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Environmental influences on mathematics performance in early childhood

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Abstract

Math skills relate to lifelong career, health, and financial outcomes. Individuals' own cognitive abilities predict math performance and there is growing recognition that environmental influences including differences in culture and variability in math engagement also impact math skills. In this Review, we summarize evidence indicating that differences between languages, exposure to math-focused language, socioeconomic status, attitudes and beliefs about math, and engagement with math activities influence young children's math performance. These influences play out at the community and individual level. However, research on the role of these environmental influences for foundational number skills, including understanding of number words, is limited. Future research is needed to understand individual differences in the development of early emerging math skills such as number word skills, examining to what extent different types of environmental input are necessary and how children's cognitive abilities shape the impact of environmental input.

Introduction

Every day people face situations requiring mathematics, from calculating a budget to following a recipe. These activities require an understanding of symbolic math—which involves number symbols (including Arabic numerals such as '8' and written or spoken number words such as 'eight')—to identify and compare numbers and perform arithmetic. Despite frequent usage of math in daily life, there is wide variability in math performance across individuals. Individual differences in math abilities predict educational attainment, income, career choice, likelihood of full-time employment, and health and financial decision-making[1,2,3,4]. These differences begin in childhood; even before children have received formal schooling there are considerable individual differences in math performance, which tend to persist throughout the school years and into adulthood[5,6]. Decades of research have focused on sources of variability in symbolic math abilities across the lifespan.

Children have a set of individual abilities and are nested in their larger family and school environment, which are nested in broader communities, culture, and socio-historical context.

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Young children's symbolic math development can be broadly explained on the basis of their ability to learn math and the opportunities provided in their environment. There are many cognitive skills related to the development of symbolic math skills, including abilities that span cognitive domains and others that are math-specific[7]. At the environment level, factors ranging from the local family and school level to the broader community and cultural levels might help support children's symbolic math development. Ongoing work implicates this environmental input in symbolic math performance, including indirect influences on the domain-general cognitive processes involved in symbolic math performance and the core systems of number processing as well as direct effects on symbolic math skills.

In this Review, we briefly summarize the abilities most frequently associated with children's symbolic math skills and then focus on the role of environmental influences in children's symbolic math development. Specifically, we review evidence suggesting that variations in language, math-related beliefs and attitudes, socioeconomic factors, engagement with math activities, and individual differences in beliefs and attitudes influence the development of symbolic math abilities. However, much of this research has focused on symbolic math abilities beginning around age three, raising questions about the role of environmental influences for foundational symbolic math skills that emerge earlier in development. Chief among these foundational skills is the acquisition of number words during infancy (0-12 months old) and toddlerhood (12-36 months old); we discuss what is currently known about environmental influences on this process. We conclude by suggesting future directions for research that examines bidirectional influences between abilities and the environment.

Abilities supporting symbolic math

Many cognitive abilities are related to the development of symbolic math skills, including abilities within numerical cognition (domain-specific) and abilities outside this domain (domain-general) (Fig. 1). Domain-general cognitive abilities relevant to early math development include executive function, attention, and reasoning. Domain-specific abilities include the two core systems of number processing that support symbolic math skills and other foundational math abilities upon which more advanced symbolic math skills are built.

Domain-general cognitive abilities

Cognitive abilities employed across domains— such as numerical cognition, language, and visual cognition— are considered domain-general. Performance on many domain-general cognitive tasks is associated with math performance starting in the early childhood years and continuing into adulthood. Executive functioning abilities including inhibition (the ability to ignore distractions and suppress a response), working memory (temporary storage and manipulation of information) and cognitive flexibility (the ability to switch between tasks or hold multiple concepts in mind simultaneously) are closely tied to a broad range of math skills in early childhood[8,9, 10,11]. These executive functioning abilities are particularly useful for learning and using symbolic math skills, as focusing on, manipulating, and holding in mind relevant information for problem solving is critical for math. Stronger executive function is typically associated with better math performance. Attention, or the ability to focus on a particular concept or sensory input, is also related to academic math performance in early childhood[10]. The efficiency of performing cognitive tasks, known as processing speed, is related to math achievement on standardized assessments in early childhood, even on assessments without a timed component[12]. Finally, logical reasoning abilities, which enable problem solving, and language abilities, including preliteracy skills such as the recognition of letters and speech sounds, are related to academic math performance in childhood[13], and math achievement on standardized assessments[8,10] throughout development.

Across these domain-general abilities, children with stronger cognitive abilities tend to perform better in math than children who struggle with these abilities, likely due to their ability to employ math skills more flexibly and easily for problem solving in the process of learning and utilizing math. Domain general abilities develop throughout the lifespan alongside math skills and likely support the acquisition of symbolic math skills and the application of math knowledge when solving math problems, although the exact causality of these links remains unknown as most evidence has been correlational in nature.

Math-specific cognitive abilities

Multiple domain-specific abilities have been studied within numerical cognition (Fig. 2). Two core systems of numerical representations present from birth have been suggested to form the basis of math abilities: the Object Tracking System and the Approximate Number System[14]. While there is considerable debate in the field regarding the existence and nature of the Approximate Number System[15,16,17,18,19], we approach our discussion from the perspective of the Approximate Number System as a core non-symbolic numerical system. The Object Tracking System and the Approximate Number System represent numerical information in different ways and as such support the development of foundational number skills that subsequently support the acquisition and execution of more advanced symbolic math processes[14].

