eAppendix for: A dynamic spatial factor model to describe the opioid syndemic in Ohio

Censoring Model

As mentioned in the Methods section, the treatment counts are subject to censoring under the confidentiality and privacy rules of the state of Ohio. Although only being used in total in this paper, the treatment counts are provided in two age groups: adults and adolescents. Counts in each age group are censored if they are between 1 and 9, and we incorporate that information into our model by adapting the approach of Famoye and Wang (2004) for interval censoring. We encounter three cases of censoring that are denoted by the following indicator:

$$c_{it} = \begin{cases} 0 & \text{total count observed} \\ 1 & \text{adolescent count censored} \\ 2 & \text{adolescent and adult counts censored.} \end{cases} \tag{1}$$

We observe both the adult and adolescent when $c_{it} = 0$ so the total treatment count is simply the sum of the adolescent and adult counts. When $c_{it} = 1$, the adolescent count is censored and so the total count must be between the adult count plus 1 and the adult count plus 9. Both the adolescent and adult counts are censored when $c_{it} = 2$ so the total count is between 2 and 18. Let $Y_{ij1}^{(1)}$ be the adolescent treatment count and $Y_{ij2}^{(2)}$ be the adult treatment count. Then, let

$$Y_{ij1} = \begin{cases} Y_{ij1}^{(1)} + Y_{ij1}^{(2)} & c_{ij} = 0 \\ Y_{ij1}^{(2)} & c_{ij} = 1 \\ 0 & c_{ij} = 2, \end{cases} \tag{2}$$

and the likelihood becomes

$$\begin{aligned} L(\lambda_{ij1}|Y_{ij1}) = &[f(Y_{ij1}|\lambda_{ij1})]^{I(c_{ij}=0)} \times [F(Y_{ij1}+9|\lambda_{ij1}) - F(Y_{ij1}|\lambda_{ij1})]^{I(c_{ij}=1)} \\ &\times [F(18|\lambda_{ij1}) - F(1|\lambda_{ij1})]^{I(c_{ij}=2)} \end{aligned} \tag{3}$$

where $I(\cdot)$ is an indicator function and $f(\cdot|\lambda_{ij1})$ is the probability mass function and $F(\cdot|\lambda_{ij1})$ is the cumulative distribution function for a Poisson distribution with mean $E_{ij1}\lambda_{ij1}$.

References

Famoye, F. and Wang, W. (2004). Censored generalized Poisson regression model. *Computational Statistics and Data Analysis*, 46(3):547–560.

