Table S1: Additional empirical results for estimators of causal effect of each additional unit exposure on outcome for the additive model

| | Method of Analysis | $\theta_2 = 0.5$ | | | | $\theta_2 = -4$ | | | |
| | | $\gamma_2 = 0.5$ | | | | $\gamma_2 = 0.2$ | | | |
		EBIAS/ABIAS	ESE	ASE	ECP	EBIAS/ABIAS	ESE	ASE	ECP
P(X=0)=0.25 $\beta_1 = log(2)$	Unadjusted	0.50	0.10	0.10	0.01	0.88	0.13	0.10	0.00
	Regression Adjustment	0.17	0.09	0.08	0.18	0.31	0.12	0.11	0.07
	Generalized Propensity Score	0.17/0.18	0.09	0.10	0.61	0.31/0.33	0.12	0.11	0.11
	Two-part Generalized Propensity Score	0.00	0.09	0.10	0.98	0.00	0.10	0.09	0.96
$\beta_1 = log(5)$	Unadjusted	0.54	0.12	0.10	0.00	0.93	0.14	0.12	0.00
	Regression Adjustment	0.18	0.11	0.08	0.39	0.34	0.19	0.11	0.03
	Generalized Propensity Score	0.18/0.19	0.11	0.10	0.64	0.34/0.34	0.19	0.12	0.07
	Two-part Generalized Propensity Score	0.00	0.12	0.10	0.94	0.00	0.14	0.11	0.95
P(X=0)=0.5 $\beta_1 = log(2)$	Unadjusted	0.52	0.12	0.10	0.02	0.87	0.13	0.11	0.00
	Regression Adjustment	0.20	0.10	0.09	0.18	0.33	0.11	0.10	0.07
	Generalized Propensity Score	0.20/0.20	0.10	0.10	0.47	0.33/0.34	0.11	0.11	0.10
	Two-part Generalized Propensity Score	0.00	0.11	0.10	0.97	0.00	0.07	0.07	0.98
$\beta_1 = log(5)$	Unadjusted	0.60	0.11	0.10	0.00	0.95	0.13	0.10	0.00
	Regression Adjustment	0.22	0.09	0.08	0.15	0.39	0.14	0.10	0.01
	Generalized Propensity Score	0.22/0.25	0.09	0.10	0.32	0.39/0.39	0.14	0.11	0.01
	Two-part Generalized Propensity Score	-0.01	0.12	0.11	0.92	0.00	0.10	0.09	0.95

EBIAS:Empirical bias/Asymptotic bias
ESE: Empirical standard error
ASE: Asymptotic standard error
ECP: Empirical coverage probability

Table S2: Additional empirical results for estimators of causal effect of each additional unit exposure on outcome for the multiplicative model

	Method of Analysis	$\theta_2 = -4$							
		$\gamma_2 = 0.5$				$\gamma_2 = 0.2$			
		EBIAS/ABIAS	ESE	ASE	ECP	EBIAS/ABIAS	ESE	ASE	ECP
P(X=0)=0.25 $\beta_1 = log(2)$	Unadjusted	-0.63	0.10	0.10	0.01	-0.81	0.11	0.09	0.00
	Regression Adjustment	-0.28	0.09	0.08	0.37	-0.31	0.12	0.09	0.05
	Generalized Propensity Score	-0.28/-0.29	0.09	0.10	0.61	-0.31/-0.32	0.12	0.09	0.05
	Two-part Generalized Propensity Score	0.00	0.09	0.10	0.98	0.00	0.08	0.08	0.90
$\beta_1 = log(5)$	Unadjusted	-0.66	0.10	0.08	0.01	0.86	0.15	0.10	0.00
	Regression Adjustment	0.31	0.11	0.08	0.01	0.37	0.23	0.10	0.01
	Generalized Propensity Score	0.31/0.33	0.11	0.08	0.01	0.37/0.35	0.23	0.11	0.01
	Two-part Generalized Propensity Score	0.00	0.09	0.07	0.95	0.02	0.15	0.09	0.93
P(X=0)=0.5 $\beta_1 = log(2)$	Unadjusted	-0.62	0.10	0.09	0.00	-0.77	0.10	0.10	0.00
	Regression Adjustment	-0.28	0.11	0.08	0.05	-0.31	0.10	0.10	0.03
	Generalized Propensity Score	-0.28/-0.29	0.11	0.08	0.05	-0.31/-0.32	0.10	0.10	0.03
	Two-part Generalized Propensity Score	0.00	0.08	0.08	0.96	0.00	0.06	0.06	0.96
$\beta_1 = log(5)$	Unadjusted	-0.68	0.10	0.09	0.00	-0.83	0.11	0.10	0.00
	Regression Adjustment	-0.33	0.10	0.08	0.00	-0.38	0.13	0.09	0.01
	Generalized Propensity Score	-0.34/-0.38	0.10	0.08	0.00	-0.38/-0.39	0.13	0.09	0.01
	Two-part Generalized Propensity Score	0.01	0.08	0.07	0.94	0.00	0.07	0.06	0.96

EBIAS:Empirical bias/Asymptotic bias
ESE: Empirical standard error
ASE: Asymptotic standard error
ECP: Empirical coverage probability

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Fig. S1: Regression diagnostic plots of the outcome model