# Supplementary Information

## Real-space observation of ergodicity transitions in artificial spin ice

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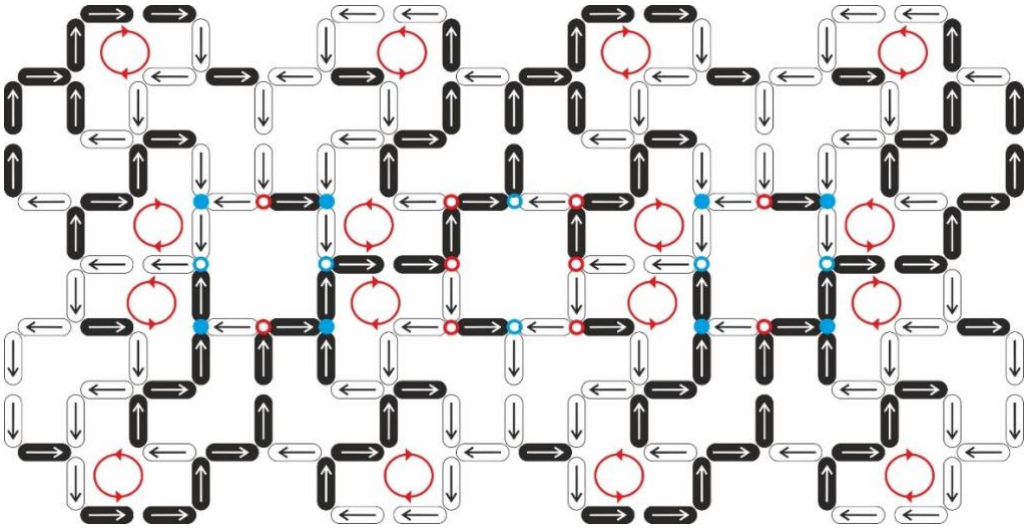
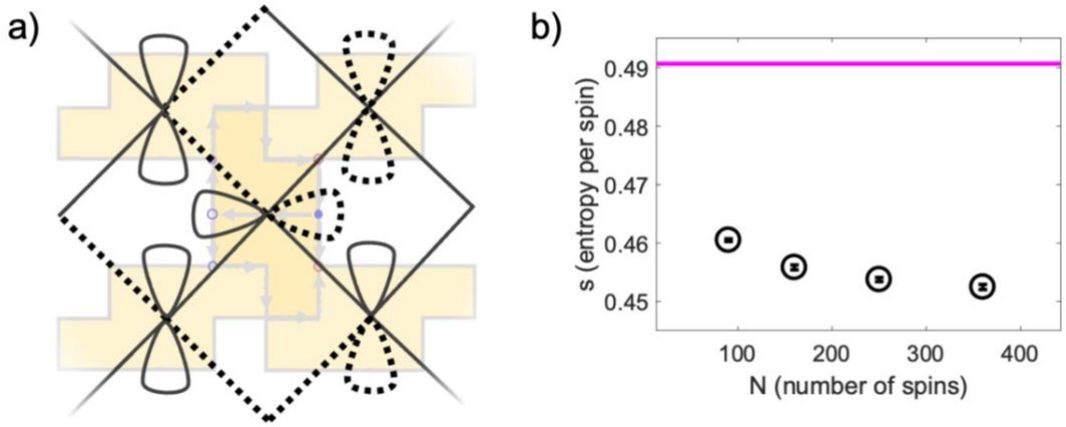
## Supplementary Figures



**Supplementary Figure 1. Nearest-neighbor ground state.** An arrangement of moments that settles each coordination two and four vertex into its local ground state while minimizing the proportion of Type B vertices, visualized by filled blue or red circles, to 25% of coordination three sites. Each coordination four vertex may independently switch orientation along with its adjacent coordination two vertices without changing local dipolar energy, resulting in an extensive bitwise entropy of $s = 1/20$ due to the size of the unit cell containing the coordination four vertex being 20.




**Supplementary Figure 2. Theoretical prediction for the entropy per spin based on the degeneracy of Type B placement. a**, A graph comprising a square grid with two loops per node that represents the potential placement of Type B vertices. In the empirically supported model, precisely two edges per node must be selected as Type B vertices. The dotted lines represent one such configuration, corresponding to the spin structure of the greyed out underlying lattice. **b,** Two estimates of the entropy per spin. The pink line is the Pauling upper bound on entropy per bit, $s = \frac{1}{10}\log_2 2\binom{6}{2}$, derived by considering an isolated node. The black circles are brute force estimates of the entropy of graphs corresponding to increasing numbers of spins. The placement of a neighboring Type B vertex may constrain another node's configurations, reducing the total entropy.