areas (Levine et al., 2010; Mix et al., 2012). Therefore, a growing number of studies have examined whether altering children's early math language experiences improves their number knowledge.

For example, one study randomly assigned preschool-aged U.S. children (who were relatively evenly divided between White and ethnic/racial minorities) and their parents (nearly all of whom were college educated) to one of two interventions or a control condition (Gibson et al., 2020). Participants assigned to the control condition were given two picture books featuring dialogic prompts that encouraged discussions about the story characters (e.g., "*Can you find the fluffy rabbit?*"). Participants in the interventions were given two books featuring the same characters and story structures. However, their discussion prompts focused on counting and labeling small number sets from one to three (e.g., "*Can you count the two rabbits?*") for the first group and large number sets from four to six for the second group. Over the study, the number knowledge of children in the small number set condition grew significantly more than that of children in the other two conditions. This suggests that talk centered on small number sets may help foster young children's number knowledge.

A similar study randomly assigned typically developing preschool-aged U.S. English-speaking children (most of whom were White) and their family caregivers (nearly all of whom were college-educated mothers) to read and discuss either three math (intervention) or nonmath (control) storybooks (Purpura et al., 2021). Participants in the intervention condition received three researcher-developed books featuring 17 core (e.g., *more, different, few*) and story-specific (e.g., *just enough*) math terms. On each page, prompts guided caregivers to discuss the math language concepts embedded within the storyline. Participants in the control condition also received three storybooks with dialogic prompts on each page. However, the content of these books and the prompts contained no mathematical language. Over four weeks, caregivers in both groups were asked to read each book four times with their children. Immediately following the intervention, children who received the math storybook intervention significantly outperformed those in the control condition on measures of mathematical language and number knowledge. Although the language effects faded over time, children who received the intervention significantly outperformed those in the control group on number knowledge tasks (e.g., set comparisons, numeral comparisons, one-to-one correspondence) eight weeks later. Therefore, the effects of language instruction may be particularly important for children's initial acquisition of mathematical content.

Results from these and other studies show that caregiver-implemented math language interventions can improve preschool-aged children's number knowledge. Indeed, verbally counting and labeling object sets supports the development of cardinal knowledge (Gibson et al., 2020). Exposing children to terms used to describe imprecise quantities (e.g., *few*) and relations (e.g., *different*) also supports growth in number knowledge (Purpura et al., 2021). Moreover, these practices can be feasibly embedded within naturally occurring home and school activities (e.g., shared reading, playing, cooking, grocery shopping; see Elliott & Bachman, 2018). However, additional research is needed with more economically diverse participants and to determine optimal dosages to ensure that the effects of these language interventions last. Research is also needed to understand how intervention effects may be moderated by individual differences in children's cognition, language, and mathematical knowledge (e.g., Silver et al., 2021).

Arithmetic

Arithmetic involves the operations of addition, subtraction, multiplication, and division. As the foundation for many subsequent mathematical concepts, it is the focus of much attention in elementary schooling. Throughout the early elementary grades, most children develop fluency with at least a basic set of arithmetic problems (e.g., 6 + 3 = 9). However, as we noted, a sizeable minority of children struggle persistently and much effort has been invested in developing interventions to help them overcome these difficulties (e.g., Fuchs et al., 2013). Recently, several U.S.-based studies have tested whether language instruction can improve arithmetic outcomes.

Two studies tested the effects of standalone math vocabulary instruction on U.S. kindergarten children's arithmetic performance. The first study randomly assigned children (who were primarily Black and Hispanic) to three conditions: 1) number sense intervention, 2) language intervention, or 3) a business-as-usual control (Jordan et al., 2012). Over 24 sessions, children in the language intervention were exposed to 43 math vocabulary terms (e.g., *before/after, big/small*) embedded in eight storybooks. At posttest, children in the intervention and control conditions did not differ significantly in arithmetic ability or knowledge of math vocabulary.

The second study randomly assigned children (who were primarily Hispanic but also included Black and White participants, and who were mostly low income as indicated by eligibility for free and reduced-price lunch) with increased likelihood for math difficulties to the same three conditions (Hassinger-Das et al., 2015). However, this time, children in the language intervention were taught 34 math vocabulary terms through dialogic reading and direct instructional activities delivered over 21 sessions. At posttest, children in the language intervention condition significantly outperformed children in the other two groups on proximal and distal vocabulary measures, but again, not on measures of arithmetic performance. Taken together, these studies show the limits of isolated language instruction on math outcomes.

