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RESEARCH

The Poisson distribution model fits UMI-based single-cell RNA-sequencing data

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Abstract

Background: Modeling of single cell RNA-sequencing (scRNA-seq) data remains challenging due to a high percentage of zeros and data heterogeneity, so improved modeling has strong potential to benefit many downstream data analyses. The existing zero-inflated or over-dispersed models are based on aggregations at either the gene or the cell level. However, they typically lose accuracy due to a too crude aggregation at those two levels.

Results: We avoid the crude approximations entailed by such aggregation through proposing an Independent Poisson Distribution (IPD) particularly at each individual entry in the scRNA-seq data matrix. This approach naturally and intuitively models the large number of zeros as matrix entries with a very small Poisson parameter. The critical challenge of cell clustering is approached via a novel data representation as Departures from a simple homogeneous IPD (DIPD) to capture the per-gene-per-cell intrinsic heterogeneity generated by cell clusters. Our experiments using real data and crafted experiments show that using DIPD as a data representation for scRNA-seq data can uncover novel cell subtypes that are missed or can only be found by careful parameter tuning using conventional methods.

Conclusions: This new method has multiple advantages, including (1) no need for prior feature selection or manual optimization of hyperparameters; (2) flexibility to combine with and improve upon other methods, such as Seurat. Another novel contribution is the use of crafted experiments as part of the validation of our newly developed DIPD-based clustering pipeline. This new clustering pipeline is implemented in the R (CRAN) package *scoipoisson*.

Keywords: Single cell; RNA-seq; Poisson distribution; Data representation

¹Background 1

²Single cell RNA-sequencing (scRNA-seq) estimates the transcriptome at the indi-
³vidual cell level. ScRNA-seq can directly measure cell-to-cell heterogeneity, which is
⁴more challenging using bulk RNA sequencing. First applied in 2009 [1], scRNA-seq
⁵has become the preferred tool to identify cell sub-populations and to investigate
⁶cellular heterogeneity [2, 3, 4, 5, 6, 7], gene regulatory networks [8, 9], stochastic
⁷fluctuations in transcription [10, 11], and so on. Due to the unique features of the
⁸data distribution in scRNA-seq, it's essential to develop statistical methods which
⁹accurately model scRNA-seq data for many important downstream analyses includ-
¹⁰ing differential expression analysis and clustering of cells. 10

¹¹Existing methods typically model the scRNA-seq data at the gene level for differ-
¹²ential expression analysis to find biomarkers, and at the sample level for clustering
¹³of cells to find cell subtypes; however they typically lose accuracy due to a too
¹⁴crude aggregation at those two levels. This aggregation has led to attempts to ex-
¹⁵plicitly model the apparent resulting zero-inflation or over-dispersion. We propose
¹⁶more precisely addressing these issues by modeling the distribution of each individ-
¹⁷ual entry of the data matrix. A major challenge is that scRNA-seq data typically
¹⁸contain a large number of zero counts for gene/cell combinations (often exceed-
¹⁹ing 90%) [12]. This is due to both biological reasons that some genes are only
²⁰expressed in a cluster of cells, and technical limitations such as low RNA capture
²¹rates, low efficiency library construction, cell disintegration and RNA degradation.
²²There also exists a severe threshold effect in detection sensitivity of gene expres-
²³sion in scRNA-seq. Typically higher expressed genes in a cell tend to have a higher
²⁴probability to be detected [13, 14, 4, 15]. These characteristics can lead to large
²⁵discrepancies among sequencing libraries for different cells, i.e. batch effects, and
²⁶render many global normalization approaches ineffective. Various approaches have
²⁷been proposed to address barriers that limit the interpretation of scRNA-seq data
²⁸[16, 17, 18, 19, 20, 21, 22, 23]. On the “wet-bench” side, unique molecular identifier
²⁹(UMI) was introduced [24]. UMI reduces biases introduced by the extreme signal
³⁰amplification that is necessary for scRNA-seq. Some researchers have argued that
³¹if the UMI technology works properly, there is no need to account for zero-inflation
³²[25, 22, 26]. This is an encouraging perspective; however, these classical probability
³³models are again only crude aggregations focusing on either cells or genes. 33

¹ To improve the accuracy of statistical modeling and gain more precise inference,¹
² we propose the novel and principled approach of studying individual entries of the²
³ gene-by-cell matrix. This approach is based on the Independent Poisson Distri-³
⁴ bution (IPD) statistical framework, where every gene in each cell follows its own⁴
⁵ Poisson distribution. Working with such a model is challenging because the max-⁵
⁶ imum likelihood estimate of each Poisson parameter is simply the corresponding⁶
⁷ count, which is too noisy to be useful. To solve this problem which presents for the⁷
⁸ validation of the IPD model we first start with several biologically homogeneous⁸
⁹ data sets derived from single clonal cell lines [27]. Next, we perform parameter⁹
¹⁰ estimation using generalized principal component analysis (GLM-PCA) [25] as a¹⁰
¹¹ noise reduction method. While this approach has clear potential to eliminate noise¹¹
¹² when keeping important biological signals, it is challenging in most applications¹²
¹³ because the critical number of GLM-PCA components is not known. However, a¹³
¹⁴ fundamental exception to this principal nicely arises in the validation of the IPD¹⁴
¹⁵ model. This is because if we can find (by trial and error) a number of components¹⁵
¹⁶ which result in a fit of the standard univariate Poisson distribution to collections of¹⁶
¹⁷ matrix entries having very similar parameters, then the goodness of fit of the IPD¹⁷
¹⁸ is verified. The fit of *Poissonicity* to sets of similar matrix entries is studied using¹⁸
¹⁹ Quantile-to-Quantile plots (Q-Q plots), together with simulated envelopes indicat-¹⁹
²⁰ ing natural variation, in addition to over-dispersion and zero-inflation hypothesis²⁰
²¹ tests. 21

²² Based on this newly proposed IPD framework which focuses on individual entries 22
²³ of the scRNA-seq data matrix, we further develop procedures based on the compu- 23
²⁴ tation of Departure from the IPD (DIPD) as a data representation to replace the 24
²⁵ scRNA-seq count data by the logistic transformation of probabilities of Departure 25
²⁶ to ensure modeling accuracy and to effectively deal with zeros. The output will be 26
²⁷ a data matrix of the same dimension of scRNA-seq with continuous values. This 27
²⁸ enables our development of other new computational approaches including cluster- 28
²⁹ ing and other downstream tasks through the novel concept of DIPD. The DIPD is 29
³⁰ initialized by a rough two-way parameter approximation of the data. Next, different 30
³¹ cell types are captured by departures from the naïve two-way approximation. Then 31
³² the data is bisected using Poisson departure as the distance measure. The cluster- 32
³³ ing algorithm terminates, when there is no significant deviation from Poissonicity 33

¹for any cell group. For some data [28] this approach gives results similar to those¹
²using other pipelines. For others [29] it shows an improvement. Overall, for addi-²
³tional downstream tasks, the DIPD matrix is proposed as a new data representation³
⁴(*model departure*).⁴

⁵ In sum, the IPD statistical framework has the potential to capture meaningful⁵
⁶biological properties at a higher resolution than prior normalization methods, with-⁶
⁷out the need for more complicated probability distributions. We demonstrate the⁷
⁸usefulness of model departure DIPD as a novel data representation by conducting⁸
⁹downstream analysis, such as clustering of cells. Our newly developed DIPD-based⁹
¹⁰clustering pipeline is validated in multiple experimental data. Another important¹⁰
¹¹contribution of this paper is the use of the novel method called *crafted experiments*¹¹
¹²for the comparison of the DIPD with other methods in a principled way. While¹²
¹³we demonstrate the value of our proposed model departure data representation for¹³
¹⁴clustering, we anticipate it will be useful for additional downstream tasks, such as¹⁴
¹⁵differential expression analysis, gene set tests and trajectory analysis, because it¹⁵
¹⁶provides a useful replacement of the conventional data matrix.¹⁶

¹⁸Results¹⁸

¹⁹Validation of Poissoneity for scRNA-seq data¹⁹

²⁰Poissoneity postulates that each matrix entry (gene by cell) comes from an in-²⁰
²¹dependent Poisson distribution. As stated in Methods, the Poisson parameter for²¹
²²each matrix entry can be estimated using GLM-PCA [25]. However, the success of²²
²³that estimation requires a good choice of the number of latent vectors L , which is²³
²⁴generally quite challenging. The model validation context we consider here allows²⁴
²⁵an unusual approach to that challenge. In particular, finding a value of L which²⁵
²⁶gives a good fit of the resulting IPD model establishes its validity. That goodness of²⁶
²⁷fit is quantified here using both Q-Q envelope visualization and formal hypothesis²⁷
²⁸testing.²⁸

²⁹ To study the Poissoneity of scRNA-seq data, we first explore the simplest case:²⁹
³⁰cells picked at random from a clonal cell line processed as a single batch (Plate 3³⁰
³¹[27]) with $L = 10$ (for the reasons given in section Methods). In Fig. 1, panels a, b³¹
³²and c display the distribution histograms. For a given Poisson parameter $\lambda = 0.5$,³²
³³ $\lambda = 2$ or $\lambda = 20$, the gold bars represent distributions based on 200 aggregated UMI³³

¹entries with the estimated Poisson parameters closest to λ . Their distributions ap-¹
²proximately follow the theoretical Poisson(λ) distributions (gray bars). In contrast,²
³the distributions from entries of genes whose gene averages are closest to λ (blue³
⁴bars), do not. 4

⁵ Fig. 1 panels d, e and f show the corresponding Q-Q envelope plots (see Methods).⁵
⁶These provide an alternative display of the distribution of the data. For all three⁶
⁷ λ choices, the gold lines (based on aggregated matrix entries) are within the gray⁷
⁸envelopes of variation, indicating good fits using the Poisson distributions. The⁸
⁹gene-level entries (blue line) do not lie within the Q-Q envelope indicating a poor⁹
¹⁰Poisson fit. Furthermore, the manner in which the blue curves leave the envelope¹⁰
¹¹show both the typically expected zero-inflation (departure below on the left) and¹¹
¹²over-dispersion (departing above on the right). This demonstrates that individual¹²
¹³raw UMI count entries follow Poisson distributions, but genes, whose averages are¹³
¹⁴often used for normalization, do not. 14

¹⁵ The hypothesis testing (based on aggregated matrix entries) have p-values $p =$ ¹⁵
¹⁶0.155 for $\lambda = 0.5$, $p = 0.056$ for $\lambda = 2$ and $p = 0.004$ for $\lambda = 20$ from over-dispersion¹⁶
¹⁷tests; and $p = 0.278$ for $\lambda = 0.5$, $p = 0.389$ for $\lambda = 2$ and $p = 1.000$ for $\lambda = 20$ ¹⁷
¹⁸from the zero-inflation tests. These are consistent with the visual representation.¹⁸
¹⁹An exception is $\lambda = 20$ (for the over-dispersion test). Here, (Fig. 1, panel f) the¹⁹
²⁰UMI-based individual entries distribution (gold) goes outside the gray variation²⁰
²¹envelope at the top for high values. This is due to a sampling effect. Relatively few²¹
²²matrix entries have parameter estimates close to $\lambda = 20$, i.e. sampled entries come²²
²³from a mixture of Poissons due to variation in the underlying parameters. If we²³
²⁴decreased the number of aggregated entries to 100, then the over-dispersion test is²⁴
²⁵not significant ($p = 0.129$) even when $\lambda = 20$. This result indicates a high quality²⁵
²⁶of fit for the IPD statistical framework and is consistent with the notion that UMI²⁶
²⁷count-based scRNA-seq data can be modeled by independent Poisson distributions²⁷
²⁸at the individual gene-cell entry level. 28

²⁹ 29

³⁰Further goodness of fit investigations 30

³¹Next, we use these goodness of fit tools (for matrix entries with similar Poisson³¹
³²parameters) to study batch variation (Fig. 2). Each plate represents a technical³²
³³replicate (batch) or different biological condition as defined in Methods. Within³³

¹each plate, we took λ ranging from 0.1 to 20, on 200 aggregated matrix entries¹
²(Poisson parameters are again estimated using GLM-PCA [25] with $L = 10$) to²
³test for Poissonity using Q-Q envelope plots and hypothesis testing. Based on³
⁴this extended data we find that: first, UMI data fall within the variation envelope⁴
⁵(gray lines) on Q-Q envelope plots, suggesting that the Poisson distribution fit the⁵
⁶matrix entries; second, inflated zeros are not detectable for UMI entries based on⁶
⁷zero-inflation tests ($p > 0.05$); third, no over-dispersion is detectable for UMI entries⁷
⁸based on dispersion hypothesis testing ($p > 0.05$). The exception is $\lambda = 20$, which⁸
⁹can be explained as a mixture of Poisson as discussed above. ⁹

¹⁰ ¹⁰
¹¹ One of the experiments deliberately violated the single cell assumption in a novel¹¹
¹²direction. Plate 8 (green) had two cells per well, i.e. per library. It shows over-¹²
¹³dispersion at $L = 10$ ($p = 0.049$ when $\lambda = 5$ and $p = 0.007$ when $\lambda = 20$). This is¹³
¹⁴consistent with the experimental design. It had a stronger signal for low abundance¹⁴
¹⁵transcripts as twice as much RNA was present per well, which resulted in more¹⁵
¹⁶biological variation. This different signal to noise ratio is handled by increasing L ¹⁶
¹⁷to 15. Compare the light ($L = 10$) and dark green ($L = 15$) curves in Fig. 2 panels¹⁷
¹⁸e and f. At $L = 15$, the curves are within the envelopes and the over-dispersion¹⁸
¹⁹tests have p-values $p = 0.882$ when $\lambda = 5$ and $p = 0.087$ when $\lambda = 20$, indicating¹⁹
²⁰no over-dispersion. ²⁰

²¹ Another experiment has an equal mixture of two different cell lines (Plates 5A ²¹
²²and 6A). In Fig. 2 panel f, the Q-Q envelope plot shows strong deviations at the ²²
²³bottom for low values at $L = 10$ when $\lambda = 20$ (orange curve; $p < 0.001$ for the ²³
²⁴over-dispersion test even decrease the number of selected entries to 100). This is ²⁴
²⁵because for this more heterogeneous data set, $L = 10$ components are inadequate ²⁵
²⁶to capture the biological variation. The fit is improved by increasing the number ²⁶
²⁷of latent vectors to $L = 20$ (the dark red curve; $p = 0.054$ for the over-dispersion ²⁷
²⁸test when $\lambda = 20$). These experiments show that deviations from cell homogeneity, ²⁸
²⁹either as a violation of the single cell assumption or as a result of a mixture of cells ²⁹
³⁰with different transcription profiles can be detected as departures from the IPD ³⁰
³¹model. This property can be compensated for by increasing the number of latent ³¹
³²variables L or it can be exploited by a clustering algorithm using Poisson model ³²
³³departure as the distance metric. This algorithm is described below. ³³

¹Poisson departure data representation ¹
² ²
³ ³
⁴Here, we introduce a novel data representation (DIPD) based on a departure from⁴
⁵the IPD. The initial step is based on a crude two-way parameter approximation,⁵
⁶where variation across cells is modeled by a cell-level parameter, and variation⁶
⁷across genes is modeled by a gene-level parameter (as defined in equation (3)).⁷
⁸This initialization step in itself does not appropriately account for cell heterogene-⁸
⁹ity. In the next step, the interesting cell structure is captured by departures from⁹
¹⁰the naïve two-way approximation in both genes and cells, and the original count¹⁰
¹¹matrix is replaced by a Poisson departure matrix. In the departure matrix, each¹¹
¹²entry is quantified by the relative location of that original count with respect to¹²
¹³the tentative Poisson distribution, whose parameter comes from the initial two-way¹³
¹⁴approximation. The departure measure is captured by a Poisson Cumulative Distri-¹⁴
¹⁵bution Function (CDF), which leaves the unexpectedly small counts nearly 0 and¹⁵
¹⁶unusually large counts close to 1. Next, the departure measure is put on a more¹⁶
¹⁷statistically amenable scale using the logit function. As a result, unexpectedly large¹⁷
¹⁸counts give large positive values and unexpectedly small counts give large negative¹⁸
¹⁹values. ¹⁹
²⁰ ²⁰
²¹ Fig. 3 shows the heatmap visualizations (two cell lines data defined in the fol-²¹
²²lowing section) based on DIPD (panel a) or Seurat after normalization and scaling²²
²³(panel b) as data representations. Note the different scale ranges. The black lines²³
²⁴in the sidebars depicted the top 2,000 most variable genes identified by Seurat. The²⁴
²⁵DIPD-based representation kept all genes, as they may become relevant for defining²⁵
²⁶sub-clusters, and also may be associated with important meta information. Such²⁶
²⁷meta-information may include drug susceptibility or the availability of a clinical²⁷
²⁸or histochemical assay to measure protein expression. The opportunity to iden-²⁸
²⁹tify genes of high clinical value is lost in approaches that select features based on²⁹
³⁰statistical properties alone. In this simple case with two distinct cell lines, both rep-³⁰
³¹resentations perform similarly as depicting the differentially expressed (DE) genes³¹
³²between the two cell lines. We will show that the DIPD-based data matrix outper-³²
³³forms Seurat normalized counts as a novel data representation in a later section. ³³