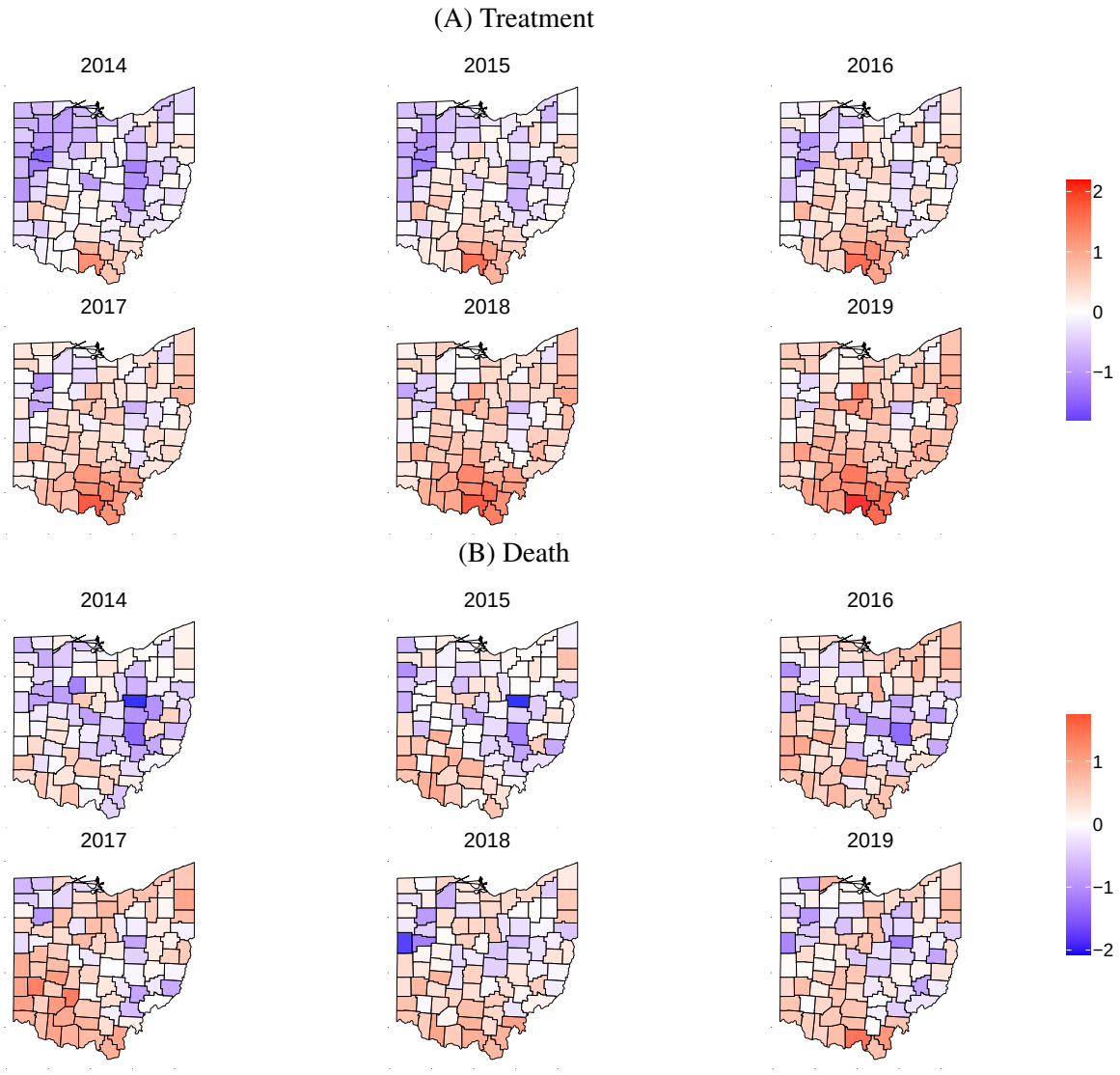


Figure 1: Observed log standardized rate ratio for treatment admissions, overdose deaths, total HCV cases, and newly diagnosed HIV infections from 2014-2019.

