[2] Morse, M., "Relations between the Critical Points of a Real Function of n Independent Variables," *Trans. Am. Math. Soc.*, **27**, 345–396 (1925).

[3] Morse, M., and Van Schaack, G. B., "The Critical Point Theory under General Boundary Conditions," *Ibid.*, **35**, 545–571 (1934).

[4] Morse, M., "The Calculus of Variations in the Large," *Am. Math. Soc. Coll. Publ.*, New York, 1934.

[5] Seifert-Threlfall, *Lehrbuch der Topologie*, Teubner, Leipzig and Berlin, 1934.

CRITERIA FOR LINEAR EQUIVALENCE

BY ANDRÉ WEIL

UNIVERSITY OF CHICAGO

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1. Equivalence criteria play an important part in the theory of algebraic surfaces; such criteria have been established by Severi[1] for surfaces without multiple points over the field of complex numbers. My work on abelian varieties[2] has enabled me to extend Severi's methods and results to arbitrary normal varieties in projective spaces. As publication of the complete proofs, which I have already written up, may be somewhat delayed, I shall give here a precise statement of the main results which have been so obtained.

Once for all, V will denote a variety of dimension n without multiple subvarieties of dimension $n - 1$, embedded in a projective space P of dimension N and not contained in any hyperplane. Let k_0 be the smallest field of definition for V; all the fields to be mentioned will be assumed to contain k_0. If H is a hyperplane in P, generic over k_0, then[3] the cycle $V.H$ is a variety of dimension $n - 1$ without multiple subvarieties of dimension $n - 2$. If L is a linear subvariety of P of dimension $N - n + 1$, generic over k_0, then $V.L$ is a curve C without multiple points which does not go through any multiple point of V; such a curve $C = V.L$ will be called a *general curve* on V; if, moreover, L is generic over a field k containing k_0, then the general curve C will be said to be generic over k.

2. Linear equivalence of divisors on V is defined as usual, and will be denoted by $\sim$. Let W be an abstract variety; let Z be a divisor on $V \times W$; if M is a simple point on W such that the cycle $Z.(V \times M)$ is defined, we write $Z.(V \times M) = Z(M) \times M$. Any divisor of the form $Z(M) - Z(N)$, where Z is as just stated and M, N are such that $Z(M)$, $Z(N)$ are defined, is said to be *algebraically equivalent to 0;* one of my lemmas states that such divisors form a group and that every such divisor can be written as $Z(M) - Z(N)$ by means of a divisor Z on the product $V \times \Gamma$ of V and of a curve Γ.

Criteria for linear equivalence can be of two kinds; those of the first kind state that a divisor X is linearly equivalent to 0 provided its trace on a subvariety of V which depends upon X is linearly equivalent to 0 on that subvariety; those of the second kind state that a divisor X is linearly equivalent to 0 provided it is algebraically equivalent to 0 and its trace on a fixed subvariety of V is equivalent to 0 on that subvariety.

3. The criteria of the first kind are as follows:

(A) *Let X be a divisor on V, rational over a field k; put $W = V.H$, where H is a hyperplane in P, generic over k. Then, if $n \geqq 3$, $X.W \sim 0$ on W implies $X \sim 0$ on V; the same is true for $n = 2$ if one assumes at the same time that there is an integer $m \neq 0$ such that $mX \sim 0$ on V.*

For $n = 2$, $X.W \sim 0$ need not imply $X \sim 0$ because of the exceptional surfaces occurring in the theorem of Kronecker-Castelnuovo; what is more, there are serious difficulties in the way of extending this theorem to arbitrary fields.[4] Nevertheless, we have the following general result:

(B) *There is a finite set of divisors D_i on V, all algebraic over k_0, with the following properties: (i) no linear combination of the D_i, other than 0, is algebraically equivalent to 0; (ii) let X be a divisor on V, rational over a field k; let C be a general curve on V, generic over k; then $X.C \sim 0$ on C if and only if X is linearly equivalent on V to some linear combination of the D_i.*

4. Let f be a mapping of V into an abelian variety A; if X is a divisor on V, and C a general curve on V such that $X.C$ is defined, I define $\bar{f}(X)$ by putting $\bar{f}(X) = S[f(X.C)]$, where S is as defined, *loc. cit.*,[2] pp. 28–29; this is independent of the choice of C; if k is a field of definition for V, A and f, and if X is rational over k, $\bar{f}(X)$ is rational over k. If $Z(M)$ is as defined above in paragraph 2, there is a mapping g of W into A such that $g(M) = \bar{f}(Z(M))$ whenever $Z(M)$ is defined.