¹Cell type clustering based on Poisson departure 1

²A major application of this data representation is cell clustering using DIPD. This²
³can be used directly as input into other algorithms. It also opens the possibility for³
⁴a novel clustering algorithm, as illustrated in Fig. 4. This algorithm, referred to as⁴
⁵*Hclust-Departure*, operates as follows: Starting with the UMI count matrix (*UMI*),⁵
⁶a very crude two-way parameter approximation (more details in Methods) is used⁶
⁷to estimate Poisson parameters ($\tilde{\Lambda}$). Cell heterogeneity is not assumed at this step.⁷
⁸Next, each UMI count is replaced by the DIPD (*D*) measure from the naïve model.⁸
⁹This DIPD-based matrix serves as the input for the clustering step. Clustering with⁹
¹⁰ $k = 2$ is applied and the two-way approximation and DIPD-based data matrix is¹⁰
¹¹recalculated separately for each of the two subclusters. This process is repeated¹¹
¹²until (a) the split is no longer statistically significant; (b) the maximum allowable¹²
¹³number of splitting steps is reached; or (c) any current cluster has less than 10 cells.¹³
¹⁴Statistical significance is calculated using Sigclust2 [30]. For a homogeneous cluster¹⁴
¹⁵of cells, all the departure entries (*D*) are similar, and therefore Sigclust2 should not¹⁵
¹⁶find significant clusters. 16

¹⁷ To investigate the performance of *Hclust-Departure*, we compared it with a com-¹⁷
¹⁸monly used package, Seurat (version 3.1.1) [31]. 18

¹⁹ 19
²⁰*Single clonal cell line* 20

²¹First, homogeneous data from a single clonal cell line (Plate 3) is tested [27]. There²¹
²²are no known clusters. This data serves as a negative control because the cells have²²
²³been maintained under optimal growth conditions to minimize variations within the²³
²⁴cell population. Applying *Hclust-Departure* to the DIPD-based matrix resulted in²⁴
²⁵no significant splits ($p = 0.933$), consistent with the experimental design (panel a²⁵
²⁶in Additional file 1). Seurat also identified only one cluster (resolution parameter²⁶
²⁷0.8, panel b in Additional file 1). 27

²⁸*Two cell lines, equal mixture* 28

²⁹Combining the data from two clonal cell lines (Plates 5A and 6A) in an equal mix-²⁹
³⁰ture provided a positive control, as the two cancer cell lines were from independent³⁰
³¹patients, but of the same lineage [27]. *Hclust-Departure* resulted in two clusters,³¹
³²consistent with the known cell lines. Seurat also identified two clusters under the³²
³³default setting (resolution parameter 0.8) as expected (Fig. 3). 33

¹*Three cell lines, unequal mixture* 1

²Next, we applied *Hclust-Departure*, to data comprised of three mixture cell lines, at ²
³a ratio of 1:3:6 [32]. *Hclust-Departure* identified three clusters (panel a in Additional ³
⁴file 2). Using the default setting, Seurat identified 7 clusters. By tuning the Seurat ⁴
⁵resolution parameter from the default 0.8 to 0.1, overfitting was resolved (panel b ⁵
⁶in Additional file 2) and both approaches identified the three biologically defined ⁶
⁷clusters. 7

⁸

8

⁹*Multiple cell lineages, unequal mixture* 9

¹⁰To increase the complexity of the data further, data from the lymphoid organs of ¹⁰
¹¹a mouse [28] was analyzed. These represent the complex lineages and populations ¹¹
¹²of the hematopoietic system: T and B cells, which mediate the adaptive immune ¹²
¹³response, as well as dendritic cells (DCs), macrophages, mast cells, etc., which me- ¹³
¹⁴diate the innate immune response as well as red blood cells (erythrocytes). Within ¹⁴
¹⁵each of these broad classes, multiple subclasses are recognized. 15

¹⁶The results are visualized using t-distributed Stochastic Neighbor Embedding ¹⁶
¹⁷(t-SNE) [33] and Uniform Manifold Approximation and Projection (UMAP) [34] ¹⁷
¹⁸in Fig. 5 panels a, c and panels b, d. *Hclust-Departure* (panels a and b) is used ¹⁸
¹⁹without dimensionality reduction or feature selection. Seurat (panels c and d) is ¹⁹
²⁰applied using the top 2,000 most variable features as defined by default. The cell ²⁰
²¹type labels are manually assigned to each cluster using known lineage markers. ²¹
²²The clusters discovered by *Hclust-Departure* are consistent with those identified ²²
²³by Seurat. Furthermore, *Hclust-Departure* identifies several significant subclusters ²³
²⁴within common Seurat labels (namely B-cells (light/dark green clusters), NK cells ²⁴
²⁵(light/dark gold clusters) and erythrocytes (light gray/black clusters)). 25

²⁶To evaluate the biological plausibility of the additional clusters identified by ²⁶
²⁷*Hclust-Departure*, we identified differentially transcribed genes using the t-test (clus- ²⁷
²⁸ter size larger or equal to 30) or the Wilcoxon rank-sum test (cluster size less than ²⁸
²⁹30) (Fig. 6). The genes colored in red are statistically significant after FDR adjust- ²⁹
³⁰ment ($p < 0.05$), and have a large mean difference. The genes colored in orange have ³⁰
³¹a significant difference but the mean difference is small. Those colored in black do ³¹
³²not differ among clusters. Known cellular identity-specific differentiation markers ³²
³³are annotated by name. Their difference in departure representation is consistent 33

¹with the existence of two functionally distinct populations as recognized by *Hclust*-¹
²*Departure*. ²

³ Fig. 6 panel a depicts two types of DCs corresponding to the coral and blue clusters ³
⁴in Fig. 5. DCs are antigen-presenting cells and are classified into two major subtypes: ⁴
⁵myeloid DCs (mDC) and plasmacytoid DCs (pDC) [35]. Cluster one downregulates ⁵
⁶the histocompatibility complex (*HLA*) class II molecules and Cystatin C (*CST3*), ⁶
⁷*LYZ*, *TMSB4X*; the other does not. Thus, the distribution of biologically defined ⁷
⁸lineage markers validated this unsupervised clustering result. ⁸

⁹ Fig. 6 panel b depicts two clusters of B cells (corresponding to the light green ⁹
¹⁰and dark green clusters in Fig. 5 panels a and b). B cells are classically known for ¹⁰
¹¹their ability to produce antibodies, yet they are capable of a variety of functions ¹¹
¹²including antigen presentation, production of several cytokines and the suppression ¹²
¹³of IL-10 secretion [36]. Comparatively high levels of lineage defining plasma B cell ¹³
¹⁴transcripts such as *MZB1* and *FKBP11* and *LTB* (an early B cell differentiating ¹⁴
¹⁵factor) differentiate the two clusters confirming that two clusters, rather than one, ¹⁵
¹⁶was consistent with the known biology. ¹⁶

¹⁷ Fig. 6 panel c focuses on Natural Killer (NK) cells (corresponding to the light ¹⁷
¹⁸and dark gold clusters in Fig. 5 panels a and b). NK cells are one of the major ¹⁸
¹⁹subpopulations of lymphocytes and components of innate immunity. Again key ¹⁹
²⁰lineage markers were differentially expressed among the two NK cells clusters such ²⁰
²¹as *CD56* and *CD16* [37]. The presence of *ZNF90*, *UBA52* and *FAU* suggests that ²¹
²²those cells were in an active transcriptional state. The absence of *TUBB* indicates ²²
²³that the cell was in a state of mature NK cell expression. ²³

²⁴ Fig. 6 panel d depicts the subdivision of erythroid cells. There are two types ²⁴
²⁵of erythroid cells: embryonic and mature. These are traditionally differentiated by ²⁵
²⁶the downregulation of several hemoglobin genes including *HBB*, *HBA2* and *HBA1* ²⁶
²⁷which are expressed during terminal differentiation [38]. The expression of *YBX1*, ²⁷
²⁸a transcriptional factor and *SERBP1*, an anti-apoptotic gene, further support the ²⁸
²⁹notion that these cells were in the early stages of erythrocytic development. ²⁹

³⁰ In sum, *Hclust-Departure* identifies biologically plausible populations from this ³⁰
³¹complex mixture of cells, establishing equivalent performance to existing scRNA-seq ³¹
³²algorithms. It also identifies additional subtypes. Obviously, other algorithms can be ³²
³³tuned to fit previously known subpopulations. However, the choice of correct tuning ³³

parameters for those methods is necessarily heuristic, specific to each data set, and not necessarily reproducible or robust. By comparison, *Hclust-Departure* has no tunable parameters, other than the significance level and neither has Sigclust2.

Hybrid Approach: model departure and Louvain clustering

A key difference between *Hclust-Departure* and other pipelines is the actual clustering algorithm. We therefore combine the DIPD data representation with the Louvain algorithm as implemented in Seurat.

To validate this combination, we used a different, very complex and very well studied data set with known ground truth. These are the Peripheral Blood Mononuclear Cells (PBMCs) data sets defined by [29]. The Zhengmix8eq data set contains 3,994 cells of eight cell types in equal proportions, some of which are quite distinct and some very similar (Fig. 7 panel a). Unsupervised clustering using Seurat with log-normalized transcription using 15 PCs and resolution parameter 0.8 recapitulate the Fluorescence-Activated Cell Sorting (FACS) labels (Fig. 7 panel b), but miss the distinction between T helper, T regulatory, and T memory cells. *Hclust-Departure* without dimension reduction performs slightly better (Fig. 7 panel c). Table 1 shows the confusion matrix. We also explored the more advanced normalization method SCTransform [18]. This method uses the residuals from Negative Binomial regression with the default parameters maintained for clustering. The results do not differ from the default normalization and are included in Additional file 3. None of the pipelines is completely consistent with the FACS labels in identifying subtypes of T cells. This may be due to the limited accuracy of the algorithms or it may be due to FACS labels not correctly signifying the underlying biological complexity, as T cell differentiation can be very fluid. Finally, DIPD-based data representation combined with Louvain clustering performs better than any of the pure pipelines (Fig. 7 panel d). The hybrid method correctly identifies the T cell subsets and subgroups of monocytes (red cluster). This result suggests that modeling UMI counts by departure from Poissonity has advantages over other normalization/transformation methods independent of the particular clustering algorithm.

To further define the performance of the hybrid approach, different parameters were explored using either DIPD-based representation (D) or log-normalized data as input. These were (a) the number of principal components (15, 20, 25 or 30)

¹and (b) the resolution parameter in the clustering step (0.6, 0.8, 1.0 and 1.2 for¹
²the larger eight cell-type data set Zhengmix8eq; and 0.05, 0.1, 0.2, 0.3, 0.5 and 0.8²
³for the other two four cell-type data sets Zhengmix4eq and Zhengmix4uneq [29]).³
⁴These experiments used the full D matrix or the top 2,000 most variable genes.⁴
⁵Performance is assessed using the Adjusted Rand Index (ARI) [39] and the purity⁵
⁶measure of [40] (Fig. 8). Except for Zhengmix4uneq (Fig. 8, panels b and e), DIPD⁶
⁷matrix D as input outperforms Seurat using normalized counts as input; however,⁷
⁸there are parameter constellations that lead to dramatic performance degradation⁸
⁹independent of the data representation. In sum, DIPD-based data representation D ⁹
¹⁰combined with Louvain clustering outperforms other normalization steps for UMI¹⁰
¹¹data. 11

¹²12

¹³*Further validation of the hybrid approach* 13

¹⁴Even though the experiments above point to DIPD-based data representation D ¹⁴
¹⁵and Louvain clustering as the optimal combination, a direct comparison between¹⁵
¹⁶algorithms that use different data representations and have multiple tunable param-¹⁶
¹⁷eters is difficult using experimental data sets with possibly unknown subpopulations:¹⁷
¹⁸overfitting cannot be decided on experimental data. An alternate approach is sim-¹⁸
¹⁹ulation based on theoretical distributions alone. This also is challenging because¹⁹
²⁰many aspects of the deep biological variation in scRNA-seq data are unknown and²⁰
²¹beyond current in silico modeling capabilities. These limitations motivate the use²¹
²²of *crafted experiments*. Here, carefully chosen perturbations are overlaid onto real²²
²³data. Crafted experiments maintain the complexity of the real data, but control the²³
²⁴signal versus noise by considering a range of perturbations from weak to strong. We²⁴
²⁵performed two different types of crafted experiments. 25

²⁶Variation in library size (total UMI counts per cell) is a driver of non-relevant 26
²⁷variation in scRNA-seq. To explore this issue we artificially magnified the library 27
²⁸size and compared different data representations (Fig. 9 panels a and b). As noted 28
²⁹above, many pipelines use multiplication and scaling to adjust for the library size 29
³⁰effects. This poses a problem for data containing many zeros. This experiment 30
³¹again used the Zhengmix4eq data. To model library size effects, cells with a large 31
³²or small library size were perturbed to be even larger or smaller (see Methods 32
³³for details). We compare data representations from DIPD (yellow), log-normalized 33

¹counts (blue) and SCTransform (green), all using the Louvain algorithm under the¹
²same parameter setting (the number of principal components was set to 15 and²
³the resolution parameter to 0.2). Note that DIPD-based data representation does³
⁴not implement feature selection, but the other methods select the most variable⁴
⁵genes (top 2,000 for log-normalized representation and top 3,000 for SCTransform⁵
⁶by default). As before, ARI and purity are used to quantitate performance, and⁶
⁷both agree. At $F < 0.5$ (weak signal), all data representations perform similarly. At⁷
⁸ $F > 0.5$ (stronger signal), performance using log-normalized data declines, whereas⁸
⁹SCTransform and model departure remain accurate. These results suggest that log-⁹
¹⁰normalization as the sole pre-processing step is sensitive to library size effects. ¹⁰

¹¹ Next, we crafted artificial clusters by perturbing some large count genes from¹¹
¹²the homogeneous luminal epithelial cell line data set (defined in [32]). Artificial¹²
¹³clusters were created by adding counts to a sub-matrix of the UMI count data¹³
¹⁴matrix (top 500 genes with the largest total counts across cells and 250 randomly¹⁴
¹⁵chosen cells (from 541 total)). For each entry of that sub-matrix, random counts¹⁵
¹⁶from the Poisson distribution with parameter $F \times \tilde{\lambda}_{gc}$ were added to the current¹⁶
¹⁷UMI count x_{gc} , where $\tilde{\lambda}_{gc}$ comes from the two-way approximation (see Methods).¹⁷
¹⁸Small (or large) values of F indicate weak (or strong) signals. These perturbed¹⁸
¹⁹cells were regarded as an artificial cluster separated from the remaining cells, where¹⁹
²⁰an accurate identification was expected for increasing values of F . The random²⁰
²¹selection was repeated ten times. Again, we used the same parameter settings for²¹
²²all data representations (15 PCs and a Louvain resolution parameter of 0.2). ²²

²³ Fig. 9 panels c and d show the mean ARI and the mean purity with standard²³
²⁴deviation. Both measures agree. For $F < 0.5$, none of the data representations dis-²⁴
²⁵tinguish the perturbed cells. For $F > 0.5$, DIPD (orange) identifies more perturbed²⁵
²⁶cells, compared to log-normalization (blue), and SCTransform (green). This may be²⁶
²⁷due to the feature selection step limiting the sensitivity at small perturbations. For²⁷
²⁸log-normalized expression, only 27.2% to 45.6% out of the perturbed 500 genes are²⁸
²⁹in the top 2,000 selected genes. Feature-selection based clustering is not as stable²⁹
³⁰as including all the genes across different randomly perturbed cells, as indicated³⁰
³¹by the larger standard deviations. The SCTransform (green) performs the worst in³¹
³²this particular experiment. This again seems to be because 36.2% to 45.8% of the³²
³³perturbed genes are among the 3,000 (default) selected genes for this method. This³³

¹experiment supports the contention that important, local information may be lost¹
²during the feature selection step. 2