The Object Tracking System enables temporary storage of information about the perceptual properties of objects ('object files'). However, the Object Tracking System is subject to memory limits: humans can exactly discriminate and represent only up to three or four objects with this system[20]. The development of symbolic math skills is thought to be aided by the one-to-one correspondence between objects in the real world and object files in memory, which enables exact representation and tracking of small quantities[21,22,23,24,25,26,27]. For example, 12-month-olds to 14-month-olds can use the Object Tracking System to track two objects by creating 'object files' to temporarily store information about each of those objects that can be used even when the objects are out of sight. Infants reliably search longer for more objects in a box if they initially saw two objects being hidden and only retrieved one than when only one object was hidden and retrieved, demonstrating that they maintain information about the second object[28]. The Object Tracking System is generally thought to be a universal system present across species[14], but it is unclear whether there is variability in this system across individuals and whether variability might be related to individual differences in symbolic math skills.

In contrast to the Object Tracking System, the Approximate Number System supports the ability to estimate large quantities without number symbols (non-symbolically). This system relies on sets of objects to represent numerical information approximately[29,30]. The Approximate Number System uses imprecise, noisy mental representations to produce estimates of set sizes. The ability to tell the difference between two sets of objects using the Approximate Number System is ratio-dependent, such that performance is more accurate for sets with a larger ratio between their sizes than for sets with a smaller ratio between their sizes. For example, 6-month-old infants are typically able to discriminate between a set of 8 and a set of 16 dots as accurately as between a set of 10 and a set of 20 dots (ratio of 1:2 in both cases), but they are unable to discriminate between a set of 8 and a set of 12 (a ratio of 2:3)[31]. The Approximate Number System becomes more precise, allowing for better acuity, or discrimination of more difficult ratios, throughout infancy, childhood, and early adulthood, reaching full maturity at approximately 30 years of age[32].

The Approximate Number System is theorized to support the acquisition of symbolic math skills through mapping its non-symbolic representations with symbolic number representations[33,34,35,36,37,38,39,40,41]. Under this view, children refine their knowledge of symbolic numbers very slowly as the imprecise representations of number in the Approximate Number System that underlie their symbolic number representations slowly become more precise. Importantly, starting in infancy, individuals vary substantially in the acuity of their Approximate Number System representations[31]. These individual differences in the Approximate Number System are related to variability in symbolic math skills from early childhood through adulthood with typically moderate effect sizes (with Cohen's d of approximately .50 or r^2 of .06)[42,43,32,39,44,40,45]. This relation is present in both typically developing and atypically developing populations, including gifted individuals and individuals with dyscalculia[46,47,39, 48].

In addition to the two core systems of number, the development and acquisition of other foundational math abilities during the infant and toddler years is associated with the development of more-advanced understanding of symbolic math. However, these abilities rely on cultural transmission and learning to a greater extent than the Object Tracking System and the Approximate Number System, which seem to be automatically employed.

Children who are better at identifying and understanding written number symbols, including the order in which those numbers appear in the count list, tend to perform better in symbolic math tasks[49,50, 51,52]. Children with better understanding and knowledge of quantifiers (math language words that express inexact quantitative information such as 'more' and 'most') [53,54,55,56] and with a greater tendency to spontaneously focus on number (that is, attend to and utilize numerical information without being prompted) [57,58,59,60] also tend to perform better in symbolic math. Finally, spatial abilities—used to manipulate the location and orientation of objects and the environment mentally and physically and to understand patterns and spatial language—are closely related to symbolic math skills such that individuals who have stronger spatial abilities also tend to perform better in symbolic math assessments[61,62,63,64,65].

Considering children's domain-general and domain-specific cognitive abilities alone cannot explain all of the variability in symbolic math performance in early childhood. Prior work has explored the relation between multiple cognitive abilities and children's symbolic math performance[8,66,67]. However, these studies are often correlational and therefore cannot indicate a causal relation, and they omit other potential sources of variability. Importantly, studies relating cognitive abilities to symbolic math skills often do not include controls for potential confounds beyond individual abilities, which may also overlap with one another. For example, including children's domain-general (IQ, working memory) and domain-specific cognitive abilities (counting, Approximate Number System acuity) as well as symbolic math skills (number symbol comparison, arithmetic) in a model only explained 52% of the variability in symbolic math performance on a general mathematics achievement measure at age 6[68]. The limited ability of children's intrinsic cognitive abilities to account for math performance suggests that extrinsic environmental factors also play a critical role in shaping children's symbolic math development.

Environmental impacts on math skills

Environmental factors such as language, culture, and socioeconomic status (SES) can influence mathematics performance (Table 1). Studies have examined differences at the community level (shared by all or most members of a particular community group) and at the individual level (specific to some individuals within a broader community). However, these classifications are based on the methodology of the studies rather than a theoretical distinction. At the community level, different languages, cultural attitudes, and beliefs are associated with symbolic math skills. At the individual level, differences in SES, math engagement, and personal attitudes and beliefs are linked to the development of math skills.

Community-level factors

Many community-level differences in symbolic math skills are thought to be driven by variations in language. For example, individuals who lack access to language input that includes exact number words (such as 'one', 'two', 'three', in contrast to inexact terms such as 'a few' and 'a lot') tend to perform more poorly in symbolic math tasks than individuals exposed to exact number words. In some cases, exposure to exact number words seems to be a prerequisite to acquire symbolic math skills. Children who are deaf or hard of hearing and lack access to fluent sign language are not exposed to exact number words from birth and typically perform worse in symbolic math (including skills like counting, arithmetic, and fractions) than their hearing peers and their peers who are deaf or hard of hearing peers with access to fluent sign language who are exposed to exact number words[69,70,71,72,73,74]. Additionally, deaf individuals who develop their own method of 'homesign' to communicate and are surrounded by a culture that uses numbers but lack access to a conventional language of number display poorer math performance than individuals with access to a language with exact number words[75]. The differences in math performance between individuals with and without language access are often moderate to large (Cohen's d between 0.50 and 0.80; r^2 between .06 and .14) [69,70,71,72,73,74]. These findings suggest that —similar to many other cognitive skills—it is difficult to develop math skills without appropriate linguistic input. Subsequently it is nearly impossible for children without language access to acquire the symbolic math skills observed in children with more extensive number language exposure.

