Supplement

Statistical Inferencing is a method to use samples from a population to draw conclusions regarding the entire population. Inferencing can be performed with three standard methods: Estimating, Prediction, and tolerance interval. The validity of inference is related to the way the data are obtained and the stationarity of the process producing the data. For instance, the least square method is one of the estimation models. Tolerance intervals are used to give a range of values that, with pre-specified confidence, will contain at least a pre-specified proportion of the measurements in the population.

One consideration in designing an experiment or sampling study is the desired precision utilizing estimators or predictors. The precision of an estimator is a measure of the estimator variability. Moreover, another equivalent way of expressing precision is the width of a level L confidence interval. For a given population, precision is a function of the sample size: the larger the sample, the greater the precision. However, if our population is not fixed, the precision of the output system is related to the system's robustness. One of the methods for approximating the output with high precision is a fuzzy system.

Fuzzy systems implement fuzzy logic, which is an extended model of standard logic. In standard logic, truth values can only be completely false or completely true (with degrees of truth equal to 0 or 1, respectively), whereas, in fuzzy logic, values can have a degree of truth between 0 and 1. This generalization provides a mathematical framework to move from discrete to continuous values. In other words, in contrast to sets in classical logic, a fuzzy set is a set without a crisp boundary. For instance, if the reference set X is a Universe of discourse for elements x, the fuzzy set A is defined:

$$A = \{ \left(x, \mu_A(x)\right) | x \in X\}$$

Where $\mu_A(x)$ is called the Membership Function (MF) for the fuzzy set A. The MF maps each element of the Universe set X to a grade between 0 and 1, i.e., a membership of 0 means that the associated element is not included, whereas a membership of 1 means a fully included element.

A fuzzy rule-based system is a modeling framework that uses the above fuzzy set theory along with a set of “if-then” rules where the antecedents and consequents are fuzzy logic propositions. This rule-based fuzzy system is used for modeling the inputs and their relationships with the output variables. A type-1 Fuzzy System (T1 FS) is a framework consisting of weighted rules, membership functions, and a fuzzy inference system. This system takes the crisp data (fuzzy singletons) or fuzzy inputs and generates fuzzy outputs based on the given if-then rules. A method of defuzzification is then used to extract a crisp value inferred from the fuzzy model.

From the commonly used membership functions such as triangular, trapezoidal, and Gaussian, this study employs the Gaussian membership function in the FIS model for the assessment of input and output variables. Further steps deal with providing the Gaussian membership functions along with fuzzy input to the neural network. The membership function A has three main features: core, support, and boundary. For any fuzzy set, the core of an MF is defined as a region characterized by full membership in the set, such that:

$$\mu_A(x) = 1$$

An FS is a framework consisting of Fuzzification, Inference Engine, and Defuzzification (Supplemental Fig. 1). This system takes the crisp data (fuzzy singletons) or fuzzy inputs and generates fuzzy outputs based on the given inference engine (if-then rules). A method of defuzzification is then used to extract a crisp value inferred from the fuzzy model. A fuzzy if-then rule has the following form:

$$\text{if } x \text{ is } A_n \text{ then } y \text{ is } B_n$$

Where A_n and B are fuzzified values defined by fuzzy sets on universes of discourse X and Y respectively. The term "x is A_n" is called antecedent and "y is B_n" is the consequent [1]. The result of this step for each rule " if x is A_n then y is B_n " is the fuzzy set B_n truncated at the level $\mu_{An}(x)$, i.e. a set $\mu_{output\ n\,|\,x}$ such that:

$$\mu_{output\ n\,|\,x}(y) = \min(\mu_{Bn}(y), \mu_{An}(x))$$

In the next step, truncated fuzzy sets corresponding to each fired rule are aggregated to provide on single fuzz set $\mu_{Mamdani\,|\,x}$ defined by the membership function:

$$\mu_{Mamdani\,|\,x}(y) = \max_n[\mu_{output\ n\,|\,x}(y)] = \max_n[\min(\mu_{Bn}(y), \mu_{An}(x))]$$

Fuzzification is the process of the above steps to convert the crisp sets to a fuzzy set [2]. FS inference engine applies the rules on the input fuzzy membership function and aggregates the output fuzzy value $\mu_{Mamdani\,|\,x}(y)$. In most applications, it is required to convert the fuzzy output value into a crisp number. This method is called "defuzzification", which can be performed with different mathematical approaches. In this developed FS, we use the "centroid" method since it has the highest correlation between crisp input and output values. The centroid method is calculated with the following formula:

$$y^* = \frac{\int \mu_{Mamdani \mid x}(y) * y \, dy}{\int \mu_{Mamdani \mid x}(y) \, dy}$$

Where y^* is the crisp output value.

