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Pricing efficiency and asymmetric multifractality of major asset classes before and during COVID-19 crisis



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ABSTRACT

We examine the impact of COVID-19 pandemic crisis on the pricing efficiency and asymmetric multifractality of major asset classes (S&P500, US Treasury bond, US dollar index, Bitcoin, Brent oil, and gold) within a dynamic framework. Applying permutation entropy on intraday data that covers between April 30, 2019 and May 13, 2020, we show that efficiency of all sample asset classes is deteriorated with the outbreak, and in most cases this deterioration is significant. Results are found to be robust under different analysis schemes. Brent oil is the highest efficient market before and during crisis. The degree of efficiency is heterogeneous among all markets. The analysis by an asymmetric multifractal detrended fluctuation analysis (A-MF-DFA) approach shows evidence of asymmetric multifractality in all markets which rise with the scales. The inefficiency is higher during downward trends before the pandemic crisis as well as during COVID-19 except for gold and Bitcoin. Moreover, the pandemic intensifies the inefficiency of all markets gains during the COVID-19 outbreak.

1. Introduction

The coronavirus (COVID-19) outbreak has caused severe damage not only on the human health causing more that 236 million affected cases and about 5 million deaths.¹ The ongoing pandemic has damaged the performance of both financial and commodity markets at a global scale (Albulescu, 2021; Baker et al., 2020; Goodell, 2020; Zhang et al., 2021), increasing the uncertainty and amplifying the negative sentiment in the markets. The equity and commodity markets have seen huge falls in late December 2019 with intensified volatilities, followed by crashes in the first quarter of 2020 (Bakas and Triantafyllou, 2020). The oil price showed significant drops, and in April 2020, the US crude oil futures even fell to negative values, crashing from \$18 a barrel to -\$38, for the first time in history, as demand for crude oil has all but dried up and stockpiles overwhelmed storage facilities, which left oil investors reeling. To the contrary, the values of the US dollar index has raised due to the demand for the US dollar and the collapse of oil prices.² This

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¹ https://www.worldometers.info/coronavirus/?utm_campaign=homeAdvegas1?%22.

 $^{^2\,}$ The dollar has increased by 3.5% against the Europe in January 2020.

phenomenon is explained by the view that in the episodes of crisis or even during recessionary periods, investors shift to the safe haven investment opportunities. With a similar reason, the gold price has been at the top of the charts since the first days of COVID-19 outbreak, and it still continues to increase especially after central banks' lowering the interest rates. Like any other assets, cryptocurrency prices have also affected by the COVID-19. The Bitcoin price has crashed in March 2020 whereas interestingly, it crossed US\$ 10,000 as of May 2020.³ The linkages among these markets has been intensified, making the investment decision complex (Zhang et al., 2021; Mensi et al., 2021b; Wang et al., 2021).

Multifractality and efficiency are two important topics for portfolio management theory. If an asset exhibits a fractal property, it indicates evidence of long-range memory and possibility of predictability of future prices. The multifractality evidence suggests a crash prediction and volatility predictability (Grech and Mazur, 2004; Wei and Wang, 2008). Cao et al. (2013) argue that long memory and fat tails are the source of multifractality. Specifically, the authors conclude that multifractality is associated to long-range correlations when the market is going up and to fat-tailed distribution when the market is going down. According to Fama (1970), a market is efficient in its weak form if past information related to asset prices is quickly and instantaneously embodied in the current prices. This indicates that market follows a random walk and impossibility to beat a market and reap abnormal profits. Besides, asymmetric information (good and bad shocks) have direct implications on the fluctuations of return volatility of commodity and financial markets. Therefore, an accurate pricing asset requires monitoring these crucial stylized facts. Besides, the effects of COVID-19 pandemic crisis have adversely affected the majority of financial and commodity prices. Therefore, the global health crisis affected the investor demand, preferences and risk appetite for financial and commodity assets. This indicates that COVID-19 crisis may alter the degree of efficiency and multifractality patterns. Kakinaka and Umero (2021) argue that COVID-19 significantly increased herding in the shortrun but not in the long-run. Lahmiri and Bekiros (2020) show that the pandemic lead to instability and irregularity in cryptocurrency market making investment riskier. In this climate, flued by the concerns of COVID-19 crisis, the uncertainty and the volatility of the prices of these assets have increased significantly, making the predictability of their prices a necessity for trading, investment, speculation and policymaking purposes. Motivated by this, we aim to examine the asymmetric multifractality, efficiency, and predictability dynamics of major asset classes, namely; the US stock market, the US dollar index, US Treasury bills, Bitcoin, crude oil, and gold, before and during the COVID-19 outbreak within a weak-form efficiency framework.

Since the outbreak of the pandemic, the literature has addressed the effect of virus on the multifractality and efficiency of financial and commodity markets. Mensi et al. (2019b) analyze the multifractality and weak-form efficiency of global, regional and GIPSI (Greece, Ireland, Portugal, Spain, and Italy) stock markets using the symmetric Multifractal Detrended Fluctuation Analysis (MF-DFA) method by Kantelhardt et al. (2002). The authors find that Greece is the highest inefficient market irrespective of the time horizon. Furthermore, global developed and emerging world indexes and regional stock markets are less efficient than GIPSI. This result is in line with the findings of Mensi et al. (2018) where they conclude that Gulf Council Cooperation stock markets are more inefficient than the global and regional stock markets.

