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**Supplemental information**

**Inferring disease architecture  
and predictive ability with LDpred2-auto**

**Florian Privé, Clara Albiñana, Julyan Arbel, Bogdan Pasaniuc, and Bjarni J. Vilhjálmsdóttir**

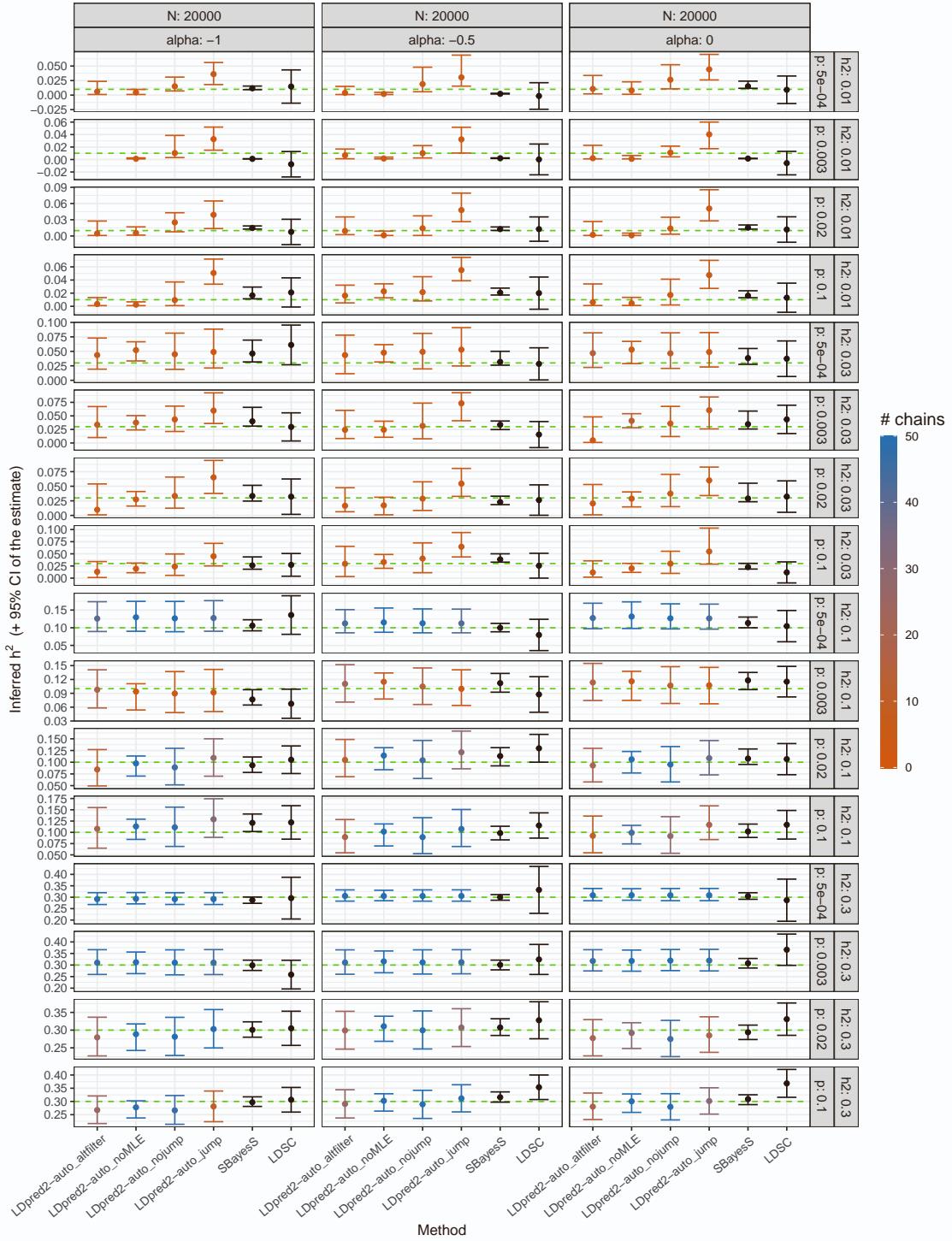


Figure S1: Inferred SNP heritability  $h^2$  in simulations with continuous outcomes and  $N=20K$ . Horizontal dashed lines represent the true simulated values. For LDpred2-auto, suffix “nojump”/“jump” refers to using `allow_jump_sign = FALSE/TRUE` (and `use_MLE = TRUE`), and “noMLE” refers to using `use_MLE = FALSE` (and `allow_jump_sign = FALSE`), and “alfilter” is similar to “nojump” but uses a different post-filtering of chains (Methods). Note that the recommended option is to use `allow_jump_sign = FALSE`<sup>1</sup>. The 95% confidence intervals for the LDpred2-auto and SBayesS estimates are obtained from the 2.5% and 97.5% quantiles of all the  $h^2$  estimates from the iterations (after burn-in) of the chains kept (note that only one chain is used and kept in SBayesS). The 95% confidence interval for the LD Score regression (LDSC) estimate is obtained as  $\pm 1.96$  of its standard error. Colors for LDpred2-auto models represent the number of chains kept (out of 50).

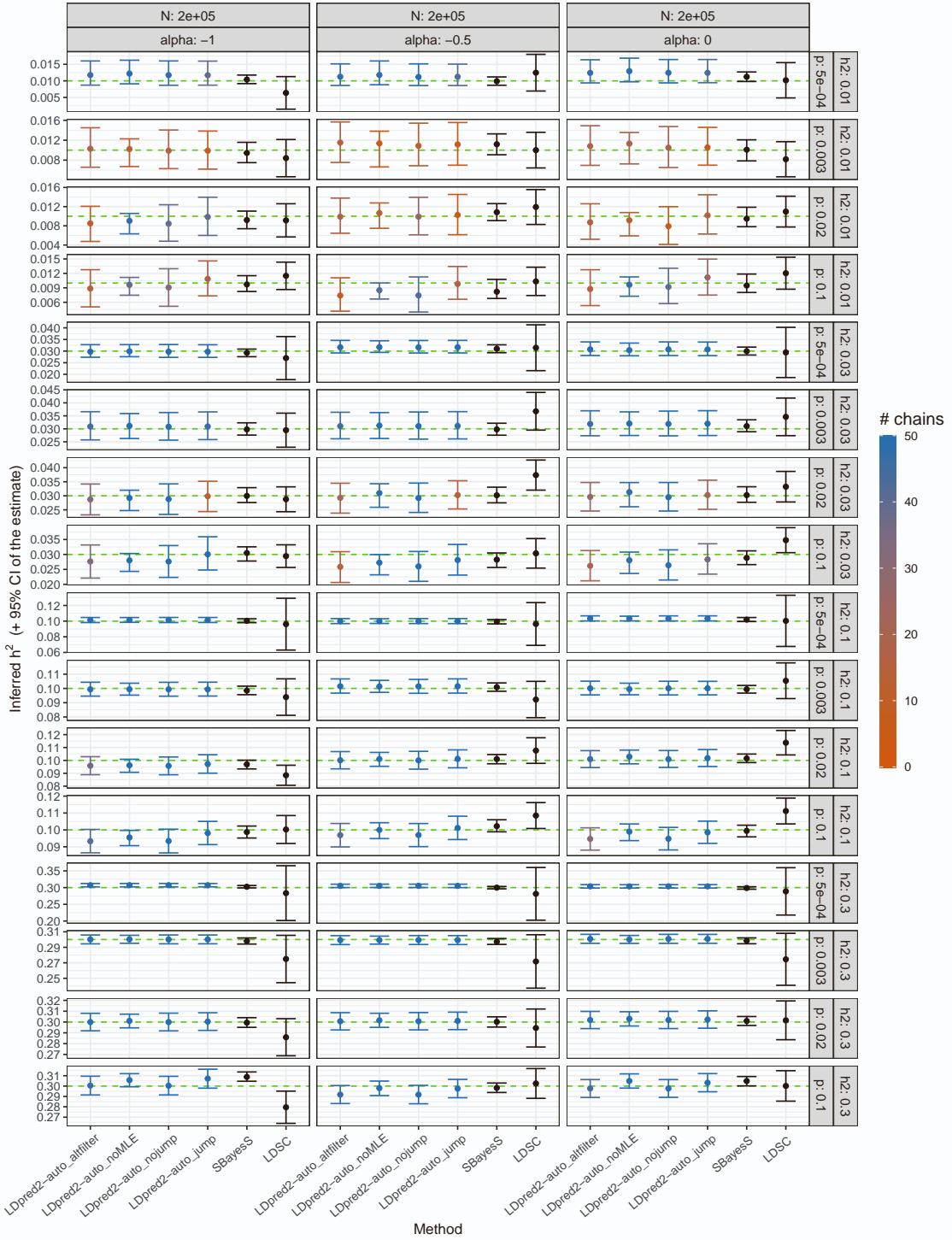


Figure S2: Inferred SNP heritability  $h^2$  in simulations with continuous outcomes and  $N=200K$ . Horizontal dashed lines represent the true simulated values. For LDpred2-auto, suffix “nojump”/“jump” refers to using `allow_jump_sign = FALSE/TRUE` (and `use_MLE = TRUE`), and “noMLE” refers to using `use_MLE = FALSE` (and `allow_jump_sign = FALSE`), and “alfilter” is similar to “nojump” but uses a different post-filtering of chains (Methods). Note that the recommended option is to use `allow_jump_sign = FALSE`<sup>1</sup>. The 95% confidence intervals for the LDpred2-auto and SBayesS estimates are obtained from the 2.5% and 97.5% quantiles of all the  $h^2$  estimates from the iterations (after burn-in) of the chains kept (note that only one chain is used and kept in SBayesS). The 95% confidence interval for the LD Score regression (LDSC) estimate is obtained as  $\pm 1.96$  of its standard error. Colors for LDpred2-auto models represent the number of chains kept (out of 50).

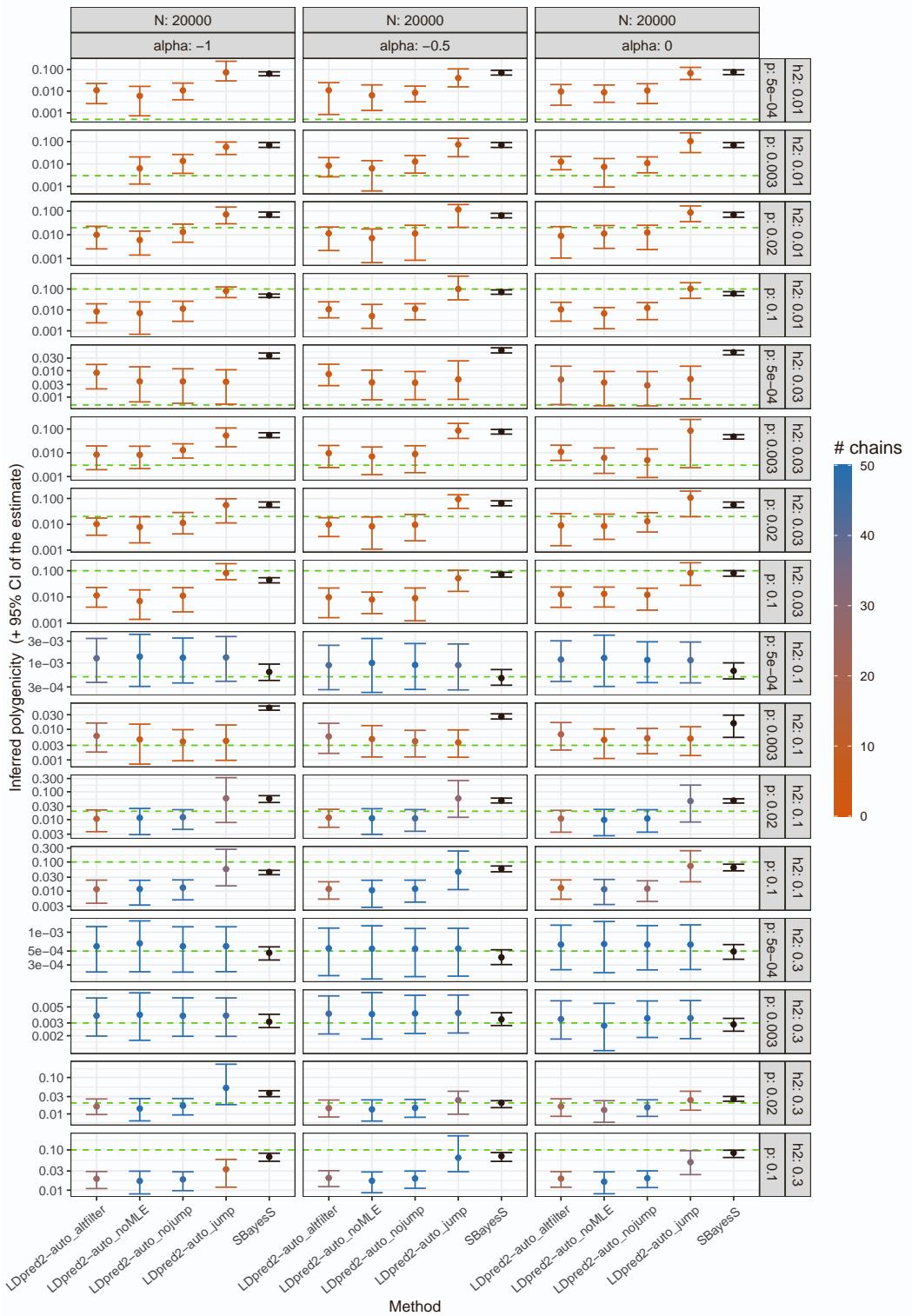


Figure S3: Inferred polygenicity  $p$  in simulations with continuous outcomes and  $N=20K$ . Horizontal dashed lines represent the true simulated values. For LDpred2-auto, suffix “nojump”/“jump” refers to using `allow_jump_sign = FALSE/TRUE` (and `use_MLE = TRUE`), and “noMLE” refers to using `use_MLE = FALSE` (and `allow_jump_sign = FALSE`), and “alfilter” is similar to “nojump” but uses a different post-filtering of chains (Methods). Note that the recommended option is to use `allow_jump_sign = FALSE`<sup>1</sup>. The 95% confidence intervals for the LDpred2-auto and SBayesS estimates are obtained from the 2.5% and 97.5% quantiles of all the  $p$  estimates from the iterations (after burn-in) of the chains kept (note that only one chain is used and kept in SBayesS). Colors for LDpred2-auto models represent the number of chains kept (out of 50).

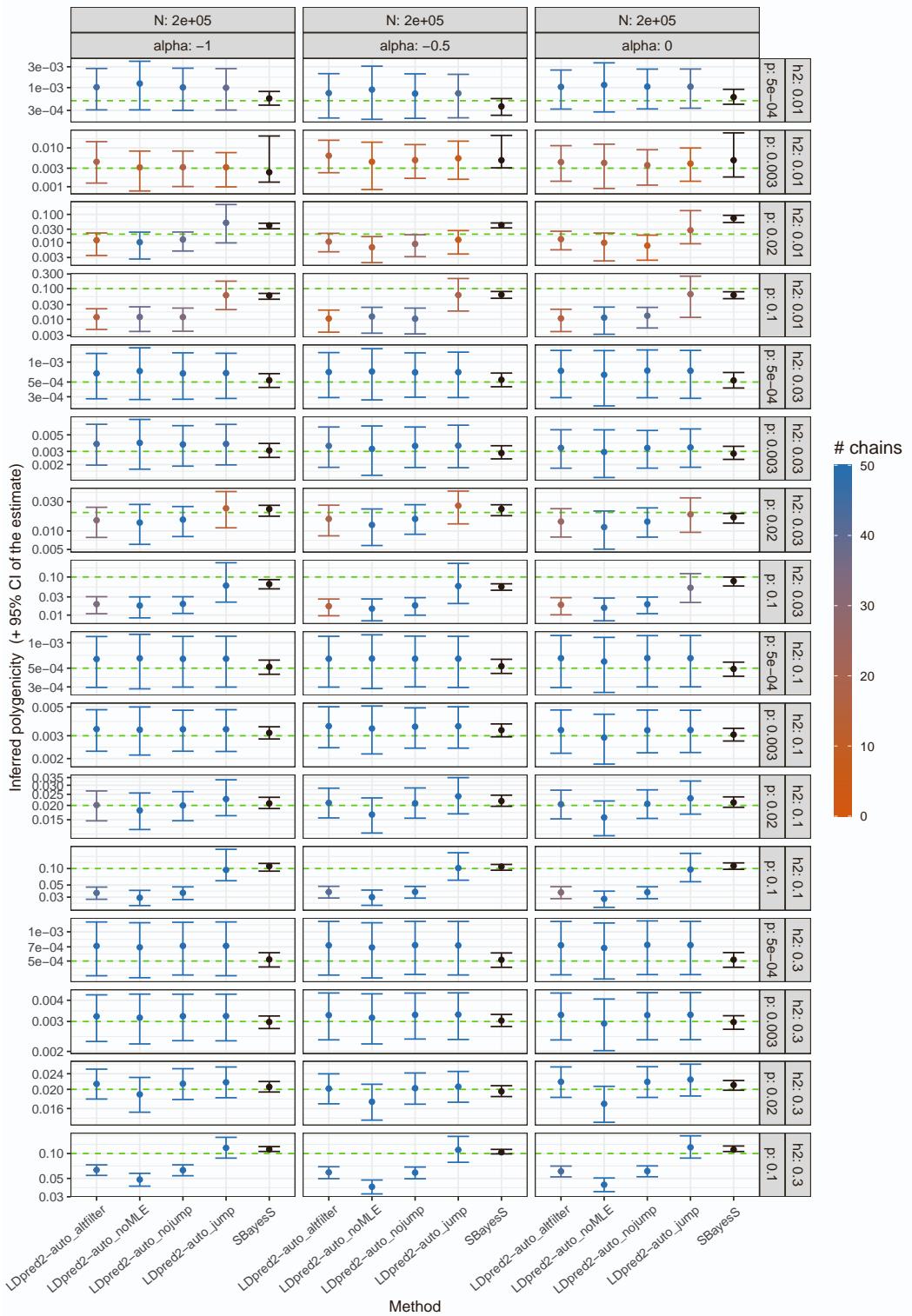


Figure S4: Inferred polygenicity  $p$  in simulations with continuous outcomes and  $N=200K$ . Horizontal dashed lines represent the true simulated values. For LDpred2-auto, suffix “nojump”/“jump” refers to using `allow_jump_sign = FALSE/TRUE` (and `use_MLE = TRUE`), and “noMLE” refers to using `use_MLE = FALSE` (and `allow_jump_sign = FALSE`), and “alfilter” is similar to “nojump” but uses a different post-filtering of chains (Methods). Note that the recommended option is to use `allow_jump_sign = FALSE`<sup>1</sup>. The 95% confidence intervals for the LDpred2-auto and SBayesS estimates are obtained from the 2.5% and 97.5% quantiles of all the  $p$  estimates from the iterations (after burn-in) of the chains kept (note that only one chain is used and kept in SBayesS). Colors for LDpred2-auto models represent the number of chains kept (out of 50).

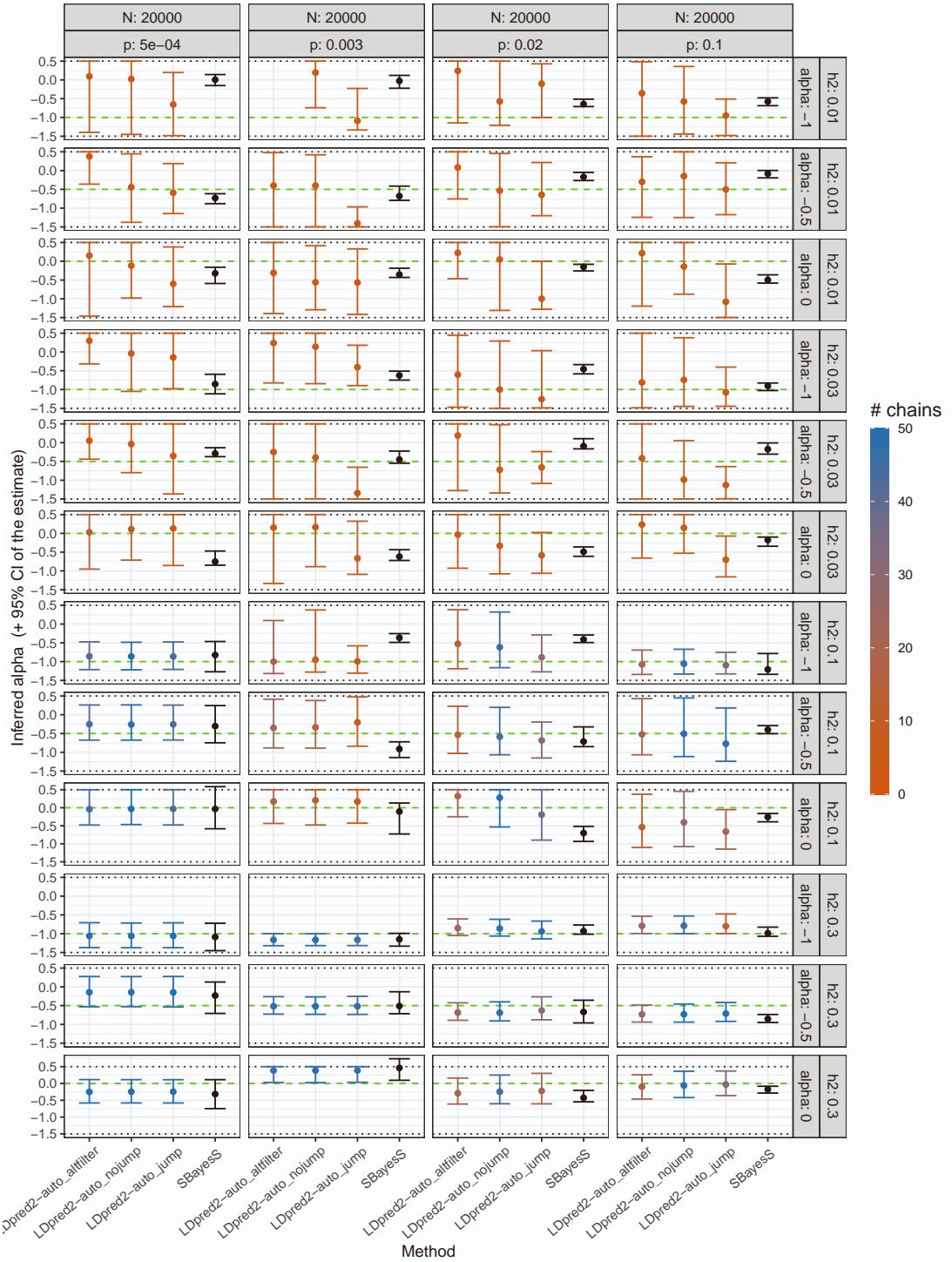


Figure S5: Inferred  $\alpha$  in simulations with continuous outcomes and  $N=20K$ . Horizontal dashed lines represent the true simulated values. Horizontal dotted lines represent boundaries imposed on the LDpred2-auto estimates. For LDpred2-auto, suffix “nojump”/“jump” refers to using `allow_jump_sign = FALSE/TRUE` (and `use_MLE = TRUE`), and “alfilter” is similar to “nojump” but uses a different post-filtering of chains (Methods). Note that “LDpred2\_noMLE” (`use_MLE = FALSE`) does not infer  $\alpha$ . Note that the recommended option is to use `allow_jump_sign = FALSE`<sup>1</sup>. The 95% confidence intervals for the LDpred2-auto and SBayesS estimates are obtained from the 2.5% and 97.5% quantiles of all the  $\alpha$  estimates from the iterations (after burn-in) of the chains kept (note that only one chain is used and kept in SBayesS). Colors for LDpred2-auto models represent the number of chains kept (out of 50).

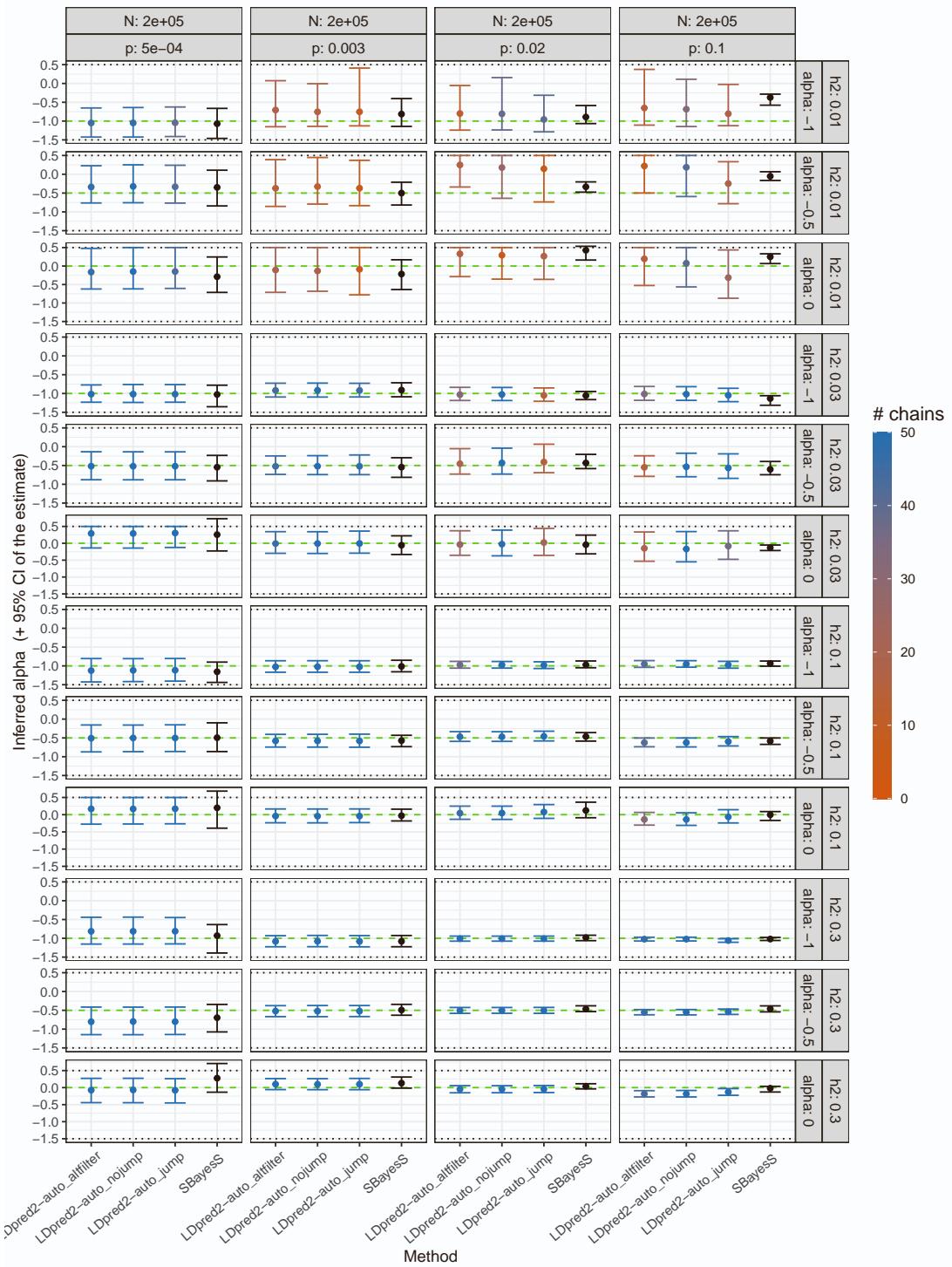


Figure S6: Inferred  $\alpha$  in simulations with continuous outcomes and  $N=200K$ . Horizontal dashed lines represent the true simulated values. Horizontal dotted lines represent boundaries imposed on the LDpred2-auto estimates. For LDpred2-auto, suffix “nojump”/“jump” refers to using `allow_jump_sign = FALSE/TRUE` (and `use_MLE = TRUE`), and “altfilter” is similar to “nojump” but uses a different post-filtering of chains (Methods). Note that “LDpred2\_noMLE” (`use_MLE = FALSE`) does not infer  $\alpha$ . Note that the recommended option is to use `allow_jump_sign = FALSE`<sup>1</sup>. The 95% confidence intervals for the LDpred2-auto and SBayesS estimates are obtained from the 2.5% and 97.5% quantiles of all the  $\alpha$  estimates from the iterations (after burn-in) of the chains kept (note that only one chain is used and kept in SBayesS). Colors for LDpred2-auto models represent the number of chains kept (out of 50).