Similarly, speakers of languages that do not contain words for exact numbers or that have restricted ways to talk about number display differences in math performance relative to speakers of languages with exact number words. For example, adult and adolescent members of the Pirahã and Mundurukú tribes, whose languages lack exact number words, perform more poorly in assessments of their non-symbolic and symbolic math skills than adults and children living in industrialized nations and speaking languages with exact number words (Box 1; [76,77]). However, members of the Mundurukú tribe who have some knowledge of Portuguese (which has exact number words) tend to perform better in these tasks than members with less or no knowledge of a second language[77].

Even within languages with exact number words and whose users are exposed to such number language from birth, there is variability in symbolic math performance across languages[78,79,80]. Children learning languages with regular structures for numbers, such as Chinese (where numbers are transparently named ‘ten-two’ instead of a unique word like ‘twelve’ in English) tend to outperform children learning languages with irregularities, with higher performance in tasks including counting, understanding the place-value system **[G]**, and other symbolic math skills[81,82,83,84,85,86]. In addition, children learning languages (such as Korean) where the concept of fractional parts is embedded in the mathematical term for fractions (for example, ‘of three parts, one’ instead of ‘one-third’) tend to outperform children whose languages do not have this vocabulary cue (such as English and Croatian [87]). Similarly, children learning languages like German with number word inversion (for example, ‘one-and-twenty’ instead of ‘twenty-one’ in English) tend to perform worse in math tasks including arithmetic, number transcoding, and magnitude comparison than those learning languages without number word inversion[88,89,90,91]. These differences in symbolic math performance due to language variability are often moderate to large (Cohen’s d between 0.50 and 0.80; r^2 between .06 and .14).

In addition to the influence of language, there are also influences of broad cultural norms regarding math on symbolic math performance[92,93,94,95,96,97]. For example, the strength of gender stereotypes and levels of gender equity tend to predict math performance on standardized tests across countries. Specifically, stronger implicit cultural beliefs that men are better than women in their science and math skills are associated with larger gender gaps in math achievement on standardized assessments[95]. Perhaps paradoxically, countries with greater gender equity in math and science opportunities tend to have larger gender gaps in performance in these domains[97]. When there is greater equity in opportunity, men and women can choose to pursue education in any field and tend to make those decisions based on personal preferences, which might reflect men’s stronger preference for math and science[97]. Similarly, countries with larger gender differences in math attitudes (how much individuals like math) show more pronounced gender gaps in math achievement on standardized assessments than countries with smaller gender differences[92]. Although the impact of cultural beliefs and attitudes in explaining the gender gaps in performance between countries are often large (Cohen’s d of 0.80, r^2 of .14), the differences between communities in overall math achievement due to societal beliefs and attitudes tend to be smaller (Cohen’s d of 0.20 to 0.50, r^2 of .01 to .09).

Individual-level factors

Within communities and language groups, additional individual-level environmental influences are associated with differences in math performance. Specifically, SES—access to resources, including family finances and parental education—is a consistent predictor of children's performance on standardized math assessments and academic math performance across development[11,98,99,100,101,102,103]. Children from households with higher SES tend to perform better in symbolic math than children from households with lower SES, with moderate to large effect sizes (Cohen's d of 0.50 to.80; r^2 between .06 and .14). The influence of family SES on children's symbolic math performance in standardized assessments has been shown within the same school and even among classmates who received the same math instruction[104]. Furthermore, the association between family SES and children's performance on standardized math assessments is present across a broad range of countries and languages[105]. However, SES has been operationalized differently, limiting comparisons across studies. Nonetheless, family income and parental education level (two of the most common indicators of SES) are typically both reliably predictive of children's symbolic math performance. Some work suggests that parental education might be a stronger predictor of symbolic math performance than family income, with large effect sizes (Cohen's d of 0.80, r^2 of .14 or larger [106]).

It remains unclear how SES shapes symbolic math performance and whether this association is due to influences on children's abilities or learning opportunities or both. On the one hand, low SES might reduce children's domain-general cognitive abilities through its impacts on health and brain development[107]. On the other hand, low SES might reduce other environmental factors related to math development such as learning resources or parental engagement in math.

Math-specific engagement, or the presence of activities and discussions using math, is another factor related to math performance. The frequency with which parents engage in math activities with young children is positively associated with children's symbolic math performance[108] on measures of arithmetic fluency **[G]**[109,110] , number facts and counting skills [111], magnitude comparison[112] and use of math language[54], as well as on standardized math assessments[113–118]. Similarly, the frequency of parental and teacher discussion of numbers and math concepts with children, regardless of the context in which this 'math talk' occurs, is related to these same symbolic math skills [119,120,121,122,123,124,125,126,127,62,119,128,129]. Children ages 2 to 12 whose parents and teachers engage in more frequent math activities and math talk tend to perform better in symbolic math than their peers who experience less math engagement, with small to moderate effect sizes (r^2 between .01 and .09). However, some studies fail to replicate the links between math engagement and symbolic math performance[130,131,117]. Regardless, more frequent math engagement is typically associated with better symbolic math performance, even above and beyond parents' overall engagement in general academic activities and conversations with their children[108,132].

Intervention studies further corroborate the association between math-specific engagement and children's symbolic math performance. Several studies that encouraged math activities at home—for example playing board games, counting more frequently, engaging in math problems when reading stories, and talking about math when grocery shopping and cooking—found an associated increase in children's subsequent symbolic math performance[133,134,135,136,137]. Similarly, studies that experimentally increase children's exposure to math talk have inferred a causal link with children's symbolic math performance. For example, children exposed to math language embedded within a storybook performed better on subsequent assessments of math language and general math knowledge than children in a preschool business-as-usual control group[62]. Furthermore, children whose parents were prompted to engage in math talk with their child showed increased attention to number outside of the parent-child interaction[120].