Cao et al. (2013) developed the asymmetric MF-DFA approach to analyze the multifractality of Chinese stock market. The results show that Shanghai stock market is characterized by long-range correlations and Shenzhen stock market by fat-tailed distribution. These are the source of asymmetric multifractality. Using the same methodology, Mensi et al. (2021a) examine the multifractality in overall, uptrends and downtrends in the top oil producers (Canada, Russia, Saudi Arabia, and USA) and oil consumers (Brazil, China, India, and Japan). The authors find higher upward multifractality than downward one. In addition, the degree of efficiency decreases during the global financial crisis in 2008 and COVID-19 pandemic. The stock markets of oil producers have high inefficient degree than the stock markets of oil consumers. Applying the same methodology for high frequency data, Mensi et al. (2020) show gold is more inefficient than oil during only downtrends pre-COVID-19 crisis. In contrast, gold is more inefficient than oil during uptrends during the pandemic period. More interestingly, the degree of efficiency enhances during the pandemic period for both gold and oil markets.

As for the literature on cryptocurrency, Naeem et al. (2021) use A-MF-DA for major cryptocurrency to examine the efficiency and asymmetric multifractality of Bitcoin, Ethereum, Litcoin, and Ripple. The authors report that COVID-19 negatively influences their degree of efficiency of cryptocurrency market. They conclude evidence of asymmetric multifractality and that Bitcoin and Ethereum are highly efficient. Using the same methodology, Kakinaka and Umero (2021) investigate the degree of efficiency of crypto markets during COVID-19 crisis and show that cryptocurrency assets become more inefficient in only the short-term. Other studies have focused on the efficiency of leading cryptocurrencies and found mixed results (Al-Yahyaee et al., 2018; Chu et al., 2019; Mensi et al., 2019a; Vidal-Tomás and Ibañez, 2018; Urquhart, 2016).

We contribute to the literature in four main ways. First, we use a high-frequency intraday data to examine the predictability and asymmetric multifractality of the prices of main asset classes namely S&P500 index, US Treasury bonds, US dollar index, Bitcoin, Brent crude oil, and gold. Various methods that test the weak-form efficiency require a significant amount of observations to apply properly. However, since COVID-19 outbreak is very recent, these methods would not allow for a robust analysis if they were applied on daily data. On the other hand, high-frequency dataset allows us to perform a robust statistical analysis since it produces enough number of observations. Second, due to the damage by COVID-19, we investigate the predictability before and during COVID-19 periods to get a complete picture on the dynamics of market prices. Moreover, we use a dynamic setup to see whether the change in efficiency has a trend or not. We follow the approach by Sensoy et al. (2017) and Sensoy (2019) to account for the dynamic level of efficiency along the time period. Third, we use a relatively new methodology, permutation entropy (PE), proposed by Bandt and Pompe (2002) which is least affected by the non-linearity and the outliers in the data compared to similar methodologies. Therefore, our results are

³ https://news.bitcoin.com/bitcoin-price-touches-10k-amid-2020s-macroeconomic-storm-and-covid-19-fears/.

statistically robust. Finally, we apply the A-MF-DFA method to test the upward and downward multifractality of the considered markets. We notice that the MF-DFA approach is based on detrended fluctuation analysis (DFA). The latter is applied to identify long-range autocorrelations and multifractality in financial and commodity markets for nonstationary time series. The A-MF-DFA is a generalization to the MF-DFA as it assumes that multifractality property behave the same under down and uptrends. However, correlation asymmetry alters the financial risk market and portfolio structure (Cao et al., 2013). Thus, price dynamics are strongly dependent to information transmitted to the markets (e.g., leverage effects).

Using permutation entropy method, we find that for all asset classes, weak-form pricing efficiency is disrupted with the COVID-19 outbreak, and this deterioration is significant. Dynamic framework further shows that this situation is not a temporary-one-off effect but has turned into a trend in worsening efficiency. For robustness, we examine not only the raw log-returns but also the conditional ones obtained via GARCH (1,1) process and reveal that results still hold. Evidently, there has been an increase in the predictability degree of all major asset classes, and the market participants should take positions accordingly. Using the A-MF-DFA approach, the results show significant upward and downward multifractality which rises with scale increases. More precisely, the size of downtrend multifractality is higher than the uptrend multifractality for S&P500 index, DYX, Brent oil and US Treasury Bond. The inverse is valid for both BTC and gold. The downward trend Hurst is higher than the upward trend Hurst for all markets except DYX as well as BTC for positive scales. Overall, we report evidence against informational efficiency of the considered markets which varies according to downward and upward movements. The COVID-19 pandemic crisis intensifies the degree of inefficiency for all markets except Bitcoin.

The remainder of this paper is organized as follows. Section 2 presents the data and summary statistics. Section 3 outlines the methodology. Section 4 discusses the results and Section 5 concludes.

2. Data and summary statistics

We use log-returns obtained from the contract-for-difference (CFD) prices for six major asset classes, namely US stock market measured by S&P500 index (S&P500), US Treasury bonds (USTBOND), US dollar index (DXY), Bitcoin (BTC), Brent crude oil (Brent), and gold. The source of our data is Dukascopy Bank SA, a Swiss forex bank and an ECN broker with its headquarters in Geneva. We use the closing prices of 30-minute interval intraday data that covers slightly more than a year period from April 30, 2019 to May 13, 2020. The sample is divided further into two periods: (i) before COVID-19 and (ii) during COVID-19 where the cutoff date for the COVID-19 is December 1, 2019. Our breakpoint is the initial day when COVID-19 spread in Hubei Province and then spread to more than 200 countries where the United States and the European Union are the most affected.

Table 1 shows that the descriptive statistics of 30-minutes price return series before and during COVID-19. The average returns have decreased significantly for all markets during COVID-19 period. Moreover, we observe that all the markets become more volatile at the same time. Brent oil is the highest volatile market, followed by Bitcoin, whereas the US dollar index is the least volatile one. The skewness values indicate evidence of asymmetric distributions of returns. The kurtosis values are different to the value of normal distribution, indicating leptokurtic behavior. The normal distribution hypothesis is rejected for all return series as showed by Jarque-Bera test. The Augmented Dickey-Fuller test (ADF) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests show that all return series follow a stationary process. The results of ARCH-LM test exhibit significant heteroscedasticity in the residuals.