³ 3

⁴ 4

⁵ **Discussion** 5

⁶We develop an alternative data representation, DIPD, for scRNA-seq data as well⁶
⁷as a clustering algorithm based on this data representation. DIPD is applicable to⁷
⁸scRNA-seq data that incorporates experimental UMI correction. With an appro⁸
⁹priate number of latent vectors in the GLM-PCA parameter estimation, the IPD⁹
¹⁰statistical framework gives reasonable fits for diverse UMI data sets. Departures¹⁰
¹¹from the IPD statistical framework (i.e. DIPD) can be incorporated into existing¹¹
¹²scRNA-seq analysis pipelines and give improved overall performance independent¹²
¹³of the particular clustering algorithm. 13

¹⁴ Working on the scale of probabilities rather than counts offers numerous advan¹⁴
¹⁵tages. First, due to the characteristics of scRNA-seq data (many zeros and low¹⁵
¹⁶counts in most matrix entries), working in probability space is a more appropriate¹⁶
¹⁷way to represent the underlying data structures. The DIPD-based data matrix, pro¹⁷
¹⁸vides a useful tool to uncover cell heterogeneity from observed counts into a model¹⁸
¹⁹departure from the hypothesized Poisson parameter matrix, as input to any sub¹⁹
²⁰sequent analyses. The large number of zeros in scRNA-seq data, which have been²⁰
²¹considered in row or column based analyses to be zero-inflation, is more precisely²¹
²²viewed as a large number of very small Poisson probabilities. Similarly, the pre²²
²³viously reported over-dispersion is explained by variation in the set of individual²³
²⁴Poisson parameters within the framework (Fig. 1). 24

²⁵ Implementing Sigclust2 in clustering provides an explicit hypothesis testing for²⁵
²⁶each cluster, which avoids parameter tuning. A direct comparison of different data²⁶
²⁷representations demonstrated that DIPD had an improved performance over con²⁷
²⁸ventional log-normalized data (Fig. 7, 8). A hybrid approach combining DIPD with²⁸
²⁹the Louvain clustering algorithm gave the best performance (Fig. 9). Using all the²⁹
³⁰data represented as model departure allowed for the detection of weaker signals³⁰
³¹compared to feature selection based clustering. 31

³² A limitation of this pipeline is computational speed because it uses the full feature 32
³³set. Computational speed vs. the number of features to be included in the model 33

¹represents a trade-off of any unsupervised learning approach. It is not specific to ¹
²this data representation. ²

³ At this point, we have only begun to identify biological scenarios that favor this ³
⁴data representation over others. It is necessary to explore additional scenarios where ⁴
⁵the DIPD and *Hclust-Departure* show differences compared to other approaches. ⁵
⁶This may identify properties of scRNA-seq data beyond over-dispersion and zero ⁶
⁷inflation. ⁷

⁸ The idea of departure based data representation could also be used for other data ⁸
⁹types based on other distributions, for example, the Assay of Transposase Accessible ⁹
¹⁰Chromatin sequencing (ATAC-seq) data based on Binomial distributions. ¹⁰

¹¹ ¹¹

¹²**Conclusions** ¹²

¹³Most of the existing scRNA-seq analysis methods suffer from a too crude aggrega- ¹³
¹⁴tion at either gene or cell level. We proposed shifting the focus from modeling counts ¹⁴
¹⁵to modeling probabilities and avoided the crude approximations by our IPD sta- ¹⁵
¹⁶tistical framework. We investigated the validity of this model using some carefully ¹⁶
¹⁷designed experiments. As a result, we achieved improved cell clustering performance ¹⁷
¹⁸using a novel data representation based on departures from the estimated Poisson ¹⁸
¹⁹distributions without prior feature selection or manual optimization of hyperpa- ¹⁹
²⁰rameters. The idea of our DIPD as data representation can also be combined with ²⁰
²¹other clustering methods, such as the Louvain algorithm implemented in Seurat. ²¹
²²This novel data representation is useful in better understanding the mechanism of ²²
²³scRNA-seq. ²³

²⁴ ²⁴

²⁵**Methods** ²⁵

²⁶Data Description ²⁶

²⁷The main performance of the Poisson independent framework for data representa- ²⁷
²⁸tion is illustrated using multiple data sets representing different scRNA-seq cate- ²⁸
²⁹gories. These are described in the next subsections. They are in increasing order ²⁹
³⁰of biological complexity: (i) single cell line data, (ii) three cell line mixture data, ³⁰
³¹(iii) normal human PBMC data, (iv) data from a mouse tissue infected with the ³¹
³²human immunodeficiency virus (HIV). The data represented a variety of technical ³²
³³platforms. ³³

¹*Single clonal cell line data* 1

² 2

³ To study a scRNA-seq data set which is as homogeneous (and thus Poisson) as 3

⁴ possible, single cell line experiments were considered. The first data set is on the 4

⁵ experiments of [27]. This data set uses flow-cytometry to place individual cells into 5

⁶ wells of a plate. This approach carefully controls the occurrence of doubletons and 6

⁷ conversely allowed us to artificially create wells containing doubletons. The exper- 7

⁸ iment is based on two cancer cell lines, which were obtained from human Primary 8

⁹ Effusion Lymphoma, called JSC-1 and BCBL-1. These cell lines are clonal and have 9

¹⁰ been in culture for many years. Based on extensive biological characterization each 10

¹¹ culture is homogeneous, and within a cell line each cell is identical. 11

¹² The overall experimental design is nested, generating different levels of batch 12

¹³ variation. Batch category one represents technical replicates called plates. Cells 13

¹⁴ within a plate are from the same cell line, collected at the same time and hence are 14

¹⁵ homogeneous in that sense. Batch category two represents data of experiment or 15

¹⁶ biological replicates. The full data set contains 10 plates, (1, . . . , 4, 5A, 5B, 6A, 6B, 16

¹⁷ 7, . . . , 10). The data were pre-processed as described in [27]. Specifically, filtering 17

¹⁸ was done such that each cell had greater than 5,000 total UMI counts and greater 18

¹⁹ than 1,500 detected cellular transcripts. Only protein coding transcripts that were 19

²⁰ detected in more than 0.5% of all cells were retained. The data set used here had a 20

²¹ total of 621 cells and 12,689 genes. 21

²² This carefully constructed data enabled us to validate the *Poissonity* under dif- 22

²³ ferent scenarios, i.e. different degrees of batch variation. The data are summarized 23

²⁴ in Table 2. For instance, Plates 1 and 2 were from the same cell line but performed 24

²⁵ on different dates (biological replicates); Plates 3 and 4 also used the same cell line, 25

²⁶ but were performed on the same date (technical replicates). They were expected to 26

²⁷ be more similar as technical variation is smaller than biological variation. Data la- 27

²⁸ beled Plate 5A and 5B represent cells where the scRNA-seq libraries from the same 28

²⁹ cell was sequenced in two independent runs. Thus these were the most similar data 29

³⁰ sets. The only variation should be due to randomness from the Poisson distribution. 30

³¹ Plates 6A and 6B were from an entirely different cell line JSC-1, and were expected 31

³² to give a radically different expression signature from the BCBL-1 cell line. Plate 8 32

³³ investigated the impact of doubletons by intentionally putting two cells per well. 33

¹*Three cell lines mixture data* 1

²This data set was generated from a mixture of three cell lines by 10X Genomics 2
³as in [41] and cleaned by [32]. There are three cell lines in this data set: human 3
⁴dermal fibroblasts-skin, breast cancer luminal epithelial cell line, and breast cancer 4
⁵basal-like epithelial cell line. These were mixed at a ratio of 1:3:6. The cell of origin 5
⁶label for each cell was retained. The data were pre-processed as discussed in [32]. 6
⁷This data set contains 2,609 cells with known labels and 21,247 genes. 7
⁸8

⁹*PBMC data* 9

¹⁰This scRNA-seq data was generated using 10X Genomics originally from [42]. Cells 10
¹¹contained in this data are peripheral blood mononuclear cells (PBMC) from Homo 11
¹²sapiens. The cells were sorted based on cell-surface markers using Fluorescence- 12
¹³Activated Cell Sorting (FACS). Randomly selected cells from this experiment were 13
¹⁴assembled by [29] as test data sets to measure the clustering performance of different 14
¹⁵software packages. In particular, three experimental data sets were assembled, each 15
¹⁶with different mixture characteristics: Zhengmix4eq (4 cell types of equal propor- 16
¹⁷tions including 3,994 cells and 15,568 genes) Zhengmix4uneq (4 cell types of unequal 17
¹⁸proportions as 1:2:4:6, including 6,498 cells and 16,443 genes) and Zhengmix8eq (8 18
¹⁹cell types of equal proportions including 3,994 cells and 15,716 genes). 19
²⁰20
²¹21

²²*Multiple cell lineages data* 22

²³This data set was based on a study by [28]. This study sampled mouse spleen tissue 23
²⁴and obtained scRNA-seq data sets using the 10X Genomics platform. We used one 24
²⁵of the mice (Sample A5) which is comprised of 1,476 cells and 12,822 genes. Seurat 25
²⁶data cleaning and cell clustering by default parameters were used in the original 26
²⁷report and provided computational cell type labels (more details in [28]). 27
²⁸28

²⁹Existing Methods 29

³⁰We first discuss the GLM-PCA algorithm, which is applied in parameter estimation 30
³¹for our assessment of the IPD framework. Then we give a brief review of the Seurat 31
³²pipeline, for data pre-processing steps and cell clustering as an example for the 32
³³state-of-the-art in RNA-seq data analysis. 33

¹*GLM-PCA algorithm* 1

²GLM-PCA is an algorithm for computing an analog of PCA in the context of ²
³generalized linear models (GLM) (see [25] for details). A typical organization for a ³
⁴scRNA-seq data set is a matrix of counts, where columns denote cells (indexed by ⁴
⁵ $c = 1, 2, \dots, C$), and rows denote genes (indexed by $g = 1, 2, \dots, G$). Let x_{gc} denote ⁵
⁶one matrix entry, and let $n_c = \sum_g x_{gc}$ denote the total counts for the cell c . The ⁶
⁷GLM-PCA calculation using the Poisson distribution treats the counts as a random ⁷
⁸variable: $X_{gc} \sim Poisson(\lambda_{gc})$, i.e. 8

9 9

$$10 \quad P(X_{gc} = x_{gc}) = \frac{e^{-\lambda_{gc}} \lambda_{gc}^{x_{gc}}}{x_{gc}!} \quad (1) \quad 11$$

12 12

13 A useful model for λ_{gc} is 13

14 14

$$15 \quad \log \lambda_{gc} = \log n_c + \alpha_g + \sum_l^L \xi_{gl} \rho_{cl}, \quad (2) \quad 16$$

17 17

18 where α_g is a gene specific parameter, where ξ_{gl} and ρ_{cl} are factor scores and 18
¹⁹loadings with latent dimension L . The scores and loadings have a similar interpre- 19
²⁰tation as in Euclidean PCA, and capture the biological variability after cell and 20
²¹gene specific offsets are removed. The relationships between the Poisson and other 21
²²count models are considered in [43]. 22

²³*Seurat algorithm* 23

²⁴Seurat (Version 3.1.1, [31]) is an R package developed for scRNA-seq data analysis. 24

²⁵It enables users to study the cell-to-cell heterogeneity from transcriptome data. 25

²⁶Seurat also integrates diverse types of single cell data sets (see more details in [23, 44, ²⁶
²⁷31]). At each step in the computation pipeline, there are multiple hyperparameters 27

²⁸to consider. These provide the users with flexibility, but are selected heuristically. 28

²⁹Recommendations for these parameters are arrived at empirically and are varied 29
³⁰depending on the input data set. Here we briefly review the standard workflow as 30
³¹described in [28]. 31

³²**quality control:** Genes with less than three positive counts overall were excluded; 32
³³cells where the unique gene counts (the number of detected genes) were above 2500 33

¹or below 200 were excluded; cells with total mitochondrial gene counts greater than ¹
²5% of the overall total were excluded. ²

³ **normalization by cell:** The gene expression for each cell (x_{gc}) was divided by ³
⁴the cell total counts (n_c) and this quotient was multiplied by a scale factor of 10,000 ⁴
⁵(default). ⁵

⁶ **transformation:** The natural log transformation was applied. ⁶

⁷ **feature selection:** The standardized variance (more details in [31]) was calcu- ⁷
⁸lated for each gene, and the top 2,000 (default) genes with the highest cell-to-cell ⁸
⁹variation were retained. ⁹

¹⁰ **scaling:** The expression of each gene was scaled to have a mean of 0 and vari- ¹⁰
¹¹ance of 1 across cells. A variation of standard scaling includes regularized negative ¹¹
¹²binomial regression, which is called SCTransform [18]. ¹²

¹³ **linear dimension reduction:** The data was represented by the first 15 principal ¹³
¹⁴components obtained by Euclidian PCA. ¹⁴

¹⁵ **clustering:** Cell clustering was done with a graph-based clustering approach using ¹⁵
¹⁶the Louvain algorithm and visualized using t-SNE or UMAP methods. ¹⁶
¹⁷the Louvain algorithm and visualized using t-SNE or UMAP methods. ¹⁷

¹⁸ **Novel Methods** ¹⁸

¹⁹ **Novel Methods** ¹⁹

²⁰In the following section, we describe the approach for assessment of the validity of ²⁰
²¹the IPD statistical framework. We propose DIPD as a novel data representation, ²¹
²²which is a measurement of the relative location of that UMI counts with respect to ²²
²³the independent Poisson distribution at the individual entry level. The cell hetero- ²³
²⁴geneity can be better reflected at the scale of continuous possibilities than in the ²⁴
²⁵original scale with excess zeros. Therefore, we further develop a departure-based ²⁵
²⁶cell clustering algorithms to identify cell subpopulations. ²⁶

²⁷ *Independent Poisson statistical framework* ²⁷

²⁸ *Independent Poisson statistical framework* ²⁸

²⁹We work with scRNA-seq data with individual matrix entries through an IPD statis- ²⁹
³⁰tical framework, where each matrix entry (x_{gc}) is a UMI count indicating expression ³⁰
³¹of gene g for cell c . In particular, we model that as a Poisson random variable X_{gc} , ³¹
³²which is independent over genes and cells. The Poisson probability function is given ³²
³³in equation (1). ³³

¹ In this framework, the maximum likelihood estimate of λ_{gc} is the UMI count x_{gc} ,
² which is not useful because of the large amount of natural Poisson variation. This
³ motivates combining information and one approach is the GLM-PCA algorithm.
⁴ The challenge to measuring the goodness-of-fit is that can not be done using only
⁵ one data point. We approach this by aggregating matrix entries x_{gc} which have
⁶ similar Poisson parameters λ_{gc} , i.e. choosing a reasonable number of entries (in this
⁷ paper we use 200, which allows assessing the “Poissonity” without introducing
⁸ too much variation in the actual underlying parameters) with estimated Poisson
⁹ parameters closest to some given values, and regard the UMI counts from these 200
¹⁰ entries as independent and identically distributed random samples generated from
¹¹ the Poisson distribution with that parameter. Such nearly homogeneous examples
¹² are considered using both Q-Q plots and hypothesis tests. Specifics for measuring
¹³ “Poissonity” are described in the next sections.

¹⁴ Note that when using formula (2) to get parameter estimates, the choice of latent
¹⁵ dimensions L was important. When L was too small, the model was not flexible
¹⁶ enough to appropriately handle biological effects such as cell cycle. So the Poisson
¹⁷ distribution did not provide a good fit to the 200 entries. When L was too large, the
¹⁸ model was too flexible and was driven by Poisson variation, resulting in overfitting
¹⁹ and thus a different poor description of the data. If our underlying IPD framework
²⁰ assumption was correct, there will be a choice of L , where we get a good fit of the
²¹ Poisson distribution. So the existence of such an L was a validation of our underlying
²² IPD framework. We approach this by attempting multiple values of L and assessing
²³ if their results were a reasonable fit. This suitable value can be different for different
²⁴ data sets.

²⁶ *Q-Q plot for small discrete counts*

²⁸ Visualization methods are useful for assessing the “Poissonity” of scRNA-seq
²⁹ data.