Finally, several indirect factors in the environment, such as the attitudes and beliefs of individuals in children's lives, are related to early symbolic math performance[138,139]. Specifically, the math anxiety levels[140,141,142,143], math attitudes[143], and beliefs about math[144,145,146,147] held by parents and teachers are related to children's symbolic math performance on standardized assessments and academic math performance in early and middle childhood. Lower levels of math anxiety, more positive math attitudes, and stronger beliefs about the importance and utility of math in parents and teachers are associated with better math performance, even above and beyond the role of parents' and teachers' beliefs and attitudes toward other academic domains. However, effect sizes for the influences of parental or teacher math anxiety, math attitudes and math beliefs on children's math performance are often small (r^2 =.01, Cohen's d of approximately 0.20).

Combined environmental impacts

Differences in environmental input at the community level in language, attitudes, and beliefs as well as at the individual level in SES, math engagement, and individual beliefs and attitudes are associated with variability in math development and performance. Community-level factors tend to account for moderate to large effects on math performance, whereas individual factors vary more dramatically in their effect sizes, ranging from small effect sizes for attitudes and beliefs to large effect sizes for SES. Studies examining community-level factors are often large cross-cultural projects that require large effect sizes to justify the use of extensive resources, whereas studies exploring individual factors might not require such large effect sizes to justify investigation. Consequently, the effect sizes reported in the literature may reflect these constraints, rather than the larger importance of community-level factors than individual factors on symbolic math performance per se. Nonetheless, each of the environmental influences reviewed here produce reliable and significant effects on symbolic math performance.

Notably, many studies of environmental influences have largely ignored the effect of children's abilities on the acquisition of symbolic math skills. In addition to the environment influencing them, children also influence their environment, potentially resulting in complex and bidirectional interactions between environmental influences and children's characteristics[128]. For example, children who have stronger non-symbolic math skills earlier in life might promote and seek out environments containing math talk and math engagement, leading to more opportunities to practice math, more feedback surrounding numbers and quantities, and the development of stronger symbolic math skills.

Number word knowledge

Despite the wealth of research on environmental influences on symbolic math broadly, little research has examined environmental influences on children's earliest symbolic math skills. Symbolic math knowledge, such as number word understanding, is foundational for learning more advanced math concepts[148–150]. Number word knowledge involves mapping the symbol for each number (the word label) to a set of items of the specified size. For example, number word knowledge is needed to label five apples with the word 'five'. Mature number word knowledge is defined by an understanding of the cardinality principle—when counting the items in a set, the last number in the count list **[G]** refers to the total number of items in the set [151,152]. This principle describes the fact that each number word refers only to an exact set of that quantity. If you count 'one, two, three, four, five apples,' there are five apples present, and one only labels a set of apples as 'five' when there are exactly five apples present. Children's number word knowledge in early childhood is one of the strongest predictors of their later symbolic math performance, beyond domain-general skills and Approximate Number System performance [153,148,149,150]. Thus, examining this process and sources of individual differences in number word performance is crucial for understanding long-term symbolic math development.

Number word acquisition

Children come to understand the meaning of exact number words very slowly. Previous work has identified a general trajectory of number words acquisition in English-speaking children (Fig. 3), which has served as a baseline for cross-cultural comparisons. This knowledge is demonstrated in number knowledge tasks where children are asked to create a set containing a certain number of objects. Around 30 months of age, English-speaking children learn the meaning of the word 'one' but lack knowledge of number words larger than one. When asked to give exactly 'one' object, a 30-month-old child will give one object, but if asked to give 'two' or any other number they will not give the correct number of objects. On average about four to five months after learning the meaning of 'one,' children reliably understand the word 'two' but not larger numbers. It takes several more months for children to display an understanding of the word 'three,' typically around 36 months old. Children do not have mature number word knowledge and mastery of the cardinality principle until at least 36-48 months old, when they can reliably count to and give the correct number of objects when asked[154,155,151, 156,157].

However, rather than adhering to a strictly stage-like developmental pattern, where children either completely do or do not understand a particular number word, number word acquisition might be a more continuous process of gradually developing number word knowledge. For instance, children display partial knowledge of number words before learning their exact meanings[25,158,159,160,161,162]. Similarly, some work suggests that even when children can reliably produce the correct number of objects when asked, they might not fully comprehend the cardinality principle[163]. These findings have encouraged continued debate about the mechanisms of number word acquisition (see [164] for a review). Whether children acquire the meaning of number words in a stage-like or more continuous process, number word acquisition is a drawn-out process across many months.

There are several theoretical accounts of how children learn number words, specifically how they transition from being a ‘subset knower’ who understands only one or a few number words to a ‘cardinal principle knower’ who understands the cardinal principle for all numbers. These theories vary in the extent to which they posit that number word acquisition relies on domain-general abilities and domain-specific abilities. One theory posits the importance of the domain-general ability to learn a language to express innate conceptual knowledge about numbers[165,166]. Other theories focus on the role of the Object Tracking System[23,24,25,26,27] and/or the Approximate Number System[33,35,36,37,40,41,167,168]. Current empirical evidence does not conclusively support one theory over another. However, previous comparisons of these theoretical accounts have largely ignored the role of environmental factors[169,151,170,27,23,171].