Table 1	
Descriptive statistics of the intraday returns.	

	GOLD	S&P500	DXY	BRENT	USTBOND	BTC		
Panel A: Pre-COVID-19								
Mean	0.000019	0.00001	0.000001	-0.000025	0.000013	0.00005		
Maximum	0.01362	0.01804	0.00525	0.12095	0.01269	0.12696		
Minimum	-0.01167	-0.01788	-0.0058	-0.04785	-0.01263	-0.11864		
Std Dev	0.00124	0.00125	0.00044	0.0034	0.00101	0.00743		
Kurtosis	15.851	27.976	21.415	266.167	25.482	67.853		
Skewness	0.116	-0.909	-0.797	6.265	0.098	0.83		
Jarque-Bera	48881.5***	174308.1^{***}	86482.5***	18417794.0***	136770.1^{***}	1273103.8^{***}		
ADF	-84.06***	-80.98^{***}	-79.56***	-79.74***	-89.05^{***}	-85.41^{***}		
KPSS	0.043	0.04	0.042	0.021	0.034	0.05		
ARCH-LM (10) test	140.01^{***}	121.73^{***}	53.71^{***}	1.33	605.94***	38.64***		
Panel B: During-COVI	D-19							
Mean	0.00003	-0.000021	0.000005	-0.000152	0.000025	0.000043		
Max	0.02608	0.04156	0.0066	0.15761	0.02886	0.12042		
Min	-0.02089	-0.05093	-0.01014	-0.24113	-0.01723	-0.15583		
Std Dev	0.00204	0.00411	0.00078	0.00951	0.0018	0.00818		
Kurtosis	22.872	32.061	17.593	120.678	39.516	92.755		
Skewness	0.424	-0.733	-0.544	-2.072	0.967	-2.927		
Jarque-Bera	87793.8***	177069.8***	41266.7***	2762066.8***	287605.1^{***}	1846256.0^{***}		
ADF	-77.25^{***}	-76.01^{***}	-67.15^{***}	-70.19^{***}	-72.93^{***}	-72.36^{***}		
KPSS	0.031	0.088	0.059	0.115	0.051	0.093		
ARCH-LM (10) test	659.2***	695.6***	411.5***	18.07	518.6***	448.3***		

Note: This table presents the descriptive statistics of the 30 min returns before and during COVID-19 periods. *** indicates the rejection of null at the 1% significance level.

3. Methodology

3.1. Permutation entropy

To measure the pricing efficiency of the main asset classes, we follow Matilla-Garcia and Marin (2008). We assume that $\{X_t\}_{t \in I}$ is a real-valued time series. For a positive integer $m \ge 2$, S_m stands for the symmetric group of order m! and π_i , called a symbol is given by $\pi_i = (i_1, i_2, \dots, i_m) \in S_m$ where m stands for the embedding dimension.

To define an ordinal pattern for a symbol π_i , we consider that the time series is embedded in an *m*-dimensional space as $X_m(t) = (X_{t+1}, X_{t+2}, \dots, X_{t+m})$ for $t \in I$. Then, it is said that *t* is of π_i type if an only if $\pi_i = (i_1, i_2, \dots, i_m)$ is the sole symbol in the group S_m verifying two conditions:

$$X_{t+i_1} \le X_{t+i_2} \le \dots \le X_{t+i_m},\tag{1}$$

$$i_{s-1} \le i_s \text{if } X_{t+i_{s-1}} = X_{t+i_s}, \tag{2}$$

where the second condition guarantees the uniqueness of the symbol π_i . π_i describes how the ordering of the dates $t+0 < t+1 < \cdots < t+(m-1)$ is converted into the ordering of the values in the time series under scrutiny.

Given a time series $\{X_t\}_{t \in I}$ and an embedding dimension *m*, one could calculate the relative frequency of a symbol $\pi \in S_m$ as follows:

$$p(\pi): p_{\pi} = \frac{\#\{t \in I \mid t \text{ of } \pi - type\}}{|I| - m + 1},$$
(3)

where |I| stands for the cardinality of set *I*. Under this setting, the permutation entropy (PE) of a time series $\{X_t\}_{t \in I}$ for an embedding dimension *m* is defined as the Shannon's entropy of *m*! distinct symbols as follows:

$$h(m) = -\sum_{\pi \in S_m} p_{\pi} \ln(p_{\pi}), \tag{4}$$

Permutation entropy h(m), is the information embodied in comparing *m* consecutive values of $\{X_t\}_{t\in I}$. By definition, $0 \le h(m) \le ln(m!)$ where the lower bound is achieved for an increasing or decreasing sequence of values, and the upper bound for a completely random system where all *m*! possible permutations appear with the same probability. More simply, higher permutation entropy means that the data-generating process is more complex and unpredictable. If a time series has a PE that is significantly low, it exhibits evidence against efficiency because the weak-form market efficiency suggests the unpredictability of future price movements. To achieve a maximum level of 1, we normalize the PEs by dividing them by ln(m!).

An independence test by using PE is proposed by Matilla-Garcia and Marin (2008). If the real-valued time series $\{X_t\}_{t \in I}$ is with |I| = T and h(m) refers to the permutation entropy of $\{X_t\}_{t \in I}$ for a fixed integer embedding dimension m > 2. If $\{X_t\}_{t \in I}$ is *.i.d.*, then the affine transformation G(m) of the PE, $G(m) = 2(T - m + 1)(\ln(m!) - h(m))$, is asymptotically $\chi^2_{m!-1}$ distributed. To test the null hypothesis that $\{X_t\}_{t \in I}$ is *.i.d.*, the decision rule at $100(1 - \alpha)\%$ confidence level is to accept the null hypothesis if $0 \le G(m) \le \chi^2_{m!-1,\alpha}$, otherwise reject the null hypothesis. Lopez et al. (2010) document that the identicalness property in the null hypothesis can be eliminated.