³⁰ In general, the Q-Q (Quantile to Quantile) plot provides a useful visualization
³¹ for comparing two distributions. These distributions can be either continuous or
³² discrete, and a common application is to compare a data set represented by its
³³ Cumulative Distribution Function (CDF), with a hypothesized probability distri-

1bution, also represented by its (theoretical) CDF. The Q-Q plot shows the respective¹
2quantiles (the input or argument of the two CDFs) on the vertical and horizontal²
3axes, corresponding to all the probabilities between 0 and 1. The closeness of the³
4graph to the 45° line indicates the closeness of the two probability distributions. ⁴

5 This is illustrated in panel a in Additional file 4 in the case of two very discrete⁵
6distributions (with very low counts of the type commonly encountered in scRNA-⁶
7seq data). Using the notation $p_i = P(X = i)$ for the distribution P on the vertical⁷
8axis, and $q_i = P(X = i)$ for the distribution Q on the horizontal axis. Note that p_i ⁸
9and q_i can either be values from a theoretical distribution such as the Poisson, or⁹
10can represent empirical probabilities derived from count data as proportions. In this¹⁰
11illustration example, define P as $p_0 = 1/3, p_1 = 1/2, p_2 = 1/6$ and Q as $q_0 = 2/3$ ¹¹
12and $q_1 = 1/3$. ¹²

13 Because of the strongly discrete nature of these distributions, the standard Q-Q¹³
14plot, shown as black dots in panel a in Additional file 4 is quite hard to visually¹⁴
15interpret. They do reflect the few integer values taken on by these random variables,¹⁵
16but essentially ignore the important probabilities driving the difference between¹⁶
17these distributions. ¹⁷

18 We provided a more informative version of the Q-Q plot by using the idea of *con-*¹⁸
19*tinuity correction*, which provides a useful bridge between continuous and discrete¹⁹
20distributions. For example, this idea was the key to the Normal approximation of²⁰
21the Binomial. The main idea was to approximate an integer valued discrete distri-²¹
22bution, with a continuous probability distribution, as seen in panel b in Additional²²
23file 4. The simple version shown there was a step function, with steps at the half²³
24integers, where the height of each rectangle was the corresponding probability. The²⁴
25CDF of a continuity corrected discrete distribution was piecewise linear with knots²⁵
26at the half integers (essentially a linear interpolation), as illustrated in panel c (for²⁶
27the distribution in panel b). The Q-Q plot comparing respective quantiles of the²⁷
28two distributions was shown as the blue curve in panel a. Because both CDFs were²⁸
29piecewise linear, this curve was as well, with knots at the union of the CDF knots.²⁹

30 In the case of checking an empirical CDF against a potential theoretical model³⁰
31CDF, a useful device for understanding the natural variation in a Q-Q plot was the³¹
32*Q-Q envelope*. This visualization identifies which observed aspects were important³²
33and which were artifacts of sampling variation. This idea modeled the hypothesized³³

¹sampling process by simulating repeated samples of the same size from the candidate¹
²theoretical distribution, and overlaying the envelope of resulting CDFs (also using²
³the idea of *continuity correction*). In the case of conventional Q-Q plots (shown as³
⁴black dots in panel a Additional file 4), this gave a useless visual impression in low⁴
⁵count discrete settings. But as seen in Results section, the continuity corrected Q-Q⁵
⁶envelopes are very useful. 6

7

7

⁸*Over-dispersion test* 8

⁹In the case of the Poisson distribution, an insightful test was the dispersion test. An⁹
¹⁰important property of the Poisson distribution was the mean equals the variance.¹⁰
¹¹However, many mixtures of Poisson, such as the Negative Binomial, have a variance¹¹
¹²which was larger than the mean, called *over-dispersion*. 12

¹³ Under the null hypothesis that $H_0 : X \sim Poisson(\lambda)$, we have $E(X) = Var(X) =$ ¹³
¹⁴ λ . The over-dispersion alternative is $Var(X) = (1+\alpha)\lambda$, ($\alpha > 0$). A test statistic was¹⁴
¹⁵derived (more details in [45]) for measuring this, which is asymptotically normal.¹⁵
¹⁶This test is conducted using the dispersion test from the R package AER (v1.2-9;¹⁶
¹⁷[46]) 17

18

18

¹⁹*Zero-inflation test* 19

²⁰A much different departure from the Poisson that can arise in certain applications²⁰
²¹was *zero-inflation*, where the number of observed zeros was larger than the expected²¹
²²number of zeros. We implemented this test with the R package vcdExtra (v0.7-5;²²
²³[47]), which was based on a score test proposed by [48] using a test statistic with²³
²⁴an asymptotic Chi-square distribution. 24

25

25

²⁶*Model departure as data representation* 26

²⁷Again, from our IPD framework, each gene expression measurement for each cell²⁷
²⁸(i.e. each matrix entry) comes from an independent Poisson distribution with pa-²⁸
²⁹rameter λ_{gc} . A naïve starting point for the application of that framework is viewing²⁹
³⁰cell and gene differences in a purely additive way, i.e. a two-way approximation,³⁰
³¹expressed as 31

32

32

$$\tilde{\lambda}_{gc} = e^{\mu + \alpha_g + \beta_c}, \quad (3) \quad 33$$

¹where g indexes gene and c indexes cell. Of course, there is much richer biological
²structure beyond this, which we will represent in terms of departures from this
³approximation of each matrix entry. 3

⁴ *Fitting of a simple two-way approximation* The model (3) is fit to the data using
⁵maximum likelihood. In order to make parameter estimation identifiable, restrict
⁶that $\sum_g e^{\alpha_g} = G$ and $\sum_c e^{\beta_c} = C$. 6

⁷ There is a closed solution, which is: 7

$$\begin{aligned} \hat{\mu} &= \log \frac{\sum_{g,c} x_{gc}}{G \times C} \\ \hat{\alpha}_g &= \log \left(\frac{\sum_c x_{gc}}{C} \right) - \hat{\mu} \\ \hat{\beta}_c &= \log \left(\frac{\sum_g x_{gc}}{G} \right) - \hat{\mu} \end{aligned} \tag{4}$$

¹⁵ It's straightforward to prove that the first derivative at parameter estimates de-
¹⁶finied above are all zero. 16

¹⁷ We used the above two-way approximation as an initial model, which gave a
¹⁸first order approximation of both library effects and also gene by gene variation.
¹⁹Phenomena, such as cell clustering, were effectively captured by studying the de-
²⁰parture from that first order approximation. In other words, features of interest
²¹were captured by the difference between the observed UMI counts and the counts
²²expected from the two-way approximation. In particular, the matrix entries that
²³showed significant departure played an important role in cell clustering. The key
²⁴idea of our departure representation of scRNA-seq data is to replace each count x_{gc}
²⁵by a number that reflects how well it is explained by the Poisson distribution from
²⁶the simple two-way approximation. Clustering such numbers is effective at finding
²⁷structure beyond the two-way fit, such as discriminating cell types. We started by
²⁸representing departure in terms of where the given count x_{gc} lay in the $Poisson(\tilde{\lambda}_{gc})$
²⁹distribution. A naïve approach to this would be to use the UMI count x_{gc} in the
³⁰CDF of the $Poisson(\tilde{\lambda}_{gc})$ distribution, i.e. $F(x_{gc}; \tilde{\lambda}_{gc}) = P(X \leq x_{gc} | \tilde{\lambda}_{gc})$. While
³¹this probability was very effective (i.e. probabilities close to zero or close to one indi-
³²cate a strong departure) for large values of $\tilde{\lambda}_{gc}$, it was less effective for small values
³³of $\tilde{\lambda}_{gc}$, because the probability had a lower bound of $P(X = 0 | \tilde{\lambda}_{gc}) = e^{-\tilde{\lambda}_{gc}} \approx 1$ 33

¹(as often encountered in scRNA-seq data). This problem was caused by the con-¹
²ventional CDF representation as $P(X \leq x)$. While it was typically not done, CDFs²
³could also be represented as $P(X < x)$, which for our purposes goes too far in the³
⁴other direction ($P(X = 0|\tilde{\lambda}_{gc}) = e^{-\tilde{\lambda}_{gc}} \approx 0$). Hence, we chose to use the average⁴
⁵form of the CDF, i.e. 5

$$\tilde{F}(x_{gc}; \tilde{\lambda}_{gc}) = \frac{P(X \leq x_{gc}|\tilde{\lambda}_{gc}) + P(X < x_{gc}|\tilde{\lambda}_{gc})}{2}. \quad 6$$

⁸By doing this, our representation of unexpectedly small UMI counts was nearly 0,⁹
¹⁰and unexpectedly large UMI counts was close to 1. 10

¹¹Another consequence of the generally skewed shape of the Poisson distribution,¹¹
¹²(at least for small values of $\tilde{\lambda}_{gc}$) was that these probabilities tend to be quite,¹²
¹³asymmetric at the two ends of the distribution. A straightforward device for more,¹³
¹⁴balanced treatment of the departures from the Poisson fit was to take the matrix,¹⁴
¹⁵entries to be the logit transform of these CDF based probabilities: 15

$$D = \text{logit}(\tilde{F}(x_{gc}; \tilde{\lambda}_{gc})) = \ln\left(\frac{\tilde{F}(x_{gc}; \tilde{\lambda}_{gc})}{1 - \tilde{F}(x_{gc}; \tilde{\lambda}_{gc})}\right) \quad 16$$

¹⁸Since exactly 0 and 1 were not allowed for the logit transformation, set any matrix¹⁸
¹⁹entries with $\tilde{F}(x_{gc}; \tilde{\lambda}_{gc})$ below 10^{-10} as $\text{logit}(10^{-10})$, and $\tilde{F}(x_{gc}; \tilde{\lambda}_{gc})$ above $(1 -$ ¹⁹
²⁰ $10^{-10})$ as $\text{logit}(1 - 10^{-10})$. 20

²¹The logit transformed data takes on very negative (or positive) values if the UMI²¹
²²count is much lower (or higher) than expected from the simple two-way approxi-²²
²³mation. The collection of cells with such novel data representation can be plugged²³
²⁴into a standard clustering algorithm (in this paper we choose hierarchical clustering²⁴
²⁵with Euclidean distance and Ward's linkage). 25

²⁶*Crafted experiments* For each matrix entry UMI count x_{gc} , we calculated the²⁶
²⁷perturbed value by generating a random count from the Poisson distribution with²⁷
²⁸parameter $\left| e^{\hat{\mu} + \hat{\alpha}_g + (1+F) \times \hat{\beta}_c} - \tilde{\lambda}_{gc} \right|$ as p_{gc} , where $\hat{\mu}$, $\hat{\alpha}_g$, $\hat{\beta}_c$ and $\tilde{\lambda}_{gc}$ are parameters²⁸
²⁹defined in the two-way approximation and estimated by equation (4). The value²⁹
³⁰for F controls the strength of the library size magnification. Then we perturbed³⁰
³¹each matrix entry as $(x_{gc} + \text{sign}(\hat{\beta}_c) \times p_{gc})_+$, where the subscript of plus denotes³¹
³²the positive part. This magnified the library size effects as the cells with originally³²
³³positive (or negative) cell effect $\hat{\beta}_c$ become even larger (or smaller). 33

¹*Cell clustering algorithm* 1

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⁷ 7

⁸ 8

⁹The proposed clustering starts with the DIPD-based matrix computed for the com-₉

¹⁰plete data set. Hierarchical clustering using Euclidean distance and Ward’s linkage₁₀

¹¹is recommended from a top-down viewpoint. At each step, we re-calculated the₁₁

¹²two-way approximation again within each subcluster, and the potential for further₁₂

¹³splitting is calculated using Sigclust2 [30], a method to assess statistical significance₁₃

¹⁴at each split based on a Monte Carlo simulation procedure. A non-significant result₁₄

¹⁵suggests cells are reasonably homogeneous and may come from the same cell type.₁₅

¹⁶In addition, to avoid over splitting, we further require setting a maximum allowable₁₆

¹⁷number of splitting steps J (default is 10, which leads to at most $2^{10} = 1024$ total₁₇

¹⁸number of clusters) and minimal allowable cluster size S (the number of cells in₁₈

¹⁹a cluster allowed for further splitting, default is 10) beforehand. Thus the process₁₉

²⁰was stopped when any of the conditions was satisfied: (1) the split was no longer₂₀

²¹statistically significant; (2) the maximum allowable number of splitting steps was₂₁

²²reached; (3) any current cluster had less than 10 cells. This process was done in₂₂

²³a recursive way. Algorithm 1 and Fig. 4 outline the procedure using hierarchical₂₃

²⁴clustering in a recursive way based on departure representation. 24

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³⁰ 30

³¹ We do not need to set the number of clusters beforehand. Thinking of the number 31

³²of clusters in a multi-scale way as in [32], a coarser scale clustering can be obtained 32

³³by stopping the clustering process at any stage in between. 33

```

1 Algorithm 1: Hierarchical Clustering using DIPD 1


---


2 Result: cluster label for every input cell 2
3 Initialize: 3
4 maxSplit  $J$  (the maximum allowable number of splitting steps, default 10) 4
5 split index  $j = 1$  5
6 6
7 splitResult  $R$  ( $C \times J$  empty matrix, with cells to cluster as rows, split index 7
8 as columns) 8
9 minSize  $S$  (the minimal allowable cluster size, default 10) 9
10 complete UMI counts data to cluster  $dat_1$  10
11 11
12 /* iterate over  $j$  in a recursive way */ 12
13 Function: 13
14 hclustDepart( $dat_j, j$ ) 14
15 Input : UMI counts sub matrix ( $dat_j$ ) with cells in a current cluster; split 15
16 index  $j$  16
17 Output : splitResult  $R$ , with  $r_{ij}$  denoting the cluster label for the cell  $i$  at 17
18 split step  $j$  18
19 set  $D_j$  to be DIPD-based data matrix calculated from the input UMI count 19
20 sub matrix  $dat_j$  20
21 apply hierarchical clustering based on  $D_j$  using Euclidean distance and 21
22 Ward's linkage 22
23 use sigclust2 to find p-value ( $p$ ) for first split 23
24 if  $p > 0.05$  or  $j > J$  or number of cells in current cluster  $\leq S$  then 24
25 | output  $R$  [all cells,  $j$ ] =  $NA$  25
26 else 26
27 | split  $D_j$  into two clusters ( $D_{1j}, D_{2j}$ ) based on hierarchical clustering 27
28 | set  $dat_{1j}$  and  $dat_{2j}$  to be corresponding UMI counts matrix of two clusters 28
29 | output  $R$  [cells in cluster1,  $j$ ] = 1;  $R$  [cells in cluster2,  $j$ ] = 2 29
30 | hclustDepart( $dat_j = dat_{1j}, j = j + 1$ ) 30
31 | hclustDepart( $dat_j = dat_{2j}, j = j + 1$ ) 31
32 end 32


---


33 33

```

¹ Declarations	1
² Ethics approval and consent to participate	2
³ Not applicable.	3
⁴	4
⁵ Consent for publication	5
⁶ Not applicable.	6
⁷	7
⁸ Availability of data and materials	8
⁹ ScRNA-seq data sets used in this study are all publicly available. The single clonal	9
¹⁰ cell line data is available at	10
¹¹ https://bitbucket.org/dittmerlab/scrnaseq_bcb11/src/master/data/	11
¹²	12
¹³ The three cell lines mixture data is available at	13
¹⁴ https://github.com/siyao-liu/MultiK/tree/main/data .	14
¹⁵ The PBMC data sets can be assessed through the DuoClustering2018 package at	15
¹⁶ https://bioconductor.org/packages/release/data/experiment/html/DuoClustering2018.html .	16
¹⁷ The mouse multiple cell lineages data is available at the Gene Expression Omnibus	17
¹⁸ (GSE148796).	18
¹⁹	19
²⁰ Code availability	20
²¹ R code used to demonstrate the fit of our IPD statistical framework and perform	21
²² clustering using <i>Hclust-Departure</i> is available as an R (CRAN) package and can be	22
²³ accessed from	23
²⁴ https://cran.r-project.org/web/packages/scpoisson/index.html .	24
²⁵	25
²⁶ Competing interests	26
²⁷ The authors declare that they have no competing interests.	27
²⁸	28
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³³ Computational Medicine Program Award 2020.	33

¹ Authors' contributions	1
² Conceptualization: YP, JSM, DW, DPD	2
³ Software: YP, JTL	3
⁴ Formal analysis: YP, DW, JSM	4
⁵ Investigation: YP, RM, JTL, DPD	5
⁶ Data Curation: YP, JTL	6
⁷ Writing (Original Draft): YP	7
⁸ Writing (Review and Editing): DPD, RM, JTL, DW, JSM	8
⁹ Supervision: JSM, DW, DPD	9
¹⁰ Funding acquisition: DPD	10
11	11
¹² Acknowledgements	12
¹³ Not applicable.	13
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18		18
19	Figures	19
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20	Tables	20
20		20
21	Additional Files	21
21		21
22	Additional file 1.pdf file	22
22	A heatmap view of the data representations of a single cell line data set (Plate 3 [27]). Data representations based	22
23	on (a) DIPD and (b) Seurat normalized and scaled counts before feature selection. The black colored lines in the	23
23	sidebars on the right represent the top 2,000 most variable genes kept by the Seurat pipeline. Visually, both data	23
24	representations demonstrate this data set is homogeneous.	24
24		24
25	Additional file 2.pdf file	25
25		25
26	A heatmap view of the data representations of a mixture cell lines data set (three mixture cell lines data [32]). Data	26
26	representations based on (a) DIPD and (b) Seurat normalized and scaled counts before feature selection. The black	26
27	colored lines in the sidebars on the right represent the top 2,000 most variable genes kept by the Seurat pipeline.	27
27	Visually, both data representations effectively demonstrate the differentially expressed genes among the three cell	27
28	lines. However, highly expressed genes within single cells, as depicted by the bright red spots, may potentially play a	28
28	role in clustering but many are filtered out by Seurat.	28
29		29
29		29
30	Additional file 3.pdf file	30
30		30
31	The UMAP plot visualizing the clustering performance in the Zhengmix8eq data set [29] using Seurat SCTransform	31
31	(15 PCs and resolution parameter 0.8). Each color represents an identified cluster. Similar as the clustering results	31
32	from Seurat with log-normalized counts, it performs well in identifying the more distinct cell types (NK cells in	32
32	green, Monocytes in red and B cells in blue), but fails to distinguish T subtypes.	32
33		33