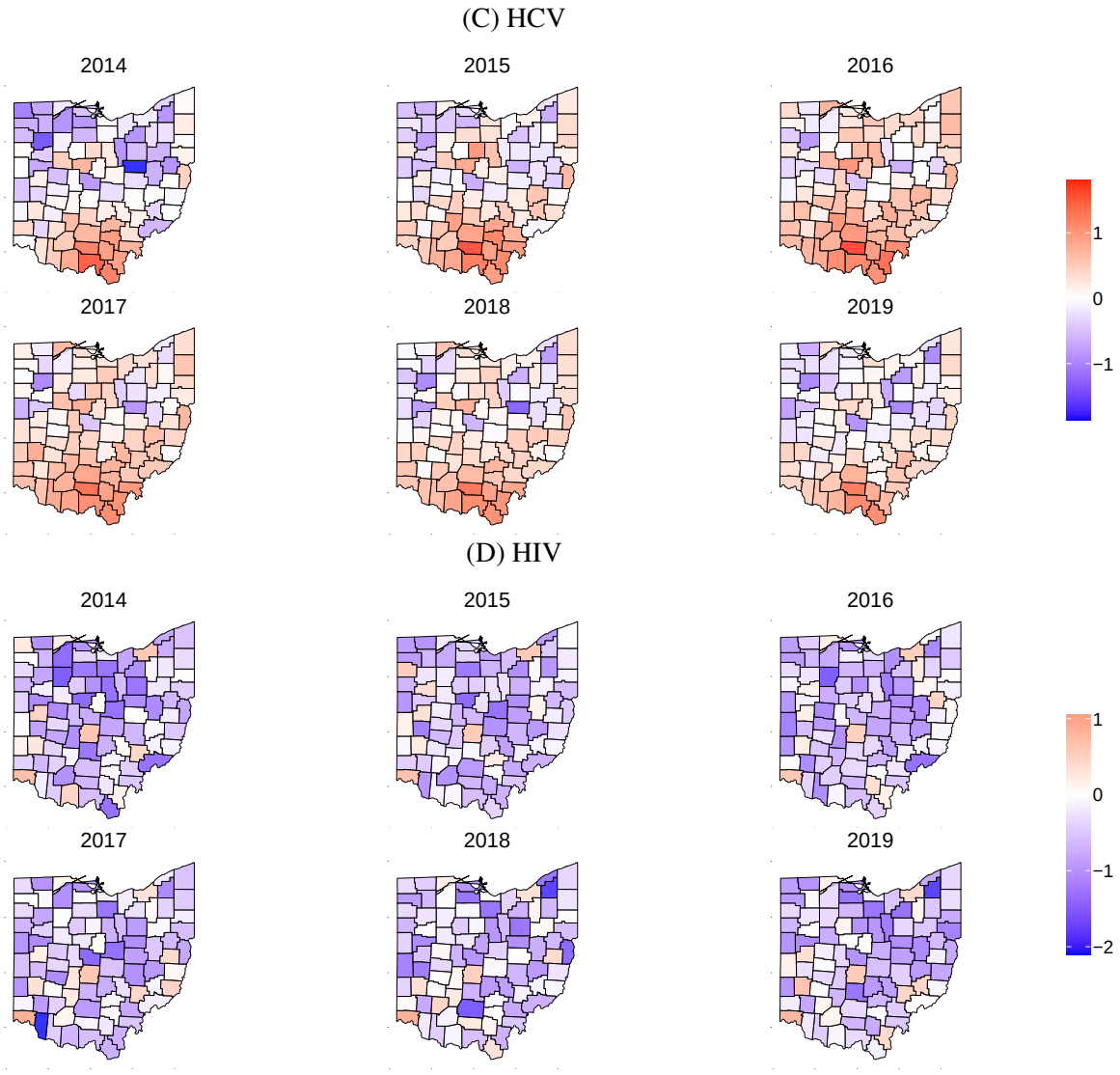


Figure 1: Observed log standardized rate ratio for treatment admissions, overdose deaths, total HCV cases, and newly diagnosed HIV infections from 2014-2019 (continued).