To select the embedding dimension, we follow Matilla-Garcia and Marin (2008) who state that, for a given data set of *T* observations, the embedding dimension *m* should be selected as the largest *m* that satisfies $5m! \leq T$.

3.2. A-MF-DFA method

We explore the asymmetric multifractal scaling behavior sing the A-MF-DFA method of Cao et al. (2013). Let the time series $X = \{x(t)\}_{t=1}^{N}$, where N is the length of the series. The A-MF-DFA method is summarized with the following steps.

Step 1: We define the profile as follows:

$$y(t) = \sum_{j=1}^{t} (x(j) - \bar{x}), j = 1, 2, \dots, N,$$

$$\bar{x} = \frac{1}{N} \sum_{t=1}^{N} x(t)$$
(5)

Step 2: We divide the time series $\{x(t)\}$ and its profile $\{y(t)\}$ into $N_s = int(N/s)$ non-overlapping sub-time series of length s. As N may not be a multiple of time series n, the length of the last segment may be shorter than s. This procedure is repeated starting from the other end of the record; thus, $2N_s$ segments are obtained. Let $S_j = \{s_{j,k}\}_{k=1}^n$ be the j^{th} sub-time series of length n and $Y_j = \{y_{i,k}\}_{k=1}^n$ be the integrated time series in the j^{th} time interval, $j = 1, 2, \dots, 2N_s$. We define the j^{th} sub-time series as follows:

$$s_{j,k} = x((j-1)n+k), y_{j,k} = y((j-1)n+k),$$
(6)

for
$$j = 1, 2, \dots, N_n$$
 and
 $s_{j,k} = x(N - (j - N_s)n + k), y_{j,k} = y(N - (j - N_s)n + k),$
(7)

for $j = N_s + 1, \dots, 2N_n$, where $5 \le n \le N/4$ is selected as recommended by Peng et al. (1994).

Step 3: For each sub-time series $S_j = \{s_{j,k}, k = 1, \dots, n\}$ and its profile time series $Y_j = \{y_{j,k}, k = 1, \dots, n\}$, the least squares line fits are defined as $L_{S_j}(k) = a_{S_j} + b_{S_j}k$ and $L_{Y_j}(k) = a_{Y_j} + b_{Y_j}k$, with $k = 1, \dots, n$, respectively. The estimated fit $L_{S_j}(k)$ is used to identify the direction of the slope b_{S_j} where the trend of the sub time series S_j is positive or negative. The linear fit $L_{Y_j}(k)$ fits to detrend the integrated time series Y_i . Therefore, we define the fluctuation functions $F_i(n)$ as follows:

$$F_j(n) = \frac{1}{n} \sum_{k=1}^n \left(y_{j,k} - L_{Y_j}(k) \right)^2.$$
(8)

For each sub-time series, $j = 1, 2, \dots, 2N_n$.

Step 4: To assess the asymmetric cross-correlation scaling properties, the average fluctuation functions are considered in cases in which x(t) exhibits piece-wise positive and negative linear trends. This trend discrimination is made by using the sign of the slope b_{S_j} ; that is, $b_{S_j} > 0$ (b_{S_j}) indicates that the time series x(t) has a positive (negative) trend in the subtime series S_j . We compute the directional *q*-order average fluctuation functions, which are defined as follows:

$$F_{q}^{+}(n) = \left(\frac{1}{M^{+}}\sum_{j=1}^{2N_{n}}\frac{sign(b_{S_{j}})+1}{2}[F_{j}(n)]^{q/2}\right)^{1/q}, \ M^{+} = \sum_{j=1}^{2N_{n}}\frac{sign(b_{S_{j}})+1}{2},$$
(9)

$$F_{q}^{-}(n) = \left(\frac{1}{M^{-}}\sum_{j=1}^{2N_{n}}\frac{-[sign(b_{S_{j}})-1]}{2}[F_{j}(n)]^{q/2}\right)^{1/q}, \ M^{-} = \sum_{j=1}^{2N_{n}}\frac{-[sign(b_{S_{j}})-1]}{2},$$
(10)

where $F_q^+(n)$ and $F_q^-(n)$ denote the upward and downward *q*-order average fluctuation functions, respectively. Assuming that $b_{x_j} \neq 0$ for all $j = 1, \dots, 2N_n$, then $M^+ + M^- = 2N_n$.

Step 5: To calculate the generalized Hurst exponent, the traditional MF-DFA is performed by computing the average fluctuation function.

$$F_q(n) = \left(\frac{1}{2N_n} \sum_{j=1}^{2N_n} \left[F_j(n)\right]^{q/2}\right)^{1/q}.$$
(11)

The scaling or power-law relationship is defined as.

$$F_{q}(n) n^{H(q)}; F_{a}^{+}(n) n^{H^{+}(q)}; F_{a}^{-}(n) n^{H^{-}(q)},$$
(12)

where H(q), $H^+(q)$, and $H^-(q)$ are the overall, upward, and downward scaling exponents, respectively. The scaling behavior of the fluctuations in Eq. (12) is determined by analyzing the log–log plots of $F_q(n)$, $F_q^+(n)$, and $F_q^-(n)$ versus n for each value q. H(q), $H^+(q)$, and $H^-(q)$ can be estimated using the ordinary least square method based on the logarithmic form. Furthermore, the correlation in the time series is persistent or long memory when $H(2)\rangle 0.5$, whereas the correlation is anti-persistent when $H(2)\langle 0.5$. If H(2) = 0.5, the time series follows a random walk process. Similarly, if $H^+(q) = H^-(q)$, the correlation in time series is symmetric, whereas if $H^+(q) \neq H^-(q)$, the correlation in time series is asymmetric. The asymmetric scaling behaviour indicates that the correlations in time series are different positive and negative trends.

