by Chin Long Chiang

A discrete-state, continuous-time Markov model of health states and state transition probabilities is outlined as the basis of a health index that would reflect the annual health distribution and expected health changes of a population. The final index is an additive one, summing expected durations of the various health states over the entire population sample. The weights attached to the expected durations are derived from the instantaneous incidence rates, or intensity functions, that define the transition probabilities. A general procedure for collecting data during overlapping 6-week periods from numerous population subsamples is described.

I.M. Moriyama in 1960 initiated a research project at the National Center for Health Statistics for the development of health indexes for the nation. The objective of the project was to publish annually an index describing the state of health of the population during each calendar year. While an annual index is yet to be published, Moriyama's effort did result in a number of scientific publications, including those by D.F. Sullivan [1–3], F.E. Linder [4], Moriyama [5], and Chiang [6]. Almost concurrently, Katz et al. [7], Lawton [8], Sverdrup [9], and others published work in the general area of measures of health for various purposes. In the late 1960s, the National Center for Health Services Research and Development, in recognition of the need for an assessment of health services in terms of the improvement of people's health, began to encourage research in developing health indexes.

The measurement of a community's health is not a new problem. Traditionally, indexes such as infant mortality, total mortality, and the expectation of life at birth have been used to measure health. The inadequacy of these measures is apparent. The declining trend of the infant mortality rate in the U.S. has slowed markedly since 1950 because of the lack of success in preventing deaths from factors associated with the birth process. Similarly, total mortality has declined as a result of successful efforts in controlling infectious diseases, but this decline has also slowed because of limited progress in reducing and

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curing chronic degenerative diseases of the aged. Thus mortality can no longer be regarded as a sensitive indicator of factors affecting the health of a community.

The increased prevalence of chronic diseases has shifted the orientation of health services to one of more concern for the health of the living. As Sullivan has pointed out [3], the economic and social consequences of illness should now receive more attention than previously in the evaluation of the importance of health problems; disease outcomes should include not only death but reduced productivity, prolonged disability, and the need for social and health services.

The traditional approaches to health measurement, apart from the use of mortality, have been through morbidity surveys and health service activity counts. Each of these has its own problems. National surveys, such as the sickness survey initiated in the United Kingdom during World War II [10] and the U.S. National Health Survey [11], have produced a wealth of information describing patterns of morbidity in their respective populations. However, much of these data are cross-sectional and do not provide information on the change of health over time. Further, although a variety of health indicators can be derived from these surveys, the problem of creating a single measure, or even a small set of summary measures, that gives a comprehensive picture of national health has yet to be clearly formulated.

A central problem facing the investigator who would measure health is the lack of a satisfactory conceptual definition. Various attempts have been made, including those by the World Health Organization [12], Goldsmith [13], Chen and Bryant [14], and Moriyama [5]. All the reviewers of these attempts to define health agree that the definitions are vague, ambiguous, and abstruse and that the definitions contain concepts that are difficult to translate into operational terms. Work on measures of health need not wait until the definitional problems are completely solved, however. Concepts are available that have heuristic value for the purposes of health measurement, and these concepts provide a useful starting point for solving the measurement problem.

# **States of Health**

A concept that is central to this model is that health is a continuum extending from some optimum state of well-being (as suggested by the WHO) to death. This concept has been proposed by Neyman [15], Merrell and Reed [16], Woolsey and Nisselson [17], Dorn [18], and others. In this article I shall use the health continuum as a basis for the construction of an index.