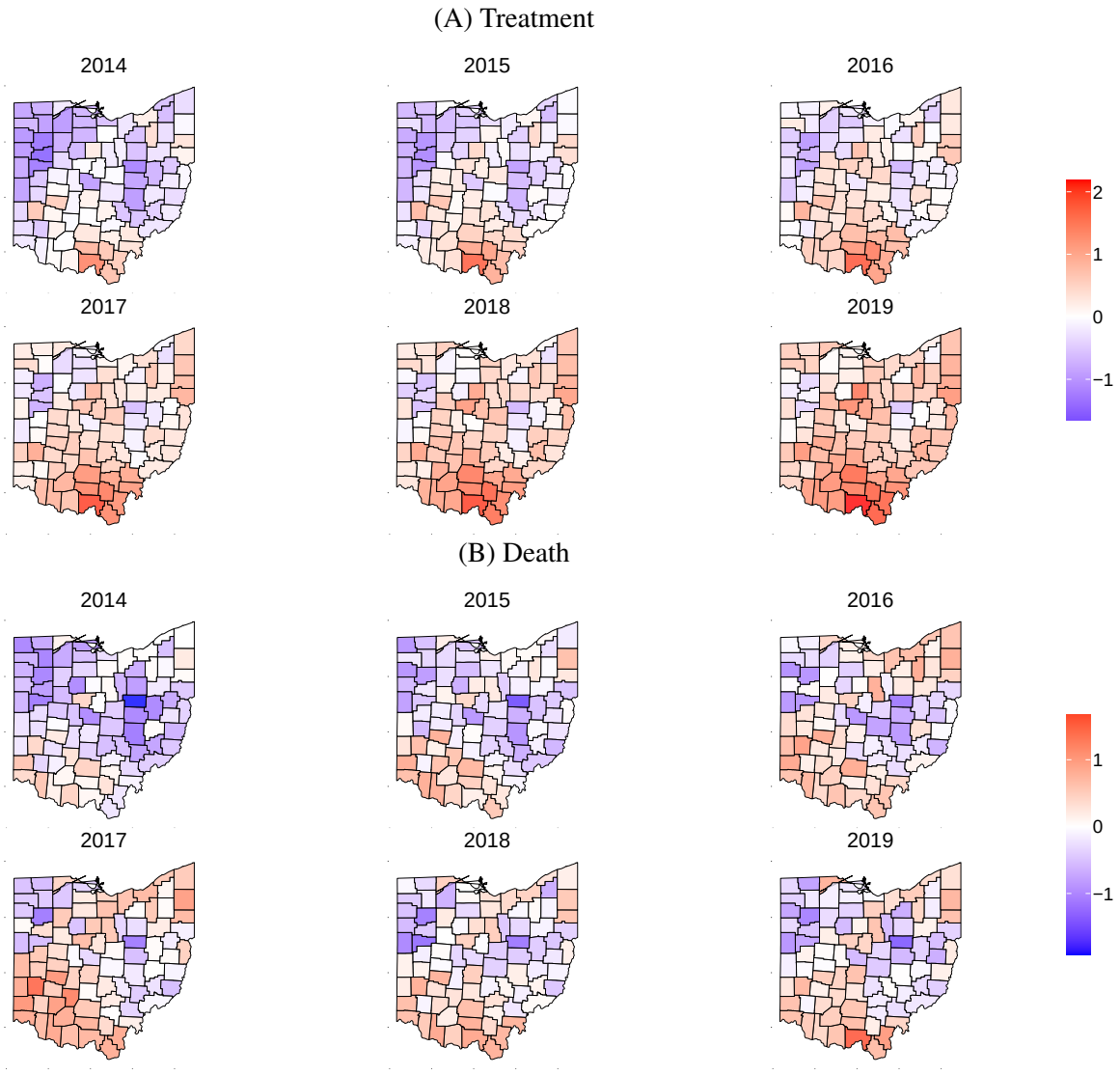


Figure 2: Posterior mean estimated log standardized rate ratio for treatment admissions, overdose deaths, total HCV cases, and newly diagnosed HIV infections from 2014-2019.

