A. Proof of Theorem 1

PROOF. To show that optimizing Equation 5 is equivalent to optimizing the CVAE loss in Equation 6, we consider the two terms in Equation 5 separately. Using the definition of mutual information, it is easy to show that

$$I(X';\theta) = \mathbb{E}_{p(x')}\left[\mathcal{D}_{\mathrm{KL}}\left[p(\theta|x')||p(\theta)\right]\right].$$

For $I(X;\theta|\tilde{\beta})$, we have

$$\begin{aligned}
& I(X;\theta|\tilde{\beta}) \\
=& H(X|\tilde{\beta}) - H(X|\theta,\tilde{\beta}) \\
=& H(X|\tilde{\beta}) + \sum_{x}\sum_{\tilde{\beta}}\sum_{\theta} p(x,\theta,\tilde{\beta})\log p(x|\theta,\tilde{\beta}) \\
=& H(X|\tilde{\beta}) + \sum_{x}\sum_{x'}\sum_{\tilde{\beta}}\sum_{\theta} p(x,x',\theta,\tilde{\beta})\log p(x|\theta,\tilde{\beta}) \\
=& H(X|\tilde{\beta}) + \sum_{x'}\sum_{x}\sum_{\tilde{\beta}}\sum_{\theta} p(x')p(x,\tilde{\beta},\theta|x')\log p(x|\theta,\tilde{\beta}) \\
=& H(X|\tilde{\beta}) + \sum_{x'}\sum_{x}\sum_{\tilde{\beta}}\sum_{\theta} p(x')p(x,\tilde{\beta}|x')p(\theta|x')\log p(x|\theta,\tilde{\beta}) \\
=& H(X|\tilde{\beta}) + \sum_{x'}\sum_{x}\sum_{\tilde{\beta}} p(x,x',\tilde{\beta})\sum_{\theta} p(\theta|x')\log p(x|\theta,\tilde{\beta}) \\
=& H(X|\tilde{\beta}) + \mathbb{E}_{p(x,x',\tilde{\beta})}\left[\mathbb{E}_{p(\theta|x')}\left[\log p(x|\theta,\tilde{\beta})\right]\right].
\end{aligned}$$

In the above derivation, we assume the conditional independence that $p(x,\tilde{\beta},\theta|x') = p(x,\tilde{\beta}|x')p(\theta|x')$. This conditional independence holds because the only common information between x and x' is contrast, and therefore given x', observing θ provides no extra information about x or $\tilde{\beta}$, and vice versa.

Combining the two terms, Equation 5 becomes

$$\begin{aligned}
\theta^* =& \arg\min_{\theta} I(X';\theta) - \lambda I(X;\theta|\tilde{\beta}) \\
=& \arg\min_{\theta} \mathbb{E}_{p(x')}\left[\mathcal{D}_{\mathrm{KL}}\left[p(\theta|x')||p(\theta)\right]\right] - \\
& \lambda\left[H(X|\tilde{\beta}) + \mathbb{E}_{p(x,x',\tilde{\beta})}\left[\mathbb{E}_{p(\theta|x')}\left[\log p(x|\theta,\tilde{\beta})\right]\right]\right] \\
=& \arg\min_{\theta} \mathbb{E}_{p(x')}\left[\mathcal{D}_{\mathrm{KL}}\left[p(\theta|x')||p(\theta)\right]\right] - \\
& \lambda\left[\mathbb{E}_{p(x,x',\tilde{\beta})}\left[\mathbb{E}_{p(\theta|x')}\left[\log p(x|\theta,\tilde{\beta})\right]\right]\right] \\
=& \arg\min_{\theta} \frac{1}{N}\sum_{i=1}^{N} \mathcal{D}_{\mathrm{KL}}\left[p(\theta|x'_i)||p(\theta)\right] - \\
& \lambda\mathbb{E}_{p(\theta|x'_i)}\left[\log p(x_i|\theta,\tilde{\beta}_i)\right],
\end{aligned}$$

where the outside expectations are approximated by the empirical mean, and N is the number of training instances.

B. Qualitative harmonization results of T_1-w images from a sagittal view

Figure 13 shows the sagittal orientation for a 10-site harmonization experiment.

C. Qualitative harmonization results of T_2-w images

Figure 14 shows the harmonization results of T_2 images.