Table 2

Comparison of static PE in pre- and during- COVID periods for both returns and GARCH-filtered standardized returns.

	GOLD	S&P500	DXY	BRENT	BOND	BTC	mean PE
Panel A: Returns							
Pre-COVID PE	0.9915**	0.9911*	0.9907	0.9913	0.9913	0.9918*	0.9911
During COVID PE	0.9879***	0.9876***	0.9879	0.9875**	0.9883**	0.9876***	0.9879
	GOLD	S&P500	DXY	BRENT	BOND	BTC	mean difference
During COVID - Pre-COVID PE difference	-0.0036	-0.0035	-0.0028	-0.0038	-0.003	-0.0042	-0.0035^{***}
Panel B: GARCH(1,1) filtered returns							
Pre-COVID PE	0.9925	0.9912	0.9904	0.9915	0.9912	0.9916 ^{**}	0.9914
During COVID PE	0.9889*	0.9886	0.9868**	0.9879	0.9881^{**}	0.9884***	0.9883
	GOLD	S&P500	DXY	BRENT	BOND	BTC	mean difference
During COVID - Pre-COVID PE difference	-0.0036	-0.0026	-0.0036	-0.0036	-0.0031	-0.0032	-0.0033^{***}

Note: PE stands for normalized permutation entropy calculated with embedding dimension m = 6. The cutoff date is December 1, 2019. For the rows that start with PRE-COVID PE and During-COVID PE, *, *** and **** denote the rejection of efficiency at 10 %, 5 % and 1 % significance levels respectively. In the rows that examine the POST-COVID – During-COVID difference, **** denotes that mean PE across assets is significantly lower in the COVID19 phase compared to pre-COVID period.

Panel A: Pre-COVID-19

Panel B: During-COVID-19





Fig. 1. Dynamic normalized PEs for each market before and during COVID-19 crisis. **Note:** Black curve denotes the estimated time varying normalized permutation entropies for gold, Brent oil, Bitcoin, S&P500, Treasury bills, and US dollar index log-return series with 30-minute sampling frequency. In the figure, m is the embedding dimension that we use to calculate PE series, and GARCH denotes GARCH (1,1)-filtered standardized return series. The blue (red) markers denote the windows that we reject efficiency at 10% (5%) significance level. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

4. Empirical results

4.1. Permutation entropy results

We start our analysis with a static approach where the cutoff date is December 1, 2019. Table 2 shows the parameter values of static PE in pre- and during- COVID periods as well as their difference ($PE_{Pre-COVID}-PE_{DuringCOVID}$). A negative value means a decrease in efficiency (increase in inefficiency level). Accordingly, we see that pricing efficiency worsens for all asset classes. Moreover, the cross-sectional mean difference between the two periods is statistically significant, showing that this deterioration in the efficiency is significant. This implies that investors can invest historical information (past returns) to predict future prices and generate abnormal returns during the pandemic. This result is explained by the investor panic, irrational trade, and herding behaviors of market actors. COVID-19 pandemic has intensified the volatility of markets under study, generating a huge panic and fear as well as overreaction.

Before starting our dynamic analysis, we want to emphasize that our sample returns exhibit volatility clustering and fat tails (see Table 1). Thus, the presence of ARCH effects may make the results of PE spurious (Matilla-Garcia and Marin, 2008). To consider this limit, we estimate the PE using not only the log-returns but also GARCH (1,1) filtered returns (i.e., standardized returns). On the other hand, our dynamic setup utilizes a rolling window approach to account for the time-varying efficiency analysis. In particular, it reveals the number of times the null hypothesis is rejected by the PE test statistic when window runs over the sample, and hence the percentage of sub-samples with an insignificant test is applied to compare the relative efficiency of the different asset classes before and during COVID-19 outbreak (Sensoy et al., 2017). At this stage, we carry out the PE under a 2-week rolling window approach (that jumps 1 day at a time) since (i) it allows us to catch the time-variation of the PEs, and (ii) it is large enough to provide satisfactory statistical significance. In this way, we have the flexibility of not being forced to impose cutoff dates which are usually subject to criticism in empirical studies.

Panels A and B of Fig. 1 show the dynamic PE for each market before and during COVID-19 outbreak for each log-returns and GARCH (1,1) filtered returns series, respectively. As we can see, the degree of dependence for all markets is time-varying and affected by COVID-19 outbreak. During Pre-COVID outbreak, the dependence is more volatile for US *T*-bond and SP500 stock market index returns, whereas Brent oil price and US dollar index returns demonstrate less volatile dynamic dependence. During COVID-19 outbreak, gold exhibits less volatile dependence. All markets experience some degrees of inefficiencies in pre- and during COVID-19 outbreak. In addition, a rough observation shows that the markets become more inefficient during the pandemic period. To statistically test this assertion, we proceed with the efficiency ratio analysis.

Table 3 presents the results of efficiency ratios pre- and during the pandemic outbreak. We show that all markets become more inefficient during the pandemic period as the efficiency ratio is higher before COVID-19 outbreak for all markets (with the exception of bond market when log-returns is used and for gold when GARCH (1,1) filtered returns is considered). This finding mostly supports that of the static analysis. Moreover, Brent oil is the highest efficient market before and during COVID-19 outbreak. This result underscores that the information is not embodied in prices instantaneously in all markets at the same rapidity. Evidently, the information diffusion process takes more time for Bond than for Brent oil. We conclude that the degree of efficiency is heterogeneous among all markets. BTC becomes more inefficient during the COVID-19 pandemic than before. This result is consistent with the findings of Wang and Wang (2021) who used the entropy-based analysis to explore the efficiency of gold, S&P 500 index, BTC, and US Dollar Index during the pandemic period.