The health of an individual is a dynamic phenomenon; it varies continuously over time. At any given moment, an individual's health may be represented by a point on the continuum and its variation over time by a path which the point travels. A collection of such paths for people in a population forms a visual description of the population's health status over the period in question. To

quantify this conceptual description of health, one may divide the continuum into a set of ordered categories and call them states of health. The order of the states will correspond to degrees of health running from excellent health to extreme illness to death, the lowest point on the continuum. This ordered set of states is the health spectrum, which is central to this effort to develop health indexes. Attempts to develop an ordered series of health levels include those appearing in publications of Bush and his coworkers [19,20], Belloc, Breslow, and Hochstim [21], and Sullivan [1,2], representing particular conceptions of health. The set of states developed by Belloc and her coworkers [21] consists of  $S_1$ : positive health;  $S_2$ : adequate health;  $S_3$ : symptomatic;  $S_4$ : single chronic condition;  $S_5$ : multiple chronic conditions;  $S_6$ : moderate disability;  $S_7$ : severe disability; and R: death.

As an alternative to severity of illness, one may use the impact of illness and impairment on the individual's daily activity, productivity, or need for medical care as a criterion to define states. In any event, the following properties of a health spectrum must be observed:

• The states must be clearly and unambiguously defined, both conceptually and operationally.

• The states must be mutually exclusive and exhaustive. At a point in time it must be possible to classify each individual's health status in exactly one of the states of the spectrum.

• The operational definitions of states associated with the health spectrum must make it possible to collect reliable and valid data.

# **Development of an Index**

Let the general health spectrum be denoted by  $(S_1, \ldots, S_s, R)$ . The symbol  $S_1$  denotes the state of optimum health, as defined by the WHO, the symbols  $S_2, \ldots, S_s$  the states of poorer health, and R the state of death. A person who is in excellent health will be said to be in  $S_1$ ; if his health declines, he enters one of the states  $S_2$  through  $S_s$ ; if he dies, he enters state R. At the beginning of a time interval (0, t) an individual must be in one of the states, and he may remain where he is or enter one of the other states, and he may enter and leave several of the states. To develop an index of health based on such a spectrum, one must define incidence rates to measure an individual's susceptibility to diseases, transition probabilities among the various states, expected duration of stay in the states, and prevalence rates indicating the distribution of a population among the states.

### **Incidence** Rates

Given an individual in state  $S_i$  at a given time t, there is a probability  $v_{ijh}$  that the individual leaves state i and enters state j in the instantaneous time element (t, t+h) for  $i \neq j$  and a probability  $u_{ih}$  that he will die, i.e., enter

state R. The two sets of parameters,  $v_{ij}$ ,  $u_i$  measure the individual's susceptibility to diseases or death and his strength of recovery at any moment of time. According to this interpretation,  $v_{ij}$  and  $u_i$  are incidence rates. For example, suppose  $S_3$  represents a state of acute illness; then the product  $v_{13}h$  is the probability that a person who is well at time t will contract an acute illness in the infinitesimal interval (t, t+h). Similarly, the product  $v_{31}h$  is the probability that a person suffering from acute illness at time t will recover in the interval (t, t+h). For notational convenience, let

$$v_{ii} = -\left(\sum_{\substack{j=1\\j\neq i}}^{s} v_{ij} + u_i\right)$$
(1)

so that  $1 + v_{ii}h$  is the probability of an individual's remaining in state  $S_i$  in a time element (t, t + h). These incidence rates are the basic parameters of the model, and all other quantities can be mathematically derived from them.

### **Transition Probabilities**

While the incidence rates measure the intensity of transition from one state to another within an instantaneous time element, the transition probabilities measure the likelihood of transition from  $S_i$  to  $S_j$  within a finite time interval (0, t). Thus  $P_{ij}(0, t)$  is the probability that an individual who is in state  $S_i$  at time 0 will be in state  $S_j$  at time t. If the interval (0, t) represents one year,  $P_{31}(0, t)$  would be the probability that a person affected with an acute illness at the start of the year will be well at the end of the year. This interpretation holds for any  $S_i$  and  $S_j$ , and the probabilities  $P_{ij}(0, t)$  are measures of prognosis. Similarly,  $Q_i(0, t)$  is the probability that a person who is in state  $S_i$  at time 0 will have died by time t.