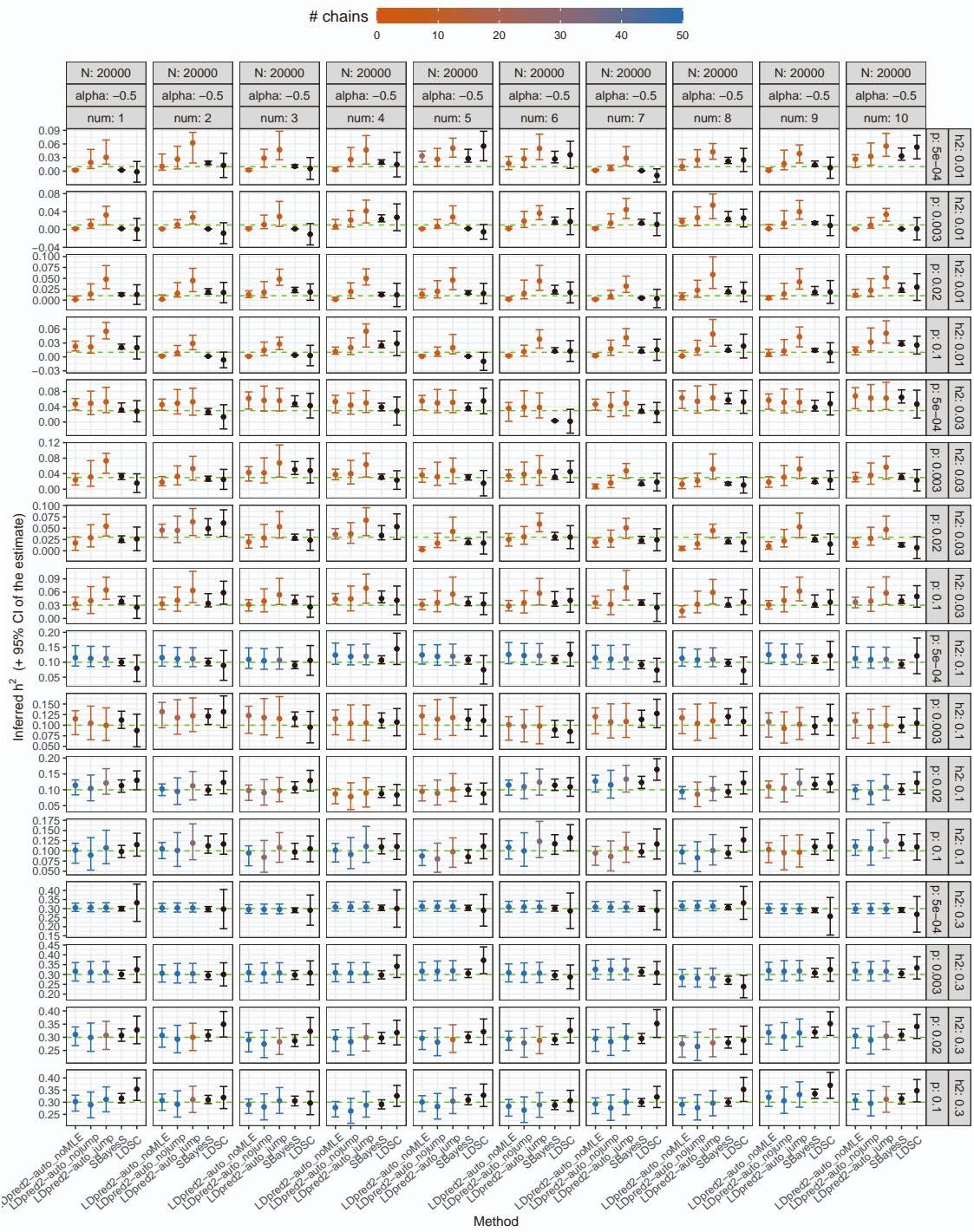


Figure S7: Inferred SNP heritability  $h^2$  in simulations with continuous outcomes and N=20K, in 10 different runs (“num”). Horizontal dashed lines represent the true simulated values. For LDpred2-auto, suffix “nojump”/“jump” refers to using `allow_jump_sign = FALSE/TRUE` (and `use_MLE = TRUE`), and “noMLE” refers to using `use_MLE = FALSE` (and `allow_jump_sign = FALSE`). Note that the recommended option is to use `allow_jump_sign = FALSE`<sup>1</sup>. The 95% confidence intervals for the LDpred2-auto and SBayesS estimates are obtained from the 2.5% and 97.5% quantiles of all the  $h^2$  estimates from the iterations (after burn-in) of the chains kept (note that only one chain is used and kept in SBayesS). The 95% confidence interval for the LD Score regression (LDSC) estimate is obtained as  $\pm 1.96$  of its standard error. Colors for LDpred2-auto models represent the number of chains kept (out of 50).

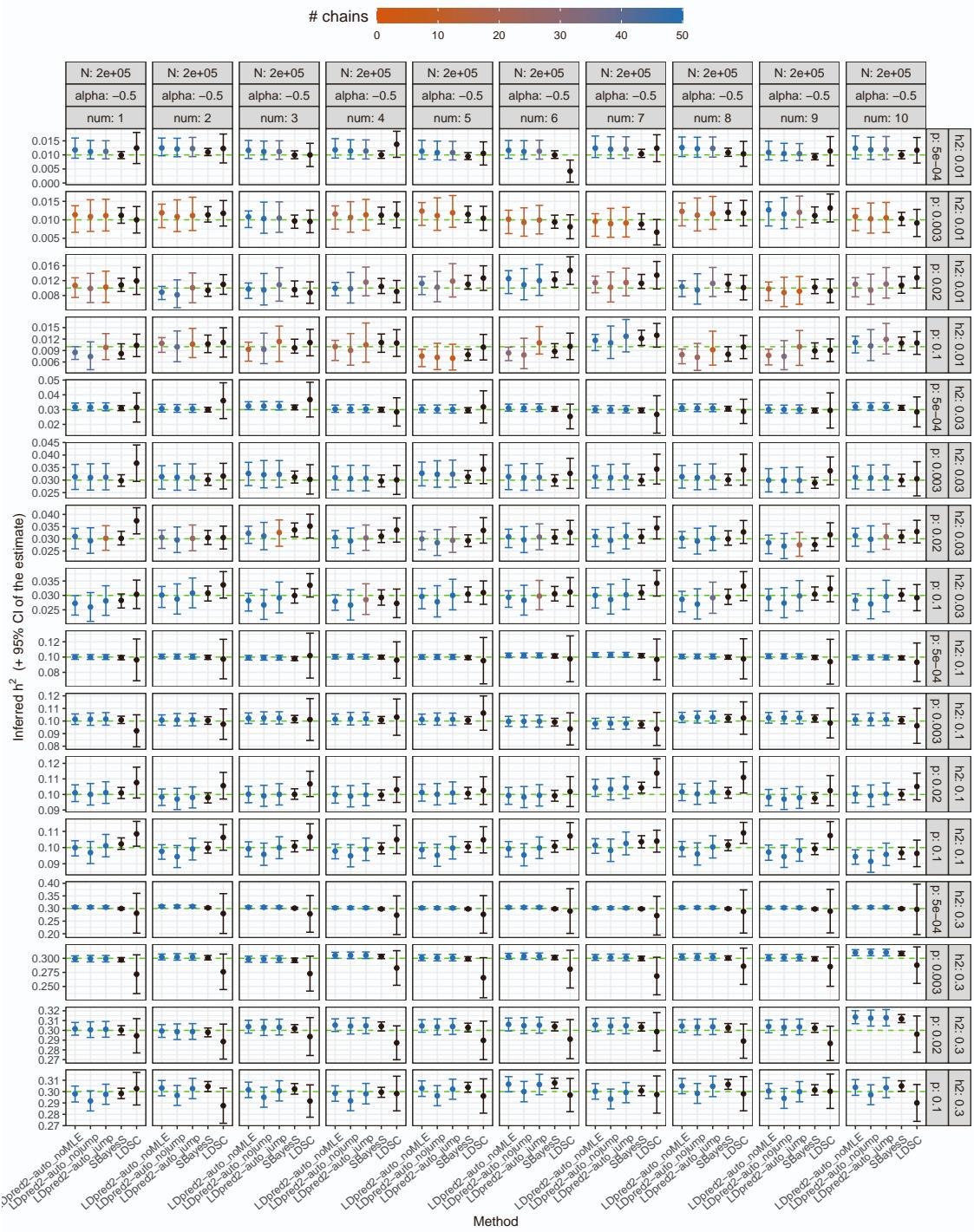


Figure S8: Inferred SNP heritability  $h^2$  in simulations with continuous outcomes and N=200K, in 10 different runs (“num”). Horizontal dashed lines represent the true simulated values. For LDpred2-auto, suffix “nojump”/“jump” refers to using `allow_jump_sign = FALSE/TRUE` (and `use_MLE = TRUE`), and “noMLE” refers to using `use_MLE = FALSE` (and `allow_jump_sign = FALSE`). Note that the recommended option is to use `allow_jump_sign = FALSE`<sup>1</sup>. The 95% confidence intervals for the LDpred2-auto and SBayesS estimates are obtained from the 2.5% and 97.5% quantiles of all the  $h^2$  estimates from the iterations (after burn-in) of the chains kept (note that only one chain is used and kept in SBayesS). The 95% confidence interval for the LD Score regression (LDSC) estimate is obtained as  $\pm 1.96$  of its standard error. Colors for LDpred2-auto models represent the number of chains kept (out of 50).

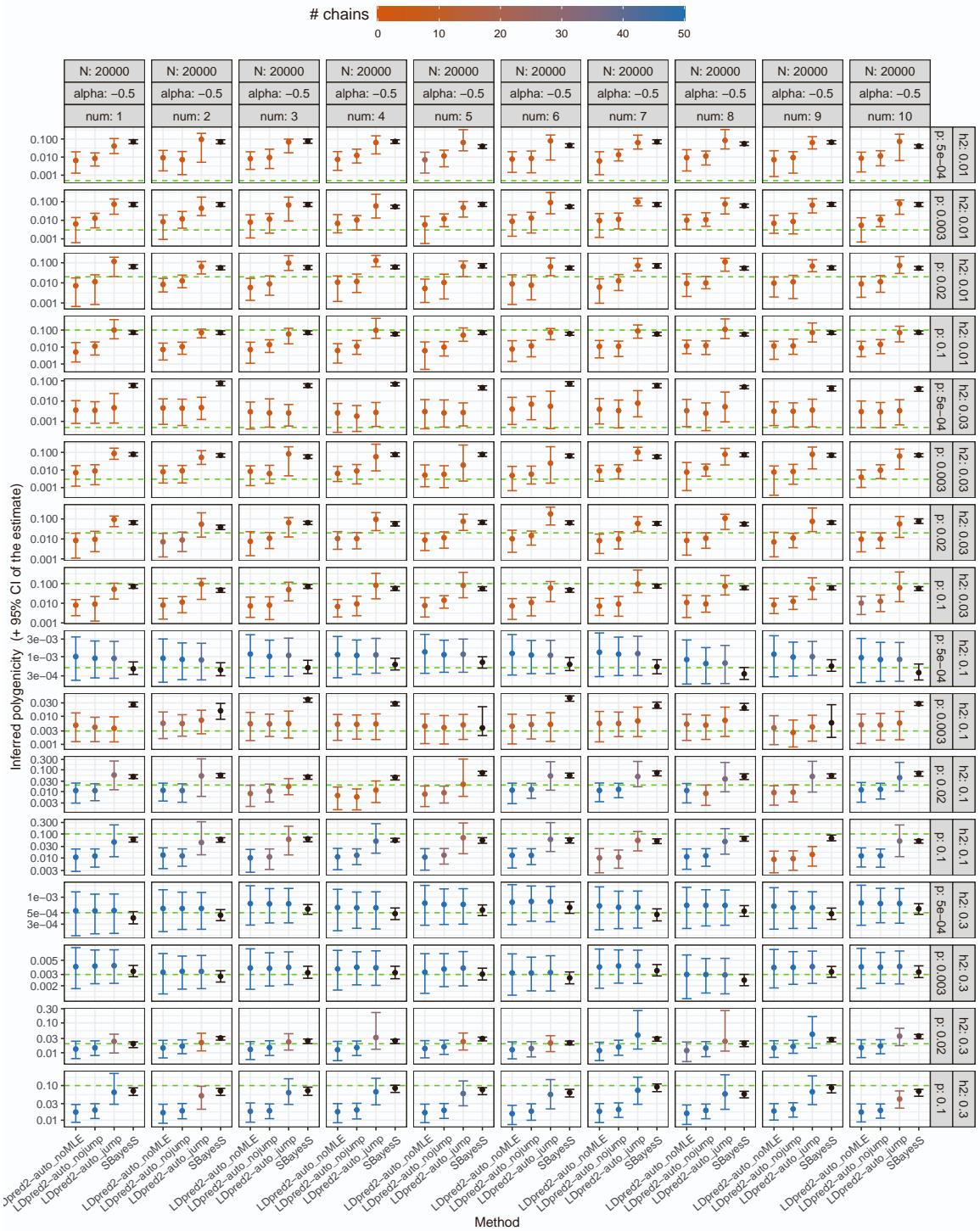


Figure S9: Inferred polygenicity  $p$  in simulations with continuous outcomes and  $N=20K$ , in 10 different runs (“num”). Horizontal dashed lines represent the true simulated values. For LDpred2-auto, suffix “nojump”/“jump” refers to using `allow_jump_sign = FALSE/TRUE` (and `use_MLE = TRUE`), and “noMLE” refers to using `use_MLE = FALSE` (and `allow_jump_sign = FALSE`). Note that the recommended option is to use `allow_jump_sign = FALSE`<sup>1</sup>. The 95% confidence intervals for the LDpred2-auto and SBayesS estimates are obtained from the 2.5% and 97.5% quantiles of all the  $p$  estimates from the iterations (after burn-in) of the chains kept (note that only one chain is used and kept in SBayesS). Colors for LDpred2-auto models represent the number of chains kept (out of 50).

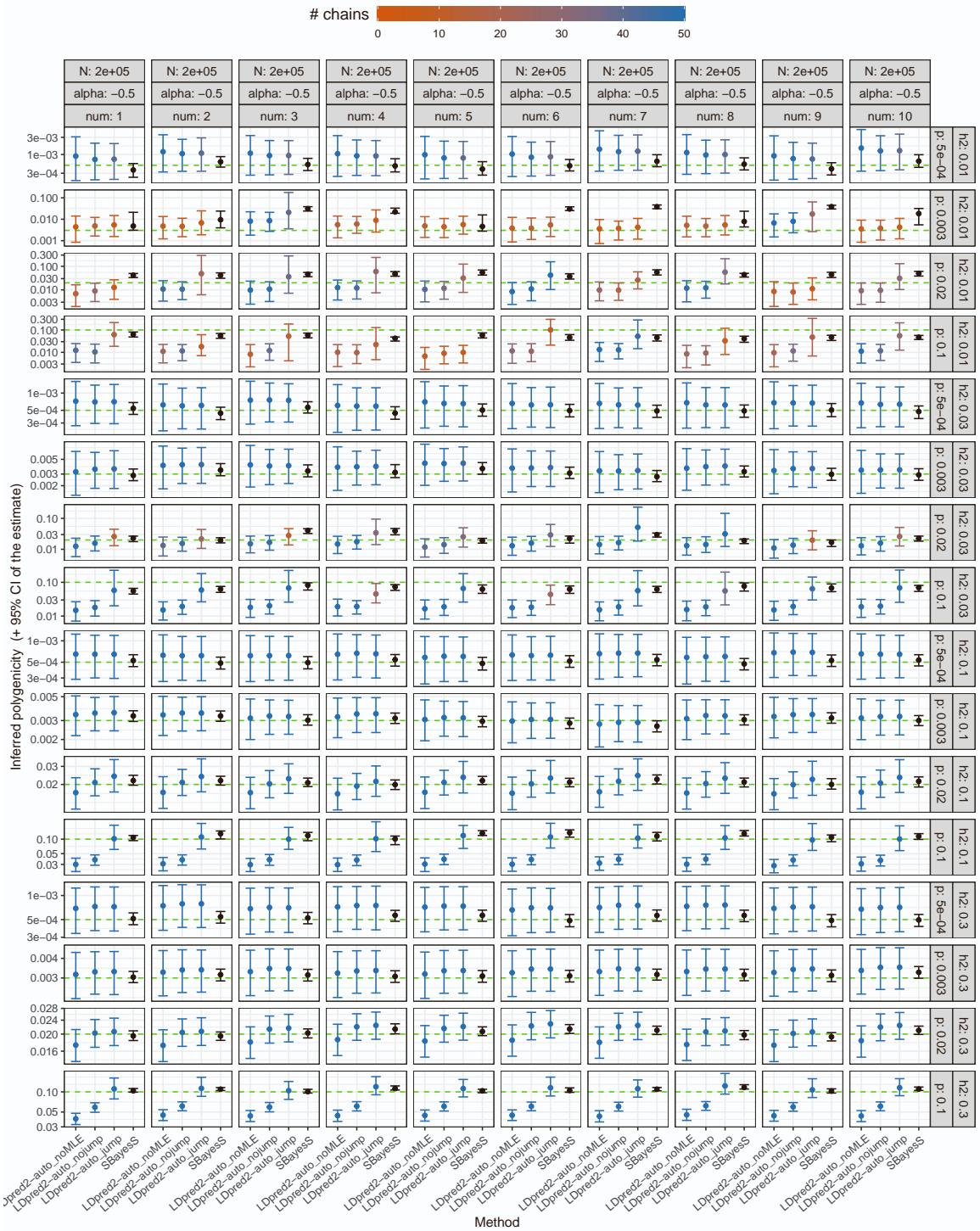


Figure S10: Inferred polygenicity  $p$  in simulations with continuous outcomes and  $N=200K$ , in 10 different runs (“num”). Horizontal dashed lines represent the true simulated values. For LDpred2-auto, suffix “nojump”/“jump” refers to using `allow_jump_sign = FALSE/TRUE` (and `use_MLE = TRUE`), and “noMLE” refers to using `use_MLE = FALSE` (and `allow_jump_sign = FALSE`). Note that the recommended option is to use `allow_jump_sign = FALSE`<sup>1</sup>. The 95% confidence intervals for the LDpred2-auto and SBayesS estimates are obtained from the 2.5% and 97.5% quantiles of all the  $p$  estimates from the iterations (after burn-in) of the chains kept (note that only one chain is used and kept in SBayesS). Colors for LDpred2-auto models represent the number of chains kept (out of 50).

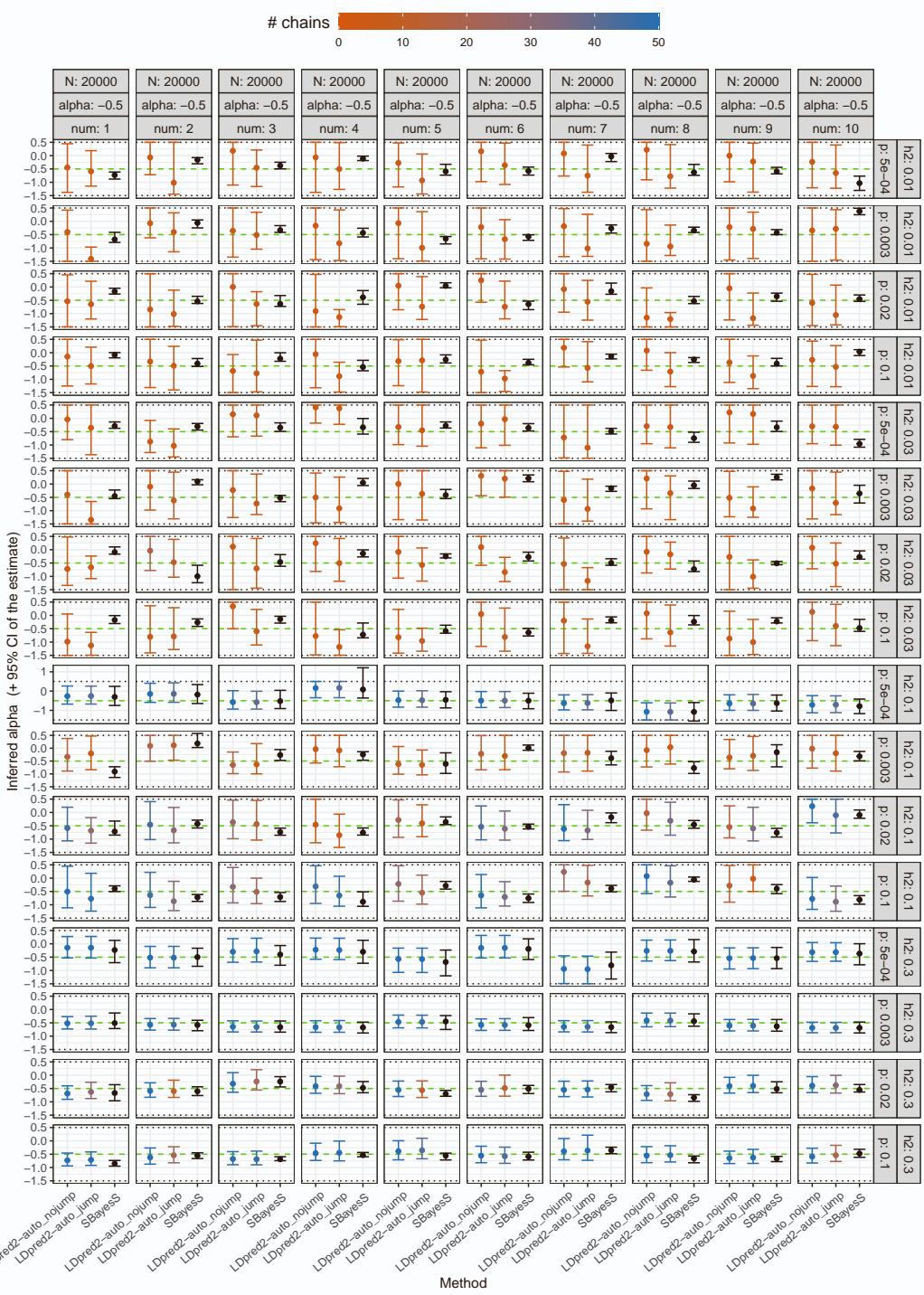


Figure S11: Inferred  $\alpha$  in simulations with continuous outcomes and  $N=20K$ , in 10 different runs (“num”). Horizontal dashed lines represent the true simulated values. Horizontal dotted lines represent boundaries imposed on the LDpred2-auto estimates. For LDpred2-auto, suffix “nojump”/“jump” refers to using `allow_jump_sign = FALSE/TRUE` (and `use_MLE = TRUE`). Note that “LDpred2\_noMLE” (`use_MLE = FALSE`) does not infer  $\alpha$ . Note that the recommended option is to use `allow_jump_sign = FALSE`<sup>1</sup>. The 95% confidence intervals for the LDpred2-auto and SBayesS estimates are obtained from the 2.5% and 97.5% quantiles of all the  $\alpha$  estimates from the iterations (after burn-in) of the chains kept (note that only one chain is used and kept in SBayesS). Colors for LDpred2-auto models represent the number of chains kept (out of 50).