For another dynamic perspective, we compare the sample PE values obtained from a rolling window approach in pre- and during-COVID19 periods. In particular, we formally test whether the mean PE value is smaller in the second phase compared to the first one. At this stage, we use a two-sample *t*-test, where we account for inequality in variances by Satterwhite approximation. The results are provided in Table 4, where we see that mean PE decreases during COVID19 period for all asset classes. Moreover, this decrease is significant for gold, S&P500 and Bitcoin (US dollar index and Bitcoin) when we use raw returns (GARCH filtered returns), supporting our earlier findings and further confirming the negative impact of the outbreak on market efficiency.

Finally, to see whether the negative impact of the outbreak is an instant response or a trend for the market efficiency, we compare the time trends of dynamic permutation entropies in pre- and during- COVID periods. The results, provided in Table 5, show evidence of worsening efficiency during the pandemic; moreover, this effect seems to have a trend during the pandemic period since almost all trend coefficients of the dynamic PEs are negative. Furthermore, negative trend coefficient is significant for S&P500, US dollar index, Brent oil and US Treasury bond (S&P500, US dollar index and Brent oil) if we use log-returns (GARCH filtered returns).

4.2. A-MF-DFA results

We show in Fig. 2 the Asymmetric MF-DFA functions $F_2(n)$ versus the time scale n of the market returns. The results reveal a significant asymmetric multifractality for all market and for various time scales. Specifically, the downward trend deviates from the upward trend for all markets, which is more pronounced at higher time scales. This exhibits that long-term investors should pay attention to this asymmetric long-range dependence. The extent of the asymmetry differs across the six markets. It is more apparent in





Fig. 1. (continued).

Dec.19

Feb.20

Mar.20

May.20





Fig. 1. (continued).

Table 3

Efficiency ratio results for raw returns and GARCH-filtered standardized returns.

		S&P50				
	GOLD	0	DXY	BRENT	BOND	BTC
Panel A: Returns						
Pre-COVID Efficiency Ratio (at 10% significance level)	0.8182	0.8455	0.8272	0.9402	0.7083	0.9118
During COVID Efficiency Ratio (at 10% significance level)	0.7789	0.7528	0.8108	0.9048	0.7473	0.6327
Pre-COVID Efficiency Ratio (at 5% significance level)	0.9015	0.9024	0.9009	0.9915	0.8333	0.9706
During COVID Efficiency Ratio (at 5% significance level)	0.8105	0.8427	0.8642	0.9286	0.8571	0.6837
Panel B: GARCH(1,1) filtered returns						
Pre-COVID Efficiency Ratio (at 10% significance level)	0.8485	0.8862	0.8468	0.9658	0.8333	0.8162
During COVID Efficiency Ratio (at 10% significance level)	0.9053	0.7978	0.7284	0.9167	0.7912	0.7245
Pre-COVID Efficiency Ratio (at 5% significance level)	0.9167	0.9024	0.9369	0.9915	0.875	0.8676
During COVID Efficiency Ratio (at 5% significance level)	0.9474	0.8652	0.7531	0.9405	0.8352	0.8469

Note: We use a rolling window with length 15 business days and that jumps 1 day at a time. In each window, we calculate permutation entropy for the selected series with an embedding dimension m = 5 and test for efficiency. When the window runs over the whole, pre- (and during-) COVID periods, we calculate the efficiency ratio by dividing the number of windows that we cannot reject efficiency by the total number of windows in the pre- (and during-) COVID periods. Higher efficiency ratio is an indicator of better market efficiency. For each time period and return type, red values indicate the least efficient market whereas shaded values stand for the highest efficient market.

Table 4

Mean PEs obtained from rolling windows in pre-COVID and during-COVID periods.

	Pre-COVID PE mean (Rolling window)	During COVID PE mean (Rolling window)	Pre-COVID mean – During COVID mean difference p-value				
Panel A: Me	an PE results (raw returns)						
GOLD	0.9825	0.9817	0.02**				
S&P500	0.9823	0.9817	0.05**				
DXY	0.9828	0.9827	0.91				
BRENT	0.9834	0.9833	0.60				
BOND	0.9812	0.9816	0.88				
BTC	0.9827	0.9799	0.00****				
Panel B: Mean PE results (GARCH(1,1)-filtered standardized returns)							
GOLD	0.9824	0.9825	0.60				
S&P500	0.9826	0.9824	0.33				
DXY	0.9816	0.9806	0.01****				
BRENT	0.9825	0.9822	0.18				
BOND	0.9827	0.9824	0.23				
BTC	0.9821	0.9809	0.00****				

Note: PE stands for normalized permutation entropy. In the first two columns, we present the average PE values obtained from rolling windows in the pre-COVID and during-COVID periods respectively. The third column presents the *p*-values for the test that examines whether the pre-COVID mean is significantly higher than post-COVID mean for each asset class. ** and **** denote the significance at 5% and 1%, respectively.

Table 5

Comparison of time trends of permutation entropies in before and during COVID crisis.

	time trend coefficient \times 10^5	<i>t</i> -stat	p-value
Panel A: Returns			
Pre-COVID			
GOLD	-0.75	-1.56	0.12
S&P500	-0.02	-0.04	0.97
DXY	2.59***	3.69	0.00
BRENT	-0.84	-1.26	0.21
BOND	-1.95^{***}	-2.65	0.01
BTC	1.79****	3.89	0.00
During COVID			
GOLD	-0.20	-0.20	0.84
S&P500	-3.26^{***}	-2.90	0.00
DXY	-5.55****	-3.72	0.00
BRENT	-2.33^{**}	-2.29	0.02
BOND	-2.16**	-2.24	0.03
BTC	0.33	0.27	0.79
Panel B: GARCH(1,1) filtered return	IS		
Pre-COVID			
GOLD	-1.35^{**}	-2.46	0.02
S&P500	3.36***	4.71	0.00
DXY	1.22**	2.03	0.05
BRENT	-0.46	-0.68	0.50
BOND	2.29****	3.42	0.00
BTC	2.94***	5.50	0.00
During COVID			
GOLD	-0.92	-1.11	0.27
S&P500	-7.59***	-7.16	0.00
DXY	-5.459****	-5.13	0.00
BRENT	-2.52*	-1.97	0.05
BOND	-0.18	-0.12	0.91
BTC	1.73*	1.98	0.05