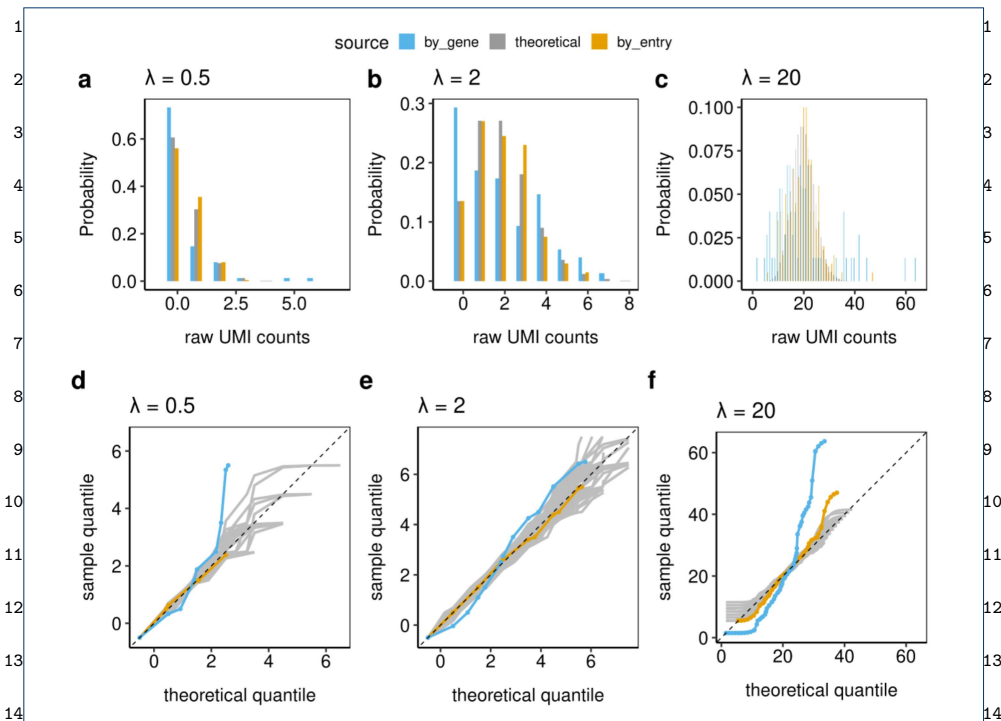


Figure 1 The distribution histograms (a, b, c) and Q-Q envelope plots (d, e, f) of raw UMI count distributions from 75 biologically clonal cells (Plate 3) as defined in section Methods. The gold bars and lines represent 200 matrix entries with estimated Poisson parameter closest to each λ ; the blue represent the entries from genes whose gene averages are closest to each λ ; and the gray represent the theoretical Poisson distributions. These plots indicate that the IPD statistical framework fits the individual matrix entries well, while working with the gene averages indicates the over-dispersion and zero-inflation may occur.

Additional file 4.pdf file

Continuity correction for point mass function (PMF) and Q-Q plot developed for small discrete counts. The black dots in panel a show all the conventional Q-Q points piled up at a few small integers. PMF for distribution P (blue shaded area as *continuous approximation*) is shown in panel b. The CDF for the same distribution P (blue shaded area as *continuous approximation*) is shown in panel c. The blue curve in panel a is the corresponding Q-Q plot comparing two discrete distributions P and Q after *continuity correction* and linear interpolation. It provides a more informative way of comparing distributions with small discrete counts.

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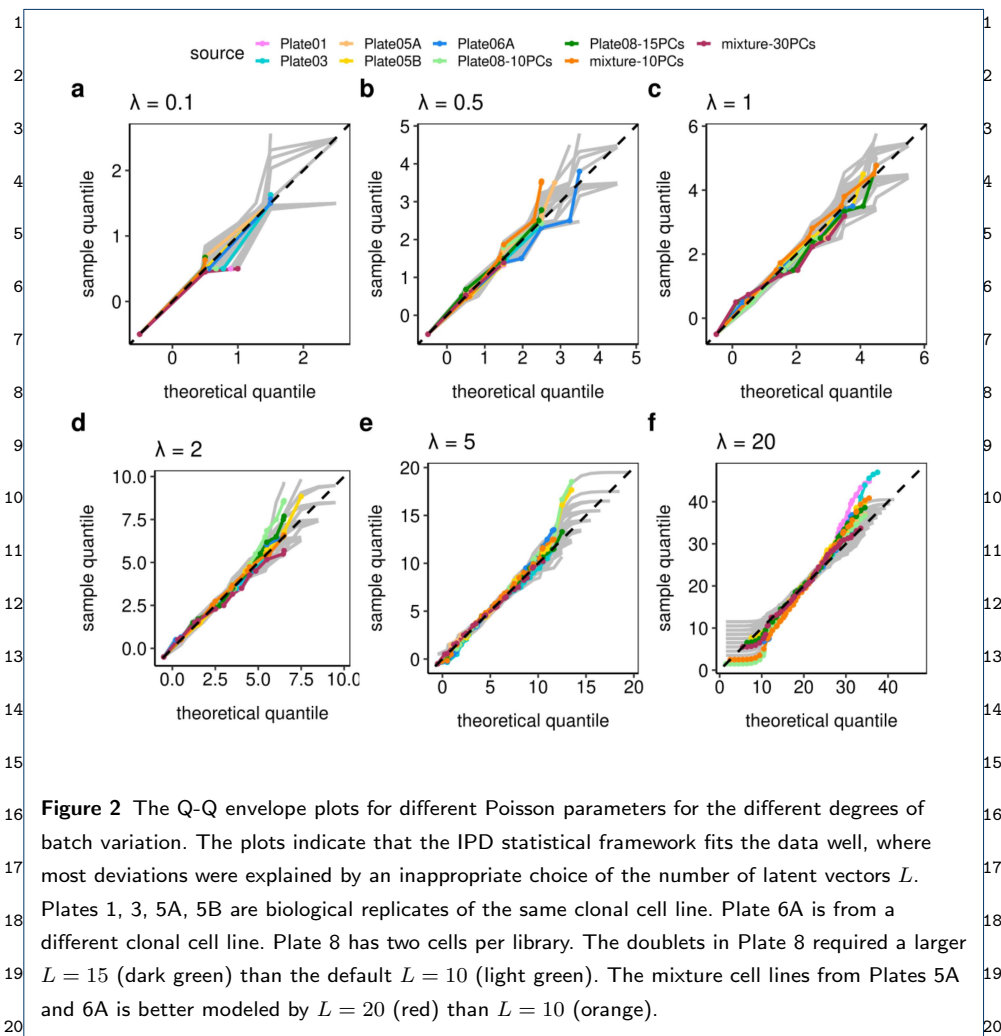


Figure 2 The Q-Q envelope plots for different Poisson parameters for the different degrees of batch variation. The plots indicate that the IPD statistical framework fits the data well, where most deviations were explained by an inappropriate choice of the number of latent vectors L . Plates 1, 3, 5A, 5B are biological replicates of the same clonal cell line. Plate 6A is from a different clonal cell line. Plate 8 has two cells per library. The doublets in Plate 8 required a larger $L = 15$ (dark green) than the default $L = 10$ (light green). The mixture cell lines from Plates 5A and 6A is better modeled by $L = 20$ (red) than $L = 10$ (orange).

Table 1 Confusion Matrix comparing clustering results with FACS labels

		Seurat									
FACS		s0	s1	s2	s3	s4	s5	s6	s7	s8	
B		0	0	0	0	418	0	0	81	0	
Monocytes		1	5	1	547	2	0	0	3	41	
NK		7	6	585	0	1	0	1	0	0	
T helper		198	180	2	0	0	0	19	0	1	
T memory		59	394	0	0	0	0	47	0	0	
Naive Cytotoxic		26	4	0	0	1	367	0	0	0	
T naive		472	19	1	0	1	2	3	1	0	
T regulatory		120	230	0	0	0	1	147	0	0	

		Hclust-Departure													
FACS		h1	h2	h3	h4	h5	h6	h7	h8	h9	h10	h11	h12	h13	h14
B		417	34	48	0	0	0	0	0	0	0	0	0	0	0
Monocytes		0	0	0	0	0	1	0	7	0	0	3	1	558	30
NK		1	0	0	0	1	0	1	5	0	3	0	589	0	0
T helper		0	0	0	214	12	103	16	52	1	0	0	1	0	1
T memory		0	0	0	80	11	108	257	28	14	1	0	1	0	0
Naive Cytotoxic		1	0	0	135	240	11	7	4	0	0	0	0	0	0
T regulatory		0	0	0	164	4	175	17	127	8	0	0	3	0	0

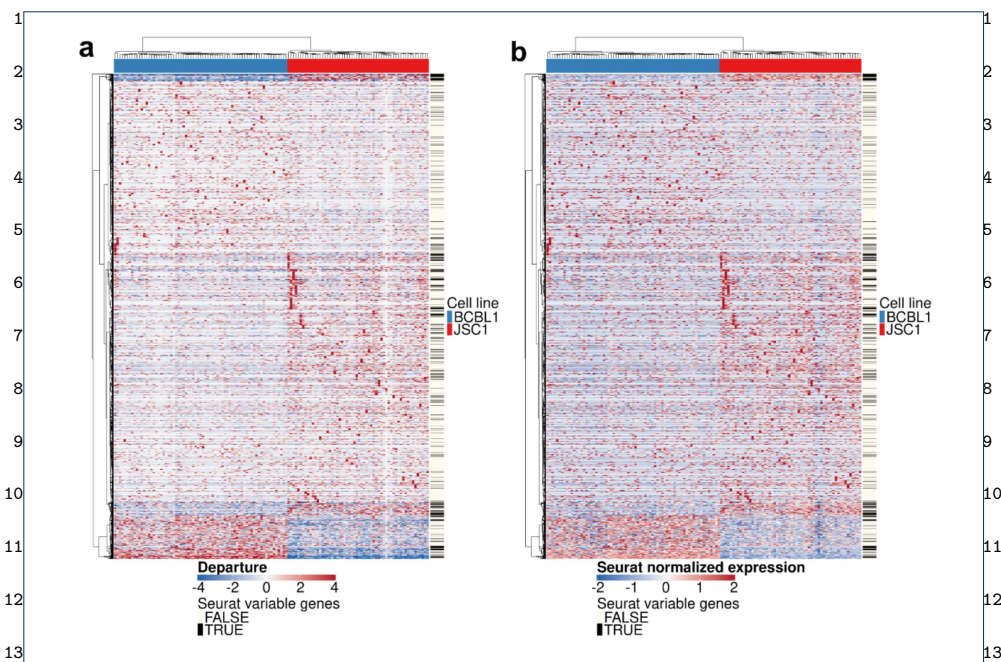


Figure 3 A heatmap view of the data representations based on (a) DIPD and (b) Seurat normalized and scaled counts before feature selection. The orders of cells and genes for both panels are based on the hierarchical clustering with Euclidean distance and Ward's linkage using model departure. The black colored lines in the sidebars on the right represent the top 2,000 most variable genes kept by the Seurat pipeline. Visually, both data representations effectively demonstrate the differential expressed genes between the two cell lines. However, highly expressed genes within single cells, as depicted by the bright red spots, may potentially play a role in clustering but many are filtered out by Seurat.

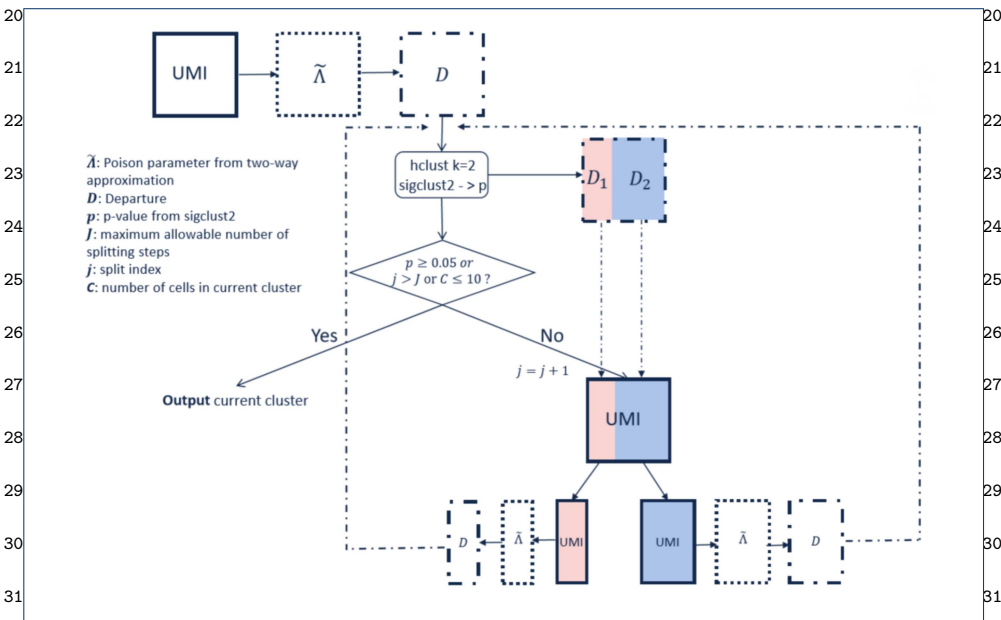


Figure 4 The *Hclust-Departure* cell clustering workflow. Hierarchical clustering is performed using Euclidean distance and Ward's linkage in a recursive way.

