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EDITORIAL

Guest Editors' Foreword

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The 36th International Symposium on Computational Geometry (SoCG) was originally planned to take place in Zürich, Switzerland, but was actually held online because of the COVID-19 pandemic, June 22–26, 2020, as part of the Computational Geometry Week (CG Week 2020). 205 papers have been submitted to SoCG 2020 from which the Program Committee accepted 70 papers for presentation at the symposium. This special issue of *Discrete & Computational Geometry* (*D&CG*) contains the full versions of ten papers from the symposium that were invited because they received a strong support from the Program Committee and provide a sample of the excellent research presented at the symposium. These papers were submitted, reviewed, and revised according to the usual high standards of *D&CG*.

We thank the authors of all the submitted papers for revising and polishing their work. We are very grateful to the anonymous referees for their dedication, expertise, and effort that ensure the high quality of the papers in this special issue. We also thank Ken Clarkson, Co-Editor-in-Chief, for his guidance through the process.

The papers in this special issue appear in alphabetical order of the names of the first authors. In the remainder of this foreword, we briefly introduce all papers arranged into broad topics.

Discrete geometry. Given two simple polygons in the 2D plane, one with $m \geq 3$ vertices, the other with $n \geq 3$ vertices, and with no two overlapping edges of the two polygons, what is the maximum number of intersections of their boundaries? This is a basic extremal geometric problem. The cases of this problem have been settled when at least one of m and n is even. When both m and n are odd, the best-known

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construction has $mn - (m + n) + 3$ intersections, and it was conjectured that this is the maximum. After several previous studies, the best-known upper bound was only $mn - (m + \lceil n/6 \rceil)$, for $m \geq n$. *Eyal Ackerman*, *Balázs Keszegh*, and *Günter Rote* prove a new upper bound of $mn - (m + n) + C$ for some constant C, which is optimal apart from the value of C.

Triangulations of point sets are commonly utilized in many applications. A natural way to provide a structure to the collection of all triangulations for a point set is to consider a graph, called *flip graph*, in which the triangulations are vertices and any two triangulations are adjacent if they can be obtained from each other by a minimal local change, called a flip operation. *Uli Wagner* and *Emo Welzl* study vertex connectivity of flip graphs for any general position point set in the plane, based on full triangulations and partial triangulations (two types of triangulations commonly considered in computational geometry or discrete geometry). They prove tight vertex connectivity bounds for these flip graphs.

Geometric algorithms. Given three point sets A, B, and C of size n each in the plane, is there a triple of points $(a, b, c) \in A \times B \times C$ that satisfies one or two prescribed polynomial equations? A well-known example of this problem, with a single (vanishing) polynomial, is to determine the existence of a collinear triple of points in $A \times B \times C$, which is a classical 3SUM-hard problem for which so far no subquadratic time solution is known. *Boris Aronov*, *Esther Ezra*, and *Micha Sharir* study this problem in the algebraic decision-tree model when some of the sets lie on algebraic curves of constant degree, and present subquadratic solutions for several challenging settings. This research combines the recent trends of using real algebraic geometry and considering the algebraic-tree model of computation.

By Tverberg's theorem, for any $k \geq 2$ and any set P of at least $(d + 1)(k - 1) + 1$ points in d dimensions, P can be partitioned into k subsets whose convex hulls have a non-empty intersection. The associated search problem (finding such a partition) lies in the complexity class CLS = PPAD $\cap$ PLS, but no hardness result is known. Its "colorful" version has also been studied and the complexity of the associated search problem is not yet resolved. *Aruni Choudhary* and *Wolfgang Mulzer* present a deterministic algorithm for finding, for any n-point set P in d-D and any $k \in \{2, 3, \ldots, n\}$, a k-partition of P in $O(nd\lceil \log k \rceil)$ time such that there is a ball of radius $O((k/\sqrt{n})\,\mathrm{diam}(P))$ that intersects the convex hull of each subset. This provides an efficient and simple new notion of approximation.