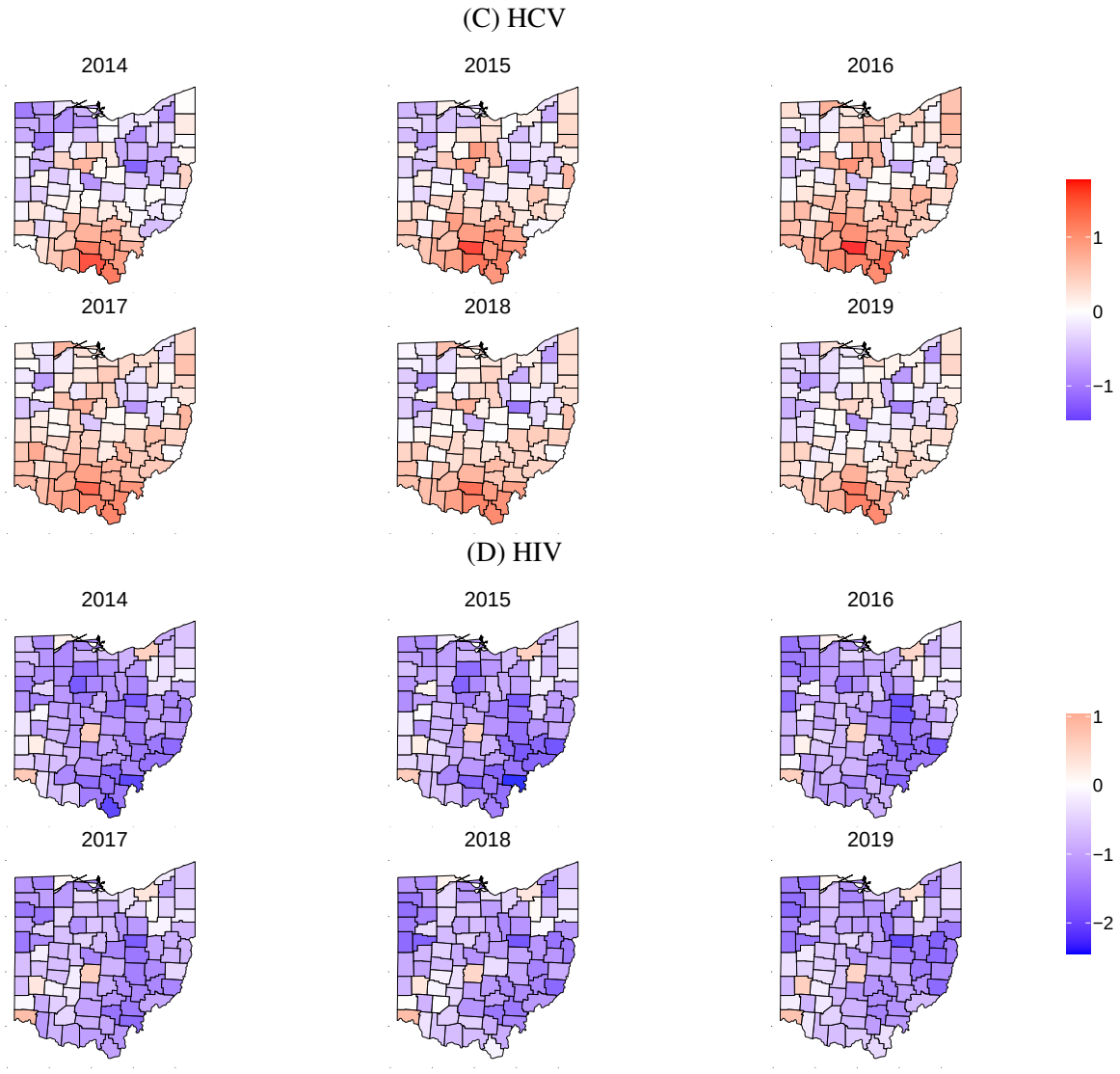
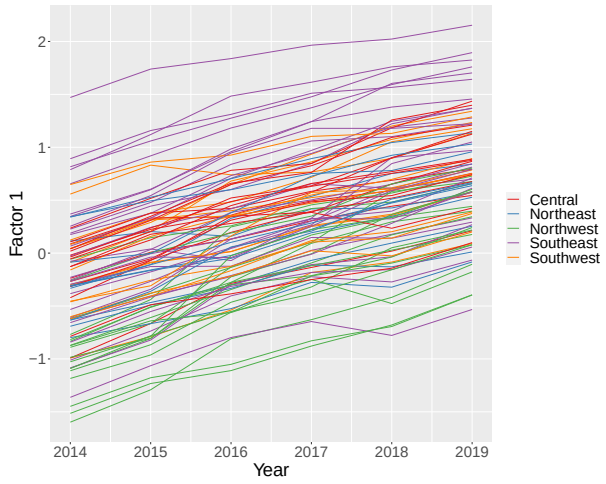
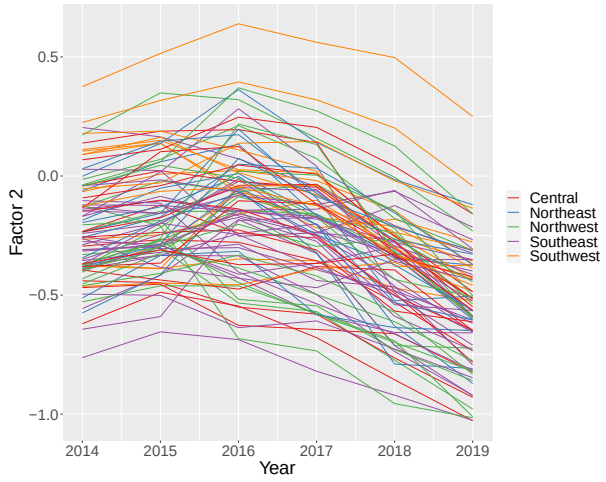


Figure 2: Posterior mean estimated log standardized rate ratio for treatment admissions, overdose deaths, total HCV cases, and newly diagnosed HIV infections from 2014-2019 (continued).