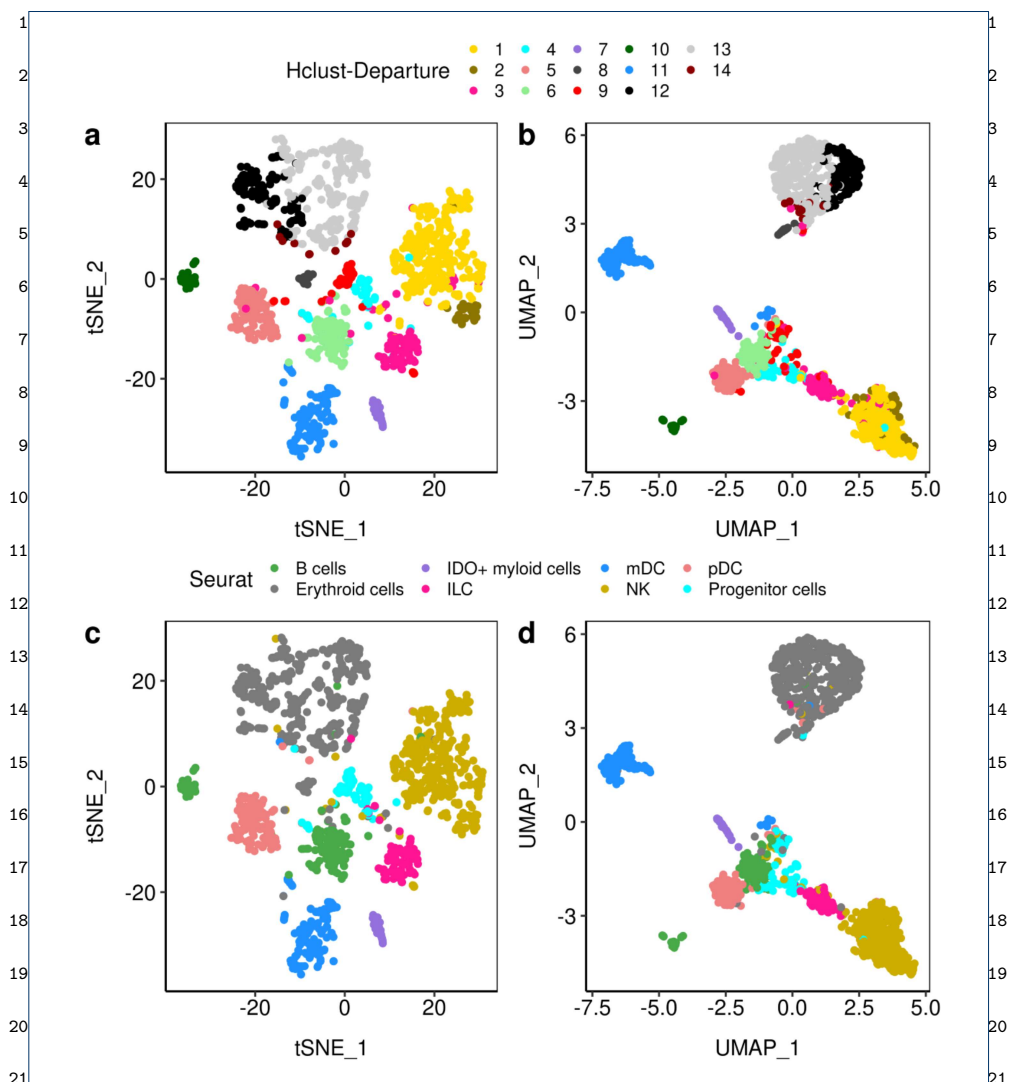
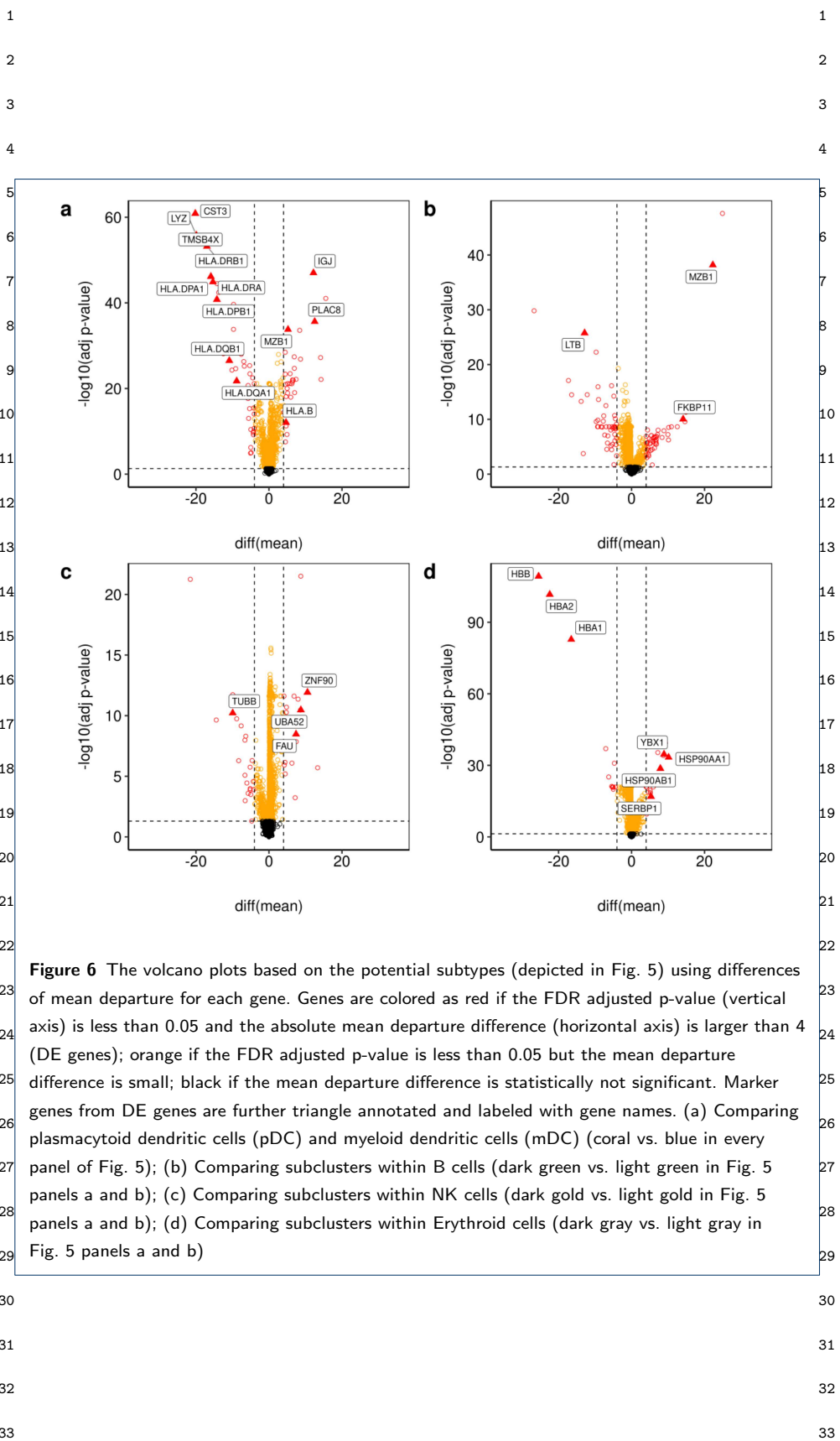


Figure 5 The t-SNE (panels a, c) and UMAP (panels b, d) visualizations of A5 sample which consists of n=1,476 cells from [28]. The top two panels (panels a, b) were based on *Hclust-Departure* using model departure as data representation. The bottom two panels (panels c, d) were labeled by cell types from the Seurat analysis of [28]. The clusters discovered by *Hclust-Departure* are consistent with those identified by Seurat. Furthermore, *Hclust-Departure* identifies several significant subclusters (namely B-cells, NK cells and erythrocytes).

Table 2 Summary of plates used

Plate	Date	Cell Line	Cells Per Well	Cells Per Plate
Plate1	2018-09-04	BCBL1	1	75
Plate2	2018-09-11	BCBL1	1	80
Plate3	2018-09-26	BCBL1	1	75
Plate4	2018-09-26	BCBL1	1	69
Plate5A	2018-09-26	BCBL1	1	71
Plate5B	2018-09-26	BCBL1	1	58
Plate6A	2018-09-30	JSC1	1	71
Plate6B	2018-09-30	JSC1	1	59
Plate8	2018-09-30	BCBL1	2	63



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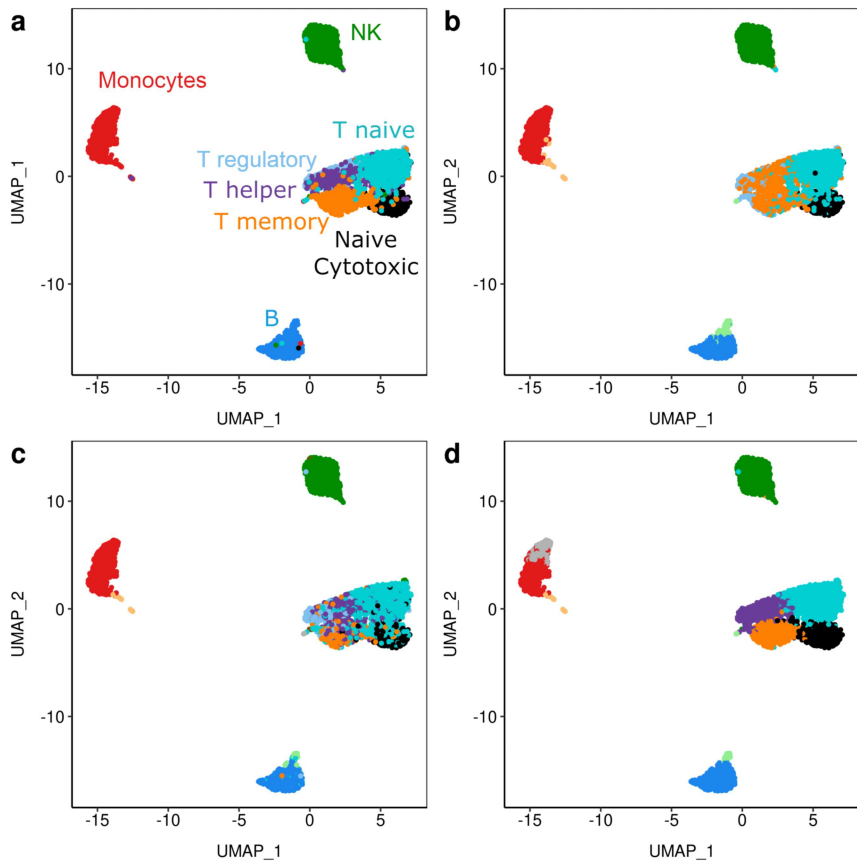


Figure 7 The UMAP plots comparing clustering performance in the Zhengmix8eq data set [29] using different data representations and clustering methods. Panel a displays the FACS labels we used as a benchmark to measure clustering performance. Both the Seurat pipeline (panel b) and our *Hclust-Departure* pipeline (panel c) correctly identify the distinct cell types but fail to distinguish the subtypes within the T cells. Panel d uses the DIPD-based data matrix as data representation combined with Louvain clustering, which is a more direct comparison with panel b since 15 PCs and a resolution of 0.8 are used in both cases. It improves the original Seurat clustering performance by better distinguishing T memory cells from T helper/regulatory cells.

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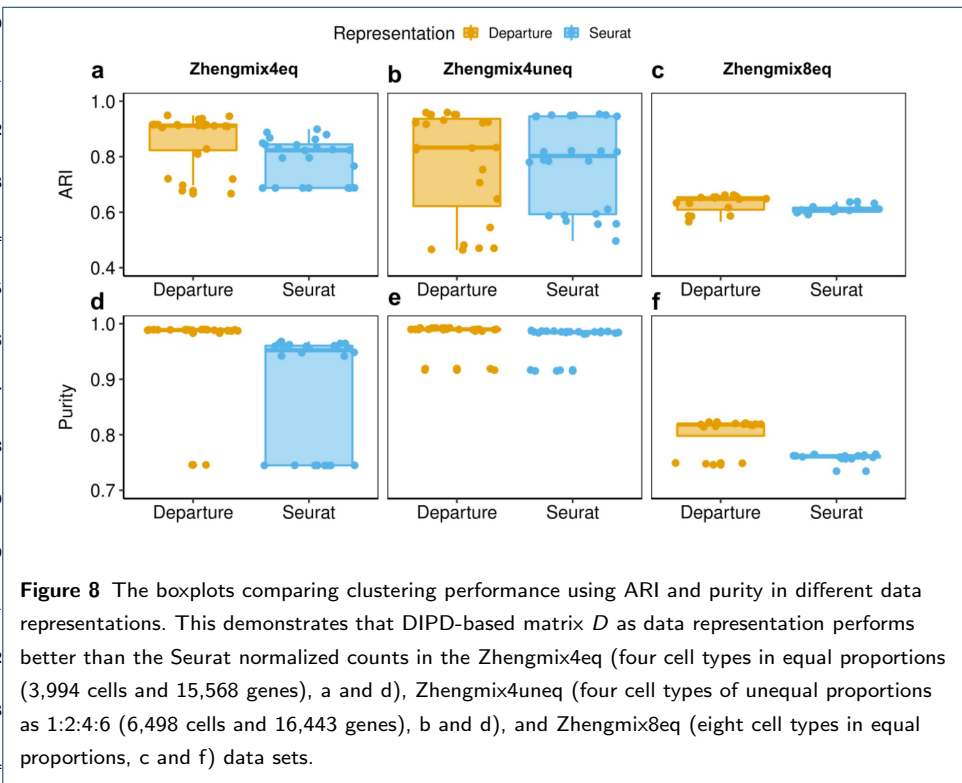
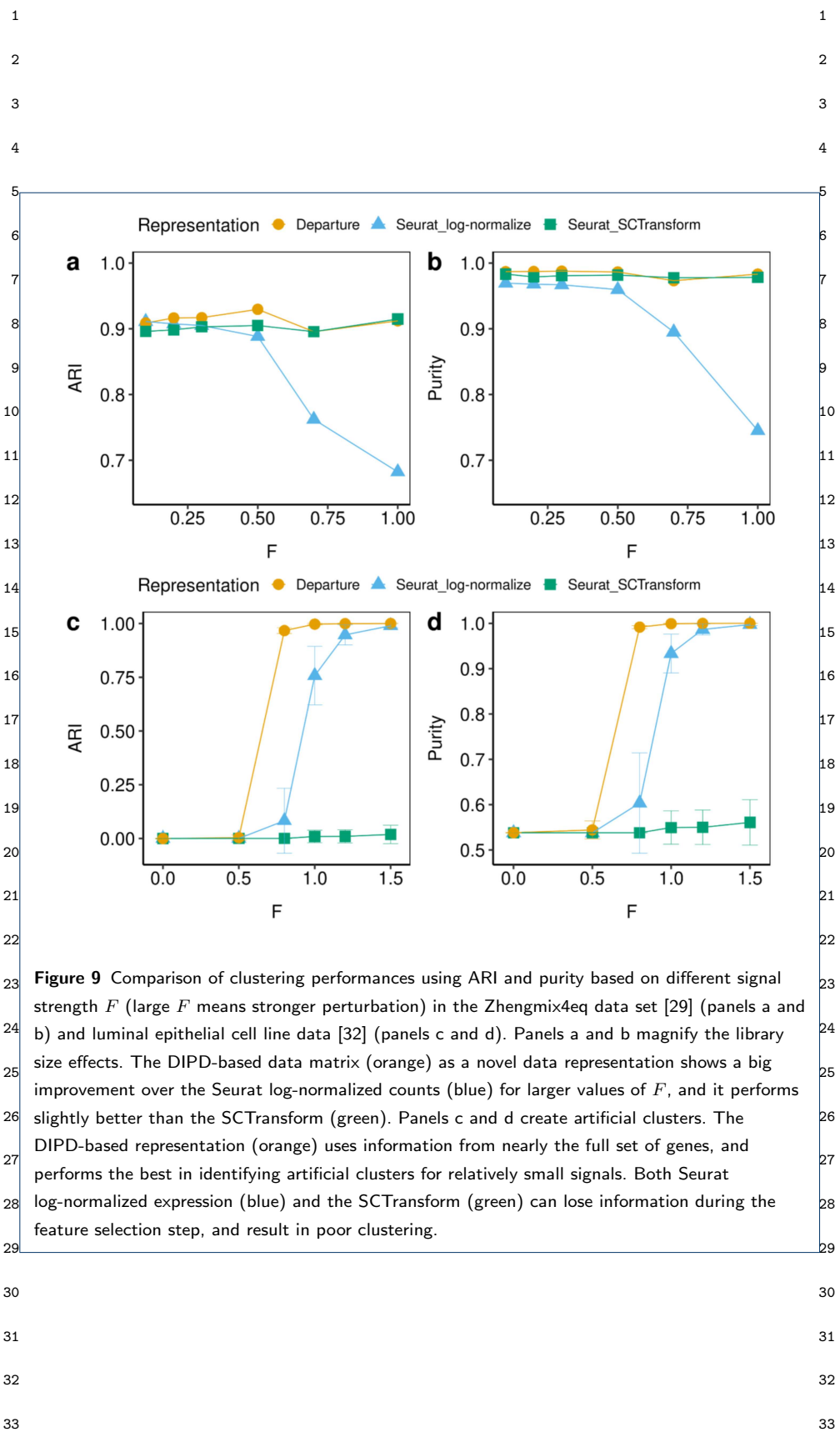


Figure 8 The boxplots comparing clustering performance using ARI and purity in different data representations. This demonstrates that DIPD-based matrix D as data representation performs better than the Seurat normalized counts in the Zhengmix4eq (four cell types in equal proportions (3,994 cells and 15,568 genes), a and d), Zhengmix4uneq (four cell types of unequal proportions as 1:2:4:6 (6,498 cells and 16,443 genes), b and d), and Zhengmix8eq (eight cell types in equal proportions, c and f) data sets.