D. An ablation study on the perceptual loss

We conducted an ablation study to show the effects of the perceptual loss. In the experiment, we kept all the hyperparameters the same, the only difference is the presence of the perceptual loss. According to our study, we found no significant difference in SSIM and PSNR of the harmonized images (see Table 3), but adding a perceptual loss helps the network converge faster, as shown in Fig. 15.

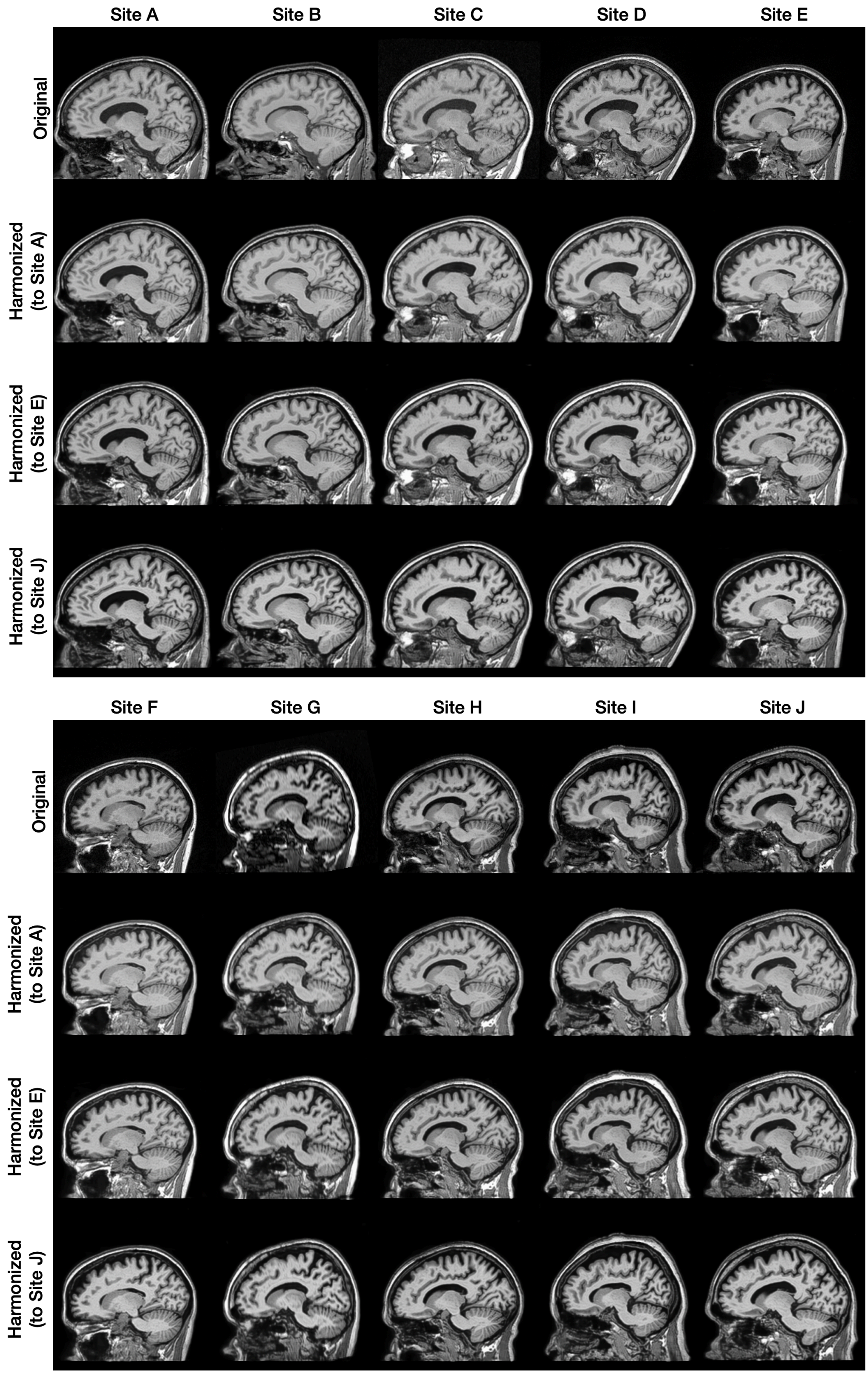

Figure 13: Shown are the original sagittal orientation of T_1-w MR images from 10 sites and their corresponding harmonized images for Sites A, E, and J.