Based on our data (Table 1), the fuzzy rules have been defined as:

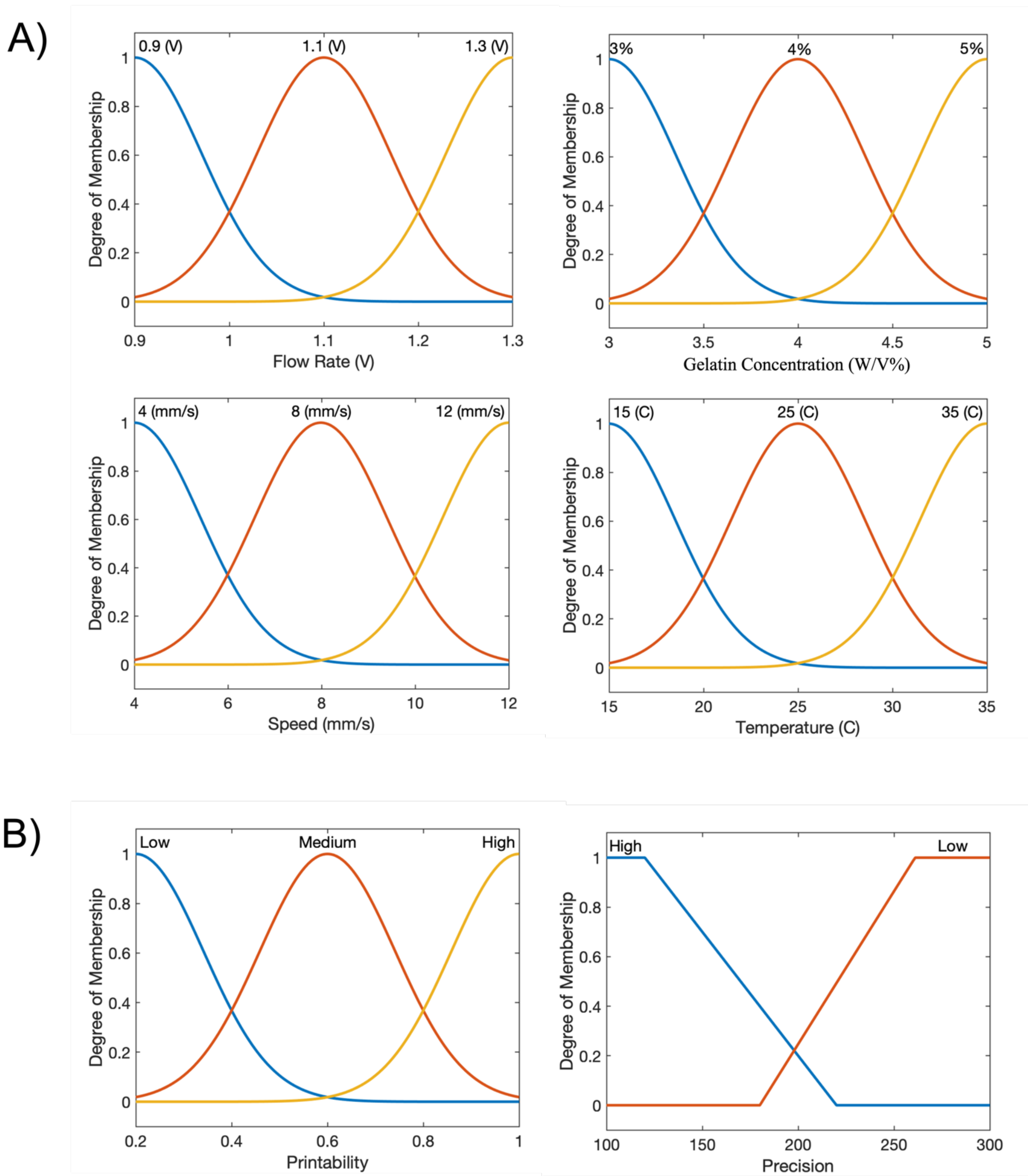
1 "Gelatin_Concentration==3 & Temperature==15 & Speed==4 & Flowrate==0.9 => Printability=Low, Precision=Low (1)"
2 "Gelatin_Concentration==4 & Temperature==25 & Speed==8 & Flowrate==0.9 => Printability=High, Precision=High (1)"
3 "Gelatin_Concentration==4 & Temperature==25 & Speed==8 & Flowrate==1.1 => Printability=High, Precision=High (1)"
4 "Gelatin_Concentration==4 & Temperature==25 & Speed==8 & Flowrate==1.3 => Printability=High, Precision=High (1)"
5 "Gelatin_Concentration==4 & Temperature==25 & Speed==12 & Flowrate==1.1 => Printability=High, Precision=High (1)"
6 "Gelatin_Concentration==4 & Temperature==25 & Speed==4 & Flowrate==1.1 => Printability=High, Precision=High (1)"
7 "Gelatin_Concentration==4 & Temperature==35 & Speed==8 & Flowrate==1.1 => Printability=High, Precision=High (1)"
8 "Gelatin_Concentration==3 & Temperature==15 & Speed==4 & Flowrate==1.3 => Printability=High, Precision=Low (1)"
9 "Gelatin_Concentration==3 & Temperature==15 & Speed==12 & Flowrate==1.3 => Printability=Low, Precision=Low (1)"
10 "Gelatin_Concentration==3 & Temperature==15 & Speed==4 & Flowrate==0.9 => Printability=Low, Precision=High (1)"
11 "Gelatin_Concentration==3 & Temperature==15 & Speed==12 & Flowrate==0.9 => Printability=Low, Precision=Low (1)"
12 "Gelatin_Concentration==3 & Temperature==25 & Speed==8 & Flowrate==1.1 => Printability=High, Precision=High (1)"
13 "Gelatin_Concentration==3 & Temperature==35 & Speed==12 & Flowrate==0.9 => Printability=Low, Precision=High (1)"
14 "Gelatin_Concentration==3 & Temperature==35 & Speed==12 & Flowrate==1.3 => Printability=Low, Precision=Low (1)"
15 "Gelatin_Concentration==3 & Temperature==35 & Speed==4 & Flowrate==0.9 => Printability=Low, Precision=High (1)"