Computational topology. Efficiently computing persistent homology of a point cloud is an important problem in computational topology. For a subset A of a point cloud X in a low dimensional Euclidean space, relative Čech persistent homology can be computed as the persistent homology of the relative Čech complex. However, this approach is not computationally feasible for larger size point clouds. *Nello Blaser* and *Morten Brun* present a new efficient method for computing relative Čech persistent homology in the d-D Euclidean space. Their approach is to embed X as Z in the $(d + 1)$-D Euclidean space, where the extra dimension encodes whether a point belongs to A or $X \setminus A$, and then introduce the concept of relative Delaunay–Čech complex of X by using an appropriate filtration of the Delaunay complex of Z.

Single-parameter persistence modules are commonly attained by applying homology to a filtered topological space. This process is called persistent homology and finds a wide range of applications. Since many applications are involved with multiple parameters, it is natural to consider multi-parameter persistence modules. *Magnus B. Botnan*, *Vadim Lebovici*, and *Steve Oudot* focus on 2-parameter persistence modules and explore two questions: (1) Can one identify a sensible class of 2-parameter persistence modules on which the rank invariant is complete? (2) Can one determine efficiently whether a given 2-parameter persistence module belongs to this class? With their class of interest being rectangle-decomposable modules, they provide positive answers to both these questions.

Given a d-chain A in a simplicial complex K and a set of weights provided by a diagonal matrix W of appropriate dimension, the Optimal Homologous Chain Problem (OHCP) seeks an L_1-norm minimal chain $\Gamma_{\min}$ homologous to A. While it is already known that OHCP is NP-hard in the classical setting, *David Cohen-Steiner*, *André Lieutier*, and *Julien Vuillamy* study a special case of OHCP, the Lexicographic Optimal Homologous Chain Problem (Lex-OHCP), for integer modulo 2 coefficients, where the optimality means a minimal lexicographic order on chains induced by a total order on simplices. They present polynomial time algorithms for Lex-OHCP, and show an application of Lex-OHCP in the context of point cloud triangulation.

When analyzing graph structures, traditional topological data analysis (TDA) has focused largely on undirected graphs. However, more and more applications are involved with directed graphs, and thus new TDA techniques for directed graphs are desired. Path-homology developed for directed graphs has been utilized to analyze directed graphs from a topological viewpoint. *Tamal K. Dey*, *Tianqi Li*, and *Yusu Wang* develop an algorithm for computing this path-homology and its persistence more efficiently for the 1-dimensional case, by exploiting various structures and their efficient computations for 1-D path-homnology. They also present experimental results for their algorithm implementation.

Geometric centers. For a convex body K (K is a compact convex set with nonempty interior) in n dimensions and a point p in the interior of K, a hyperplane h passing through p is called *barycentric* if p is the barycenter (also called the centroid) of $K \cap h$. A proof in 1963 stated that for any convex body K in n-D, there exists an interior point p of K through which there are at least $n+1$ distinct barycentric hyperplanes, where $p = p_0$ is the point of maximal depth in K. *Zuzana Patáková*, *Martin Tancer*, and *Uli Wagner* show that one of the auxiliary claims in the 1963 proof is incorrect by providing a counterexample. This re-opens the question on barycentric cuts of a convex body. They also prove that four distinct barycentric cuts through p_0 are guaranteed if $n \geq 3$.

Given a set S of n points in the Euclidean plane, the 2-center problem seeks to find two congruent disks of the smallest radius whose union covers all the points of S. This is a classical problem in Computational Geometry that has attracted considerable attention and for which most results were obtained two to three decades ago. *Haitao Wang* presents a deterministic $O(n \log^2 n)$ time algorithm for this problem, improving the previous best-known result over 20 years ago. The paper also presents a deterministic $O(n \log n \log\log n)$ time algorithm for the case when the points of S are in convex position, that is, when every point of S is a vertex of the convex hull of S. To achieve the main results, Wang develops new techniques to dynamically maintain circular hulls of points, a setting that appears in several other geometric problems.