Explicit formulas that relate the probabilities  $P_{ij}(0, t)$  and  $Q_i(0, t)$  to the incidence rates  $v_{ij}$  and  $u_i$  are fundamental to the stochastic study of the health phenomena of a population. These formulas were derived in refs. 22 and 23; they are reproduced here for completeness.

If the incidence rates  $v_{ij}$  are written in matrix notation:

$$\mathbf{V} = \begin{bmatrix} \boldsymbol{v}_{11} \dots \boldsymbol{v}_{1s} \\ \vdots & \vdots \\ \boldsymbol{v}_{s1} \dots \boldsymbol{v}_{ss} \end{bmatrix}$$
(2)

and one introduces the characteristic matrix of V:

$$\mathbf{A}(\rho) = \begin{bmatrix} \rho - v_{11} & -v_{12} \dots & -v_{1s} \\ -v_{21} & \rho - v_{22} \dots & -v_{2s} \\ \vdots & & \vdots \\ -v_{s1} & -v_{s2} \dots & \rho - v_{ss} \end{bmatrix}$$
(3)

Setting  $|\mathbf{A}(\rho)| = 0$ , one obtains *s* roots,  $\rho_1, \ldots, \rho_s$ . For each root  $\rho_l$ ,  $l = 1, \ldots, s$ , there is a corresponding matrix

$$\mathbf{A}(\rho_{l}) = \begin{bmatrix} \rho_{l} - v_{11} & -v_{12} \dots & -v_{1s} \\ -v_{21} & \rho_{l} - v_{22} \dots & -v_{2s} \\ \vdots & & \vdots \\ -v_{s1} & -v_{s2} \dots & \rho_{l} - v_{ss} \end{bmatrix}$$
(4)

with each cofactor (j, i) of  $\mathbf{A}(\rho_l)$  denoted by  $\mathbf{A}_{ji}(\rho_l)$ . It can be shown that

$$P_{ij}(0,t) = \sum_{l=1}^{s} \frac{\mathbf{A}_{ji}(\rho_l)}{\prod_{\substack{m=1\\m \neq l}}^{s} (\rho_l - \rho_m)} e^{\rho_l t} \qquad i, j = 1, \dots, s$$
(5)

and

$$Q_{i}(0,t) = \sum_{j=1}^{s} \sum_{l=1}^{s} \frac{\mathbf{A}_{ji}(\rho_{l})u_{j}}{\prod_{\substack{m=1\\m\neq l}}^{s} (\rho_{l} - \rho_{m})\rho_{l}} (e^{\rho_{l}t} - 1) \qquad i = 1, \dots, s$$
(6)

Therefore the transition probabilities  $P_{ij}(0, t)$  and  $Q_i(0, t)$  can be computed from Eqs. 5 and 6 for a given set of incidence rates  $v_{ij}$ .

#### **Expected Duration of Stay**

The duration and level of well-being enjoyed by an individual forms the basis for judgment of his health. Thus the health of an individual during a year should be judged not by his condition at a particular moment, but by the length of time he spends in each of the health states. Corresponding to this intuitively appealing idea, there is an important concept for the theoretical formulation of the problem: the expected duration of stay in each of the health states during a calendar year. Formulas for the expected duration of stay will be devised and this concept used in the development of a health index.

The time spent in each state by an individual during the year depends on the state he occupies initially and on the probability of transition. The length of time a person can expect to spend in state  $S_j$  is denoted by  $E_{ij}(0, t)$  if he is initially in state  $S_i$ .