Figures

source ■ by_gene ■ theoretical ■ by_entry

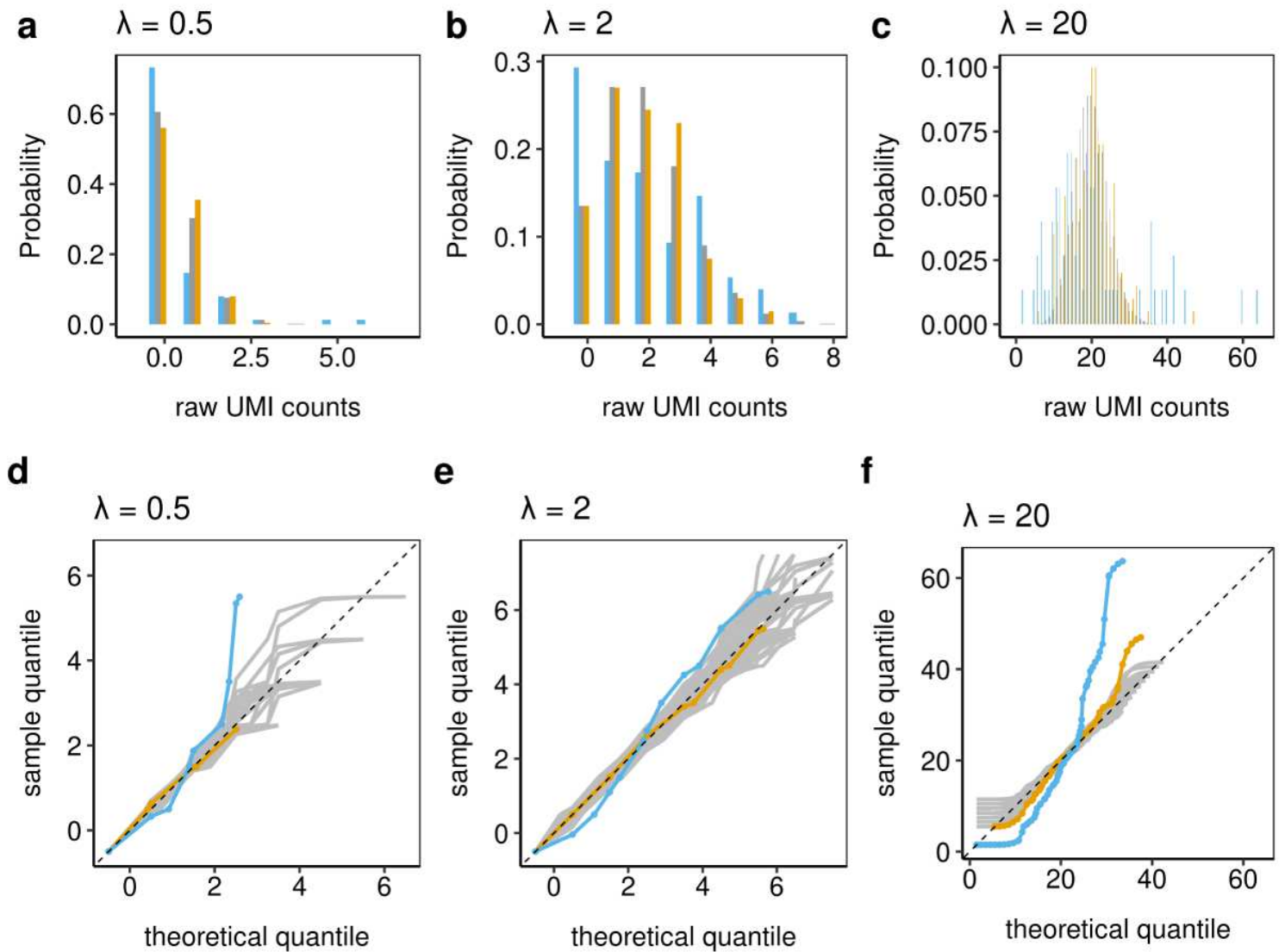


Figure 1

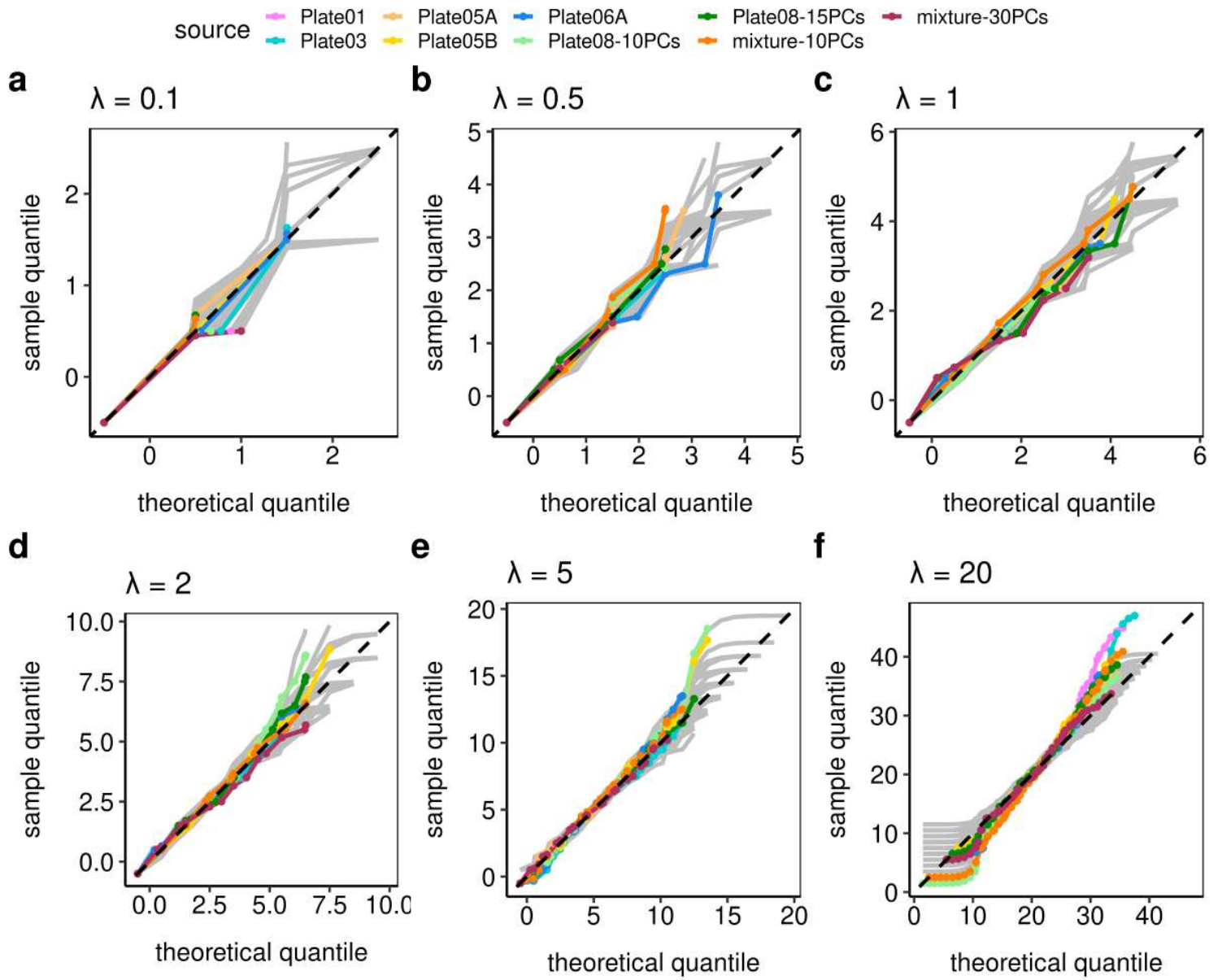


Figure 2

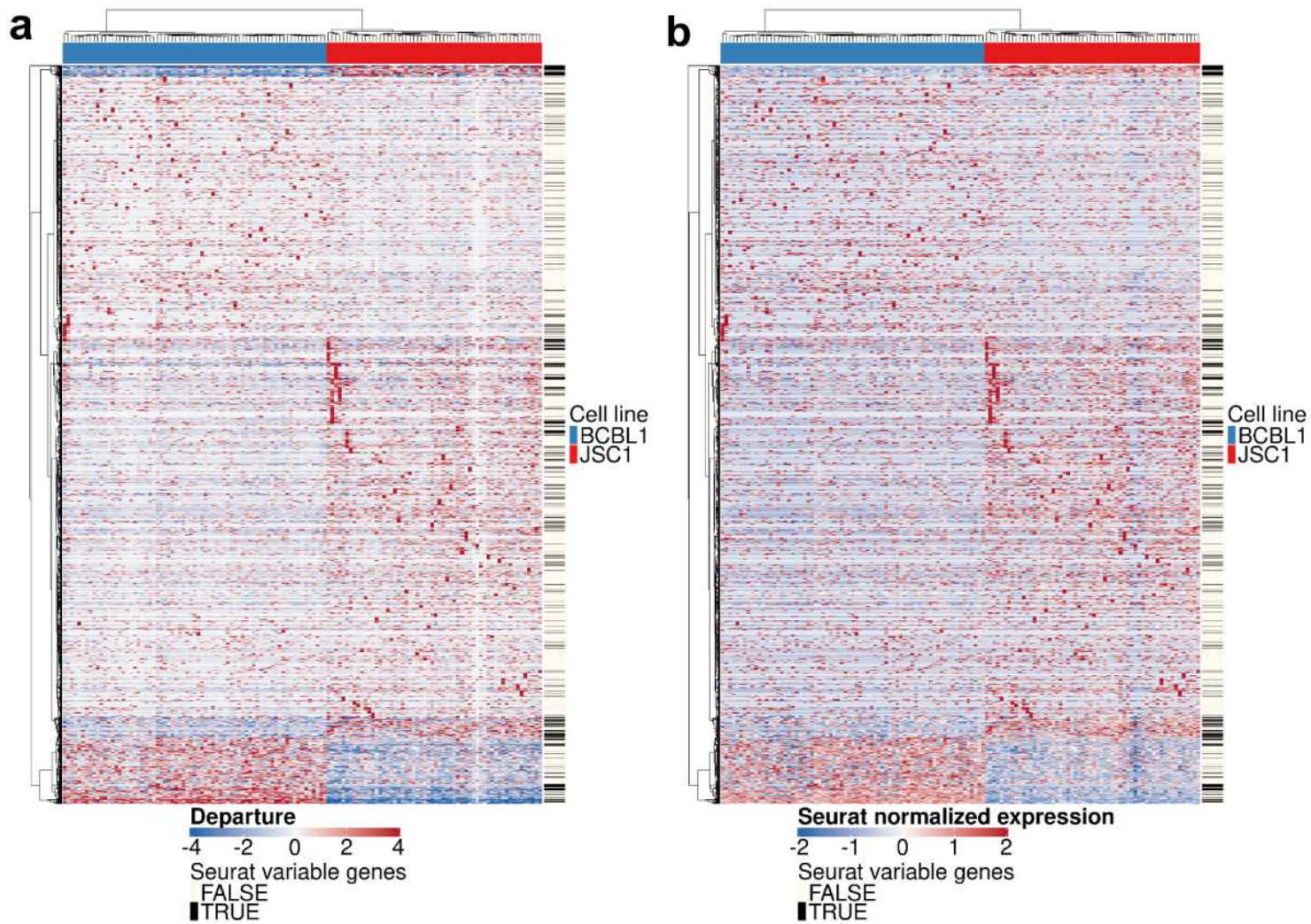


Figure 3

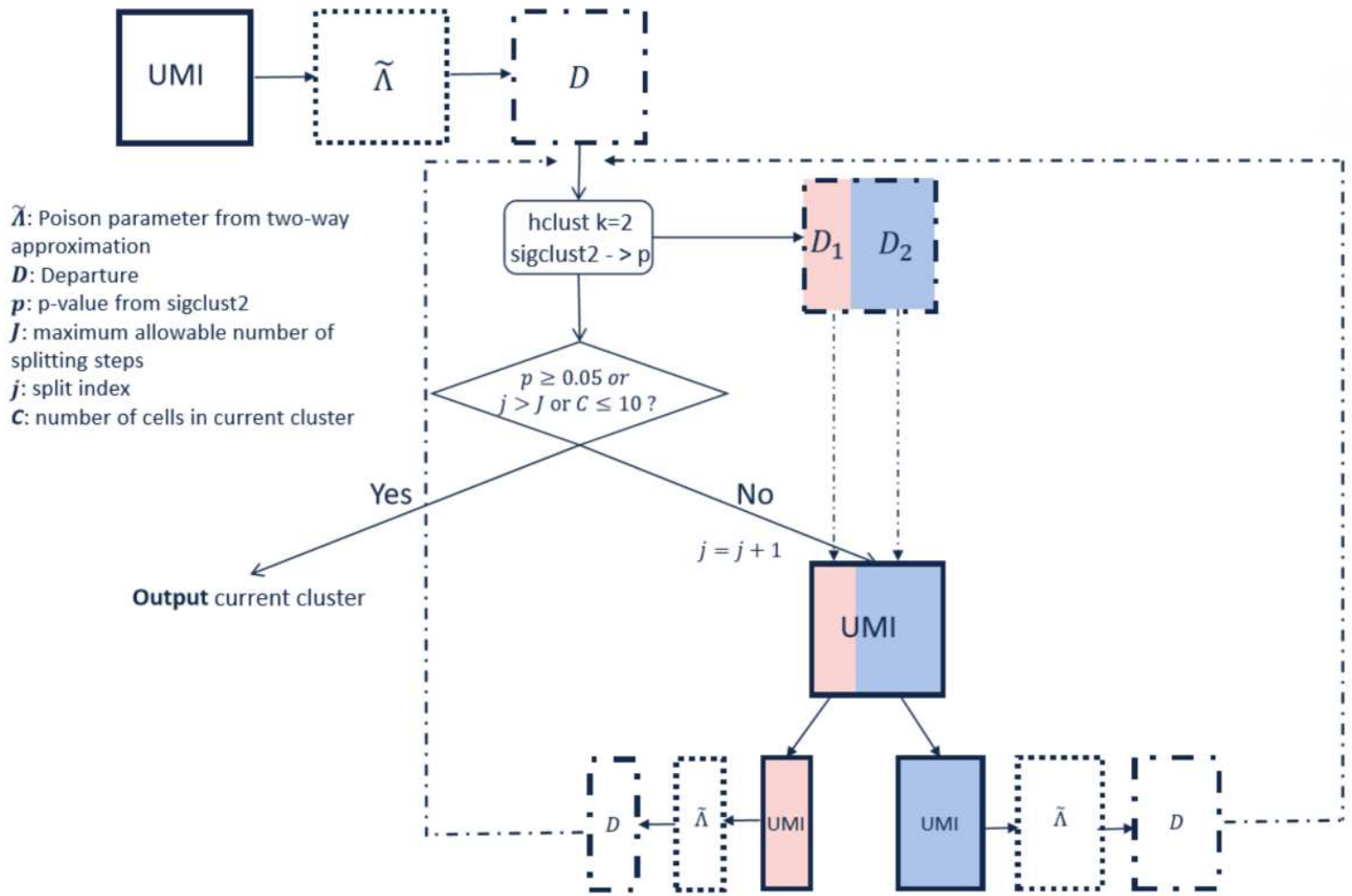


Figure 4

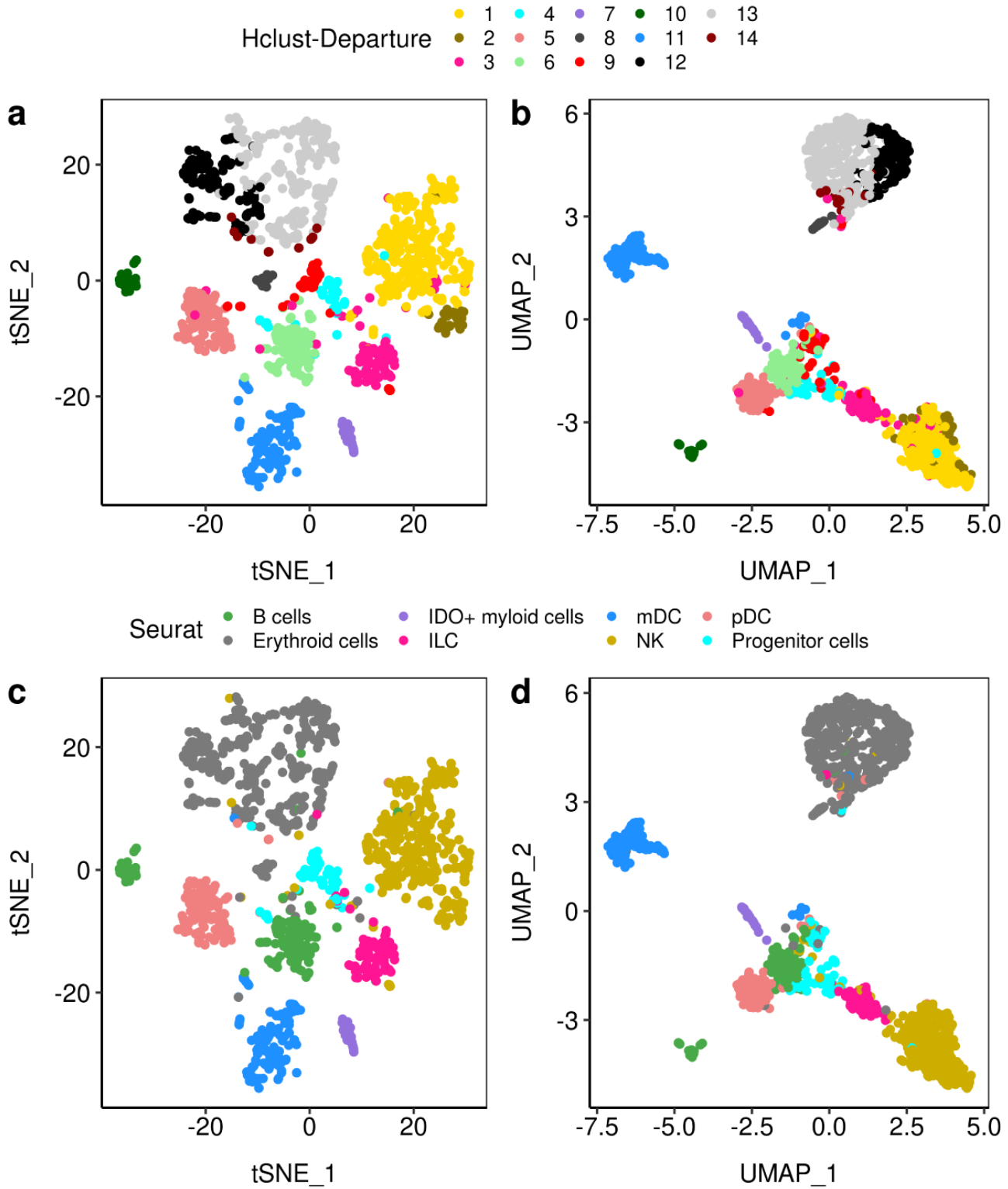


Figure 5

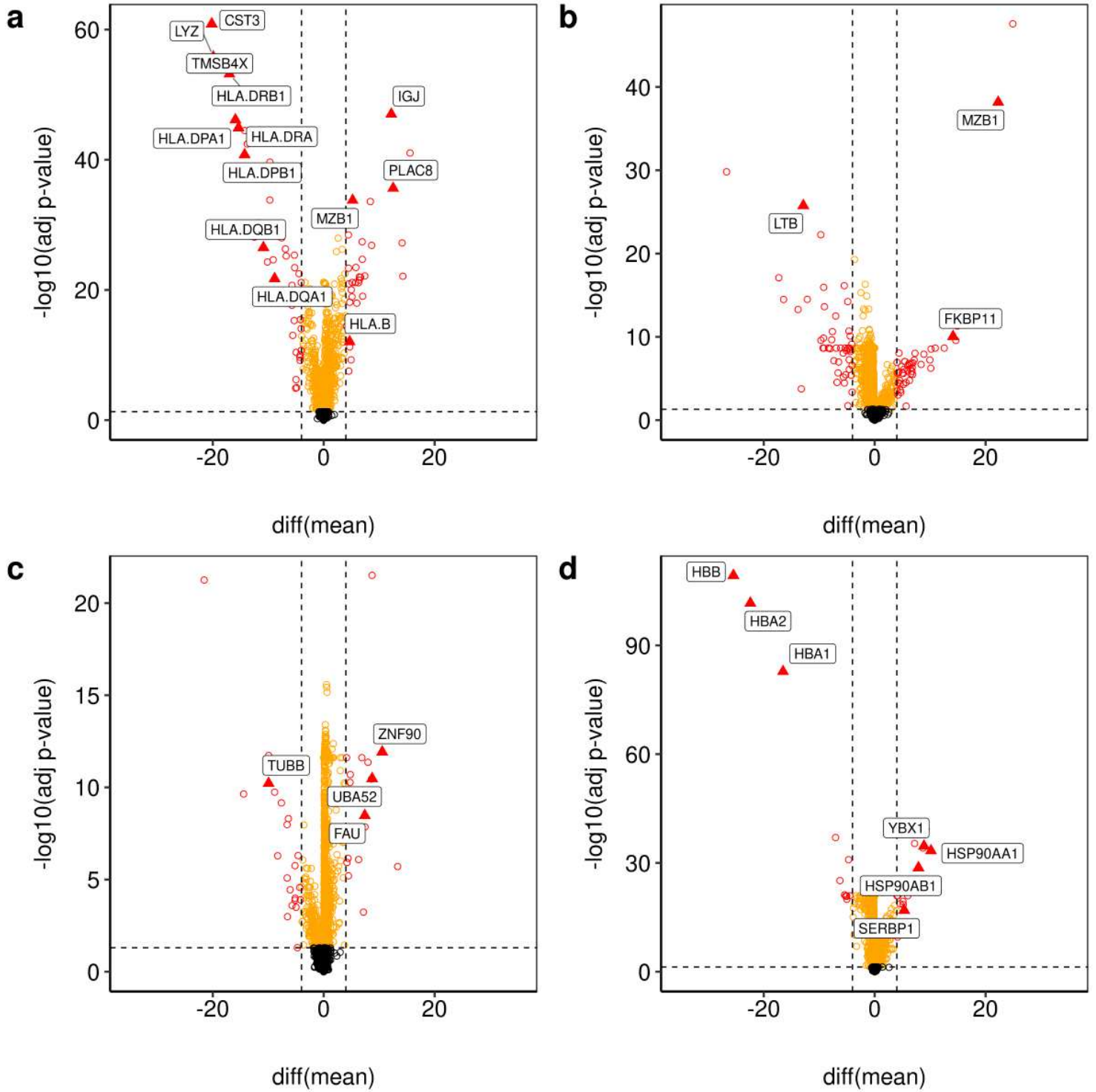


Figure 6