16 "Gelatin_Concentration==3 & Temperature==35 & Speed==4 & Flowrate==1.3 => Printability=Low, Precision=Low (1)"
17 "Gelatin_Concentration==5 & Temperature==15 & Speed==4 & Flowrate==0.9 => Printability=Low, Precision=Low (1)"
18 "Gelatin_Concentration==5 & Temperature==15 & Speed==12 & Flowrate==1.3 => Printability=Low, Precision=High (1)"
19 "Gelatin_Concentration==5 & Temperature==15 & Speed==4 & Flowrate==1.3 => Printability=Medium, Precision=Low (1)"
20 "Gelatin_Concentration==5 & Temperature==25 & Speed==8 & Flowrate==1.1 => Printability=High, Precision=High (1)"
21 "Gelatin_Concentration==5 & Temperature==35 & Speed==12 & Flowrate==0.9 => Printability=Medium, Precision=High (1)"
22 "Gelatin_Concentration==5 & Temperature==35 & Speed==4 & Flowrate==1.3 => Printability=Medium, Precision=Low (1)"
23 "Gelatin_Concentration==5 & Temperature==35 & Speed==4 & Flowrate==0.9 => Printability=Low, Precision=High (1)"
24 "Gelatin_Concentration==5 & Temperature==35 & Speed==12 & Flowrate==1.3 => Printability=Low, Precision=Low (1)"

Since we utilized the RSM method to design the study, one of the printing parameter sets has been repeated (3-9). However, we only utilized one of them for the rule implementation. We have used “Minimum” as the “And Method”, “Maximum” as the “Or Method”, “Minimum” as the “implication Method”, and “Maximum” as the “Aggregation Method”.

References:

1. Alayón S, Robertson R, Warfield SK, Ruiz-Alzola J. A fuzzy system for helping medical diagnosis of malformations of cortical development. J Biomed Inform [Internet]. 2007 Jun [cited 2020 Nov 5];40(3):221–35. Available from: /pmc/articles/PMC2099697/?report=abstract
2. Izquierdo SS, Izquierdo LR. Mamdani fuzzy systems for modelling and simulation: A critical assessment. JASSS [Internet]. 2018 Jun 30 [cited 2021 Mar 6];21(3). Available from: http://jasss.soc.surrey.ac.uk/21/3/2.html



Supplemental Figure 1 – FIS Membership Functions. A) Membership functions of the model inputs (gelatin concentration, speed, flowrate, temperature). B) Membership functions for the model outputs (printability and precision).