For each individual in  $S_i$  at time 0, there is a probability  $P_{ij}(0, \tau)$  that he will be in  $S_j$  at time  $\tau$  and once in  $S_j$  he will remain there for the duration  $(\tau, \tau + d\tau)$ . The summation over all possible values of  $\tau$  in the interval (0, t) is the expected duration of stay in  $S_j$ . This is the heuristic justification of the mathematical relationship between  $P_{ij}(0, t)$  and  $E_{ij}(0, t)$ :

$$E_{ij}(0,t) = \int_0^t P_{ij}(0,\tau) \, d\tau \tag{7}$$

Substituting Eq. 5 for the probability  $P_{ij}(0, t)$  and simplifying, one obtains a more explicit formula for  $E_{ij}(0, t)$  in terms of the transition probability  $P_{ij}(0, t)$  and the incidence rates  $v_{ij}$ :

$$E_{ij}(0,t) = \sum_{l=1}^{s} \frac{A_{ji}(\rho_l)}{\prod_{\substack{m=1\\m\neq l}}^{s} (\rho_l - \rho_m)\rho_l} (e^{\rho_l t} - 1) \qquad i, j = 1, \dots, s$$
(8)

Using similar reasoning, one can express the expected duration of stay in state R as:

$$\epsilon_i(0,t) = \int_0^t Q_i(0,\tau) d\tau \qquad i = 1,\ldots,s$$

Thus if (0, t) represents one year,  $\epsilon_i(0, t)$  is the expected fraction of the year that a person starting the year in state *i* will be in state *R*.

### **Prevalence** Rates

For every state  $S_i$ , the proportion  $\pi_i(t)$  of people in that state is a measure of the prevalence in the population at time t of the health condition represented by  $S_i$ . Collectively, the set of prevalence rates  $\pi_1(t), \ldots, \pi_s(t)$  describes the population distribution with respect to health status in time t. The prevalence rates at time t = 0, denoted by  $\pi_1, \ldots, \pi_s$ , are of particular interest in the formulation of health indexes. The expected duration of stay  $E_{ij}(0, t)$  depends on state  $S_j$  and the initial state  $S_i$  an individual occupies at time 0. To remove the dependence on the initial state  $S_i$ , one applies the prevalence rates  $\pi_i$  to  $E_{ij}(0, t)$  to find the weighted average

$$E_{i}(0,t) = \pi_{1} E_{1i}(0,t) + \ldots + \pi_{s} E_{si}(0,t)$$
(9)

Since  $\pi_i$  is the probability that a person picked at random at time 0 will be in  $S_i$ , Eq. 9 gives the expected duration of stay in  $S_j$  during the interval (0, t) regardless of the initial state. For j = 1,  $E_1(0, t)$  is the expected length of time within (0, t) that an individual will be in excellent health. For the state of death R, the equivalent expression is:

$$\boldsymbol{\epsilon}(0,t) = \boldsymbol{\pi}_1 \, \boldsymbol{\epsilon}_1(0,t) + \ldots + \boldsymbol{\pi}_s \, \boldsymbol{\epsilon}_s(0,t)$$

For a period of one year, the symbol (0, 1) may be dropped and the expected durations of stay written as:

and

$$E_{j} = \pi_{1} E_{1j} + \ldots + \pi_{s} E_{sj}$$
 (10)

$$\boldsymbol{\epsilon} = \boldsymbol{\pi}_1 \, \boldsymbol{\epsilon}_1 + \ldots + \boldsymbol{\pi}_s \, \boldsymbol{\epsilon}_s$$

# **A** Health Index

The quantity  $E_j$  in Eq. 10 is the fraction of a year that an individual is expected to be in state  $S_j$ . An index of health for a current population should be based on these fractions. Specifically, the following linear function of the expected durations of stay is proposed as a health index:

$$H = w_1 E_1 + w_2 E_2 + \ldots + w_s E_s + w_r E_r \tag{11}$$

The weights  $w_1, \ldots, w_s$ , and  $w_r$  signify the relative health status of people in  $S_1, \ldots, S_s$ , and R. If a weight  $w_r = 0$  is assigned to the death state R and a weight  $w_1 = 1$  assigned to excellent health for people in state  $S_1$ , the health index becomes:

$$H = E_1 + w_2 E_2 + \ldots + w_s E_s \tag{12}$$

### **Determination** of Weights

The weights  $w_2, \ldots, w_s$  in Eq. 12 are measures of the health of individuals in states  $S_2, \ldots, S_s$  relative to the health of individuals in  $S_1$ . By setting  $w_1 = 1$ , one is assuming that the individuals in  $S_1$  receive a perfect score for their health condition. Since people in  $S_2, \ldots, S_s$  are of poorer health, the weights  $w_2, \ldots, w_s$  should be assigned numerical values between zero and one. Determination of the exact values for the weights, however, is a problem. Attempts have been made in the literature to determine weights in a similar situation. For example, psychometric techniques of scaling, as described by Torgerson [24], have been used by Patrick and Bush [25] in their search for an operational definition of health. Dorothy Rice [26] introduces the concept of direct and indirect costs of suboptimal health. Many criteria have been considered for the determination of weights; the following are a few examples:

• The economic impact on the individual in terms of income foregone and expenses incurred

- The demand for health services and medical care
- The loss to society of an individual's productivity and services

• The intangible aspects of physical and mental impairment, such as pain and loss of leisure and companionship.

While each of these criteria has its merit, their adoption does not resolve the problem of assigning numerical values to the weights. Furthermore, when factors other than health are used for the determination of weights, the relevance of such factors to health must be assessed, and the choice of one factor (or factors) over the others must be made.

The most direct approach to determining weights is assessing the health of individuals in the states  $S_2, \ldots, S_s$  relative to those in the state  $S_1$ . The health condition of an individual who is in a state  $S_i$  may be measured by the degree of rapidity with which he returns to perfect health, that is, to state  $S_1$ . The incidence rate  $v_{i1}$ , on the other hand, is a measure of instantaneous recovery of an individual's health from  $S_i$  to  $S_1$ . Therefore the weight  $w_i$  should be proportional to the incidence rate  $v_{i1}$ , for  $i = 2, \ldots, s$ . Letting c be the constant of proportionality, one has the health index

$$H = E_1 + c v_{21} E_2 + \ldots + c v_{s1} E_s \tag{13}$$

When c is set equal to 1, the health index becomes

$$H = E_1 + v_{21}E_2 + \ldots + v_{s1}E_s \tag{14}$$

and the index can be readily computed on the basis of the information gathered for the population in question.

If everyone in the population is perfectly healthy, i.e., is in state  $S_1$ , throughout the year the index of health assumes the value 1.0. In this case the fraction of the year that each person is healthy is one and the fraction of the year that he is in poorer health is zero; that is,  $E_1 = 1$  and  $E_2 = E_3 = \ldots = E_s = 0$ . When persons in the population have experienced varying degrees of health and illness  $E_2 \ldots E_s$  will have positive values less than one, and the index for the population will be less than one.

It may be noted that since the rates  $v_{i1}$  are directly associated with the "position" of the state  $S_i$  on the health continuum relative to  $S_1$ , the weights so determined may help to adjust for variation in the definition of states. The mortality intensity function  $u_i$  might be included with  $v_{i1}$  as a factor in the weight  $w_i$ , so that both recovery rate and probability of death are taken into account. For example,  $w_i$  might be a linear function of the two:

$$w_i = a + bv_{i1} + cu_i \qquad i = 2, \ldots, s$$

If the coefficients  $a = b = \frac{1}{2}$  and  $c = -\frac{1}{2}$ ,

$$w_i = \frac{1}{2} [v_{i1} + (1 - u_i)]$$
  $i = 2, ..., s$ 

Then  $w_i$  would be large, and the person in state  $S_i$  would be considered quite healthy, if the recovery rate  $v_{i1}$  is high and the chance of dying,  $u_i$ , is low.