Another study examined the effects of embedding math vocabulary instruction in an arithmetic intervention (Powell & Driver, 2015). Researchers randomly assigned an ethnically and racially diverse sample of U.S. first-graders with math difficulties to three conditions: 1) arithmetic instruction, 2) arithmetic and math vocabulary instruction, or 3) a business as usual control. In both intervention conditions, students received 15 sessions of the same arithmetic intervention. In the combined condition, students were also taught 13 mathematical terms (e.g., *number, less, add, equation*), which were introduced or reviewed through tutor-led discussions and explanations (e.g., *"What does it mean to compare two numbers?"*). At posttest, children in both interventions significantly outperformed those in the control condition on measures of mathematical vocabulary and addition fluency. However, students in the two intervention conditions performed comparably on both measures. This indicates that arithmetic instruction alone was sufficient for developing mathematical vocabulary knowledge. Moreover, math vocabulary instruction offered no benefit over arithmetic instruction for improving children's fluency in addition.

Word-Problem Solving

Schema-based instruction is an approach to word-problem solving that has proven effective for children across a range of ages and ability levels (Cook et al., 2020; Peltier & Vannest, 2017). This type of instruction teaches children to categorize word problems based on the underlying structure of the problem (e.g., total, difference, combine). For example, consider the following problem: "Diego and Ariel have 9 markers. Diego has 4. How many markers does Ariel have?" With schema-based instruction, a child learns to categorize this as a total problem and more specifically as a problem in which one of the parts is the unknown quantity. Once the problem has been categorized, the child builds the number sentence that follows the schema's problem model sentence (e.g., P1 + P2 = 4 + P2 = 9) and then solves for the unknown quantity (e.g., P2 = 5; see Fuchs et al., 2013).

Researchers examined the added value of embedding instruction in language comprehension in a validated schema-based word-problem solving intervention (Fuchs et al., 2021). They randomly assigned a low-income (as indicated by eligibility for free and reduced-price lunch) and ethnically and racially diverse sample of U.S. first-graders with an increased likelihood for math difficulties to four conditions: 1) schema-based instruction, 2) schema-based instruction + language comprehension instruction, 3) number knowledge instruction only, or 4) a business as usual control. In each of the three intervention conditions, children received 45 sessions of one-to-one tutoring. Children in the condition that incorporated language comprehension instruction were taught a variety of mathematical vocabulary terms and syntactic constructions. For example, when learning about combine problems, they learned terms such as *altogether* and *in all*, and about superordinate concepts and vocabulary (e.g., *dogs* and *cats* are *animals*). For compare problems, instruction focused on terms and constructions such as *more, fewer*, and - *er*, and for change problems, children were introduced to cause-and-effect conjunctions (e.g., *then*), verbs relating to quantity change (e.g., *ate*), and time passage (e.g., *the next day*). Also, children were explicitly taught how and why linking taught words to operations often produces incorrect solutions (e.g., *more* is associated with addition approximately only half the time). At posttest, children in the schema-based instruction + language comprehension condition significantly outperformed those in the other conditions, including those in the schema-based instruction without language instruction group, on word-problem solving tasks. This provides strong evidence for a causal role for language in word-problem solving.

Summary, Implications, and Directions

As the developmental links between language and math have become clearer, a growing body of math language interventions has been developed and rigorously tested. In this literature, compelling evidence suggests that certain forms of language instruction can strengthen young children's number knowledge (Gibson et al., 2020; Purpura et al., 2021) and performance in word-problem solving (Fuchs et al., 2021). But currently, there is no evidence that standalone or embedded language instruction can improve children's arithmetic performance (Hassinger-Das et al., 2015; Jordan et al., 2012; Powell & Driver, 2015). Although the generalizability of these findings is tempered by the limited socioeconomic, linguistic, and cultural diversity of the samples, we find support for incorporating at least some forms of language instruction into early math education. Moving forward, much additional research is needed to understand more thoroughly what forms of language instruction work, for whom, and under what conditions.

Specifically, the studies reviewed in this article only scratch the surface of the voluminous mathematical content students are expected to master over their educational careers. They also consider only a narrow range of the language comprehension dimensions and none of the language production dimensions (e.g., classroom talk, writing) that have been linked to mathematical performance into adolescence (Schleppegrell, 2007). Going forward, researchers should examine the effects of instruction in a much wider range of language dimensions and mathematical domains (e.g., algebra, geometry, statistics). Such work is needed to chart the linguistic pathways in mathematical development more completely and to determine if forms of language instruction enhance acquisition of the more advanced mathematical concepts and procedures introduced in the upper elementary and secondary grades (e.g., algebra, geometry, statistics).

Language interventions can vary widely in their targeted modalities and domains (e.g., comprehension), purposes (e.g., skill development), methods of delivery (e.g., whole class), forms (e.g., decontextualized), and teaching techniques (e.g., questions; Denman et al., 2021). Along these dimensions, a lingering question concerns the characteristics of effective language instruction for improving mathematical performance. The answer undoubtedly depends on the mathematical content being addressed (e.g., arithmetic), the targeted population (e.g., children with math difficulties), and the learning context (e.g., home, school). Researchers need to systematically manipulate language interventions along these and other dimensions (e.g., dosage, duration, timing of intervention) and consider at the front end which time, capital, and interest factors most frequently affect implementation (Century & Cassata, 2016).

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