There are also individual differences in the process of number word acquisition. Children with more advanced knowledge of the count list and number symbols, who can correctly count higher and identify more number symbols, or who have stronger quantity discrimination skills or higher IQ tend to become cardinal principle knowers earlier than their peers with less advanced skills in these areas[172]. Individual differences in acquisition of number word knowledge predict individual differences in other math skills. Children who learn the cardinality principle earlier than their peers tend to display better performance in counting, number comparison, number symbol identification, arithmetic, and understanding number lines[149]. Children who showed more advanced understanding of number words at age 3 tended to perform better in symbolic math at age 6, relative to their peers who displayed less advanced number word knowledge[149].

Environmental impacts on number words

The environmental influences associated with individual differences in broader symbolic math performance can be tested for their influence on the earlier acquisition of number words. As before, we group the influences according to scale, examining community-level and individual-level influences (Table 1).

Considering linguistic influences, variation in the number word vocabulary and linguistic structure of number words are associated with different developmental progressions in learning number words[173,174,175,176,177]. For example, children learning number words in languages like English and Russian that have ways of signaling whether a word is singular or plural tend to learn the word ‘one’ sooner than children learning languages that do not have this distinction (for instance, Japanese and Mandarin) [176,178]. Similarly, children learning number words in languages with a singular-plural distinction and an additional way of distinguishing pairs from larger quantities (dual-marking, found in Slovenian and Saudi Arabic) tend to learn the meaning of ‘two’ faster than children learning languages without dual marking [173]. Broader differences in the linguistic cues (how number words are used in sentences and the broader context surrounding their usage) associated with number word input might also relate to children’s learning of number words[179]. Using number words in conjunction with other cues to the quantity being discussed within the sentence may help signal the numerosity and aid in children's learning of numbers. For example, hearing 'three blickets' in a sentence, where the number word is used as referent of the noun that occurs before the noun, and the noun includes a plural marking, may help indicate the number referred. Insomuch as languages may differ in the usage and placement of referents and plural indications, these differences may be associated with differences in children's learning of number words.

Few studies have examined how other community-level variations relate to children's number word acquisition. In particular, little is known about how societal differences in attitudes or beliefs might relate to children's number word acquisition, or how individual-level and community-level influences might interact.

Turning to individual-level influences, children from lower-SES households acquire the cardinality principle later than peers from higher-SES households, implicating the home environment in number word acquisition[180,102]. For example, children from middle-income and low-income families understood cardinality nearly six months later than children from higher-income families[180]. In line with this pattern, children from lower-SES backgrounds tend to perform worse in number knowledge tasks assessing cardinality principle understanding than their peers from higher socioeconomic backgrounds, with moderate to large effect sizes (Cohen's d between 0.50 and 0.80; r^2 between.06 and.14) [124,126,53,181]. However, it is unknown whether these impacts of SES on number word learning are due to influences on children's skills or learning opportunities; more work is needed to unpack these associations.

The influence of parents' verbal input during parent-child interactions on number word learning has received more attention than many other influences. Children whose parents engage in more frequent discussion of numbers and number concepts (number talk) tend to have better number word knowledge than children who hear less number talk [121,124,126]. For example, children whose parents used more number talk during everyday interactions while they were 14–30 months of age displayed a better understanding of the number words between 'one' and 'six' at 46 months of age, even when controlling for SES and general quantity of parent talk[126]. Similarly, children whose preschool teachers use more frequent number talk tend to have better number word knowledge than those who hear less number talk [182]. Some types of number talk—including counting and labeling sets of present objects, particularly in larger numbers greater than three or four—might be most beneficial for number word learning [121,124]. Toddlers whose parents engage in these types of number talk tend to have better number word knowledge than their peers, even years later in childhood. This work suggests that in addition to the overall frequency, the quality of the environmental input likely plays a role in children's number word acquisition.

In addition to these observational and correlational findings, a few studies have been conducted that provide some causal evidence for the role of number talk in the number word acquisition process. For example, interventions that promote parents' and teachers' counting and labeling of set sizes are associated with better cardinal principle understanding [123,127,183]. Books and games that promote and encourage parent and child number talk are associated with greater learning of number words than books that promote other types of math talk or non-math talk. In one study, 24–48 month-old children whose parents read picture books including numbers to them every day for four weeks showed larger increases in number knowledge than children whose parents read picture books without these number prompts. This effect was particularly strong for books including small numbers (η^2_p =.118; [123]). Although encouraging parents to count and label sets of objects within the context of these interventions led to subsequent improvements in children's number knowledge, parents very infrequently spontaneously engage in these behaviors with their children[127]. Thus, it remains unclear whether the natural home and school environment— if parents are not prompted to engage in specific activities—provides enough number talk to benefit number word learning.

It also remains unclear how other environmental factors typically studied in relation to older children's and adults' math performance might relate to children's number word learning. For example, no work to date has evaluated how parent or teacher attitudes and beliefs might shape children's number word acquisition.

Summary and future directions

The development of symbolic math skills is shaped by children's domain-general and domain-specific abilities as well as the opportunities provided in their environment. Variability in environmental factors, ranging from community-level differences in language and beliefs to individual-level differences in resources, engagement, and attitudes, is closely tied to symbolic math performance. However, more studies measuring both cognitive abilities and environmental factors are needed to identify the unique effects of each, the possible bidirectional association between the environment and children's cognitive abilities, and the consequences of these complex associations for symbolic math development.