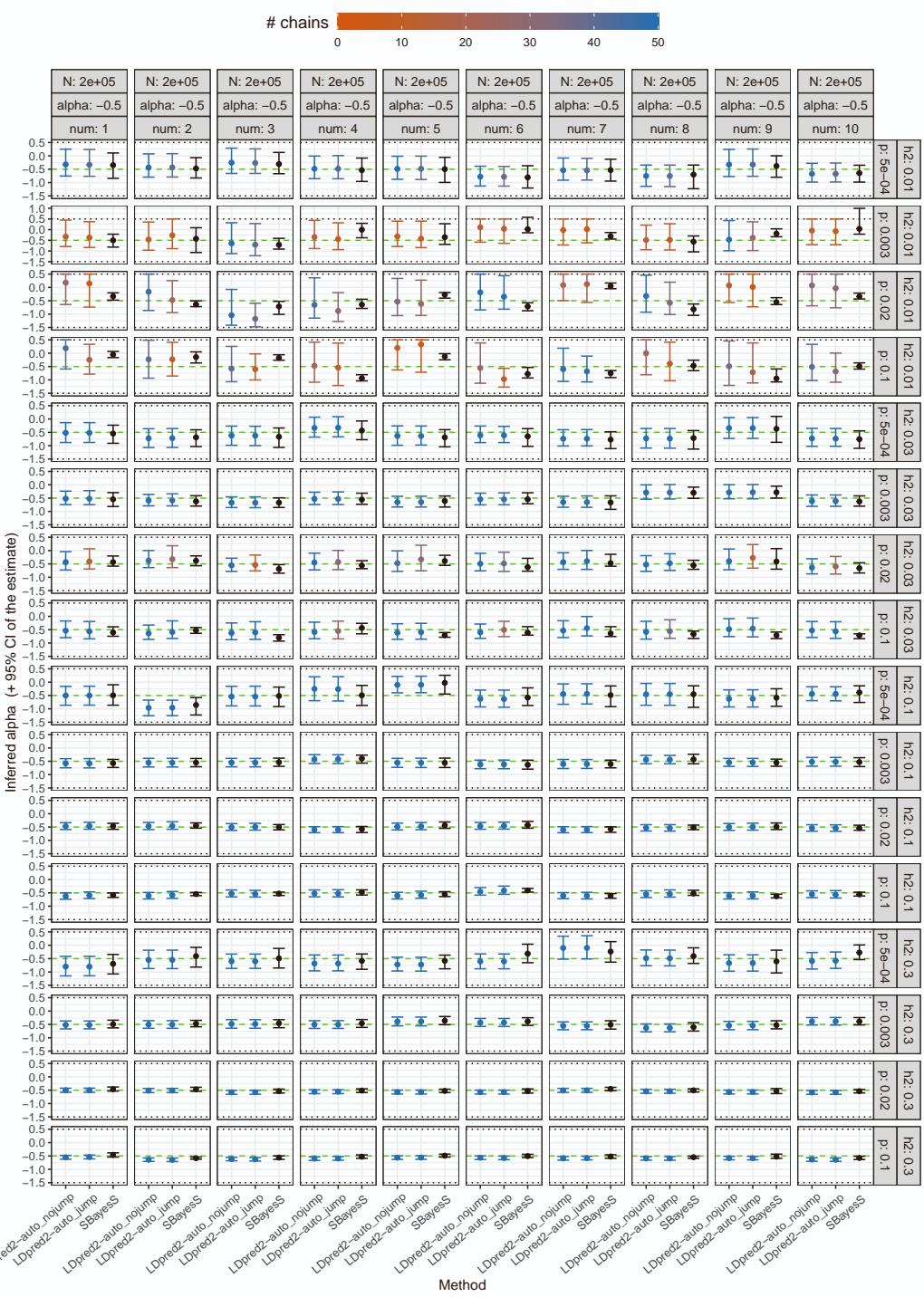


Figure S12: Inferred  $\alpha$  in simulations with continuous outcomes and  $N=200K$ , in 10 different runs (“num”). Horizontal dashed lines represent the true simulated values. Horizontal dotted lines represent boundaries imposed on the LDpred2-auto estimates. For LDpred2-auto, suffix “nojump”/“jump” refers to using `allow_jump_sign = FALSE/TRUE` (and `use_MLE = TRUE`). Note that “LDpred2\_noMLE” (`use_MLE = FALSE`) does not infer  $\alpha$ . Note that the recommended option is to use `allow_jump_sign = FALSE`<sup>1</sup>. The 95% confidence intervals for the LDpred2-auto and SBayesS estimates are obtained from the 2.5% and 97.5% quantiles of all the  $\alpha$  estimates from the iterations (after burn-in) of the chains kept (note that only one chain is used and kept in SBayesS). Colors for LDpred2-auto models represent the number of chains kept (out of 50).

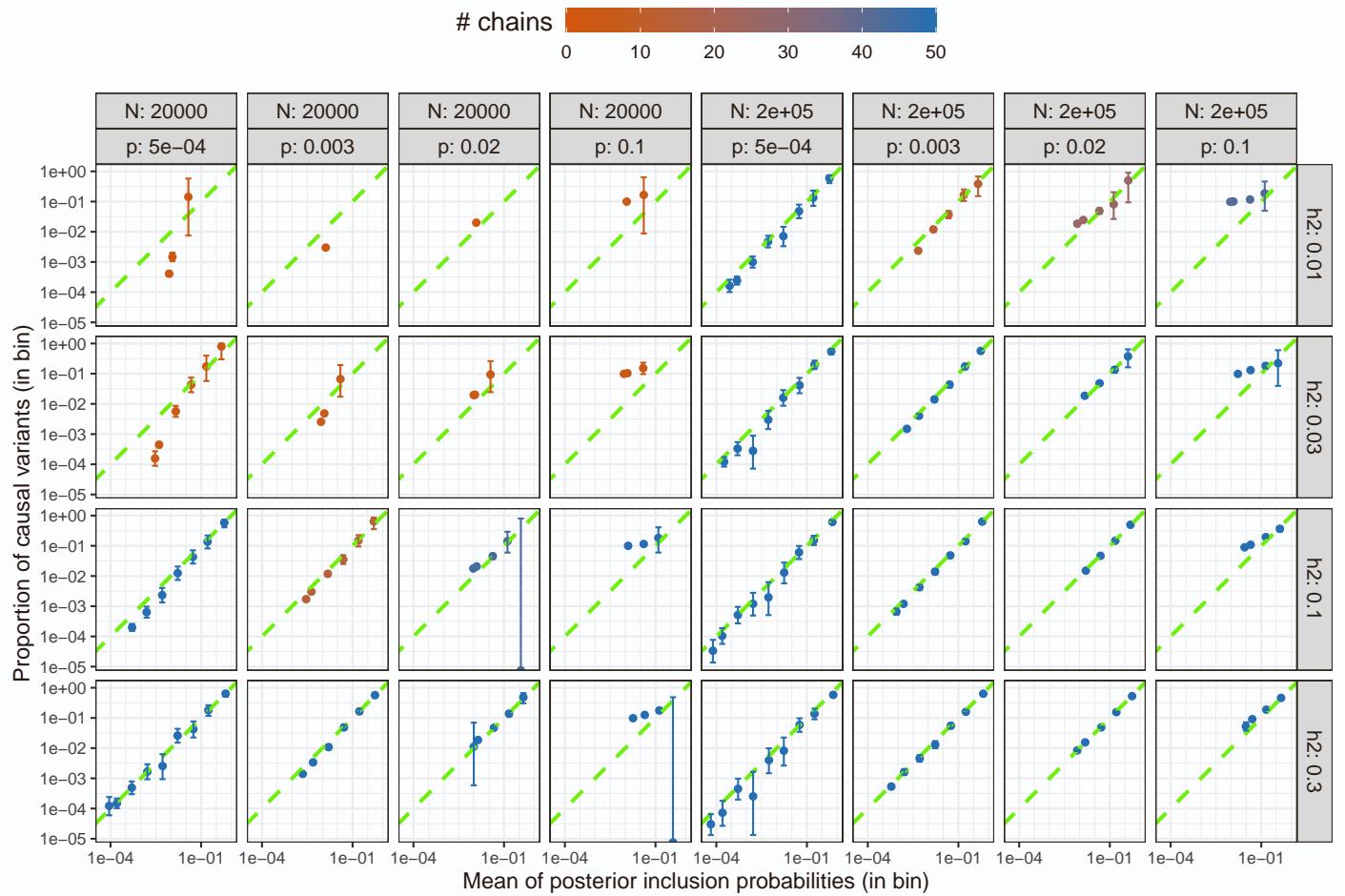


Figure S13: LDpred2-auto calibration of per-variant posterior probabilities of being causal (also known as posterior inclusion probabilities) in simulations with continuous outcomes. These probabilities are binned on a log-scale, and mean in each bin is compared to the proportion of simulated causal variants in the bin (with 95% CI). Green dashed lines represent the 1:1 line. Only results for  $\alpha = -0.5$  and `allow_jump_sign = FALSE` are represented. Colors represent the number of chains kept (out of 50).

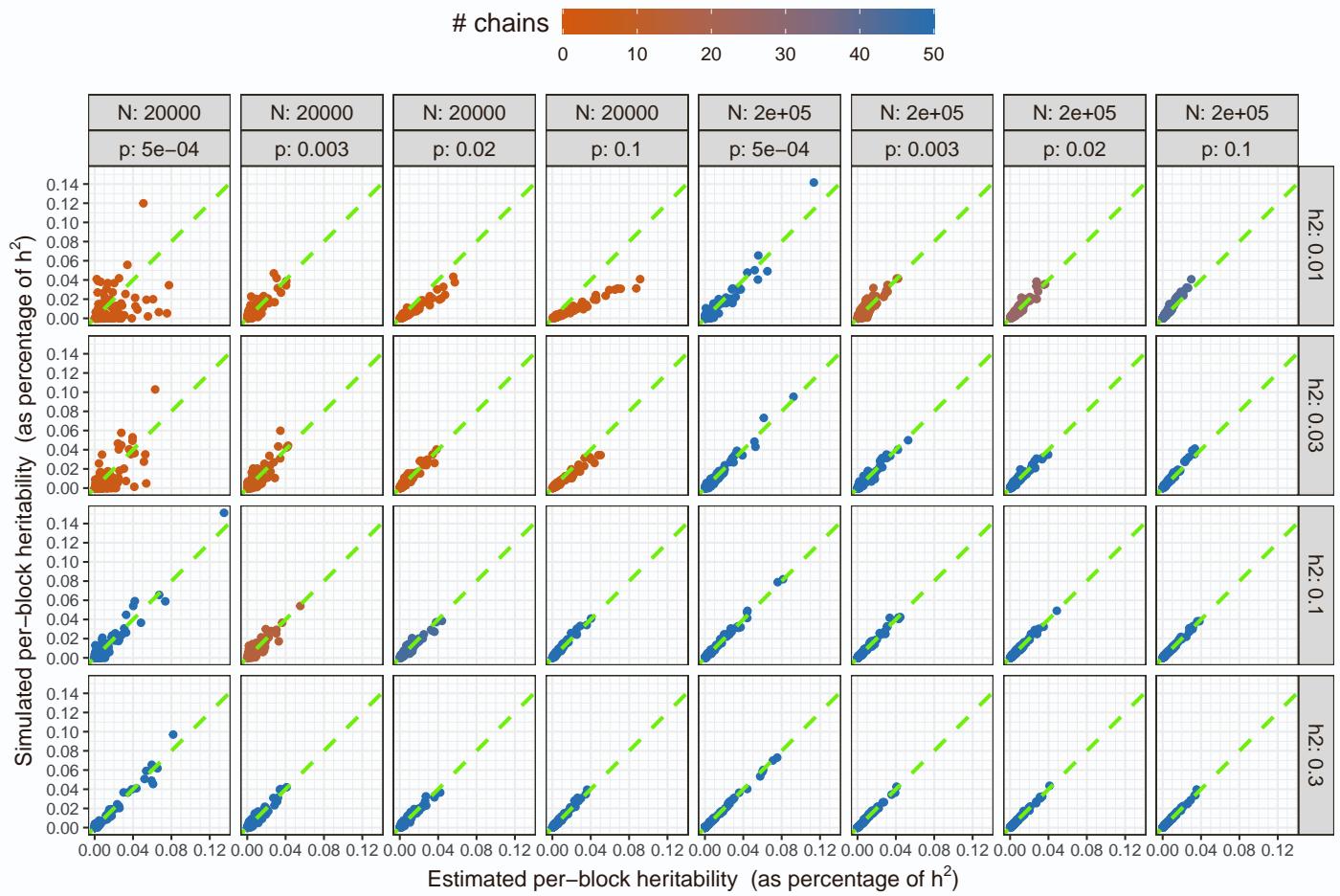
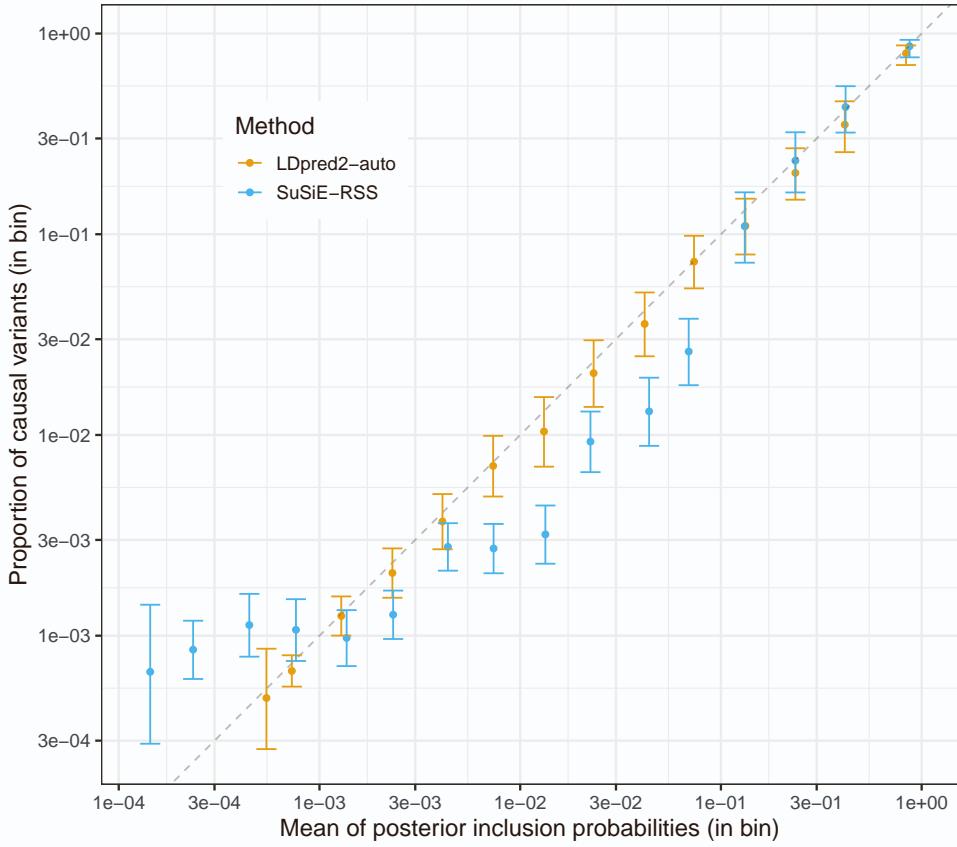
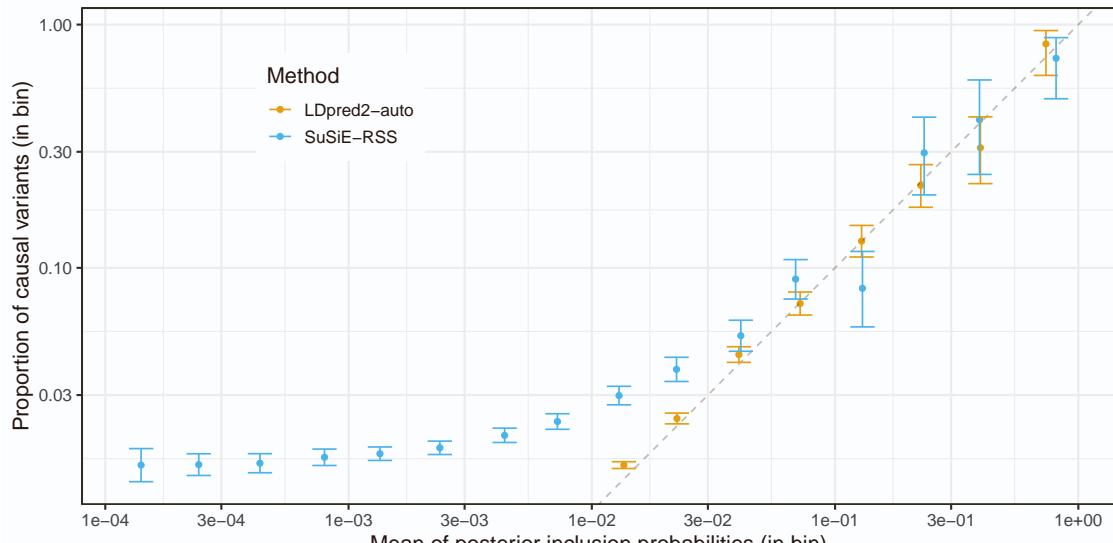


Figure S14: LDpred2-auto calibration of per-block heritability estimates in simulations with continuous outcomes. Local  $h^2$  estimate for each block is compared to the true simulated local heritability in the block. Green dashed lines represent the 1:1 line. Only results for  $\alpha = -0.5$  and `allow_jump_sign = FALSE` are represented. Colors represent the number of chains kept (out of 50).



(a) With  $p = 0.002$ .



(b) With  $p = 0.02$ .

Figure S15: Calibration of per-variant posterior probabilities of being causal in two simulations with continuous outcomes, assuming  $h^2 = 0.1$ ,  $\alpha = -0.5$ , and  $N = 10^5$ .

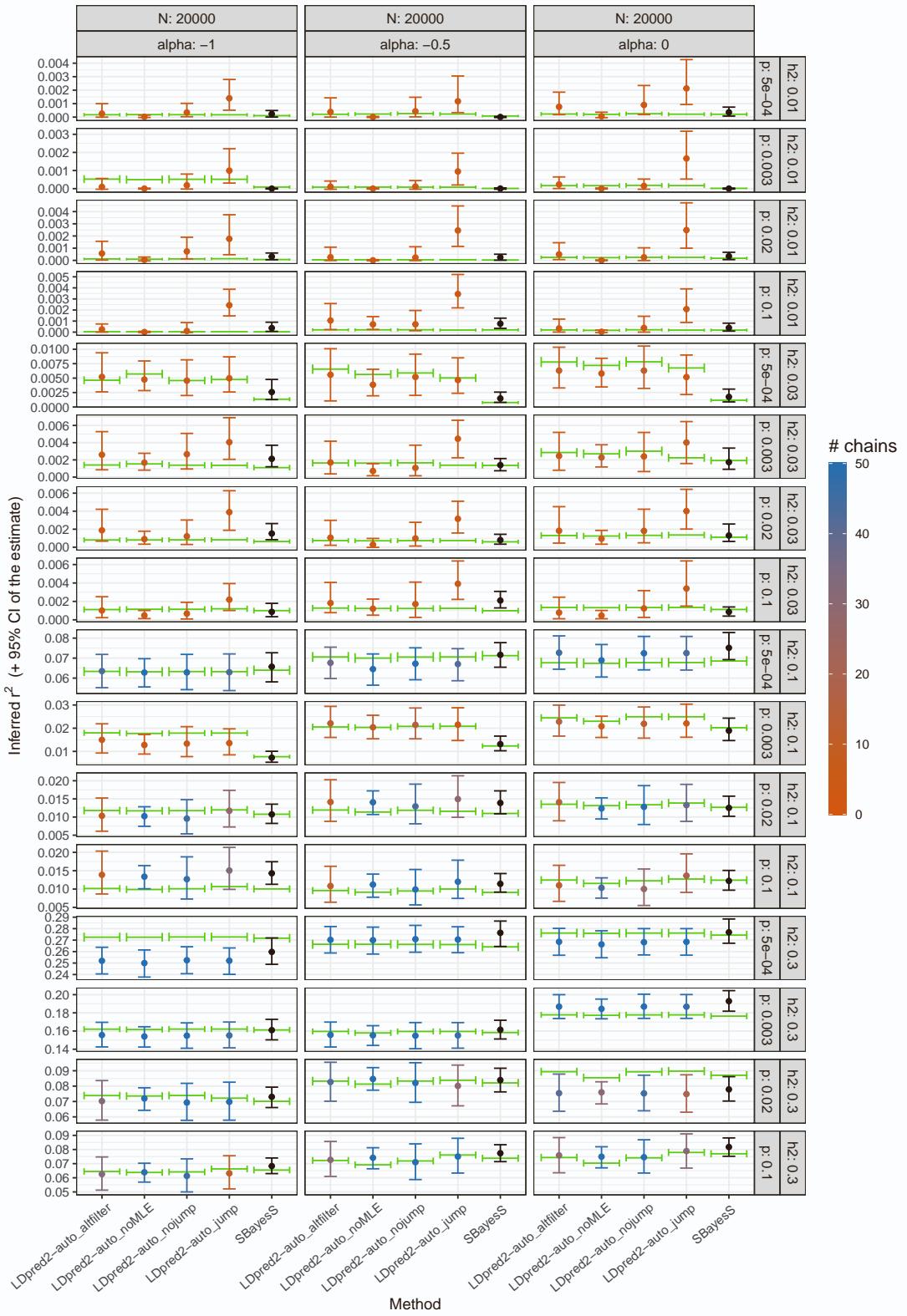


Figure S16: Inferred predictive performance  $r^2$  in simulations with continuous outcomes and  $N=20K$ . Green segments represent  $r^2$  in the test set. For LDpred2-auto, suffix “nojump”/“jump” refers to using `allow_jump_sign = FALSE/TRUE` (and `use_MLE = TRUE`), and “noMLE” refers to using `use_MLE = FALSE` (and `allow_jump_sign = FALSE`), and “altfilter” is similar to “nojump” but uses a different post-filtering of chains (Methods). Note that the recommended option is to use `allow_jump_sign = FALSE`<sup>1</sup>. Colors for LDpred2-auto models represent the number of chains kept (out of 50).

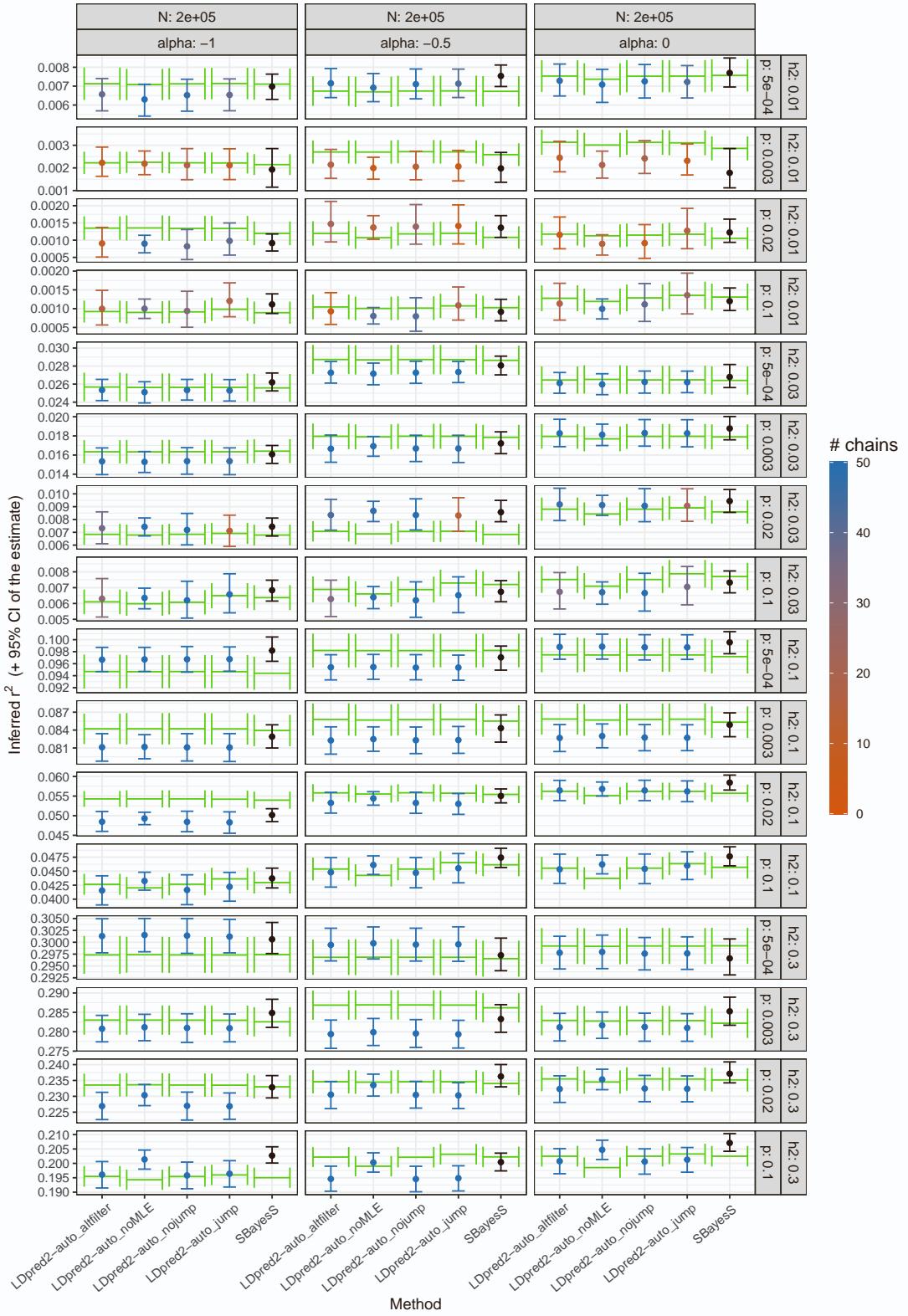


Figure S17: Inferred predictive performance  $r^2$  in simulations with continuous outcomes and  $N=200K$ . Green segments represent  $r^2$  in the test set. For LDpred2-auto, suffix “nojump”/“jump” refers to using `allow_jump_sign = FALSE/TRUE` (and `use_MLE = TRUE`), and “noMLE” refers to using `use_MLE = FALSE` (and `allow_jump_sign = FALSE`), and “altnfilter” is similar to “nojump” but uses a different post-filtering of chains (Methods). Note that the recommended option is to use `allow_jump_sign = FALSE`<sup>1</sup>. Colors for LDpred2-auto models represent the number of chains kept (out of 50).