Note: PE stands for normalized permutation entropy. For each asset, we fit a simple linear model to the time-varying PEs obtained from rolling windows in pre- COVID and during- COVID periods respectively, i.e., PE = a + bt, where *a* is the intercept, *t* is the time trend and *b* is the corresponding trend coefficient. A positive (negative) trend coefficient is an indicator of improving (deteriorating) market efficiency. *, **, and *** denote significance at 10 %, 5 % and 1 %, respectively.

the S&P500 and Brent oil markets. Moreover, we find that the extent of downtrend multifractality is higher than the uptrend multifractality for S&P 500 index, DYX, Brent oil and US Treasury Bond. In contrast, the multifractality in uptrends is higher than those in downtrends for BTC and gold. The evidence of the crossover reveals that the behavior of investors is heterogeneous, depending on horizons. In addition, the investment strategies are sensitive to multifractal behaviors. In addition, the difference is the size of asymmetric multifractality can be attributed to the depth, maturity, and the development of each market.

Fig. 3 plots the excess asymmetry in multifractality for markets under study. The value of excess asymmetry in multifractality $(\Delta h(q))$ is the difference between the upward Hurst value $(h^+(q))$ and the downward Hurst value $(h^-(q))$. When $\Delta h(q)$ is zero, it indicates evidence of symmetric multifractality that is upside multifractality is equal to the downside multifractality. Conversely, when $\Delta h(q)$ is different to zero and the higher, it is the more asymmetric multifractality exist in the price return series. The graphs show a significant scale-dependent excess asymmetric in multifractality for all return series. This result shows the importance of disentangling downtrends from uptrends and thus the advantage of A-MFDA method relative to symmetric MF-DFA. More importantly, the trajectory of the excess asymmetry in multifractality differs across markets. For gold market, we find a positive value of excess multifractality, indicating that the cross-correlation exponent is higher when the price returns have a positive trend than when it has a negative trend. The excess asymmetry in multifractality has a positive value in most periods, implying that the multifractality is much stronger in upward price movements. Similar result is found for BTC and US bonds. This reveals that the positive trends generate higher cross-correlations compared to negative trends. In contrast, S&P500, Brent oil and US dollar index exhibit a negative excess asymmetry in multifractality in downward S&P500, Brent oil price and US dollar index movements. Overall, our result reports evidence against informational efficiency of the considered markets. The large excess multifractality confirms the appropriateness of A-MF-DFA method.

To confirm this result, we illustrate in Fig. 4 the overall, downward, and upward Hurst exponent values. We observe that the Hurst exponent differs for overall, downward, and upward trends. Looking at both gold, Brent oil, US dollar index and US bonds, the graph shows that the deviation of upward Hurst and downward Hurst is important for positive scales. This suggests that the gold, Brent oil, bonds and US dollar index markets are more inefficient during upward trends than in its overall or downward trends. These threes markets experience anti-persistence behavior for high scales as the Hurst value is below 0.5. They follow a mean reverting process for positive scales. For S&P500 and BTC the deviation of upward and downward Hurst is higher under negative scales, suggesting a higher efficiency during downward trends. The inefficiency is higher under upward trends for US dollar index irrespective to the scale. In



Fig. 2. Asymmetric MF-DFA functions $F_2(n)$ vs the time scale (n). Note: This figure represents the plot of $log_{10}(F_2(n))$ vs $log_{10}(n)$ for each intraday returns.

contrast, the inefficiency higher under downward trend for different scales for gold, S&P500, Brent oil, and bonds. This result is consistent with Mensi et al. (2022) for the case of gold. For BTC, the inefficiency is higher under downward trends for negative scale and under upward trends for positive scales. Moreover, we show a significant persistence or long-range memory under negative scales



Fig. 3. Excess asymmetry in multifractality for intraday returns. **Note:** The x-axis represents the time scale n, which varies from 5 to N/4 (where N is the number of observations in the time series). The y-axis represents the difference between $log_{10}(F_2^+(n))$ and $log_{10}(F_2^-(n))$.



Fig. 4. Plots of Hurst exponents for stock markets. **Note:** This figure shows the trend of overall H(q), upwards $H^+(q)$, and downwards $H^-(q)$ versus q (q = -10, -9, ..., 9, 10).

and anti-persistence for positive scales. We notice that the values of Hurst exponent decrease with the increase of q, implying high correlations under small fluctuations than large fluctuations.

Fig. 5 depicts the asymmetric multifractal spectrum. The graphical evidence shows that the multifractality of the gold, Brent oil and



Fig. 5. Asymmetric multifractal spectrum.

BTC has a very large width compared to the remaining markets. This suggests evidence of strong multifractality in these markets. More importantly, a very large width of multifractality for downward trends compared to those of the upward trends is observed for BTC market. In addition, we observe that the spectrum exhibits an inverted parabola shape for all markets. This inverse U-pattern supports previous findings of asymmetric multifractality. In addition, we show that S&P500, gold and BTC have a spectrum centered at α , which is equal to 0.5. In addition, the width of multifractal spectrum in downtrends is wider than those in uptrends. This implies that down multifractality is higher than upper multifractality.