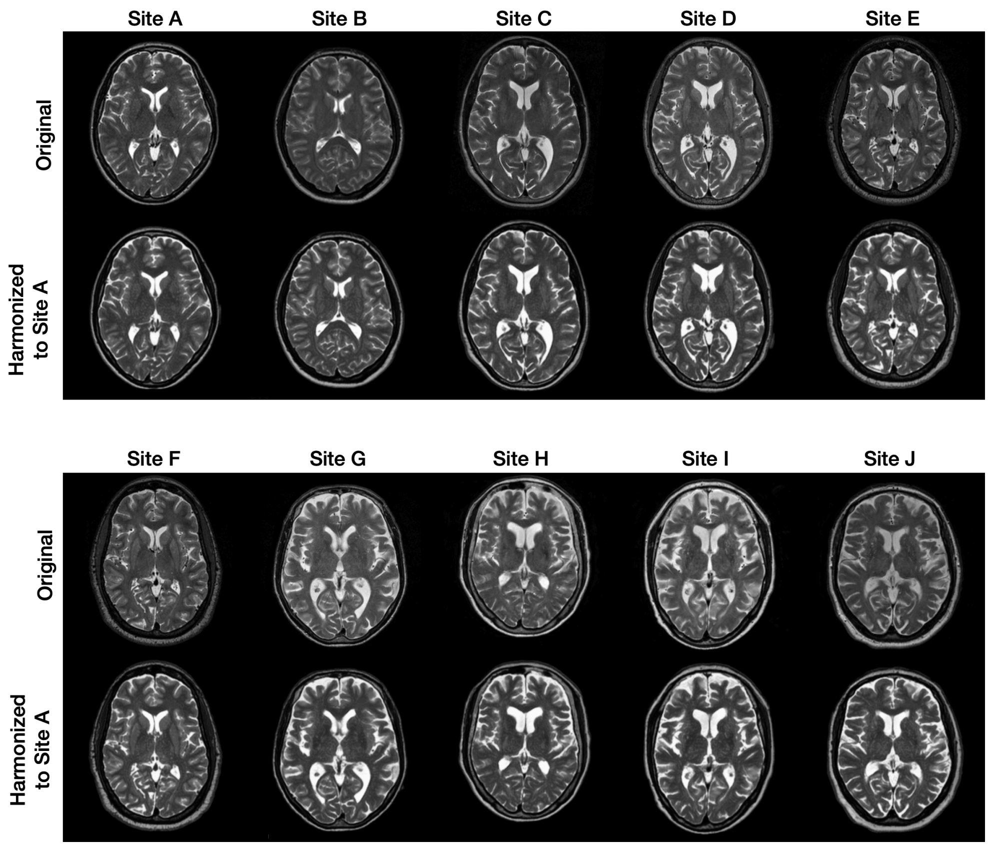

Figure 14: Shown are the original T_2-w MR images from 10 sites and their corresponding harmonized images for Sites A.

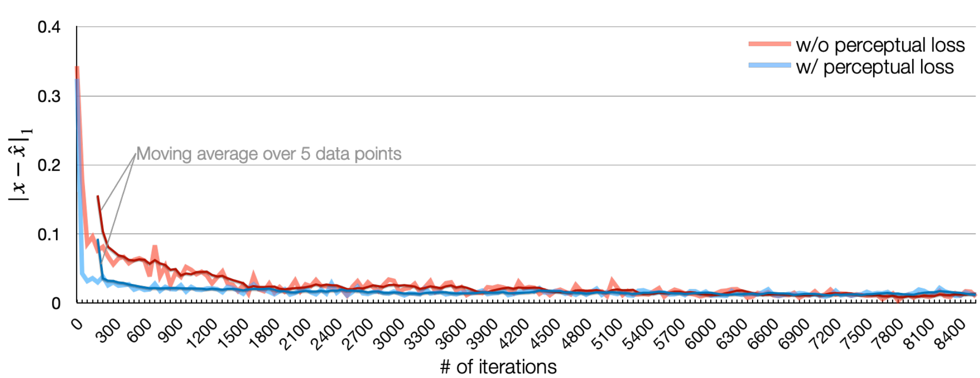

Figure 15: The l_1 reconstruction error with respect to the number of training iterations.