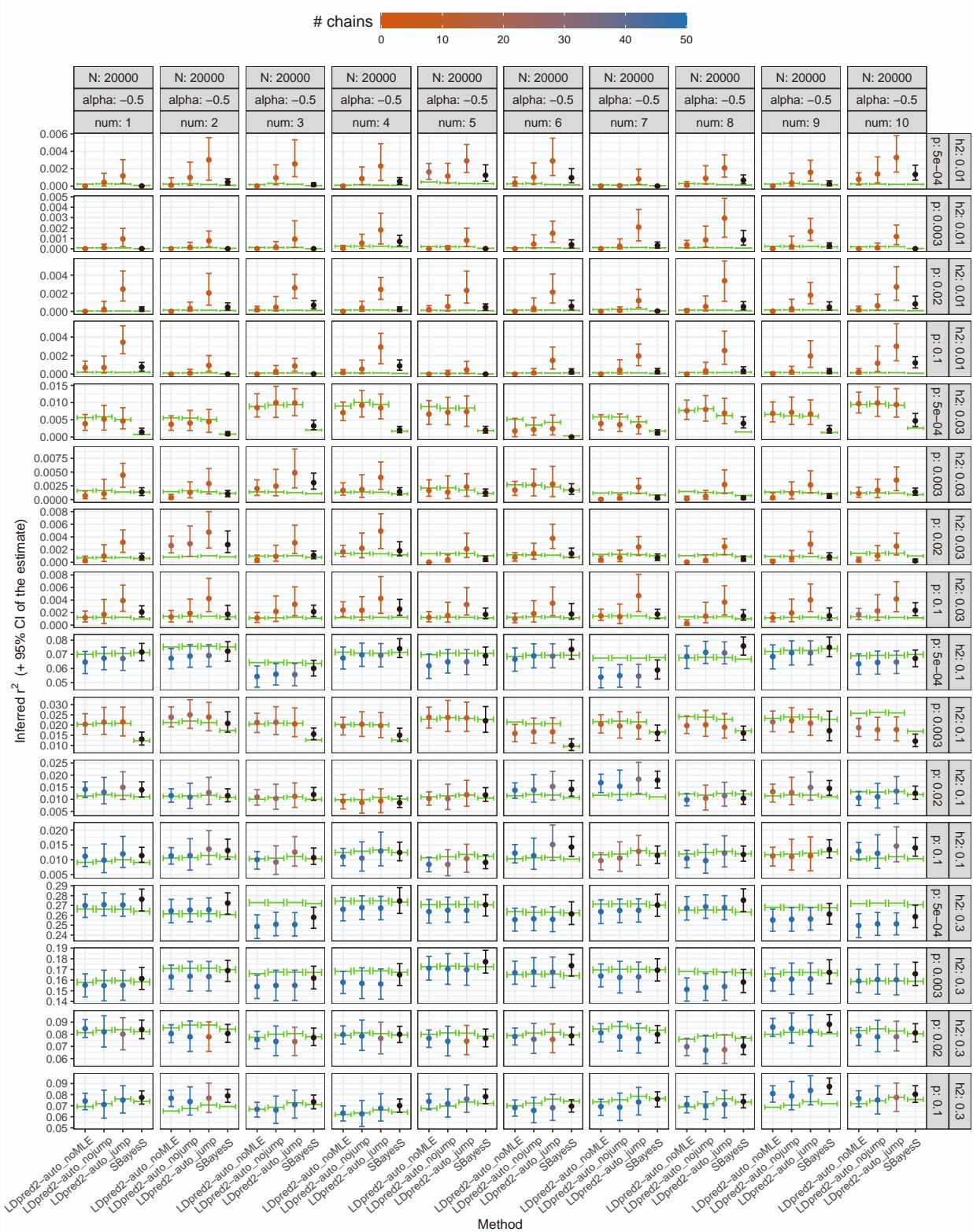


Figure S18: Inferred predictive performance  $r^2$  in simulations with continuous outcomes and N=20K, in 10 different runs (“num”). Green segments represent  $r^2$  in the test set. For LDpred2-auto, suffix “nojump”/“jump” refers to using `allow_jump_sign = FALSE/TRUE` (and `use_MLE = TRUE`), and “noMLE” refers to using `use_MLE = FALSE` (and `allow_jump_sign = FALSE`). Note that the recommended option is to use `allow_jump_sign = FALSE`<sup>1</sup>. Colors for LDpred2-auto models represent the number of chains kept (out of 50).

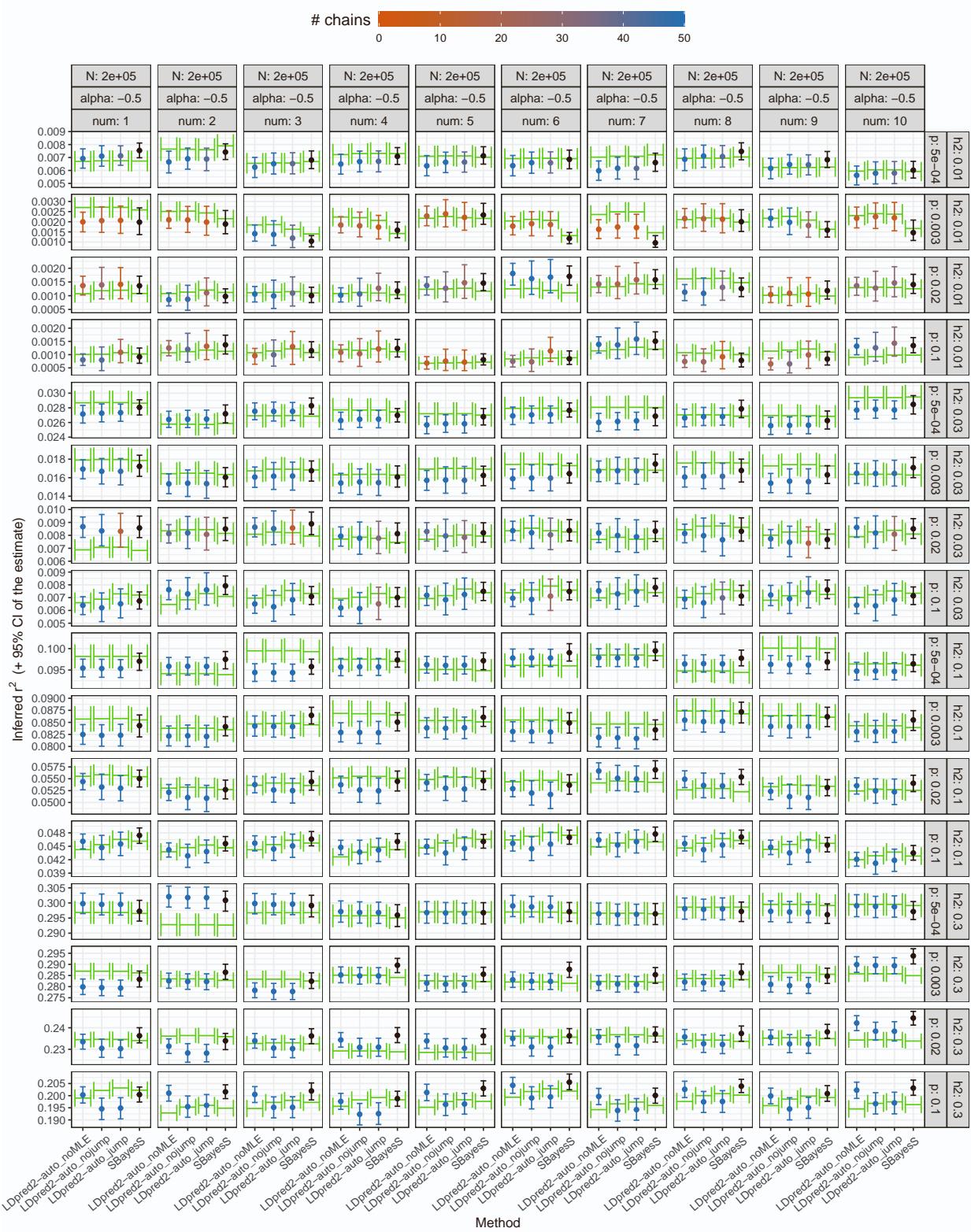


Figure S19: Inferred predictive performance  $r^2$  in simulations with continuous outcomes and N=200K, in 10 different runs (“num”). Green segments represent  $r^2$  in the test set. For LDpred2-auto, suffix “nojump”/“jump” refers to using `allow_jump_sign = FALSE/TRUE` (and `use_MLE = TRUE`), and “noMLE” refers to using `use_MLE = FALSE` (and `allow_jump_sign = FALSE`). Note that the recommended option is to use `allow_jump_sign = FALSE`<sup>1</sup>. Colors for LDpred2-auto models represent the number of chains kept (out of 50).

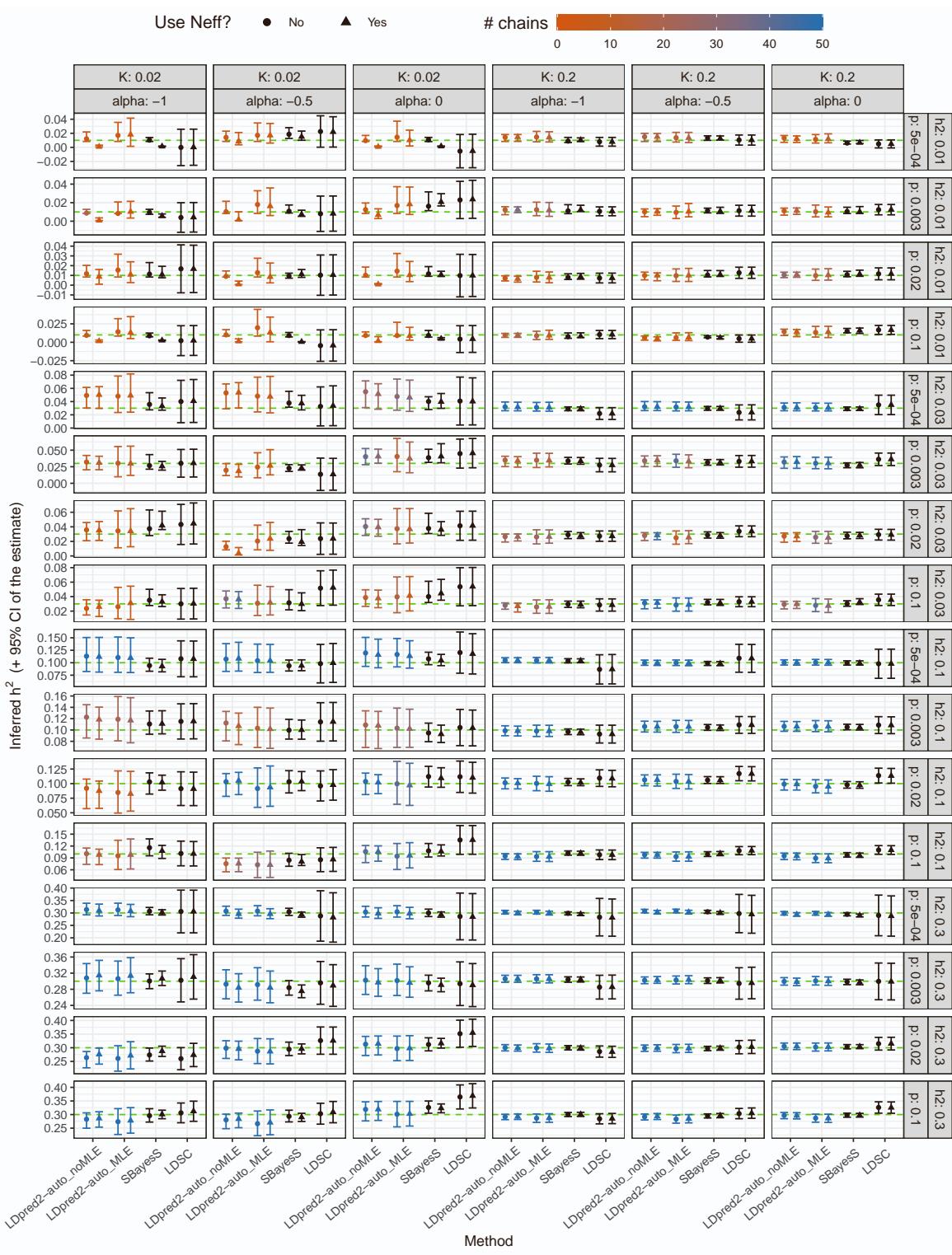


Figure S20: Inferred SNP heritability  $h^2$  in simulations with binary outcomes. Horizontal dashed lines represent the true simulated values. The 95% confidence interval for the LDpred2-auto estimate is obtained from the 2.5% and 97.5% quantiles of all the  $h^2$  estimates from the iterations (after burn-in) of the chains kept. The 95% confidence interval for the LD Score regression estimate is obtained from  $\pm 1.96$  times its standard error. Colors for LDpred2-auto models represent the number of chains kept (out of 50). Option `Neff` controls whether a logistic regression is used for the GWAS with the effective sample in LDpred2, or a linear regression and then the total sample size. All  $h^2$  estimates are transformed to the liability scale with `K_pop=K` and `K_GWAS` either `K` (the simulated prevalence) or 0.5 (when using `Neff`) using function `coef_to_liab` from R package `bigsnpr`<sup>2,3</sup>.



Figure S21: Inferred polygenicity  $p$  in simulations with binary outcomes. Horizontal dashed lines represent the true simulated values. Colors for LDpred2-auto models represent the number of chains kept (out of 50). Option  $\text{Neff}$  controls whether a logistic regression is used for the GWAS with the effective sample in LDpred2, or a linear regression and then the total sample size.

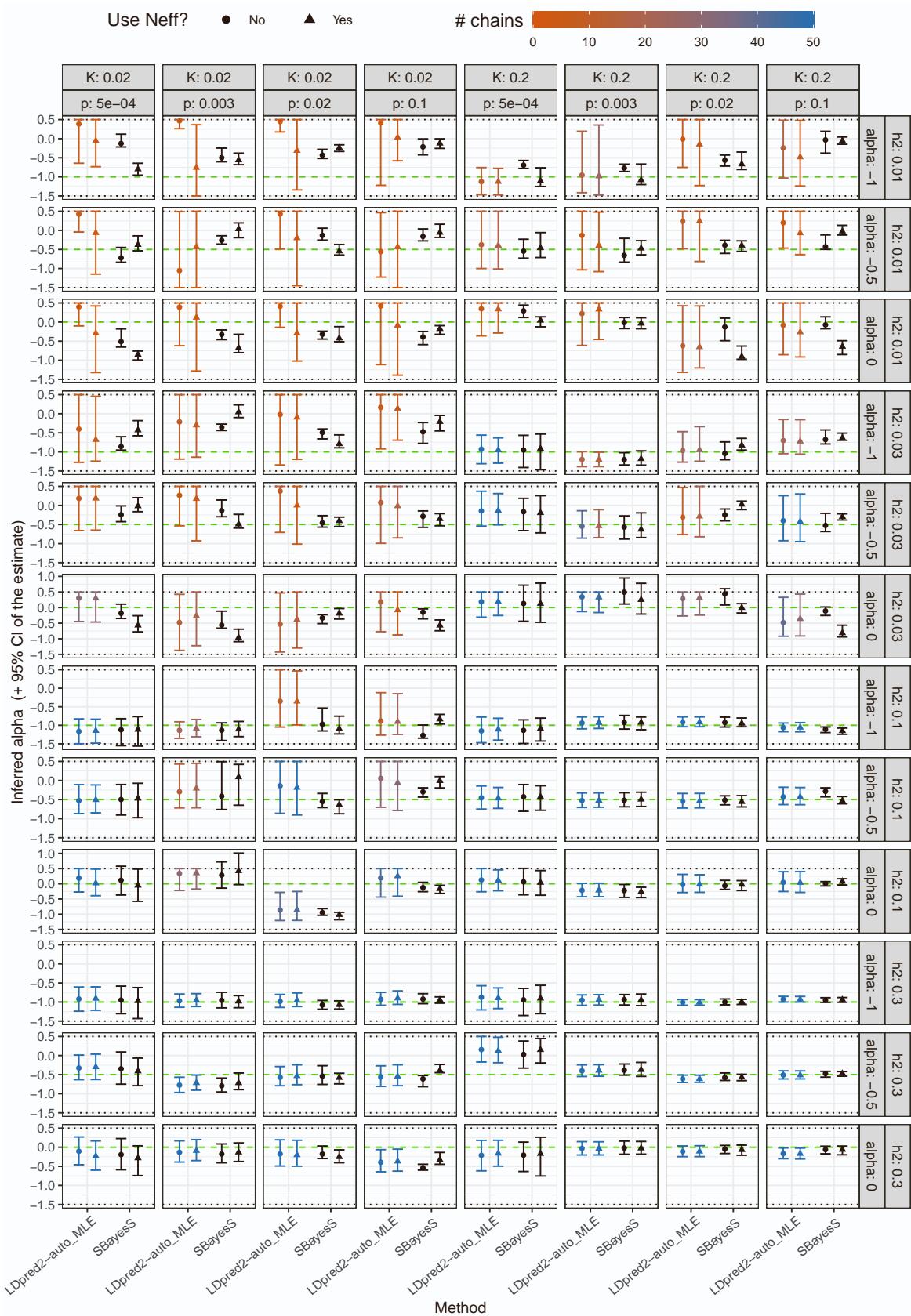


Figure S22: Inferred  $\alpha$  in simulations with binary outcomes. Horizontal dashed lines represent the true simulated values. Horizontal dotted lines represent boundaries imposed on the estimates. Colors for LDpred2-auto models represent the number of chains kept (out of 50). Option  $N_{eff}$  controls whether a logistic regression is used for the GWAS with the effective sample in LDpred2, or a linear regression and then the total sample size.

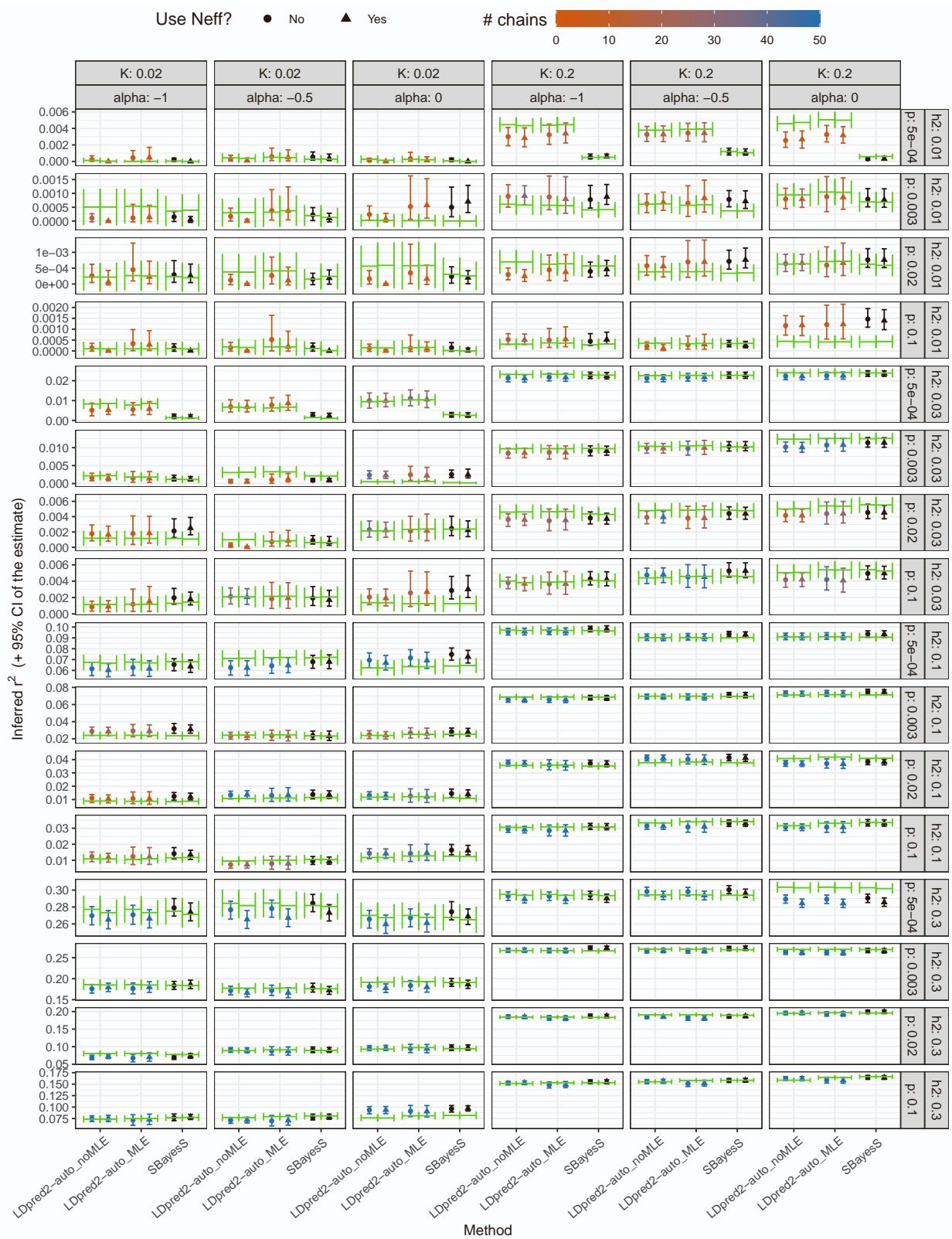


Figure S23: Inferred predictive performance  $r^2$  in simulations with binary outcomes. Green segments represent  $r^2$  in the test set. Colors for LDpred2-auto models represent the number of chains kept (out of 50). Option Neff controls whether a logistic regression is used for the GWAS with the effective sample in LDpred2, or a linear regression and then the total sample size. All  $r^2$  estimates are transformed to the liability scale with  $K_{\text{pop}}=K$  and  $K_{\text{GWAS}}$  either  $K$  (the simulated prevalence) or 0.5 (when using Neff) using function `coef_to_liab` from R package `bigsnpr`<sup>2,3</sup>; the ones from the test set are transformed with  $K$  and  $K$ .

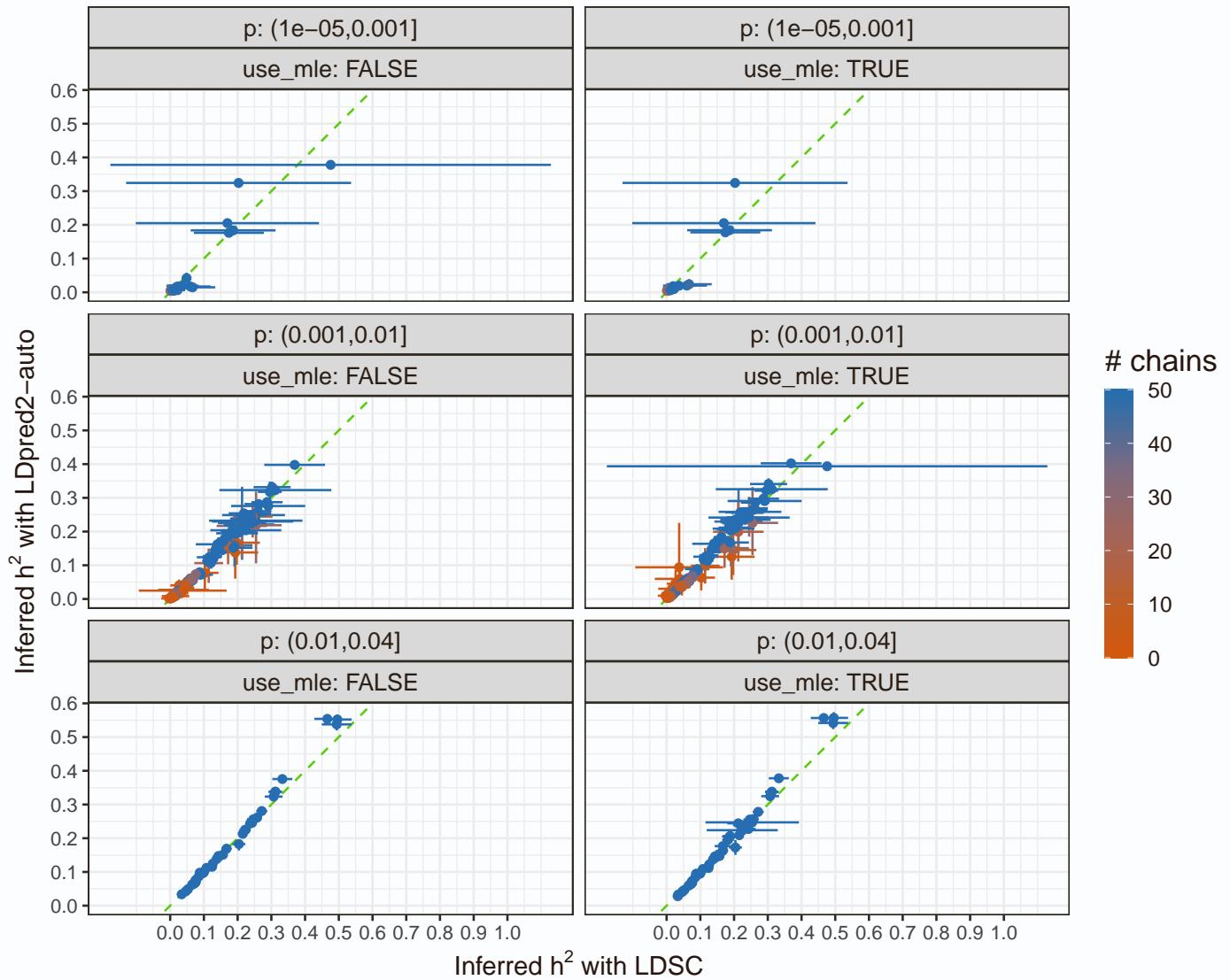


Figure S24: SNP heritability estimates from either LDpred2-auto or LD Score regression for all 248 phenotypes defined from the UK Biobank. These are stratified by the polygenicity estimated from LDpred2-auto. Green dashed lines represent the 1:1 line. The 95% confidence interval for the LDpred2-auto estimate is obtained from the 2.5% and 97.5% quantiles of all the  $h^2$  estimates from the iterations (after burn-in) of the chains kept. The 95% confidence interval for the LD Score regression estimate is obtained from  $\pm 1.96$  times its standard error. “use\_mle: TRUE” corresponds to using the extended 3-parameter model and sampling scheme (Methods). Colors represent the number of chains kept for LDpred2-auto (out of 50).