Prior work has focused on identifying the various environmental factors associated with symbolic math skills across childhood and adulthood, with little attention to potential relations with the precursors of these skills. Understanding number words provides a critical foundation for later symbolic math learning and number word acquisition is a complex process spanning multiple years [155], with ample opportunity for environmental influence. The limited previous work on this topic suggests that language, the availability of more socioeconomic resources, and increased frequency of number talk shape the rate at which children learn number words. Minimal work has examined how community attitudes and beliefs, or individual differences in the attitudes and beliefs of teachers and parents might be related to children's developing number word knowledge. Future research is needed to understand how environmental influences and potential interactions between the environment and children's abilities impact number word learning. Furthermore, the proposed theories for number word acquisition likely require expansion to consider other aspects of cognition, including knowledge of the logical vocabulary that relates to numbers (for example, quantifiers; [184]), the ability to switch from viewing collections of objects as a bunch of individuals to viewing them as a set [185,186], and the ability to group objects into hierarchical sets[187].

Future work should also investigate whether environmental influences are related to children's number word learning to the same extent across communities. To date, most of the work that has investigated the role of environmental influences has focused on the impact of number talk on children learning English number words (Box 2). Additionally, it remains unknown whether broader societal beliefs and attitudes are related to children's number word learning, or whether these influences are only found on more advanced symbolic math skills [95]. Specifically, future work might examine whether societal gender stereotype beliefs and attitudes are related to differences in number word knowledge and whether these are indirectly related to number word learning via math engagement. It will be important to examine whether parental and teacher attitudes and beliefs, which are closely related to more advanced symbolic math skills[140,144], are also related to children's learning of number words. Distal factors such as attitudes and beliefs might relate to why individuals talk about numbers with children and inform interventions to promote engagement in beneficial types of number talk.

Similarly, identifying why SES is related to number word learning and symbolic math skills can inform interventions to support children's number word acquisition in lower SES families. Further work examining how the environment, particularly SES, shapes and supports domain-general and domain-specific cognitive skills, and the subsequent impact on number word learning and more complex symbolic math skills, will undoubtedly prove useful for teasing apart these mechanisms.

It is also important to consider the potential for interrelations and confounds between cognitive and environmental factors. Although we have largely reviewed the isolated impact of individual environmental influences on math performance, there are likely close relations between environmental factors. For example, there might be broad socioeconomic disparities or different societal beliefs and attitudes between communities using different languages. Thus, the contributions of linguistic differences cannot be teased apart from these larger community differences, nor can the associations between societal beliefs and attitudes or SES be examined completely independently of linguistic influences. Similarly, even within communities, there is likely interplay between SES, individuals' beliefs and attitudes, and engagement in math activities with young children. Future work must consider these interrelations and the dynamic interactions between environmental factors.

Adding to the complexity of environmental influences, age might play a role in determining the impact of environmental input. For example, very young (less than 36 months) infants and toddlers might be unaffected by their environment and only after a certain point would they begin to benefit from specific types of environmental input. Similarly, the degree of environmental influence might itself change with development. Finally, to understand individual differences in symbolic math performance more broadly, it will be necessary to consider how environmental influences might relate to the development of other types of foundational math skills.

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Glossary

Place-value system	a system of symbolic number notation in which the position of a digit within a number string denotes its power, and the quantity is represented by the symbol
Arithmetic fluency	the ability to solve arithmetic problems accurately and efficiently
Count list	the list of number words in the order they appear when counting

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Box 1

Methodological considerations in working with diverse communities.

When testing math skills in diverse samples, especially where the researchers are not members of the local community, researchers should take care to ensure that their research is both respectful and reliable. For example, the format of the assessments needs to be chosen carefully. As an illustration, Brazilian children who work as street vendors are able to solve arithmetic problems when they are presented as monetary transactions (for example, asking how much 10 coconuts each costing 35 cents will be in total) but not when presented in a more traditional academic format (for example, 35 x 10 = ?)[192]. This work suggests the importance of the method of assessment and familiarity of the participants with the testing format. These factors should be taken into account when observing performance differences in cross-cultural studies.

Similarly, in communities whose members lack access to number symbols or rarely employ symbolic number (for example, members of the Pirahã and Mundurukú tribes and deaf homesign communities), it is necessary to ensure that any differences in performance are not due to familiarity with the testing methods or compatibility in number and math representations with the testing format. In an attempt to overcome these issues, researchers have used predominantly non-symbolic methods of assessing math skills, including tasks that require matching large sets of objects to a sample, comparing two quantities of dots to determine which has more, performing approximate arithmetic using sets of dots, or summing two small sets of objects, none of which require extensive numerical vocabulary[76,77,193,194,75]. Even on these non-symbolic tasks, adults' performance is often better when strategies involving symbolic skills like counting are used and differences are often seen between communities with languages including formal number systems and those without. However, this pattern has not been found for children[193,194]; children's performance is often equivalent regardless of their language's number system (though this might be due to immature use of symbolic skills by children from languages with formal number systems). For tasks that involve symbolic calculations, it is not surprising that users of languages lacking formal number symbols or participants who have never received formal education in symbolic number systems perform more poorly than those whose languages contain number symbols and who presumably have more familiarity and exposure to the types of symbolic questions typically assessed in those measures.

Box 2.

Unknowns in the influence of number talk on number word acquisition.

The majority of prior work examining environmental influences on number word learning has focused on the role of number talk. Past work has tended to examine overall frequency of number input, without much attention to the quality of that input or the contexts in which it occurs. For example, it is likely that hearing number talk in different contexts might provide different benefits for number word learning. Some activities might incorporate number talk and encourage thinking about cardinality in more effective ways than others. Having a set of items physically present while discussing the cardinal value of the set is especially beneficial for number word knowledge[124] and therefore it is likely that activities that include physical items that can be counted would better support number word learning than activities that require more abstract discussion of sets, at least at initial stages of number word learning.

Additionally, the linguistic context of the number talk might also play a role. It is possible that number talk in different sentence types might provide different benefits, and some types of utterances including number talk might be more useful than others. Previous work suggests that the use of prompts and questions is particularly useful for children’s vocabulary learning more broadly[195,196], and their symbolic math skills more specifically[197], but it is unknown whether similar trends are found for number word learning. Similarly, it remains an open question how hearing number talk in the presence of other math concepts (for instance, quantifiers) influences number word learning.