The criteria of the second kind can now be formulated:

(C) *Let X be algebraically equivalent to 0 on V. Then, if $\bar{f}(X) = 0$ for every mapping f of V into an abelian variety, there is an integer $m \neq 0$ such that $mX \sim 0$ on V.*

(D) *Let W be an abstract variety; let Z be a divisor on $V \times W$. Assume that, for every mapping f of V into an abelian variety, the mapping g of W into the same variety defined by $g(M) = \bar{f}(Z(M))$ is constant. Then all the divisors of the form $Z(M)$ are linearly equivalent to one another on V.*

(E) *Let C be a general curve on V. Then, if a divisor X is algebraically equivalent to 0 on V and $X.C$ is defined and ~ 0 on C, X is linearly equivalent to 0 on V.*

Only one consequence of the above results will be mentioned here: If k is any field of definition for V, and a divisor X_0 is rational over k, then every divisor X satisfying a relation $mX \sim X_0$, where m is an integer other than 0, is linearly equivalent to some divisor X_1 which is algebraic over k.

On the other hand, it will be clear to everyone who is familiar with this subject that the above results, and particularly (E), open the way to a theory of the so-called Picard varieties of arbitrary varieties.[5]

[1] Severi, F., *Ann. di Mat.* (III), **12**, 55–79 (1905); *Rend. Pal.*, **21**, 257–282 (1906); *Rend. Acc. Lincei* (V), **30**, 328–332 (1° sem. 1921). Cf. also O. Zariski, *Algebraic Surfaces*, Chelsea, New York, 1948, pp. 88–89 and pp. 126–128.

[2] Weil, A., *Variétés abéliennes et courbes algébriques*, Act. Sc. et Ind. no. 1064, Hermann et Cie, Paris, 1948.

[3] Matsusaka, T., *Kyoto Math. Mem.*, **26**, 51–62 (1950).

[4] I am, however, informed by P. Samuel that he has been able to avoid all such difficulties by the use of a suitable birational transformation.

[5] Weil, A., *Colloque d'Algèbre et Théorie des Nombres*, Centre Nat. de la Rech. Scient., Paris, 1950, pp. 125–127. I have been informed by T. Matsusaka, and also by A. Néron and P. Samuel, that they have independently developed theories of the Picard varieties. By means of the device referred to in footnote 4, Néron and Samuel have been able to derive the basic results on the existence of the Picard variety from the most elementary form of the criterion of the first kind.

RAPID EFFECTS UPON THE RENAL CIRCULATION PRODUCED BY NEPHROTOXIC GLOBULIN ADMINISTRATION IN THE RAT

BY RICHARD W. LIPPMAN, HELEN U. MARTI AND E. ELMO JACOBS

INSTITUTE FOR MEDICAL RESEARCH, CEDARS OF LEBANON HOSPITAL, LOS ANGELES, CALIFORNIA*

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Although rabbit anti-rat-kidney serum has been shown to contain antibodies that localize in the glomeruli,[1] renal tubule cells also are damaged by the administration of this serum or of rabbit anti-rat-kidney gamma globulin (nephrotoxic globulin, NTG). Indeed, the damaging effect of NTG on renal tubule cells has been shown in tissue culture explants, where indirect circulatory effects are excluded.[2] It has been suggested that the tubular damage produced *in vivo* by administration of NTG might result from either a direct effect upon the tubule cells or circulatory disturbances subsequent to NTG administration, or from a combination of these two factors.[3]

Various methods have been used to study circulatory changes in the kidney. The injection of foreign materials, such as neoprene or India ink, may produce distortions of the vascular pattern, as a result of mechanical or pharmacologic effects. Such a quasi-physiologic method as the measurement of renal blood flow by means of para-aminohippurate or diodrast clearances cannot be used after NTG administration, since the