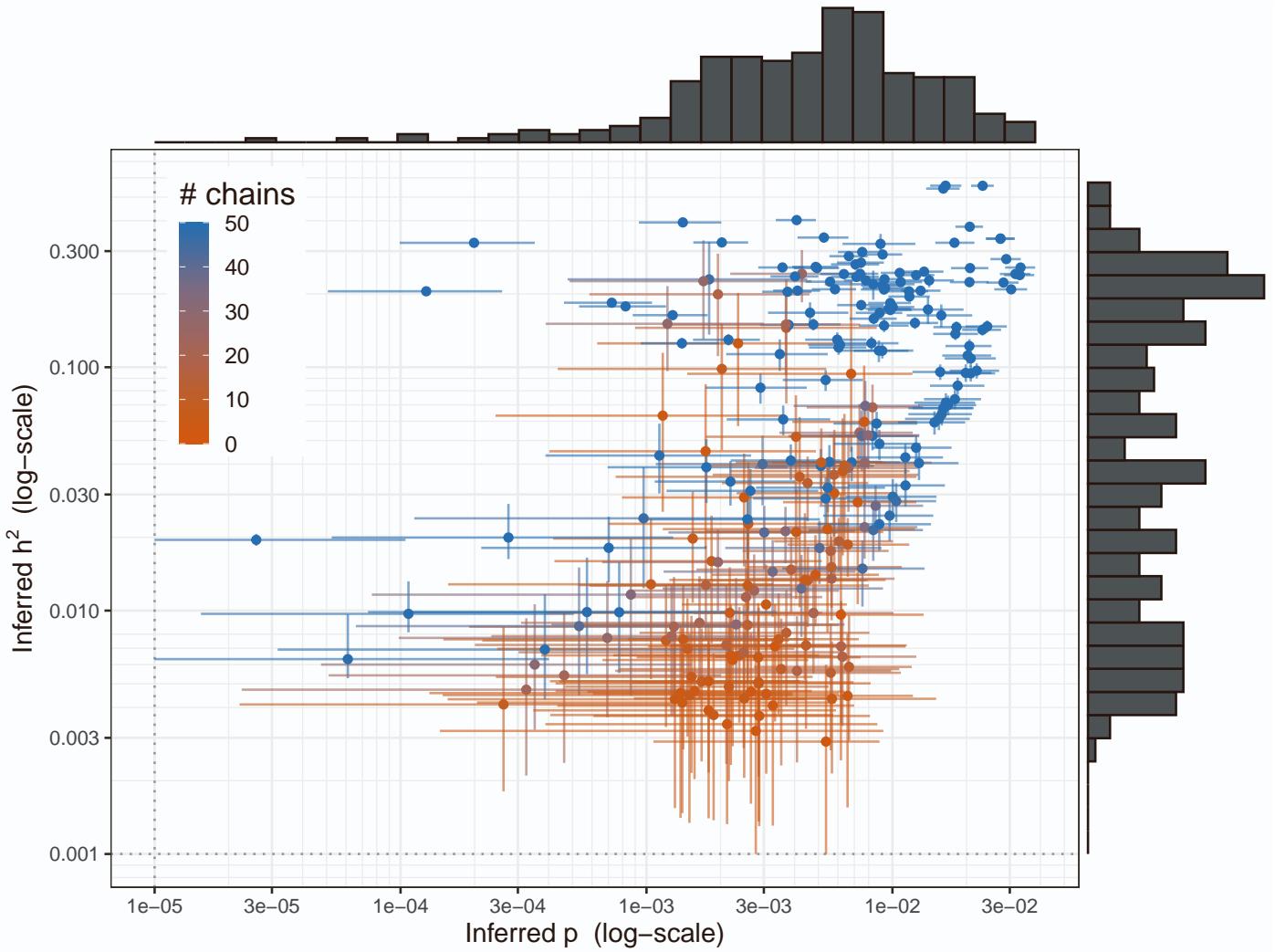


Figure S25: Estimates from LDpred2-auto for either the SNP heritability  $h^2$  or the polygenicity  $p$  for all 248 phenotypes defined from the UK Biobank. Estimates of  $h^2$  are constrained to be at least 0.001, and at least  $10^{-5}$  for  $p$ . The 95% confidence interval for the LDpred2-auto estimate is obtained from the 2.5% and 97.5% quantiles of all the estimates from the iterations (after burn-in) of the chains kept. Colors represent the number of chains kept for LDpred2-auto (out of 50).

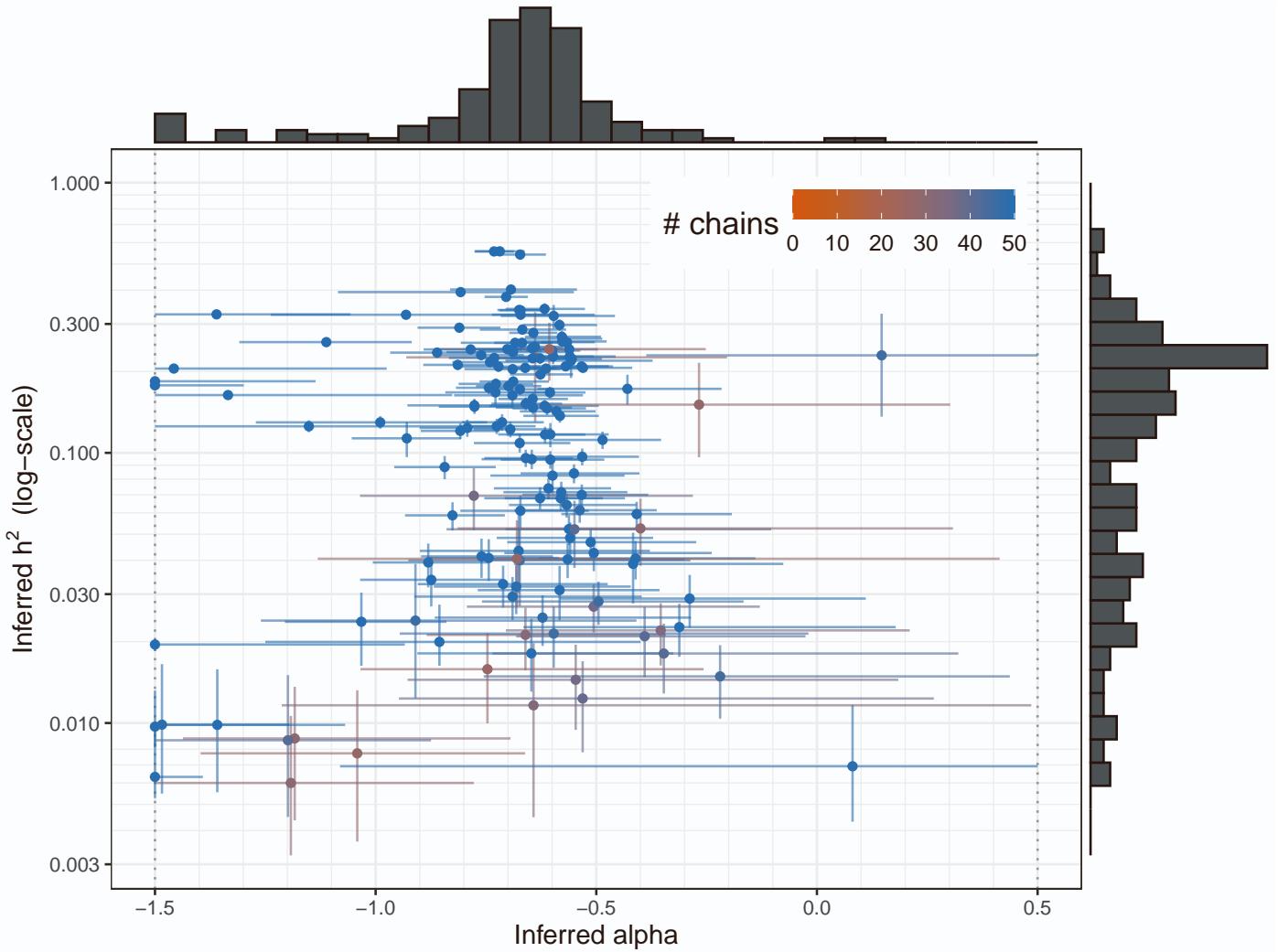


Figure S26: Estimates from LDpred2-auto for either the SNP heritability  $h^2$  or  $\alpha$  for all 248 phenotypes defined from the UK Biobank. The 95% confidence interval for the LDpred2-auto estimate is obtained from the 2.5% and 97.5% quantiles of all the estimates from the iterations (after burn-in) of the chains kept. Colors represent the number of chains kept for LDpred2-auto (out of 50). We only show phenotypes for which there are more than 25 chains kept, because simulations have shown that  $\alpha$  estimates are unreliable when a small number of chains is kept (Figure S5).

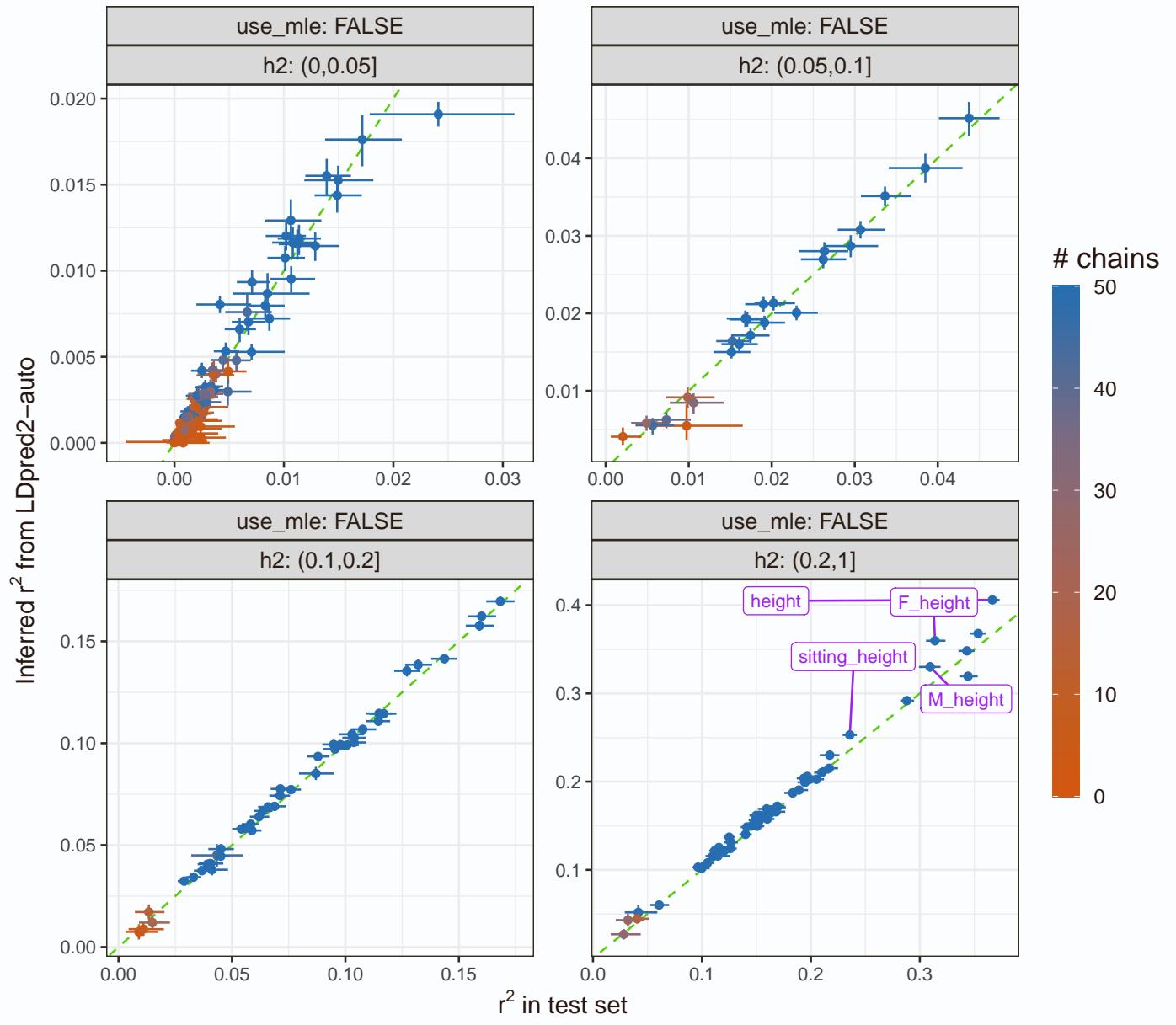


Figure S27: Inferred predictive performance  $r^2$  from the Gibbs sampler of LDpred2-auto versus the ones obtained in the test set, for all 248 phenotypes defined from the UK Biobank. These are stratified by the polygenicity estimated from LDpred2-auto. Green dashed lines represent the 1:1 line. The 95% confidence interval for the LDpred2-auto estimate is obtained from the 2.5% and 97.5% quantiles of all the  $r^2$  estimates from the iterations (after burn-in) of the chains kept. The 95% confidence interval for  $r^2$  in the test set is obtained from bootstrap. Colors represent the number of chains kept (out of 50). “F\_height” and “M\_height” use females and males only, respectively (in both GWAS and test sets).

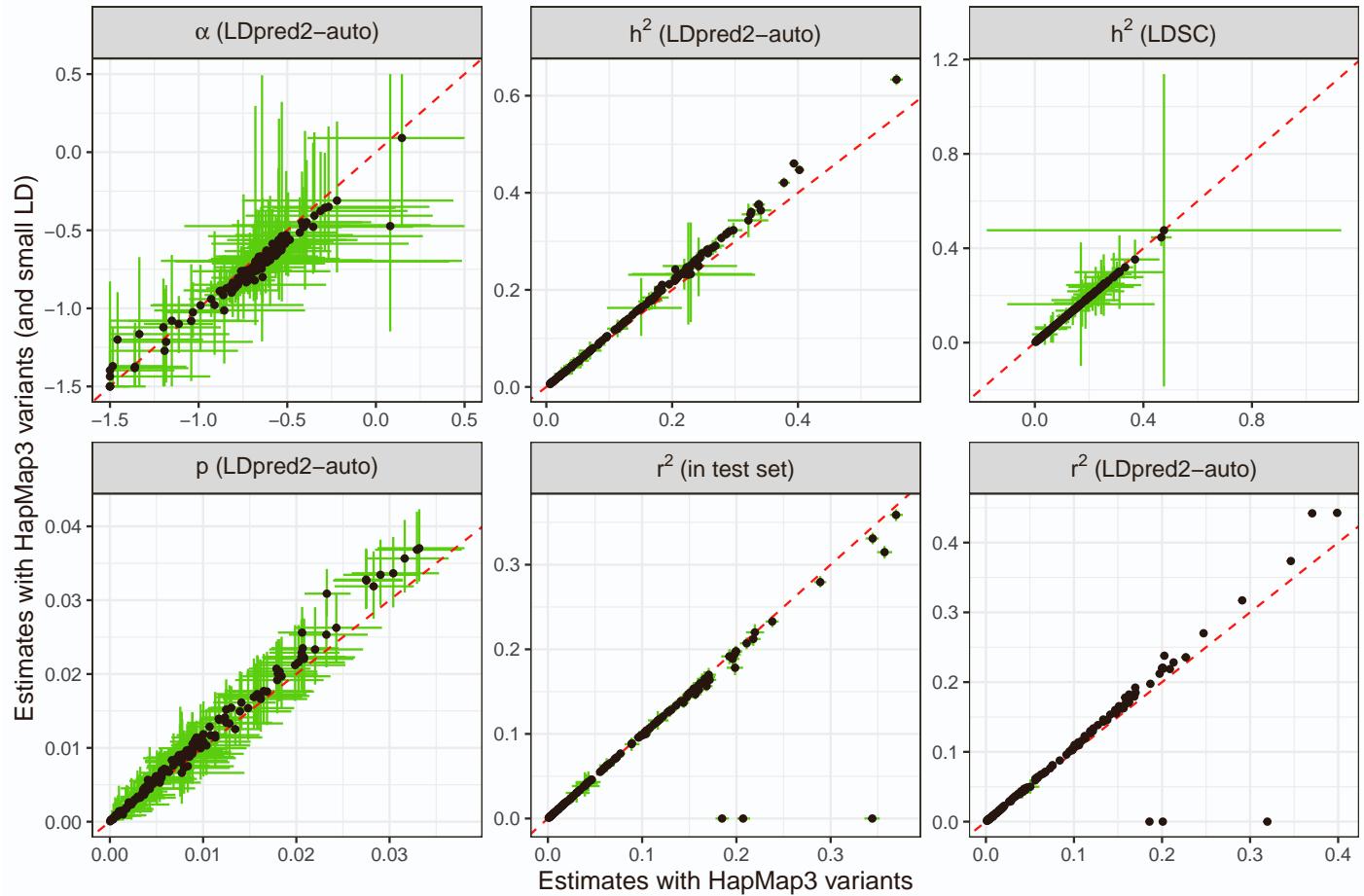


Figure S28: LDpred2-auto estimates for UKBB phenotypes with either a small or a large LD reference. Only 154 phenotypes with more than 25 chains kept when using the large LD reference are represented here. Red dashed lines represent the 1:1 line. The 95% confidence interval for the LDpred2-auto estimate (in green) is obtained from the 2.5% and 97.5% quantiles of all the estimates from the iterations (after burn-in) of the chains kept. The 95% confidence interval for  $r^2$  in the test set is obtained from bootstrap.

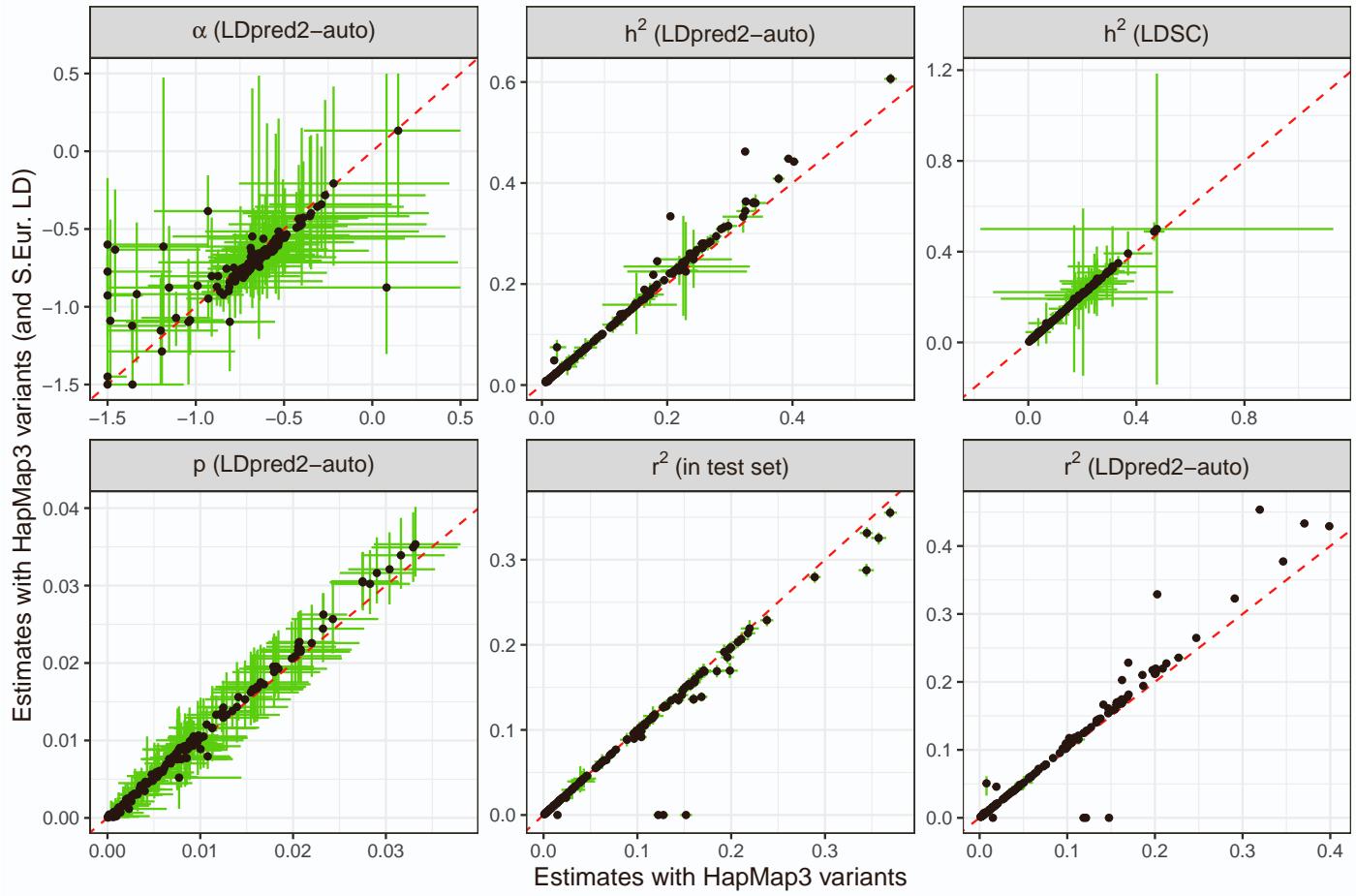


Figure S29: LDpred2-auto estimates for UKBB phenotypes with either a N.W. European or a S. European (“altpop”) LD reference. Only 154 phenotypes with more than 25 chains kept when using the N.W. European LD reference are represented here. Red dashed lines represent the 1:1 line. The 95% confidence interval for the LDpred2-auto estimate (in green) is obtained from the 2.5% and 97.5% quantiles of all the estimates from the iterations (after burn-in) of the chains kept. The 95% confidence interval for  $r^2$  in the test set is obtained from bootstrap.

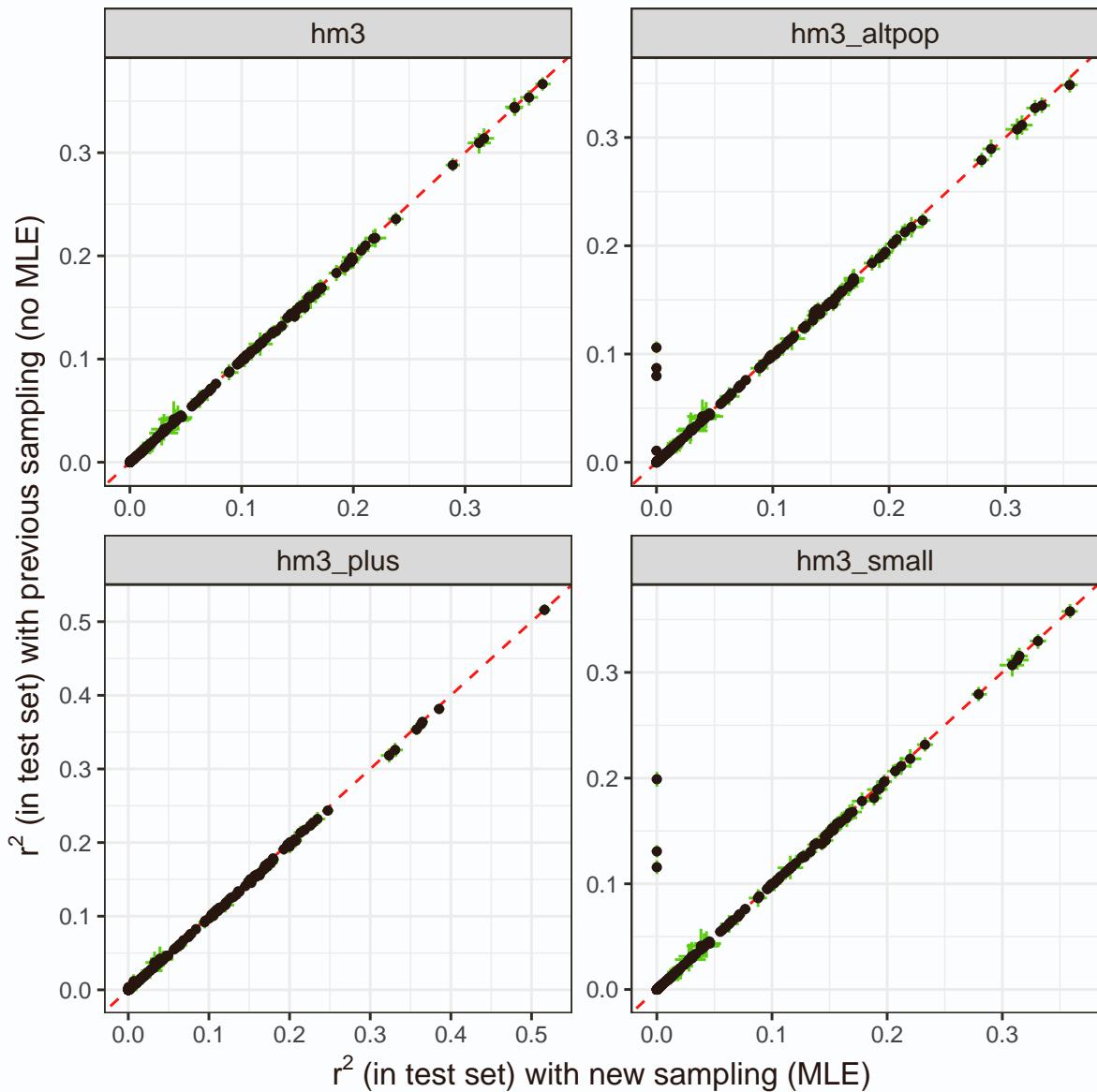


Figure S30:  $r^2$  estimates (in the test set) for UKBB phenotypes, using LDpred2-auto with either the extended model and sampling (MLE, 3-parameter model) or the previous one (2-parameter model). Red dashed lines represent the 1:1 line. The 95% confidence interval for  $r^2$  in the test set is obtained from bootstrap.