Finally, the source of the number talk might also play a role. Previous work has typically examined parents’ and teachers’ input to children, but no work has examined whether hearing number talk from different people leads to different outcomes. For instance, it is unknown whether it is differently beneficial to hear the same input from multiple people (for example, from parents and teachers) or solely from one person (for example, multiple times from only a teacher). Furthermore, it is unknown whether there are some people whose number input is more useful than others (for example, perhaps people who are less familiar to the child are less effective at helping them learn).

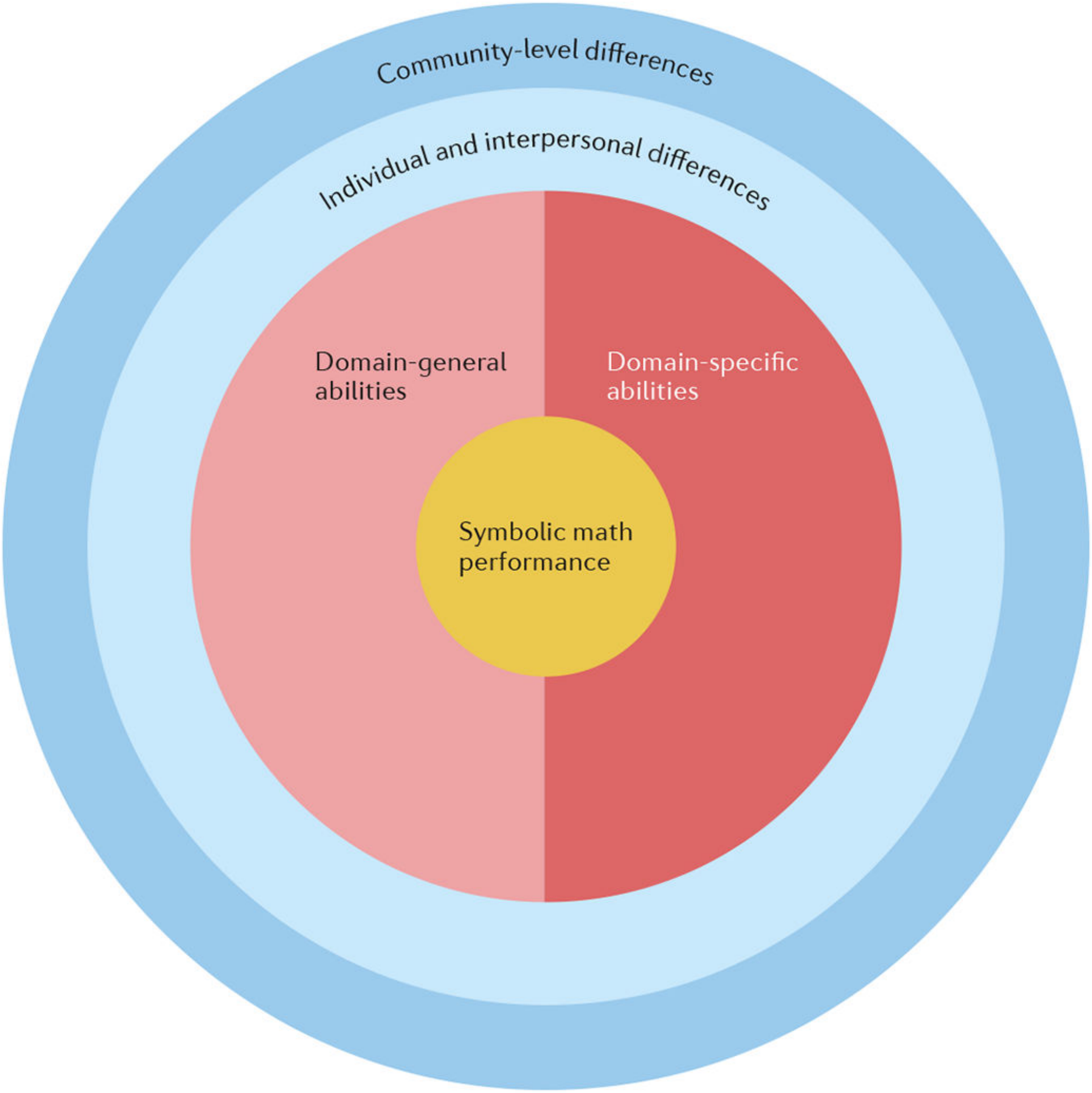

Figure 1. Abilities and environmental factors influencing symbolic math performance.
Environmental factors (blue) might influence symbolic math performance (yellow) directly as well as via indirect effects on domain-general and domain-specific cognitive skills and abilities (red). Meanwhile, children’s abilities might influence symbolic math performance directly by influencing how much children benefit from the learning opportunities in their environment.