(A) Factor 1



(B) Factor 2



(C) Factor 3

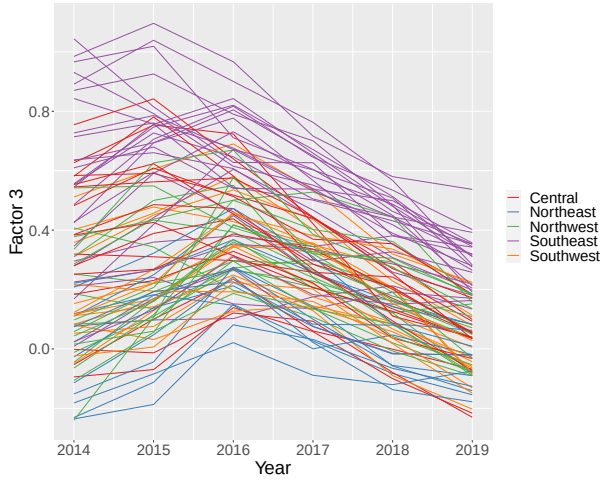


Figure 3: Posterior mean estimates of each of the factors over time for Ohio's 88 counties. Counties are colored by region.

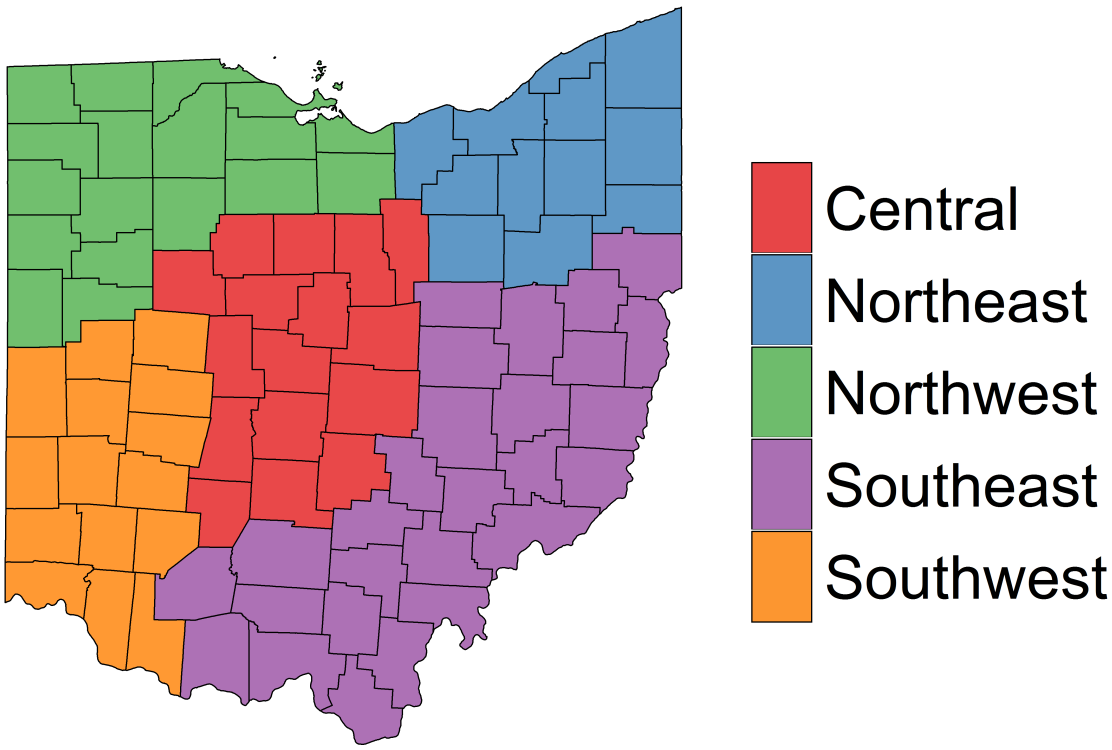