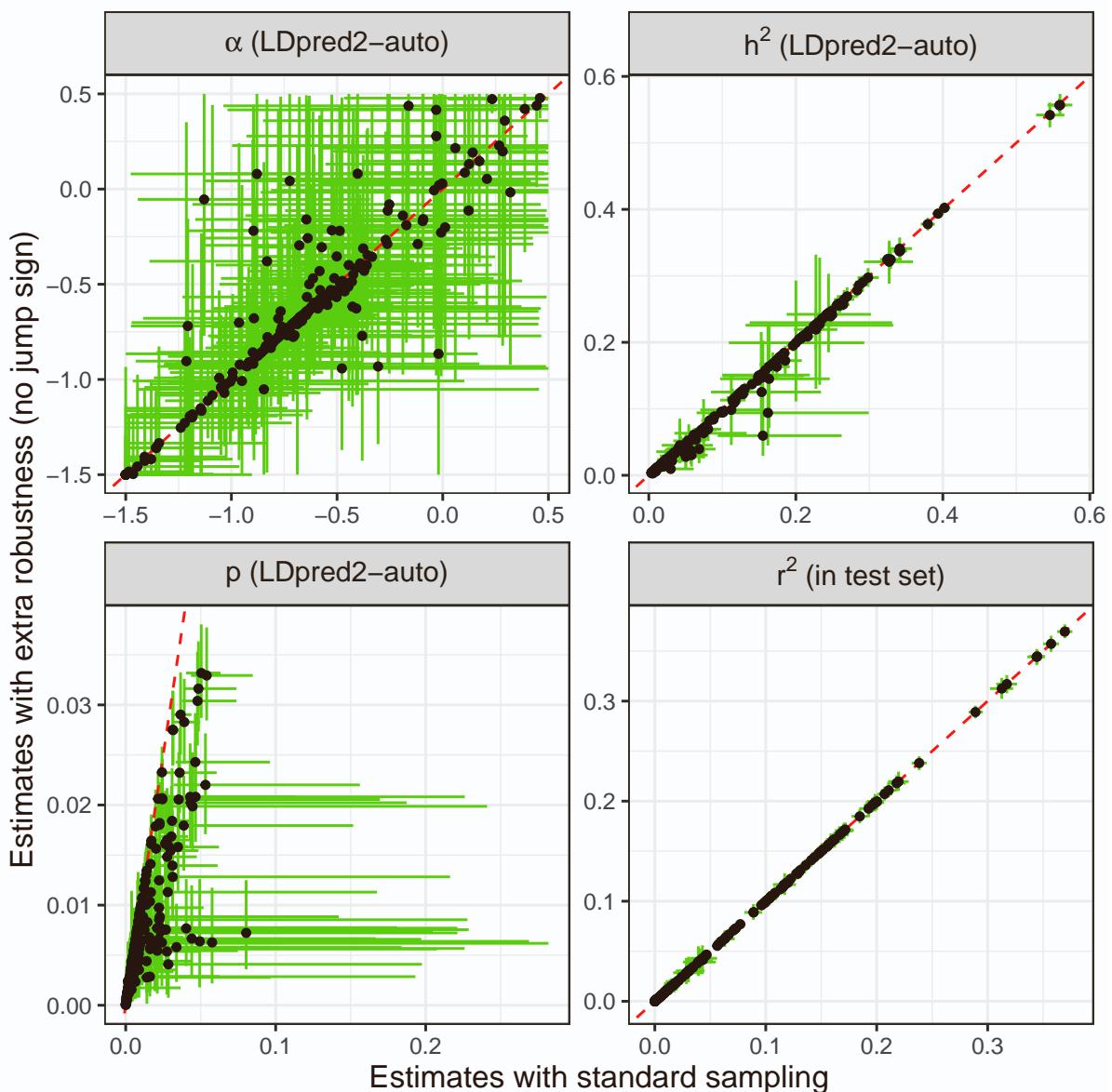


Figure S31: LDpred2-auto estimates for 248 UKBB phenotypes, with or without option ‘allow\_jump\_sign’ enabled. Red dashed lines represent the 1:1 line. The 95% confidence interval for the LDpred2-auto estimate (in green) is obtained from the 2.5% and 97.5% quantiles of all the estimates from the iterations (after burn-in) of the chains kept. The 95% confidence interval for  $r^2$  in the test set is obtained from bootstrap.

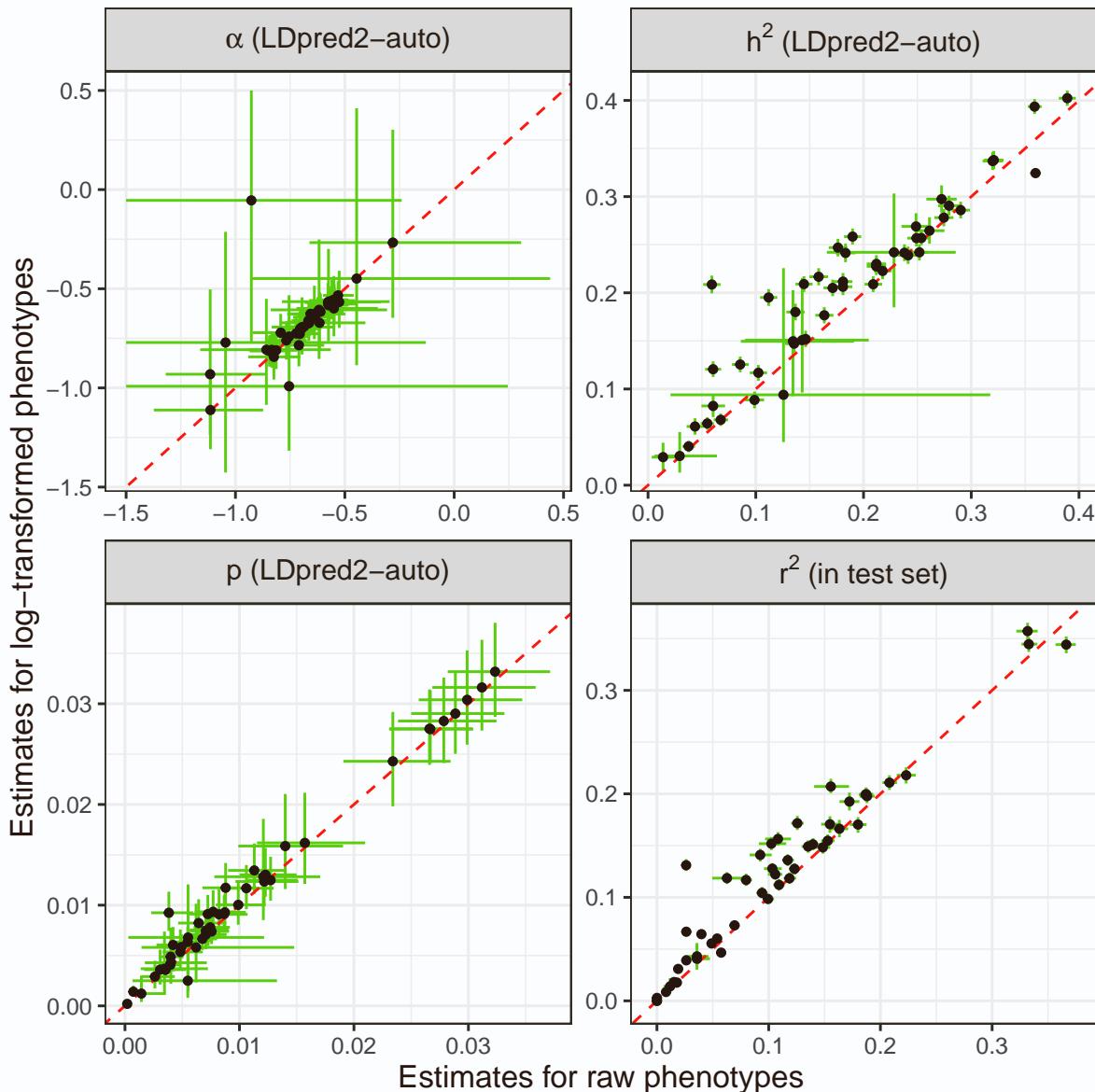


Figure S32: LDpred2-auto estimates for 49 log-transformed UKBB phenotypes, versus for their raw versions. Red dashed lines represent the 1:1 line. The 95% confidence interval for the LDpred2-auto estimate (in green) is obtained from the 2.5% and 97.5% quantiles of all the estimates from the iterations (after burn-in) of the chains kept. The 95% confidence interval for  $r^2$  in the test set is obtained from bootstrap.

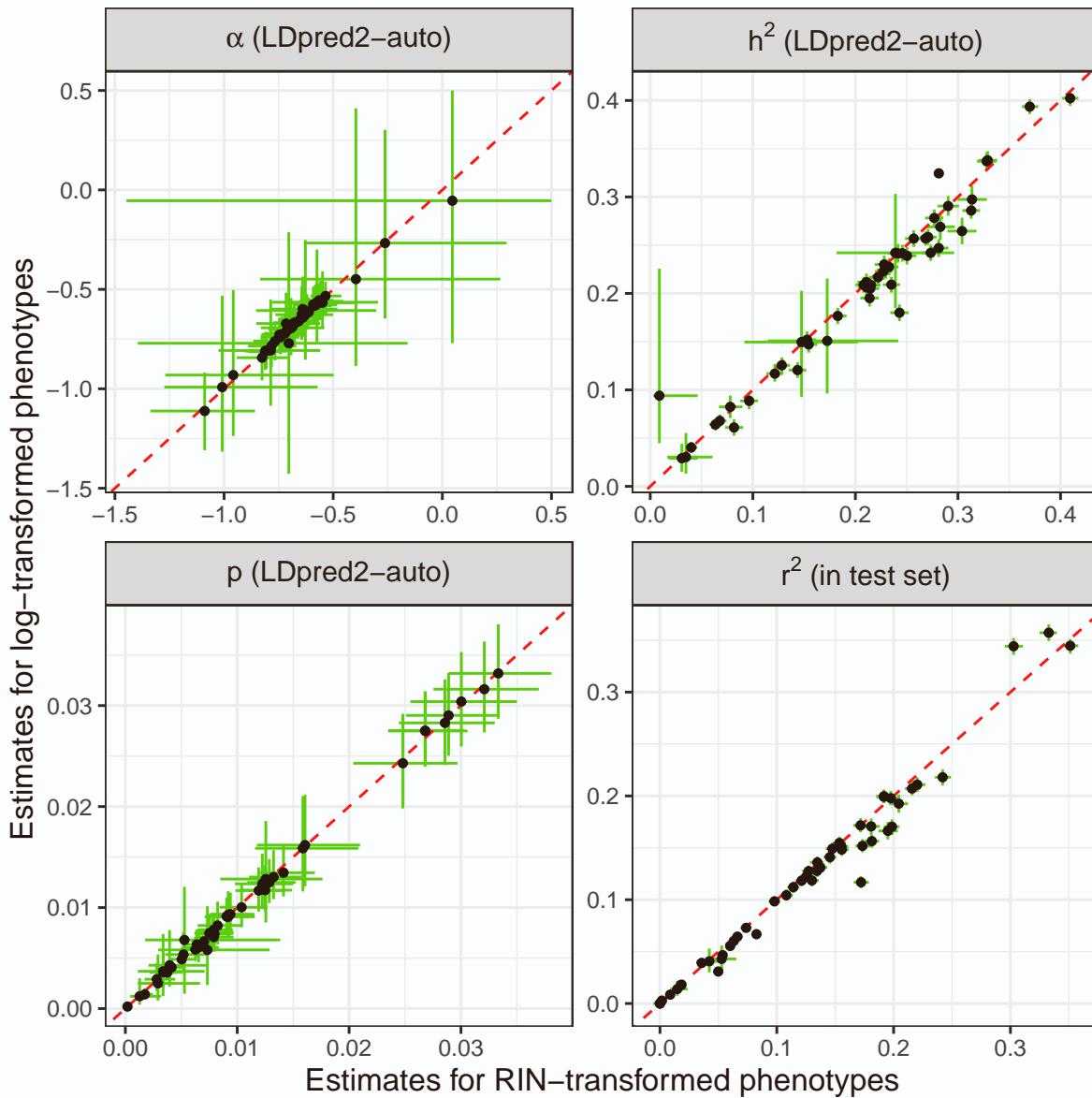


Figure S33: LDpred2-auto estimates for 49 log-transformed UKBB phenotypes, versus for their rank-based inverse normal (RIN) transformed versions. Red dashed lines represent the 1:1 line. The 95% confidence interval for the LDpred2-auto estimate (in green) is obtained from the 2.5% and 97.5% quantiles of all the estimates from the iterations (after burn-in) of the chains kept. The 95% confidence interval for  $r^2$  in the test set is obtained from bootstrap.

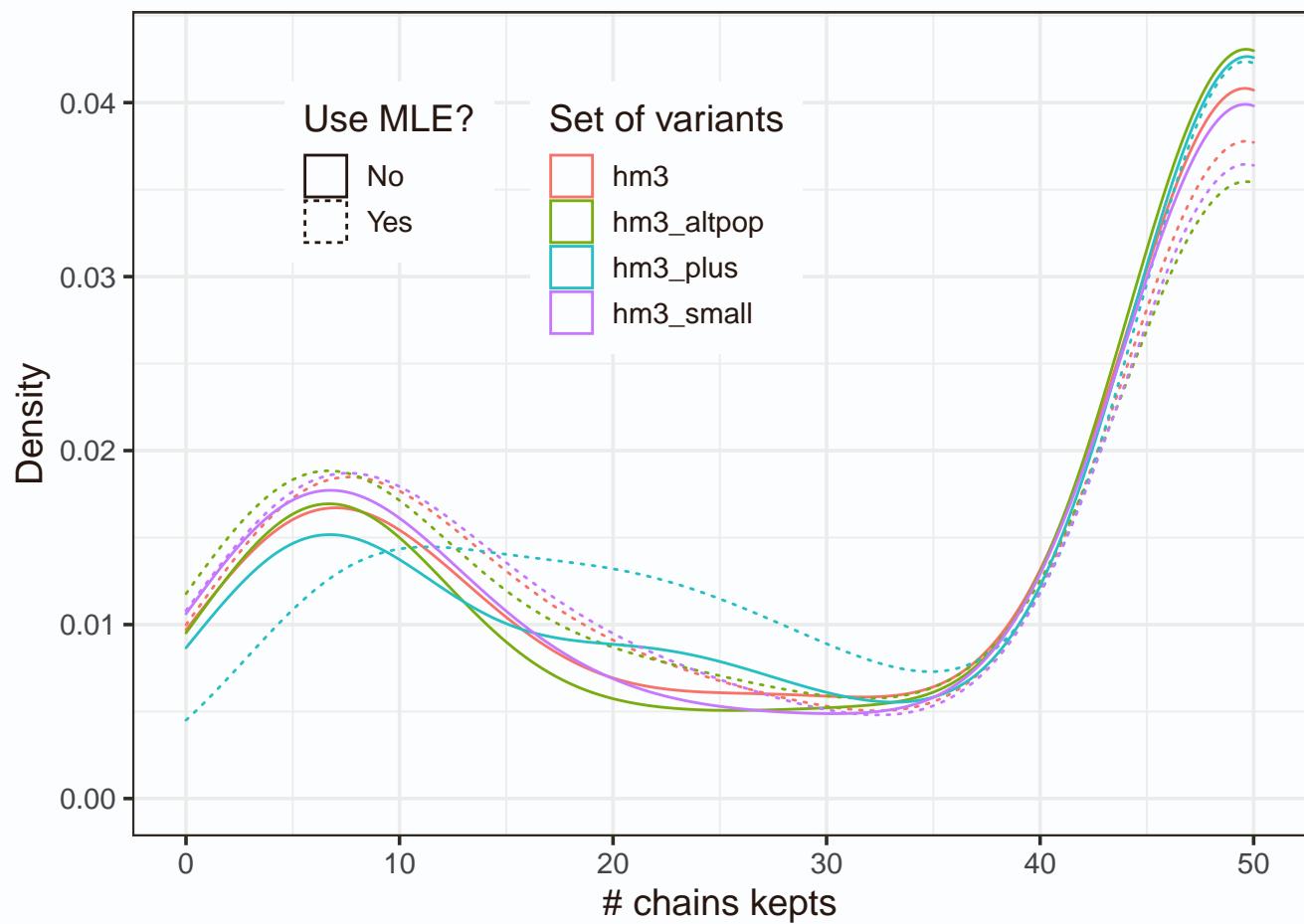


Figure S34: Distribution of the number of LDpred2-auto chains kept across 248 UKBB phenotypes. “Use MLE” corresponds to using the extended 3-parameter model and sampling scheme (Methods).

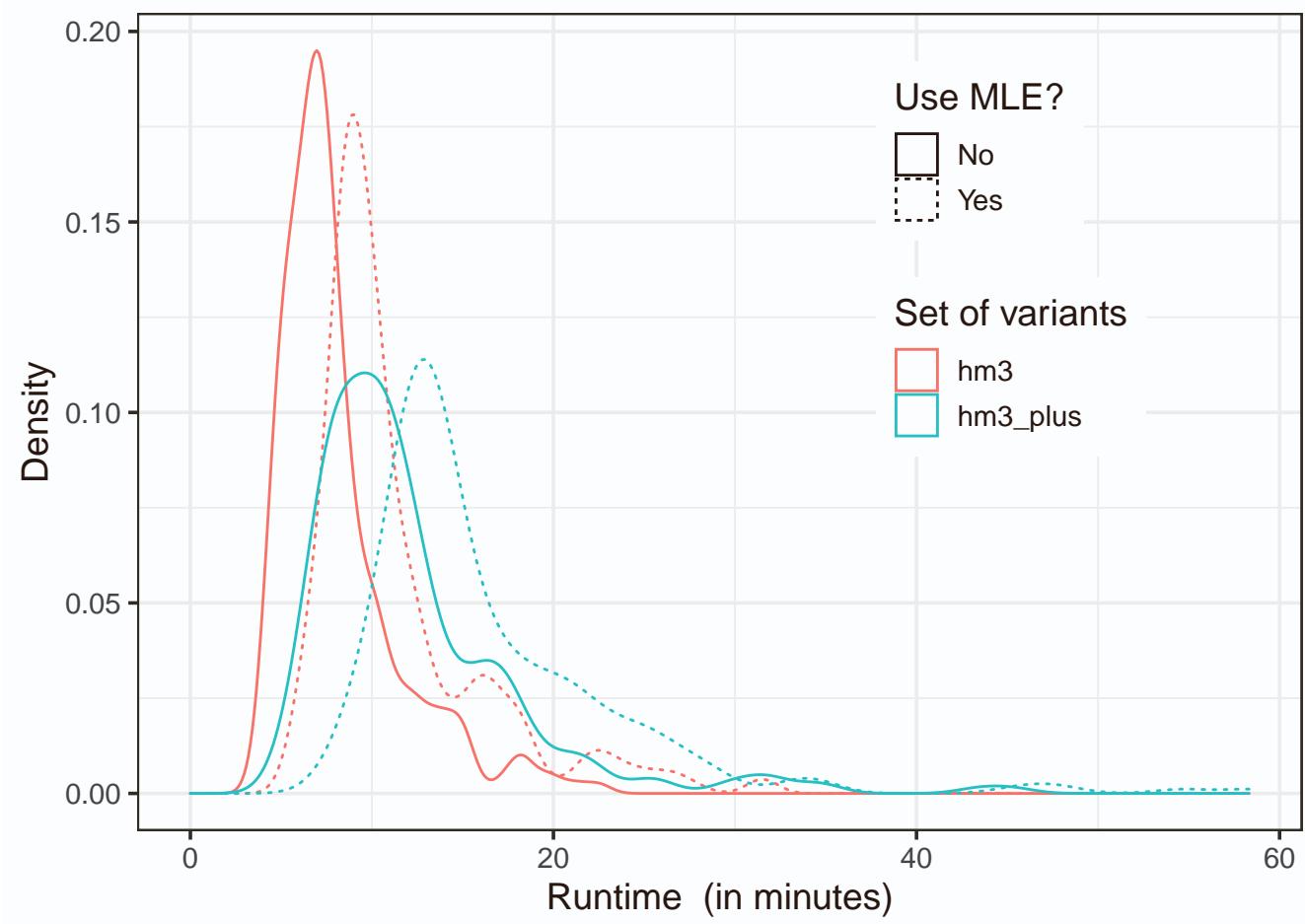


Figure S35: Distribution of LDpred2-auto runtimes across 248 UKBB phenotypes. “Use MLE” corresponds to using the extended 3-parameter model and sampling scheme (Methods). For each phenotype, 50 chains are used, parallelized over 13 cores.

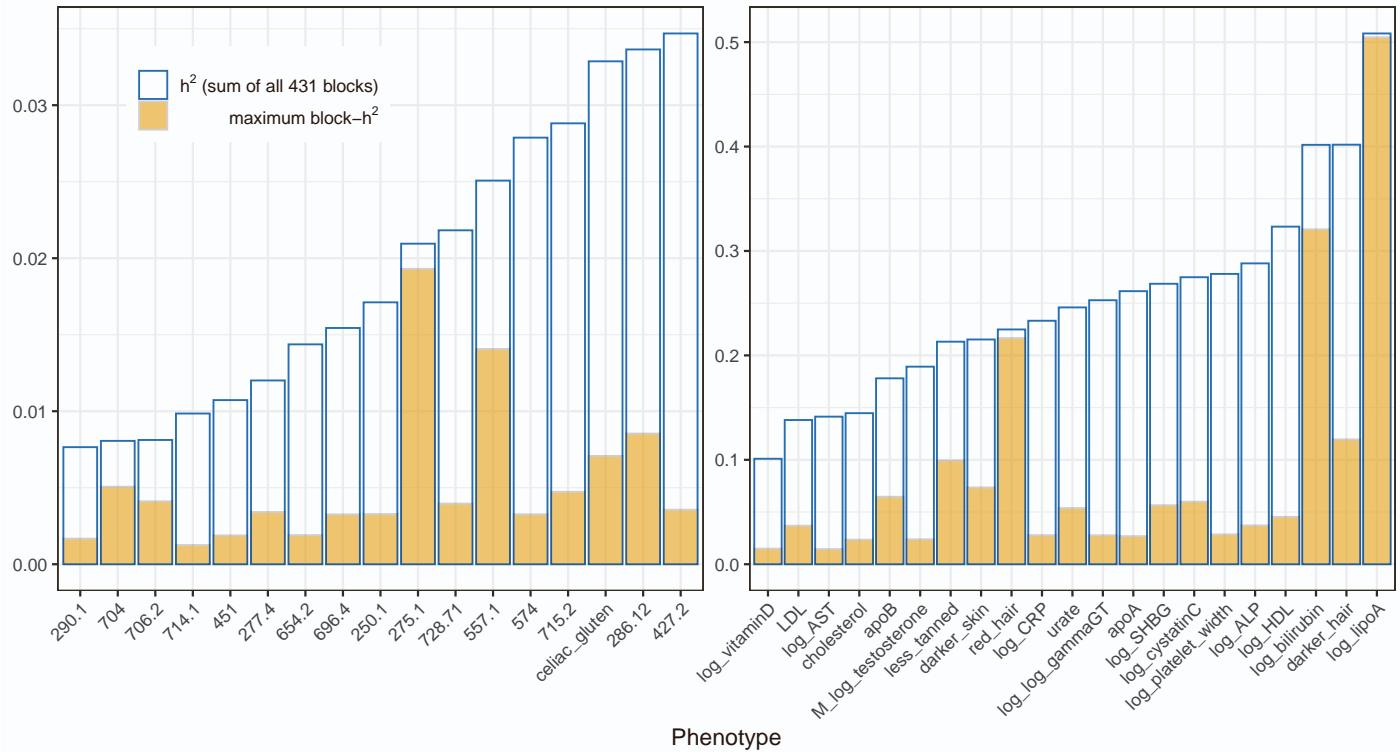


Figure S36: Genome-wide and per-block heritability estimates from LDpred2-auto for UKBB phenotypes. The HapMap3+ variants are used here. The maximum local  $h^2$  is the maximum heritability estimate across all 431 independent LD blocks defined for this set of variants. Only phenotypes for which this represents at least 10% of the total heritability are represented.

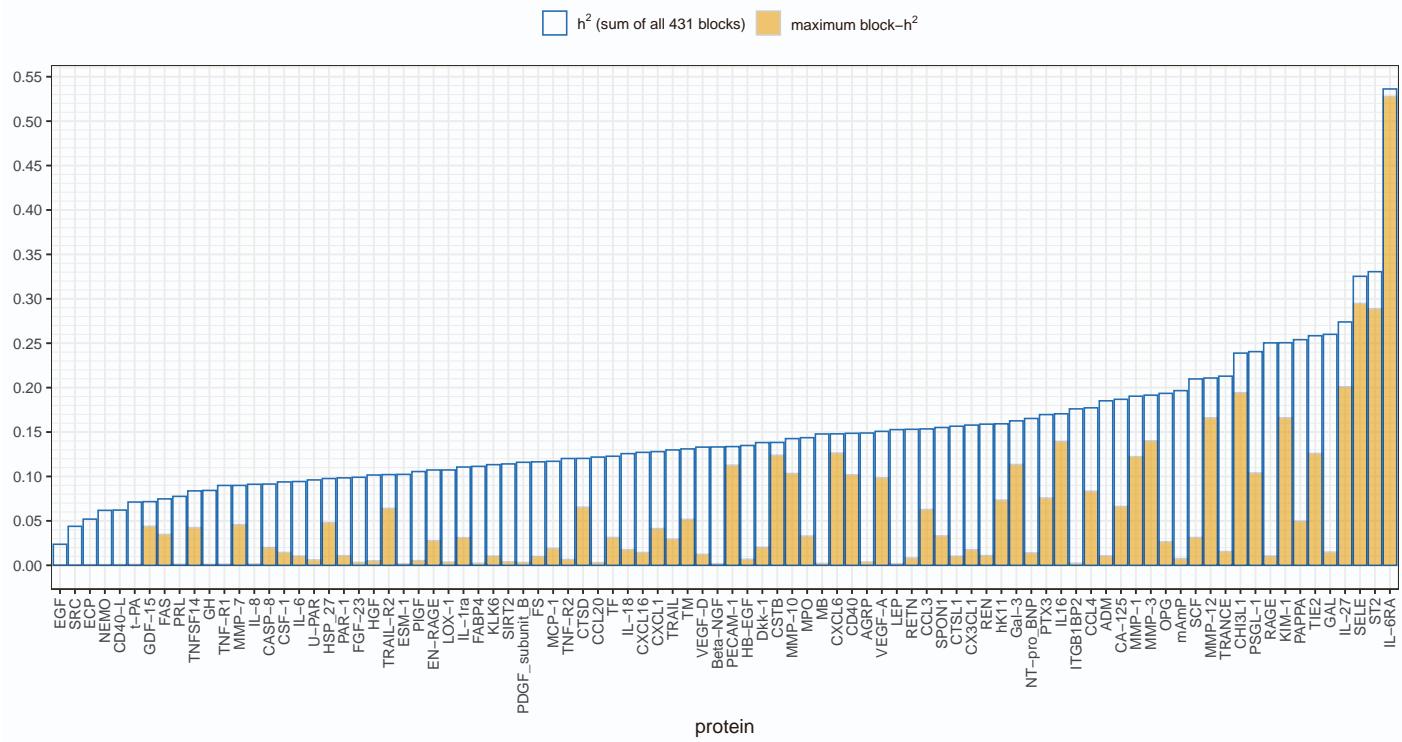


Figure S37: Genome-wide and per-block heritability estimates from LDpred2-auto for 90 protein concentrations<sup>4</sup>. The HapMap3+ variants are used here. The maximum local  $h^2$  is the maximum heritability estimate across all 431 independent LD blocks defined for this set of variants.

