17 Supplemental Figures
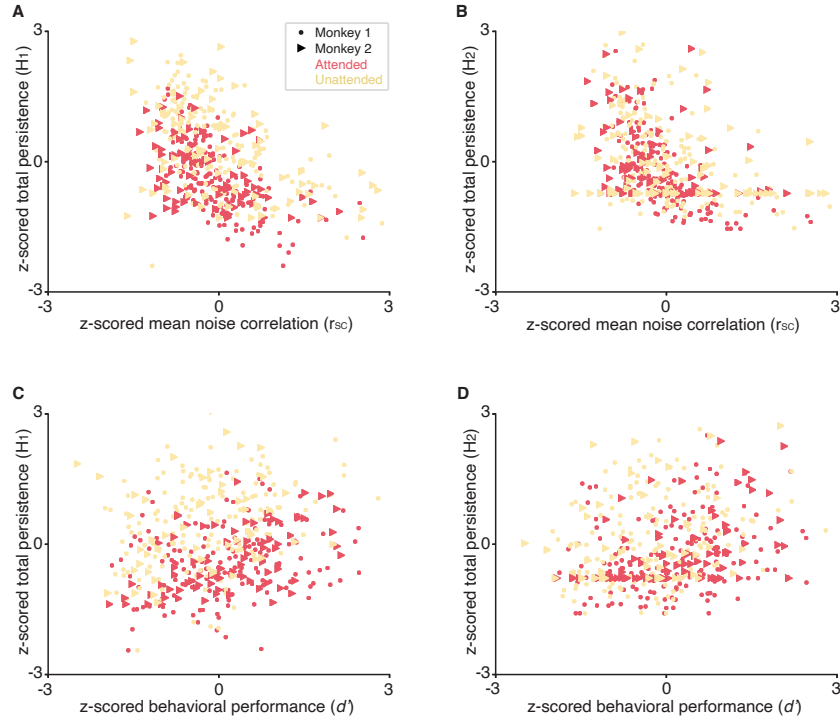
Supplemental Figure 1: Other topological descriptions of correlated variability are also strongly related to mean noise correlations and behavior. (A). Total persistence for the $1st$ homology group (tracking circular features) as a function of mean noise correlation. Total persistence is defined as the sum divided by the difference between the distance thresholds at which each hole begins and ends (termed birth and death times of the persistence features in the language of algebraic topology). Both the noise correlation values and total persistence values were z-scored for each animal, and the lines were fit for each attention condition. Symbols were assigned based on monkey. (attended: $r = -0.54$, $p = 5.14e-20$; unattended: $r = -0.5$, $p = 1.3e-16$; Paired T-test (Attended and Unattended,total persistence):$p = 7.3e-26$). (B). Same, for the $2nd$ homology group (spherical features; attended: $r = -0.45$, $p = 2.8e-14$; unattended: $r = -0.41$, $p = 2.7e-11$; Paired T-test (Attended and Unattended,total persistence):$p = 1.5e-7$). (C) Relationship between total persistence and behavioral sensitivity d' for the $1st$ homology group (circular features). Conventions and z-scoring as in (A). (attended: $r = 0.32$, $p = 2.7e-7$; unattended: $r = 0.27$, $p = 5.7e-5$) (D) Same, for the $2nd$ homology group (spherical features; attended: $r = 0.31$, $p = 1.5e-6$; unattended: $r = 0.41$, $p = 6.8e-6$).
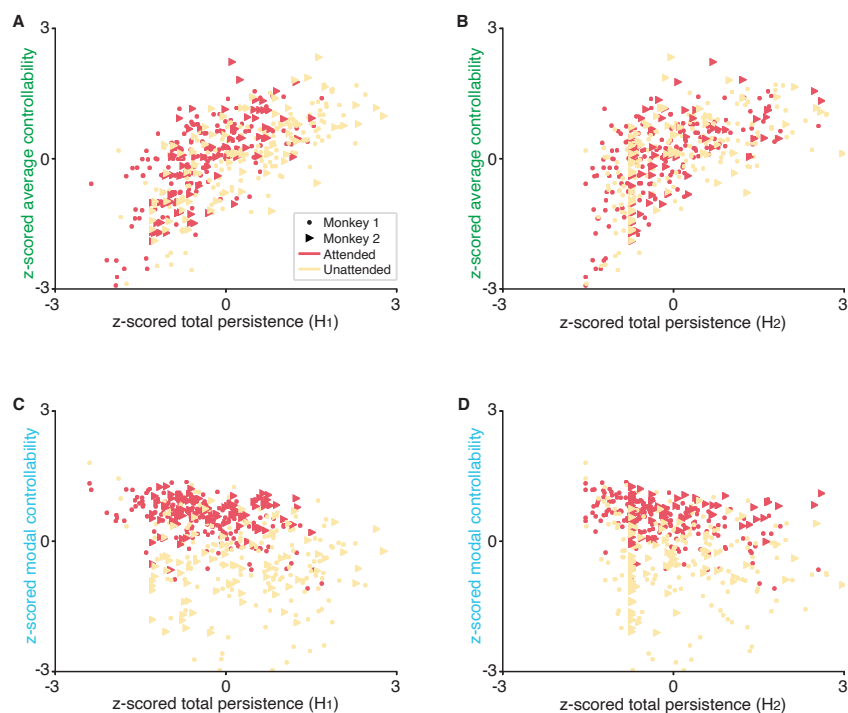
Supplemental Figure 2: Total persistence has similar relationships to average and modal controllability as the peak Betti value. A). Total persistence for the $1st$ homology group (circular features) as a function of average controllability. Z-scoring and other conventions the same as Supplementary Fig. 1. (attended: $r = 0.68$, $p = 5.19e-35$; unattended: $r = 0.7$, $p = 6.86e - 37$). (B) Same, for the $2nd$ homology group (spherical features; attended: $r = 0.59$; $p = 1.64e-24$; unattended: $r = 0.57$, $p = 4.36e - 22$) (C). Total persistence for the $1st$ homology group (circular features) as a function of modal controllability. Conventions as in (A). (attended: $r = -0.38$, $p = 3.91e - 10$; unattended: $r = -0.18$, $p = 5.2e - 3$). (D). Same, for the $2nd$ homology group (spherical features; attended: $r = -0.37$, $p = 2.07e - 9$; unattended: $r = -0.18$, $p = 4.7e - 3$).