To assess the degree of market efficiency, we follow Wang et al. (2009) to compute the market efficiency measure (MDM) as follows:

$$MDM = \frac{1}{2} (|h(-4) - 0.5| + |h(4) - 0.5|).$$
(13)

A market is efficient if all kinds of fluctuations, such as small fluctuations (q = -4) and large fluctuations (q = +4), will follow a random walk process. Therefore, the MDM value will be zero for an efficient market. Conversely, it will have high value for an inefficient market.

Table 6 reports the market deficiency measure (MDM) of the overall, upward, and downward Hurst exponent trends for the whole period, before and during COVID-19 crisis. As we can see in Panel A of Table 6, gold, Brent oil, and US treasury bonds are more inefficient during upward market conditions whereas the US dollar index, BTC, and S&P500 index are more inefficient during downward market conditions. Before the COVID-19 (Panel B), all markets are more inefficient during downward trend. However, we find that gold and BTC are more inefficient during upward trend during the pandemic spread (Panel C). Conversely, Brent oil, US bonds, US dollar index and SP500 index are more inefficient during downward trend. Overall, we observe that the inefficiency is higher for gold during the pandemic spread under upward movements, implying large upside inefficiency than the downside inefficiency. This indicates that the predictability of future prices of gold is higher during upside trends. The inefficiency is higher during the pandemic for S&P500, DXY, US bond markets irrespective of movement trends. Conversely, BTC is less inefficient during the COVID-19 crisis. A for Brent oil market, it is more inefficient during the pandemic under downward movements. On the other hand, S&P500 is the least inefficient market before COVID-19 period for overall and upward trends and DXY for downward trend. During the pandemic, DXY is the least inefficient market for overall and downward trends and Brent for upward trend. This inefficiency of BTC market indicates the possibility of predicting future cryptocurrency returns based on past information. The predictability becomes more pronounced during the COVID-19 and particularly for downward movements. The long-range memory and fat tails are the common source of multifractality. These are due to the uncertainty, different investor's risk appetite, investor heterogeneity, macro shocks. These variables lead to over-speculation, generating inefficiency of financial and commodity markets.

In sum, the transition in the efficiency degree is explained by the heterogeneous responsiveness to the global health crisis. The different patterns of inefficiency explain the complexity of price dynamics of financial and commodity markets. Besides, the rise in the values of up/down MDM during the pandemic crisis reveals the presence herding behaviors of market actors. The external economic and financial stocks are the driver of upside and downside inefficiency of the analyzed indexes. Moreover, it is worth noting that the difference in the inefficiency level may be attributable to the fact that prices of some assets like US dollar index, treasury bonds and gold experience an upside trend during economic or health crises, playing the role of a safe haven asset. In contrast, the price of S&P500 and Brent oil experience a downside pattern during times of financial and energy crises. Therefore, the efficiency of the markets under investigation is sensitive to the market conditions (bearish, tranquil, and bullish market scenarios). This finding reveals that financial, energy, and precious metals are not a single asset class and have different risk-return pattern. To monitor the panic and fear during the ongoing pandemic, the policymakers can issue various regulations including restrictions of positions and increased margins, reducing market-wide position limits for volatile scripts.

5. Conclusion

This study is the first to examine the impact of COVID-19 outbreak on the degree of informational efficiency and asymmetric

	COLD	SDEOO	DVV	DDENIT	LIETROND	PTC		
	GOLD	3F300	DAI	BRENI	USIBOND	ыс		
Panel A: Whole perio	bd							
Overall	0.3254	0.1011	0.1062	0.1317	0.2189	0.1722		
Upward	0.3268	0.1262	0.1208	0.2584	0.3248	0.1160		
Downward	0.2842	0.1976	0.1808	0.2421	0.2664	0.2196		
Panel B: Pre COVD-	-19							
Overall	0.3653	0.0928	0.1319	0.1180	0.1159	0.2231		
Upward	0.2896	0.0552	0.1265	0.1424	0.1024	0.2184		
Downward	0.4112	0.2412	0.1413	0.1450	0.1554	0.2263		
Panel C: During COVID-19								
Overall	0.2442	0.1553	0.1414	0.1473	0.1762	0.1583		
Upward	0.3717	0.2018	0.1371	0.1312	0.2038	0.1927		
Downward	0.2178	0.4438	0.1509	0.2034	0.4152	0.1773		

Table 6

Note: The bold values indicate the most inefficient market in each intraday returns.

multifractality in different asset classes, including gold, oil, Bond, Bitcoin, US dollar index, and S&P500. We apply the permutation entropy and the asymmetric MF-DFA methods with a rolling window approach to intraday data.

Accordingly, both static and dynamic analysis show a deterioration in pricing efficiency for almost all markets during COVID-19 outbreak, and in many cases, this deterioration is significant. Moreover, trend analysis shows that the worsening in pricing efficiency is not a one-off instant response to COVID-19 outbreak but a drift, suggesting that the deterioration in the efficiency of these asset classes will continue in the near future. On the other hand, we show using the A-MF-DFA approach significant asymmetric multifractality which rises with scale increases. The magnitude of downtrend multifractality is higher than the uptrend multifractality for S&P 500 index, DYX, Brent oil and US Treasury Bond whereas for both BTC and gold the uptrend multifractality exceeds the downward multifractality. Besides, the downward trend Hurst is higher than the upward trend Hurst for all markets except DYX as well as BTC for positive scales. COVID-19 crisis intensifies the degree of inefficiency for all markets except Bitcoin.

In terms of implications, findings suggest that there exist exploitable patterns in asset prices that makes these markets more speculative in the COVID-19 period. Regulators and policymakers should be more careful and even be pro-active in the pandemic period. Moreover, all sample assets in our paper serve as underlying to many derivative products. Many models of derivatives pricing assume the randomness of the underlying assets' prices. However, our findings reveal that this is not the case during COVID-19 period. Therefore, it is possible that mispricing occurs more than there used to be in the derivatives markets around these days, which might lead to new risk measurement-related problems in financial markets.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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