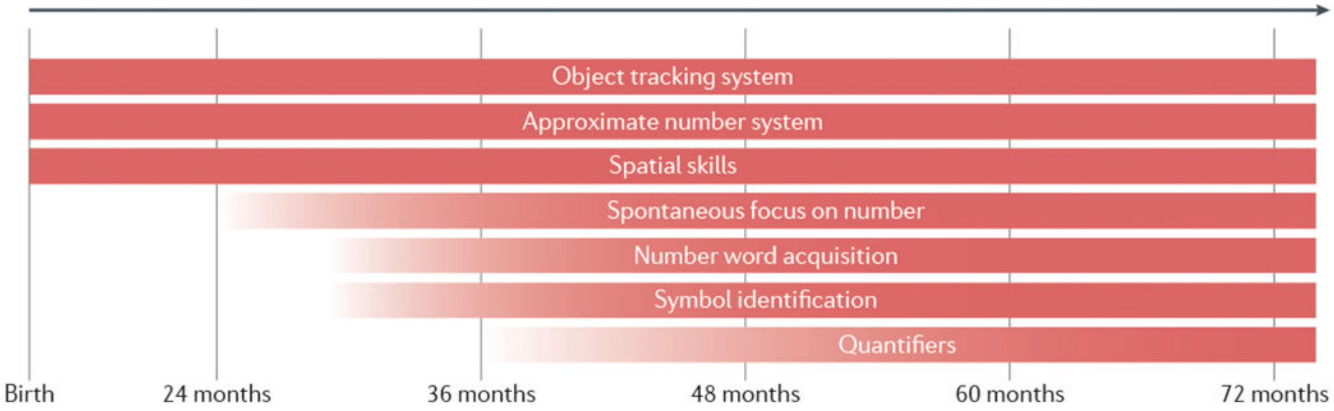

Figure 2. Typical timeline for emergence of foundational math skills in English-speaking children.

The Object Tracking System and Approximate Number System, two core number processing systems, are present from birth [14]. Spatial skills are also present in infancy [188], and spontaneous focus on number has been documented as young as 24 months of age [189]. Around 30 months of age, children begin acquiring number words [155], and begin learning to identify number symbols [190]. By around 36 months of age, children begin understanding and correctly using quantifiers [191].

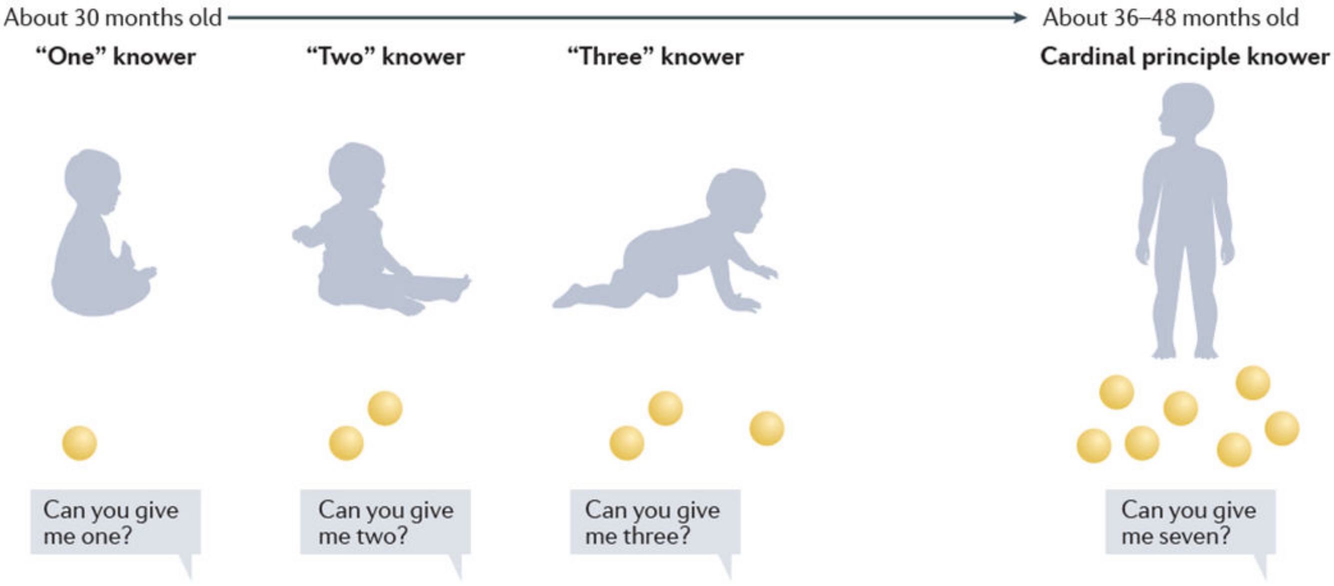

Figure 3. Average timeline of the number word acquisition process for English-speaking children.

Children's number word knowledge is typically measured via tasks that ask them to produce sets of a particular size (for example, 'Can you give me one?'). Children's knower-level is the highest number at which they can reliably produce the correct set of objects. It takes months for children to progress from being a 'one' knower to being a 'cardinal principle' knower.

Table 1:

Summary of evidence for environmental influences on symbolic math skills.

Level of influence	**Environmental factor**	**Impact on symbolic math skills (relative to lack of exposure)**	**Impact on number word acquisition (relative to lack of exposure)**
Community-level	Language	Children exposed to a language with exact number words have stronger performance in non-symbolic and symbolic math tasks [69–77]	Children exposed to language with singular-plural distinction learn ‘one’ faster [176,178]
		Children exposed to language with regular structures (embedded fractional parts and non-inverted number words) have stronger performance in symbolic math tasks [78–91]	Children exposed to language with an additional distinction for pairs learn ‘two’ faster [173,175]
	Societal attitudes and beliefs	Individuals from communities with stronger gender stereotypes and gender equity have larger gender gaps in symbolic math performance [92–97]	Unknown
Individual-level	Socioeconomic status (SES) and education	Children from higher SES households perform better in symbolic math tasks [11,98–103]	Children from higher SES households learn number words faster [102,124,180]
	Math engagement	Children with exposure to frequent math activities and math talk have stronger performance in symbolic math tasks [62,108–117,119,120,122,125,128–131]	Children exposed to frequent number talk (especially counting and labeling sets) learn number words faster [121,123,124,126,127,182,183]
	Attitudes and beliefs of parents and teachers	Children with exposure to caregivers with more positive attitudes, lower math anxiety, and stronger beliefs about math importance have stronger performance in symbolic math tasks [138–147]	Unknown