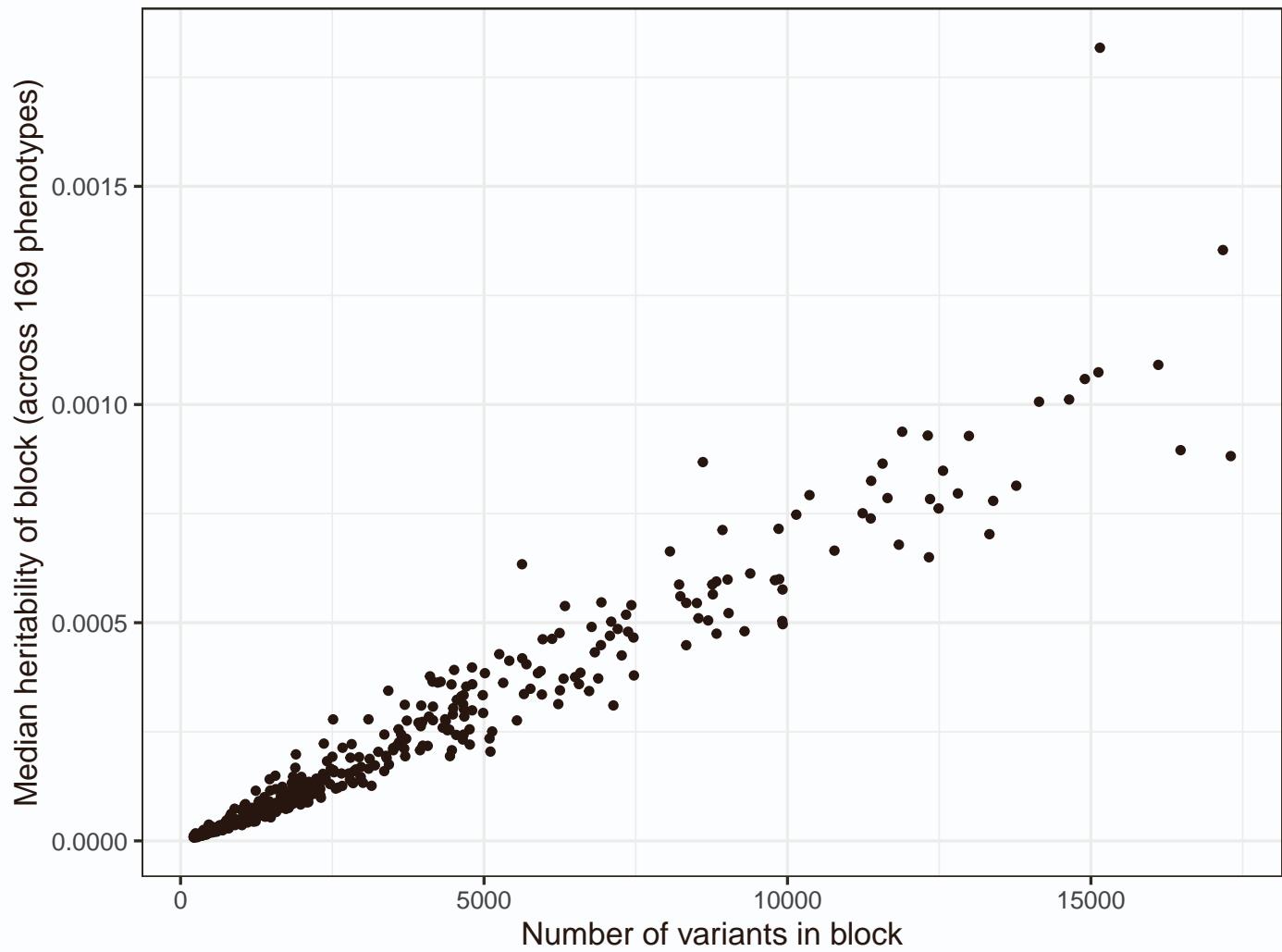


Figure S38: Per-block median heritability across 169 UKBB phenotypes. The HapMap3+ set of variants is used, with 431 independent LD blocks. Only phenotypes with more than 25 chains kept are used here. The top block is on chromosome 6 [22.1-41.4 Mb], which contains the HLA region.

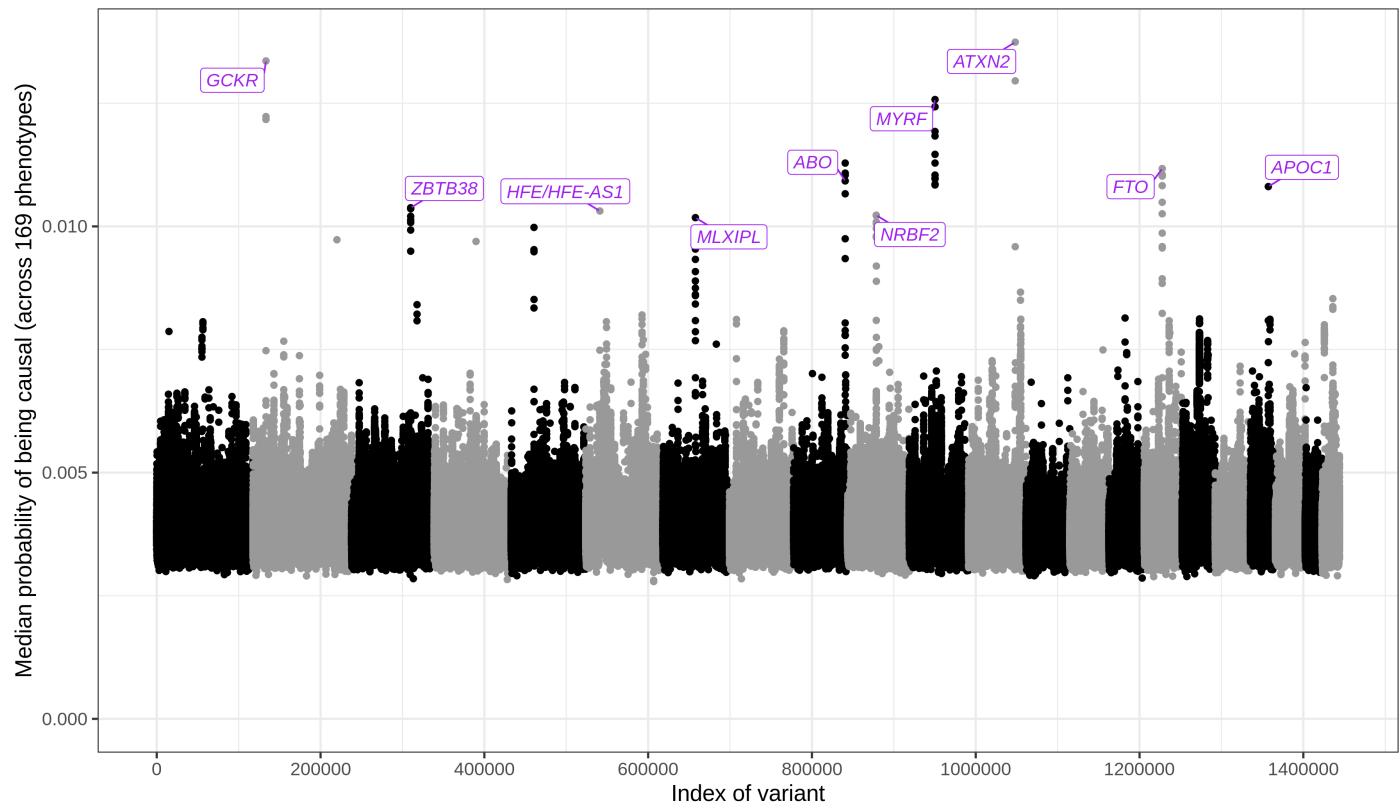


Figure S39: Per-variant median probabilities of being causal across 169 UKBB phenotypes. Variants with a median probability larger than 0.01 were mapped to genes using R package rsnps (only the largest per peak). The HapMap3+ set of variants is used. Only phenotypes with more than 25 chains kept are used here.

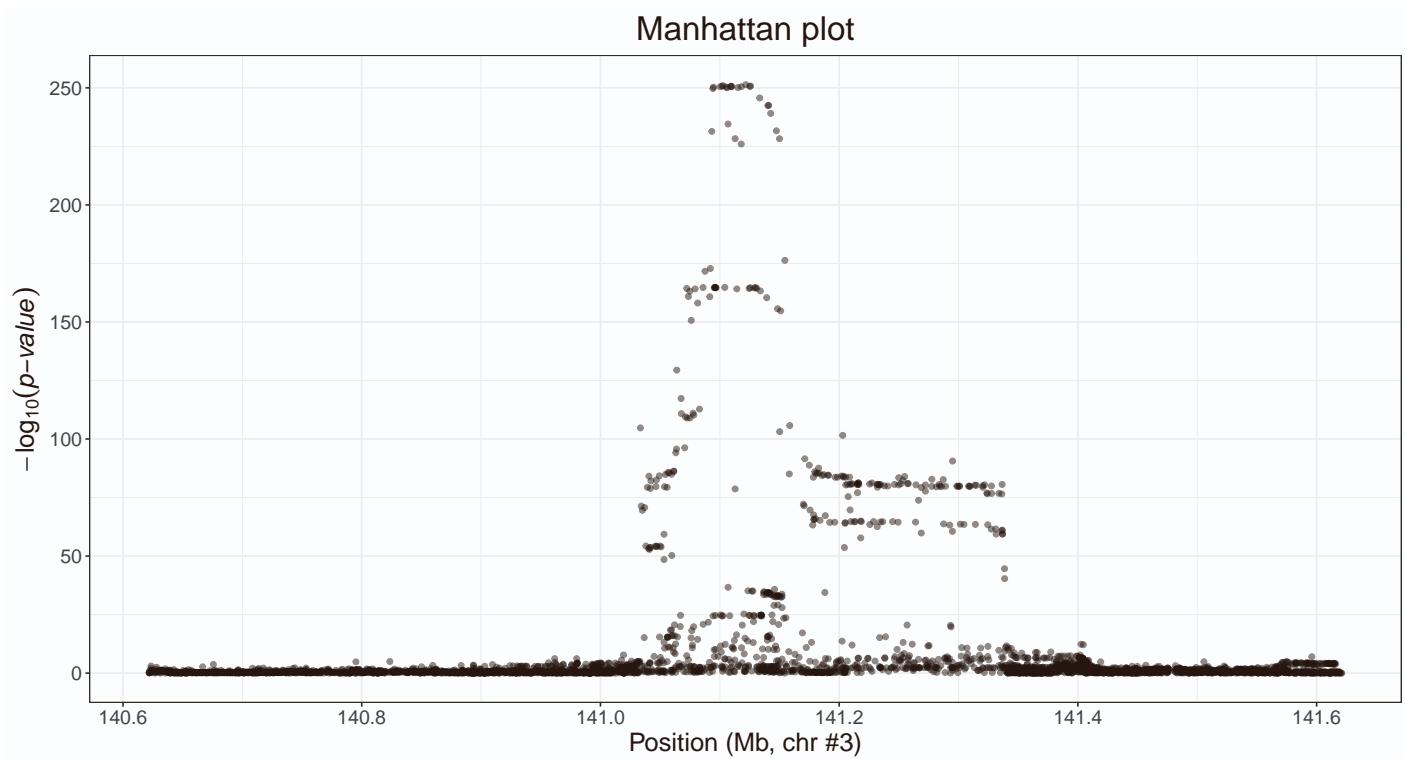


Figure S40: GWAS of height in a restricted region containing the most significant HapMap3+ variant for height.

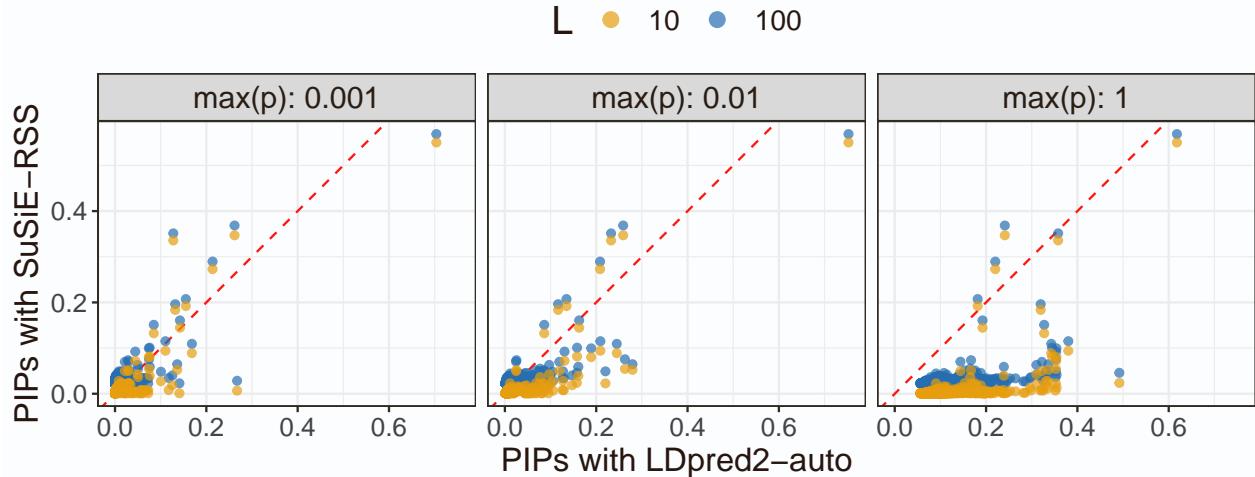
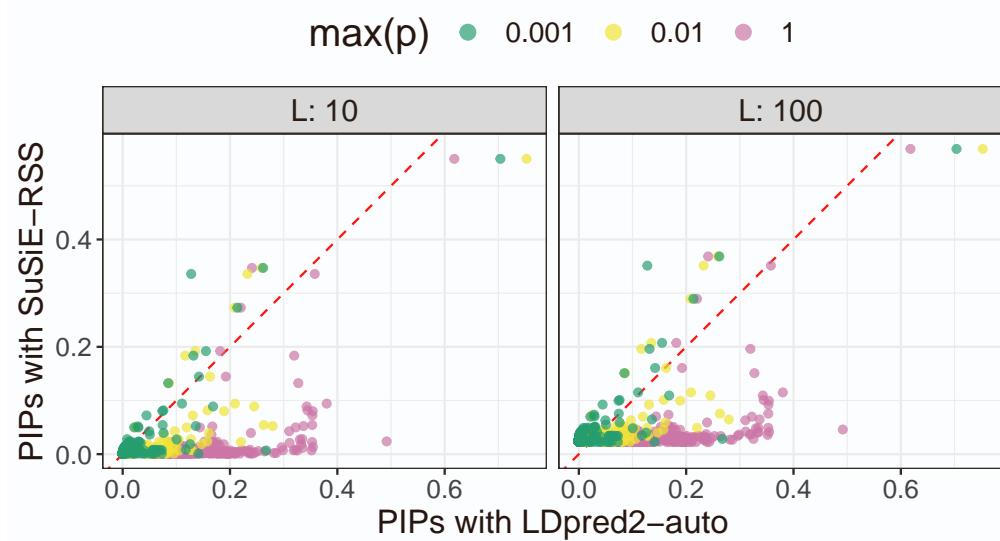
**A****B**

Figure S41: Posterior inclusion probabilities (PIPs) for the variants in Figure S40.  $L$  is the maximum number of causal variants in SuSiE-RSS, and  $\text{max}(p)$  is the maximum value of the estimated  $p$  in LDpred2-auto. Both panels A and B represent the same data, but colors and facets are changed.

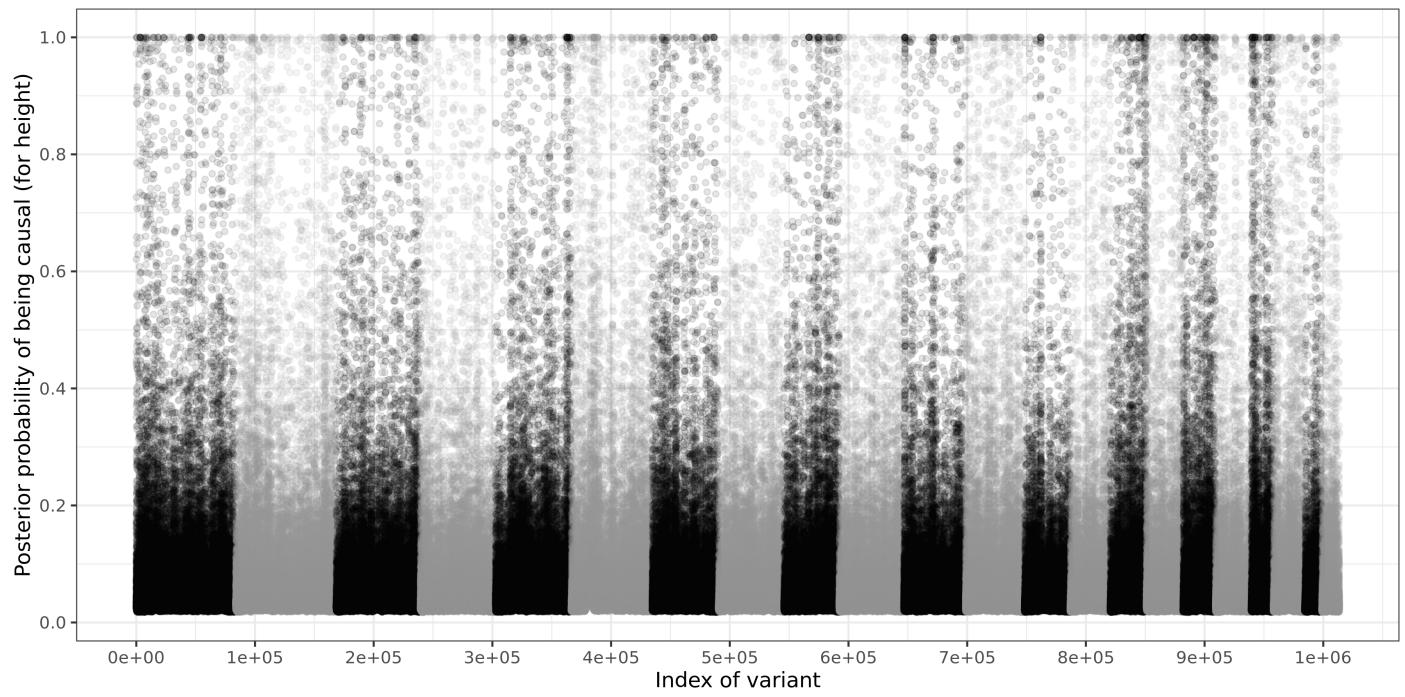


Figure S42: Per-variant probabilities of being causal for height. These are provided by LDpred2-auto when using the external GWAS summary statistics of 1.6M European individuals from<sup>5</sup>.

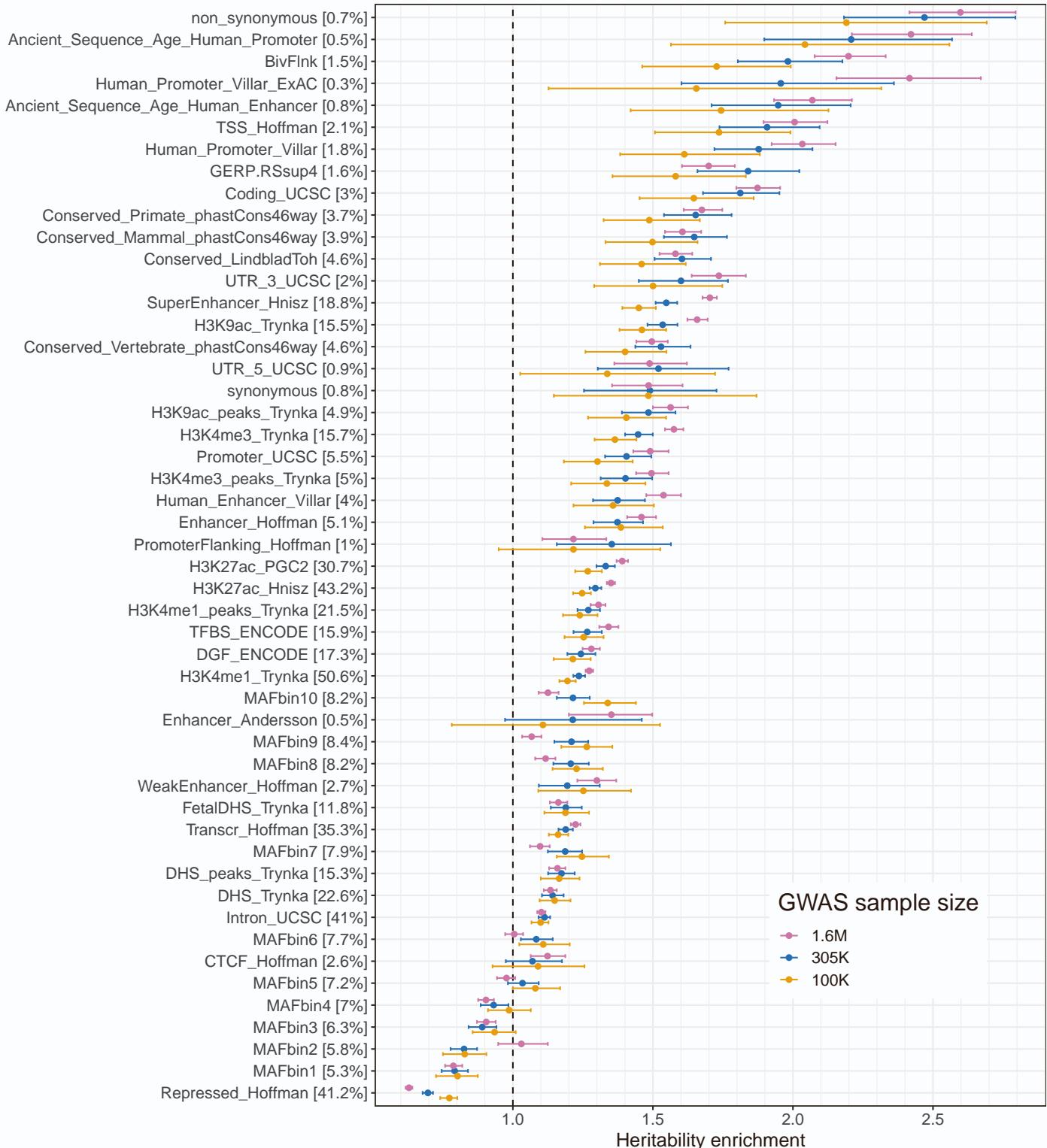
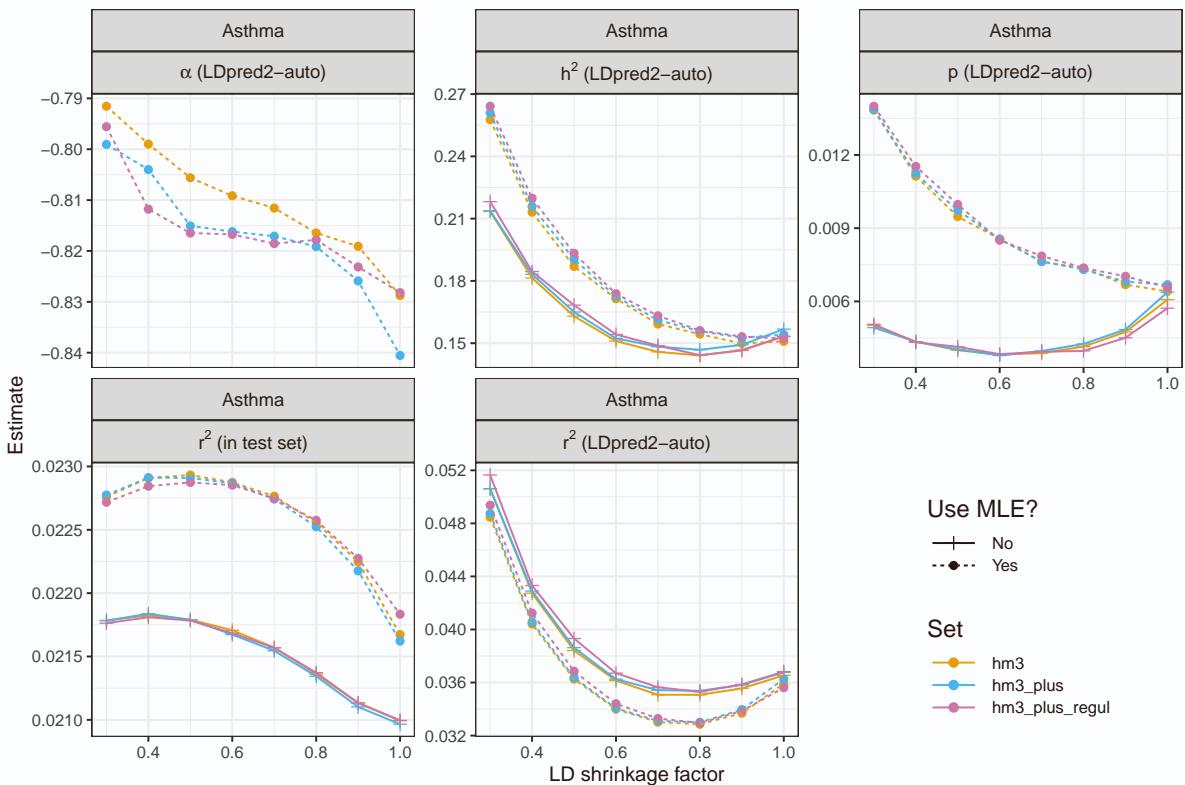
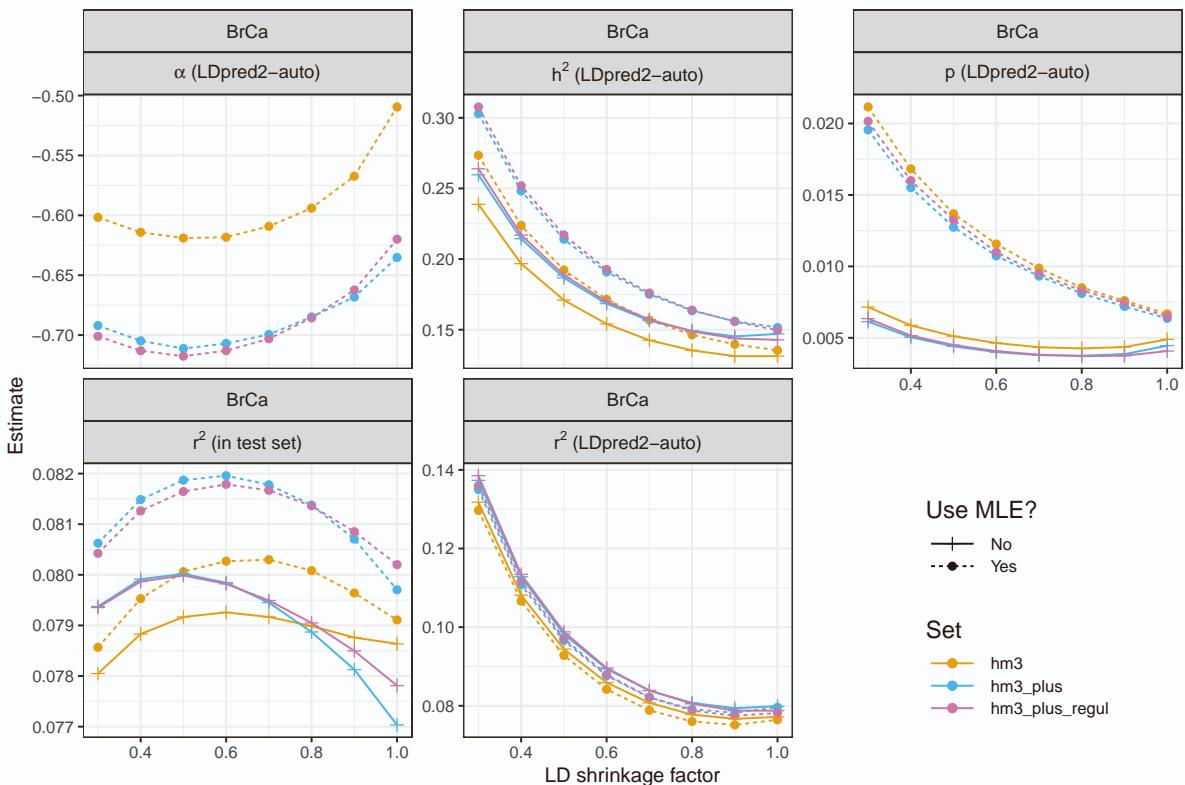