Figure 4: Regions of Ohio

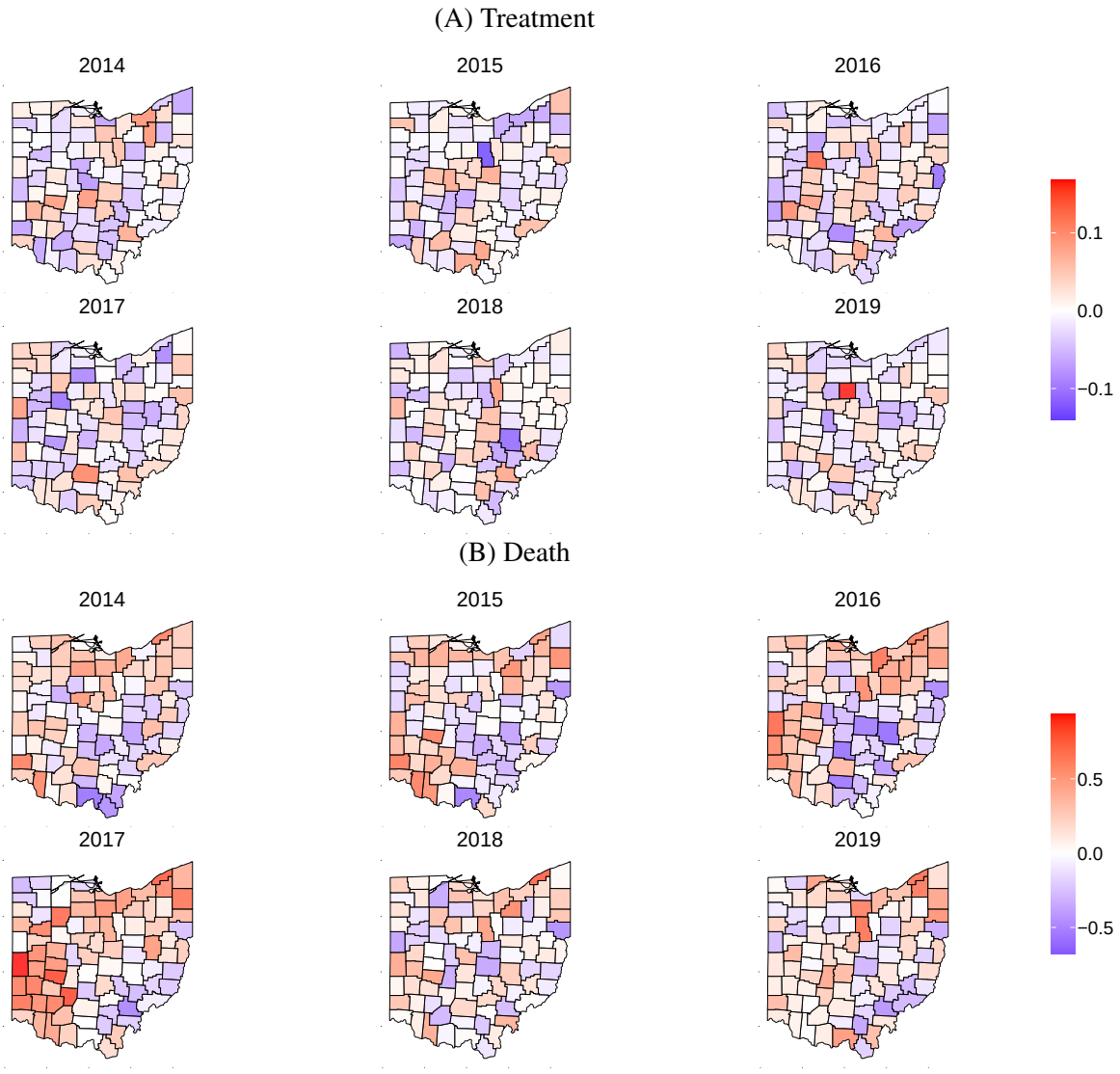


Figure 5: Posterior mean estimated uncorrelated heterogeneity for treatment admissions, overdose deaths, total HCV cases, and newly diagnosed HIV infections from 2014-2019.