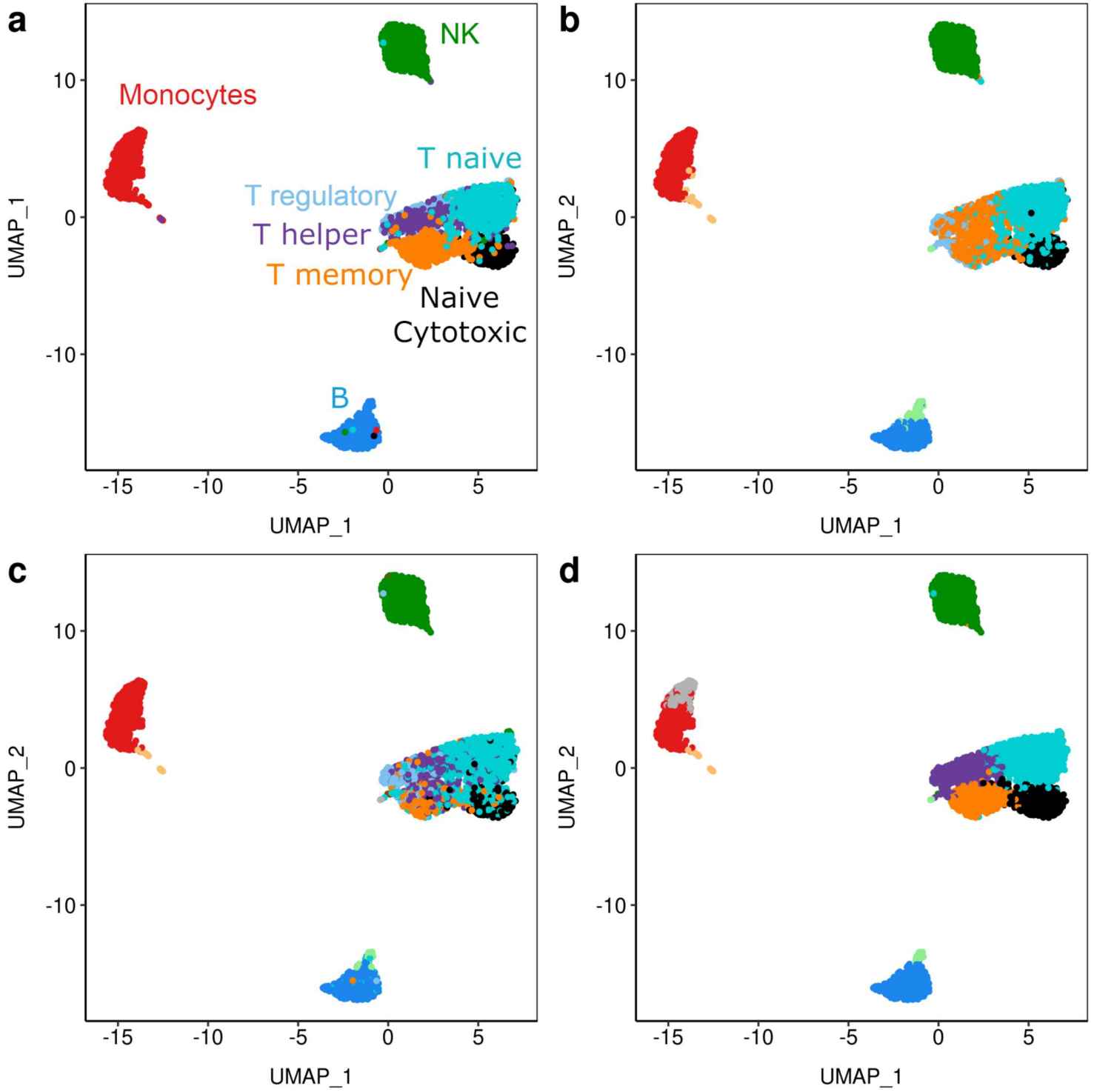


Figure 7

Representation ■ Departure ■ Seurat

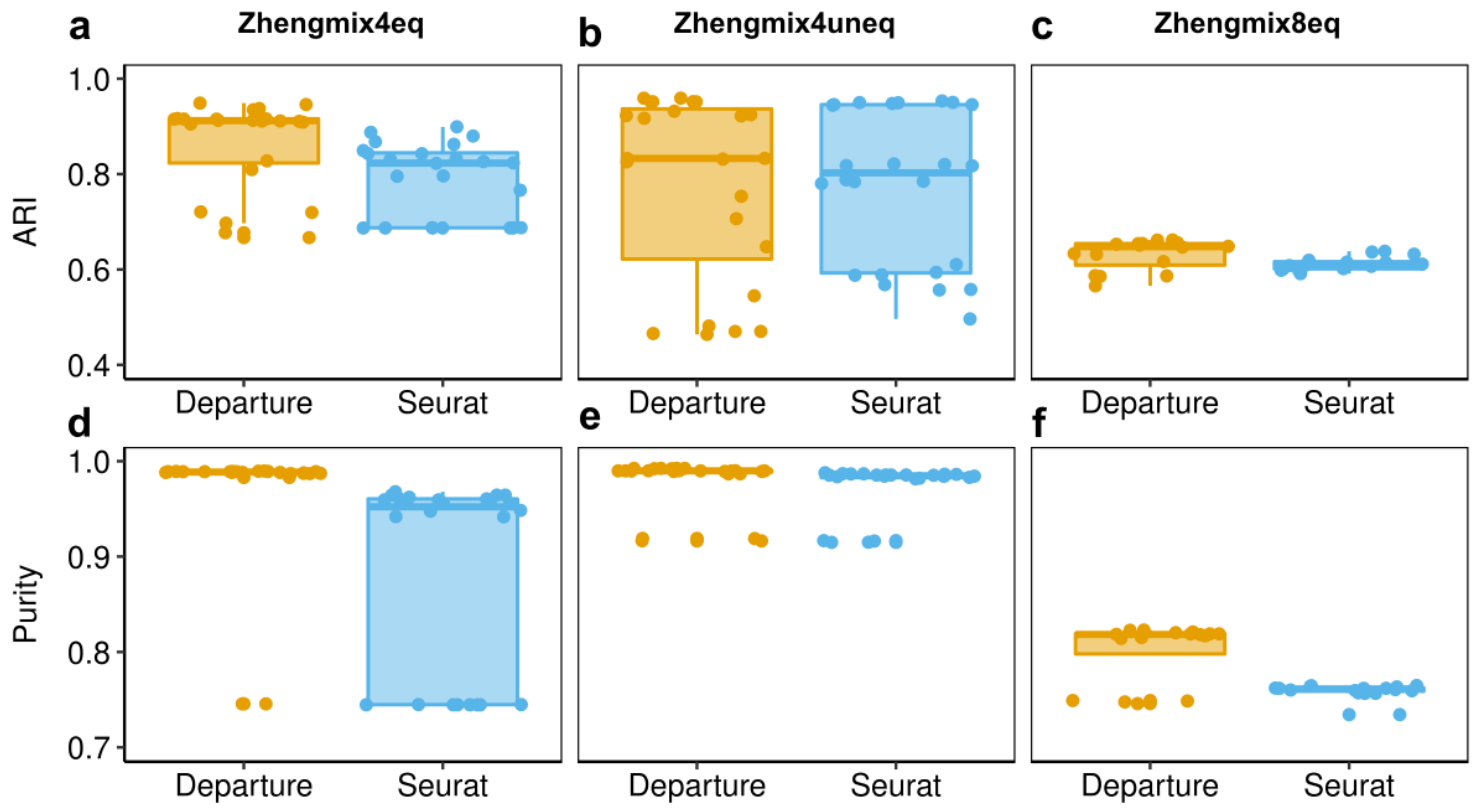


Figure 8

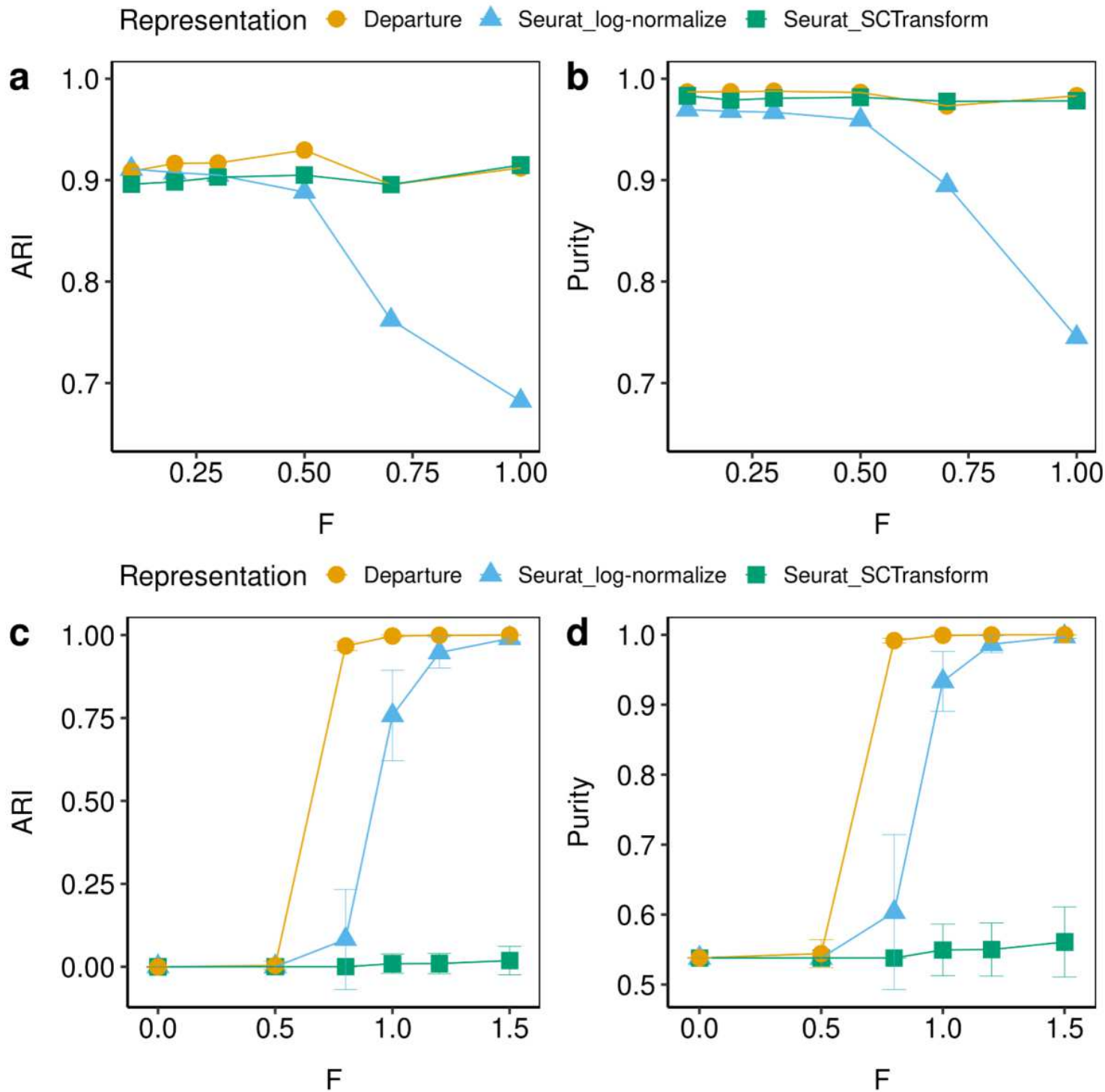


Figure 9

Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.

- [Table1.docx](#)
- [Table2.docx](#)

- [Additionalfile1.pdf](#)
- [Additionalfile2.pdf](#)
- [Additionalfile3.pdf](#)
- [Additionalfile4.pdf](#)