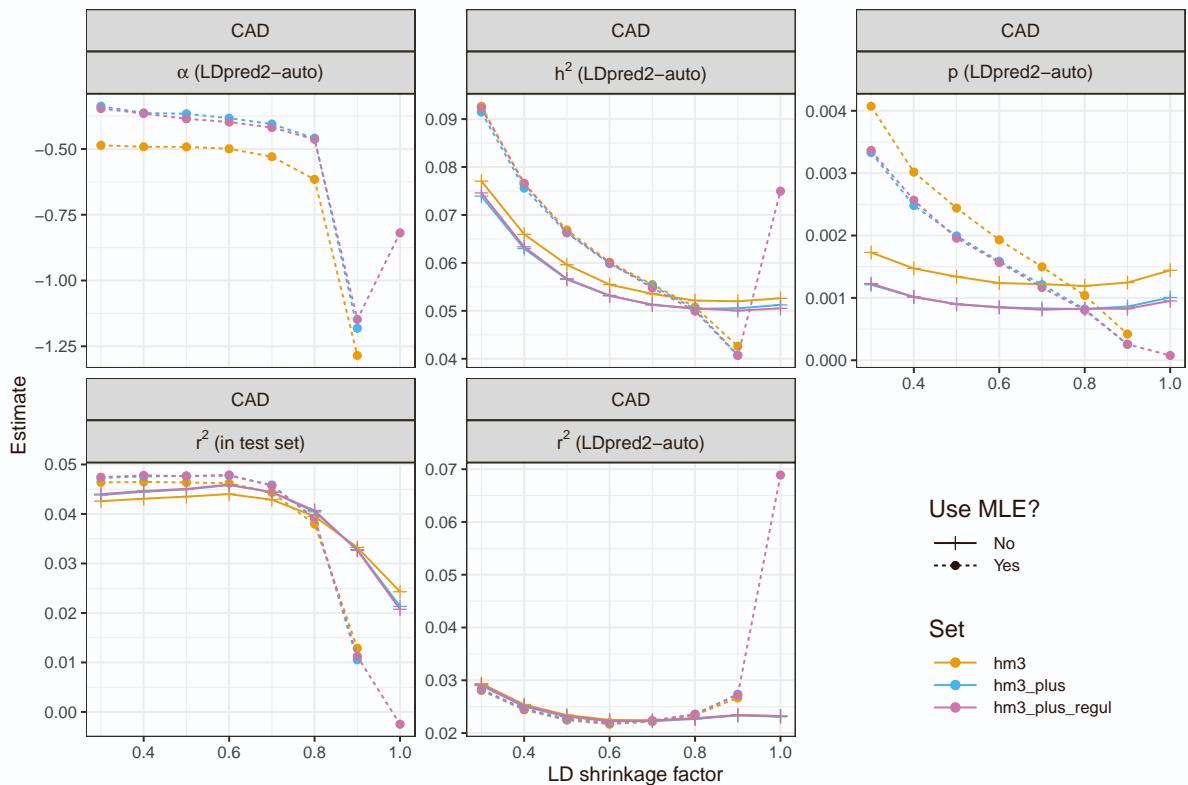
Figure S43: Heritability enrichment from LDpred2-auto for height across 50 functional annotations. The 95% confidence interval for the LDpred2-auto estimate is obtained from the 2.5% and 97.5% quantiles of all the estimates from the iterations (after burn-in) of the chains kept.



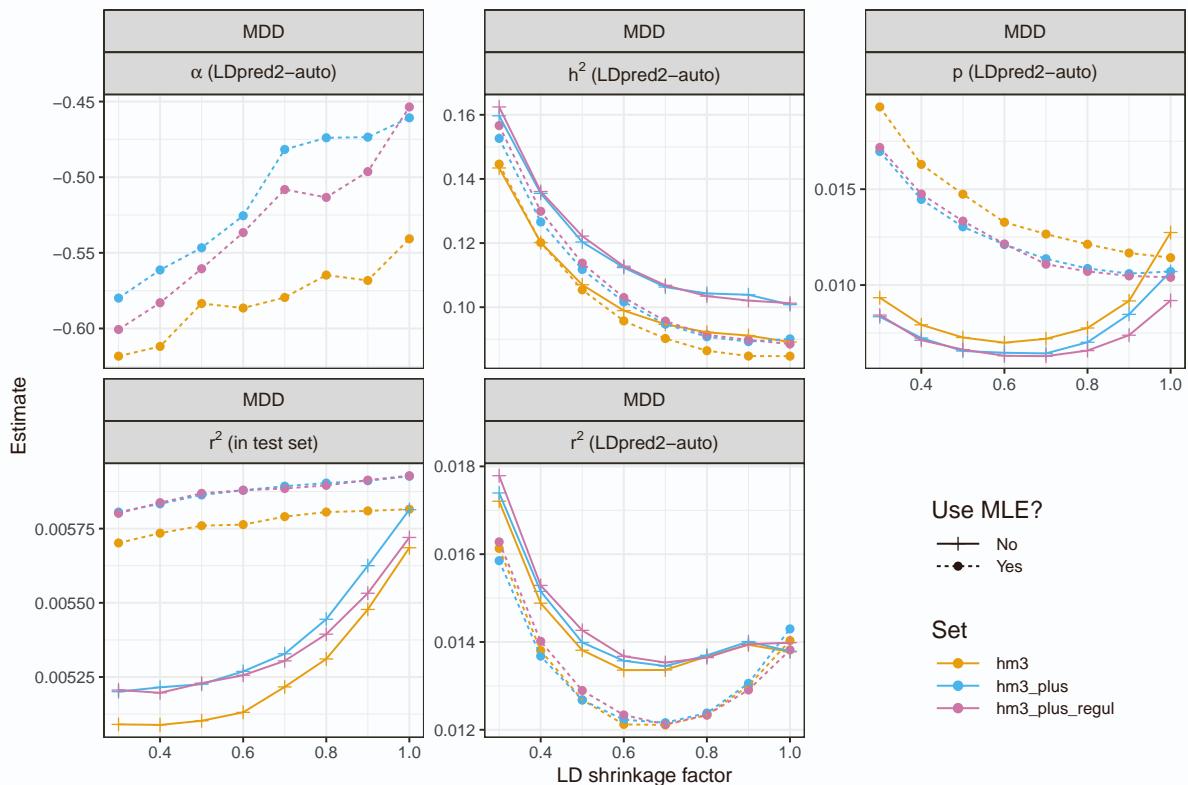
(a) Asthma



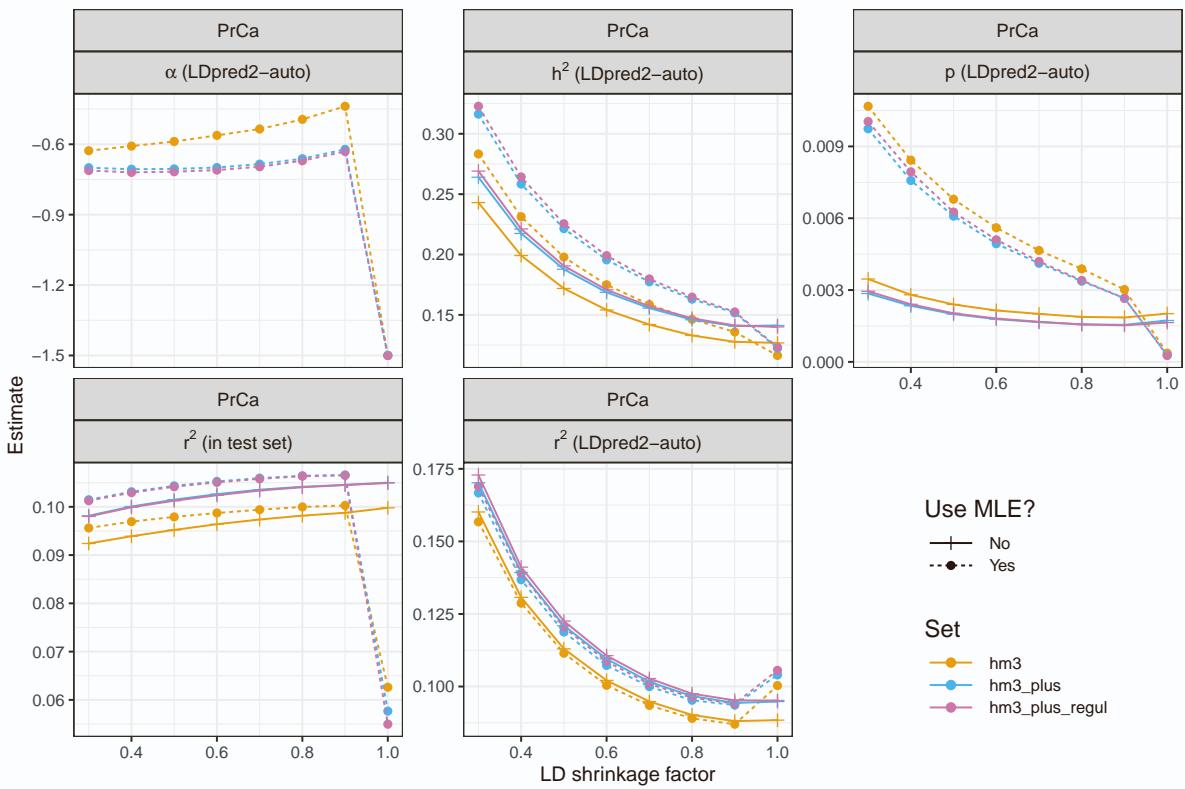
(b) Breast cancer



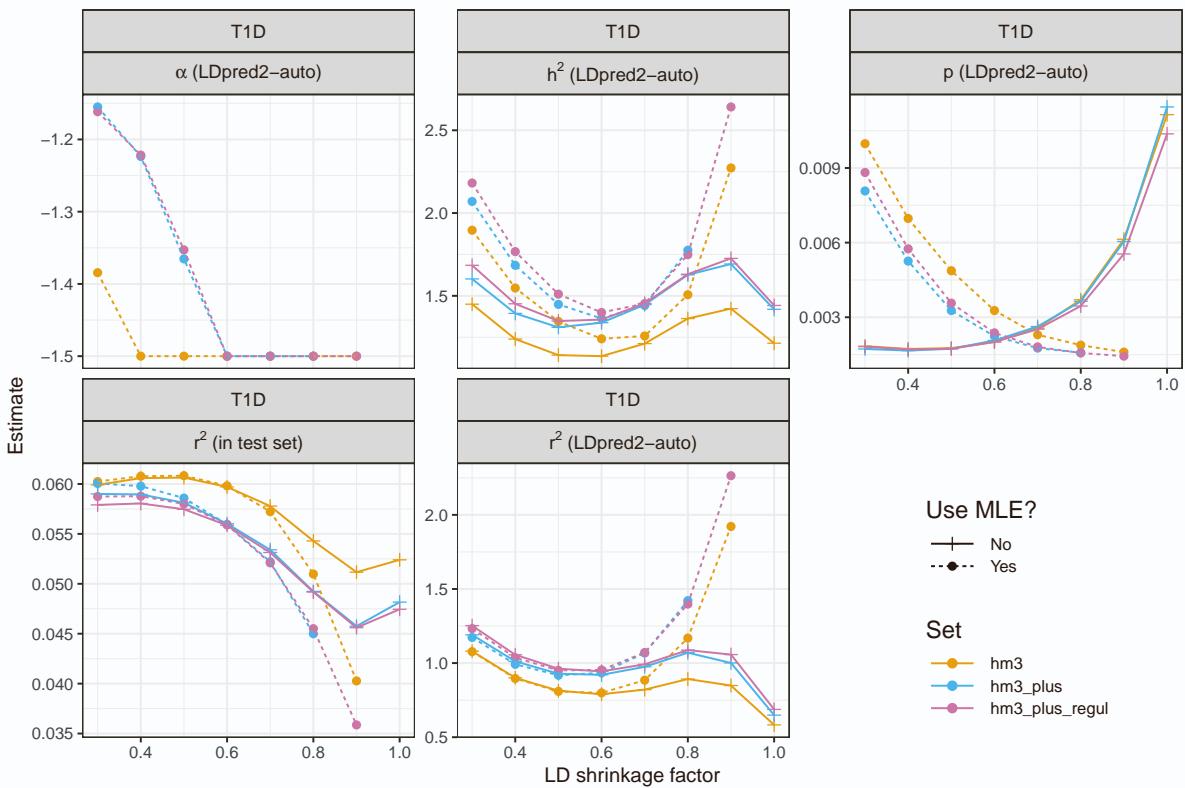
(c) Coronary artery disease



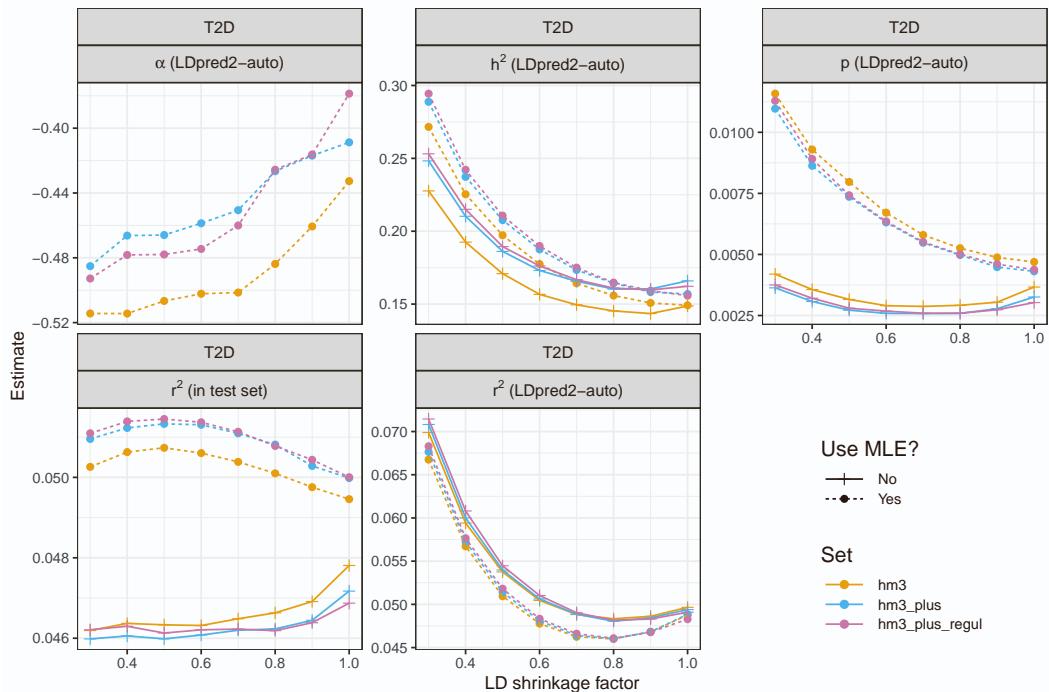
(d) Depression



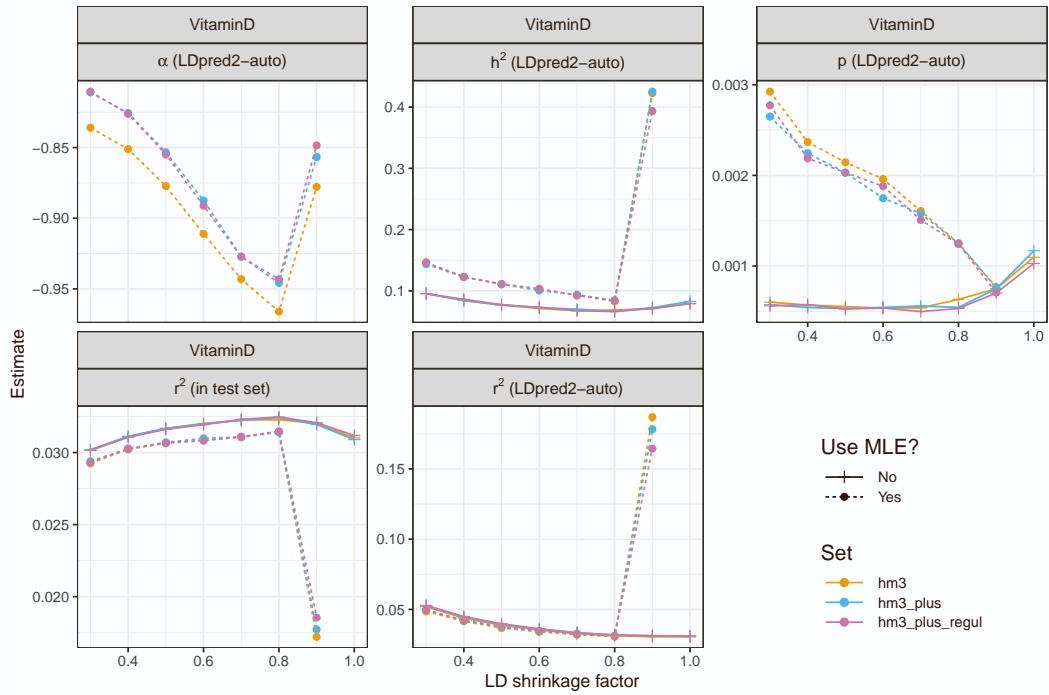
(e) Prostate cancer



(f) Type 1 diabetes



(g) Type 2 diabetes



(h) Vitamin D

Figure S44: LDpred2-auto results with external GWAS summary statistics. We run LDpred2-auto using either the HapMap3 or HapMap3+ variants, with either the extended or previous model and sampling (via parameter `use_MLE`, where setting to TRUE uses the extended model), and explore multiple values for parameter `coef_shrink` (multiplicative coefficient for shrinking/regularizing off-diagonal elements of the LD matrix). The UK Biobank is used as test set to compute  $r^2$ .

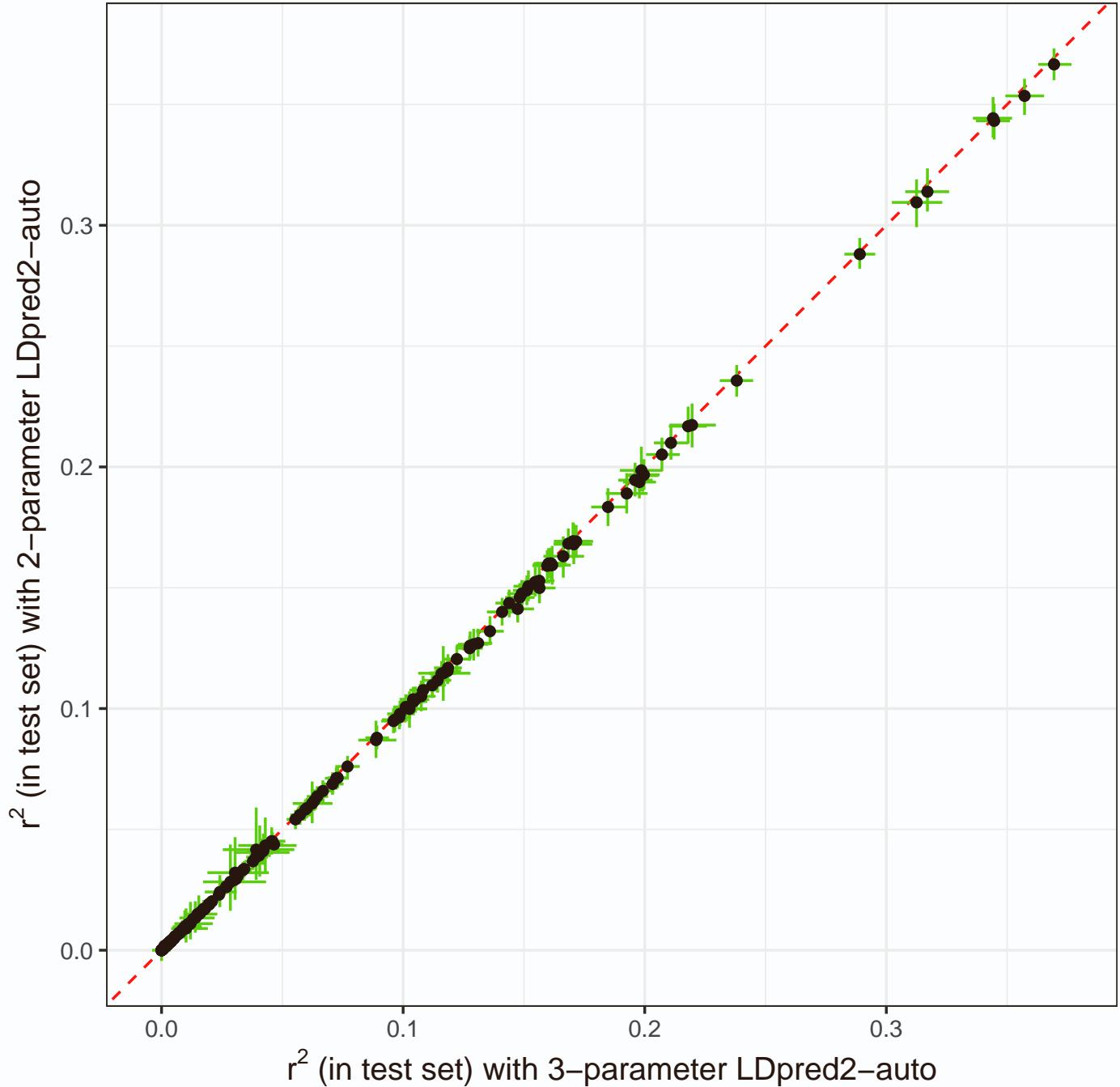


Figure S45: Predictive performance  $r^2$  obtained in the test set, for all 248 phenotypes defined from the UK Biobank. The red dashed line represent the 1:1 line. The 95% confidence interval for  $r^2$  in the test set is obtained from bootstrap.

## References

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- [2] Lee, S. H., Goddard, M. E., Wray, N. R., and Visscher, P. M. (2012). A better coefficient of determination for genetic profile analysis. *Genetic Epidemiology* 36, 214–224.
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- [4] FolkerSEN, L., Gustafsson, S., Wang, Q., Hansen, D. H., Hedman, Å. K., Schork, A., Page, K., Zhernakova, D. V., Wu, Y., Peters, J., et al. (2020). Genomic and drug target evaluation of 90 cardiovascular proteins in 30,931 individuals. *Nature Metabolism* 2, 1135–1148.
- [5] Yengo, L., Vedantam, S., Marouli, E., Sidorenko, J., Bartell, E., Sakaue, S., Graff, M., Eliasen, A. U., Jiang, Y., Raghavan, S., et al. (2022). A saturated map of common genetic variants associated with human height. *Nature* 610, 704–712.