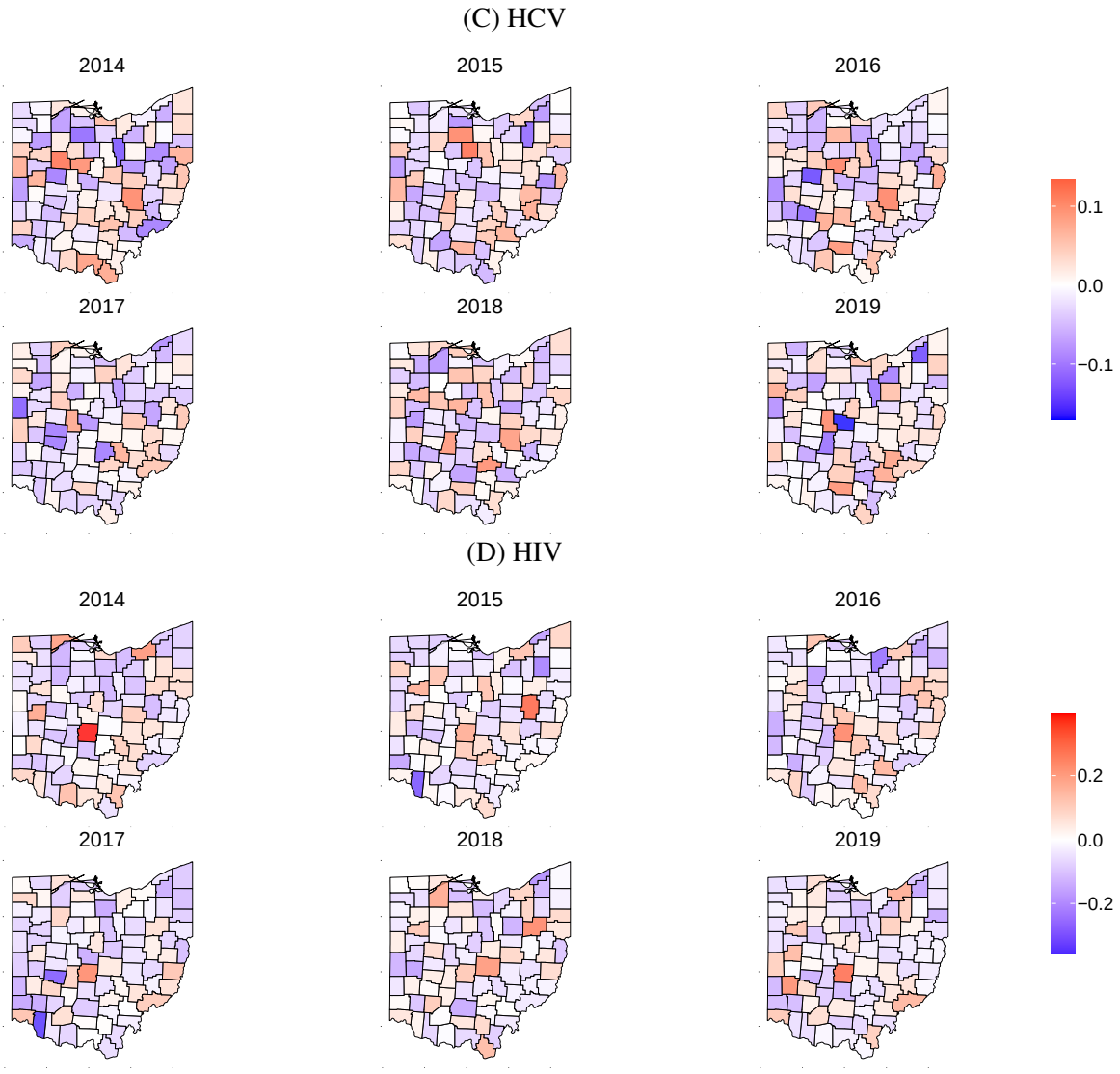


Figure 5: Posterior mean estimated uncorrelated heterogeneity for treatment admissions, overdose deaths, total HCV cases, and newly diagnosed HIV infections from 2014-2019 (continued).