### **Estimation of Incidence Rates**

Computing a health index for a population by the present method requires knowledge of the prevalence rates  $\pi_1, \ldots, \pi_s$  and the incidence rates  $v_{ij}$ . Prevalence rates can be determined from population data without much theoretical difficulty. Estimating incidence rates from a sample, however, requires some explanation. Let N be the number of individuals in a sample from a population for which a health index is to be computed. Health condition and changes in health status of each individual in the sample during a short time period of length T are completely determined. The duration of stay in each state and the number of transitions from one health state to another are ascertained. The information obtained for each individual is then expressed as a function of the incidence rates  $v_{ij}$ ,  $u_j$ . This function, which is known as the likelihood function, is of the following form:

$$\prod_{j=1}^{s} \prod_{i=1}^{s} e^{v_{i}it_{i}} v_{ij}^{n_{ij}} u_{j}^{d_{j}}$$
(15)

where  $t_i$  is the total length of time an individual stays in state  $S_i$ 

 $n_{ij}$  is the number of transitions from S<sub>i</sub> to S<sub>j</sub>

 $d_j = 1$  if an individual enters the state of death R from  $S_j$ .

Each  $t_i$  is a real number greater than or equal to 0, while  $n_{ij}$  is a nonnegative integer. The likelihood function L for the entire sample is the product of the likelihood functions for the N individuals:

$$L = \prod_{j=1}^{N} \left( \prod_{j=1}^{s} \prod_{i=1}^{s} e^{v_{i,i}t_i} v_{ij}^{n_{ij}} u_j^{d_j} \right)$$
(16)

From Eq. 16, one has maximum likelihood estimates of the incidence rates:

$$\hat{\boldsymbol{v}}_{ij} = \sum_{i=1}^{N} n_{ij} / \sum_{i=1}^{N} t_i \qquad j \neq i \qquad i, j = 1, \dots, s$$
(17)

and the corresponding variances

$$(S_{\hat{v}_{ij}})^2 = \sum_{j=1}^{N} n_{ij} / \left(\sum_{j=1}^{N} t_i\right)^2 \qquad j \neq 1 \quad i, j = 1, \dots, s$$
 (18)

Similarly, the estimate of  $u_i$  is given by

$$u_i = \sum_{i=1}^{N} d_i / \sum_{i=1,\ldots,s}^{N} t_i \qquad i = 1,\ldots,s$$
(19)

the estimate of  $v_{ii}$  is derived from Eq. 1:

$$\hat{\boldsymbol{v}}_{ii} = -\left(\sum_{\substack{j=1\\j\neq i}} \hat{\boldsymbol{v}}_{ij} + \hat{\boldsymbol{u}}_i\right) \qquad \boldsymbol{i} = 1, \dots, \boldsymbol{s}$$
(20)

As soon as the values of  $t_i$  and  $n_{ij}$  are determined from a sample, the estimates  $\hat{v}_{ij}$  and  $\hat{u}_i$  can be determined from Eqs. 17, 19, and 20. Using these estimates one computes, successively, the transition probabilities  $P_{ij}(0, t)$  from Eq. 5, the expected durations of stay  $E_{ij}$  from Eq. 8, and  $E_j$  from Eq. 9. Finally, one uses Eq. 14 to determine the health index of the population under study.

The computation of a health index as described above is applicable to any defined population. For the entire nation, where the people in different age and sex categories have different susceptibility to diseases, the population should be divided into subpopulations according to age and sex, and possibly also race. A health index can then be determined for each subpopulation. For subpopulation k,  $H_k$  is the health index for a calendar year and  $P_k/P$  is the proportion of the total population that is in subpopulation k. The health index for the nation, denoted by H, is a weighted average of  $H_k$  with the proportions  $P_k/P$  applied as weights:

$$H = \sum_{k} H_k P_k / P \tag{21}$$

## **Discussion: Sampling and Data Collection**

It is tacitly assumed in the preceding section that a sample of T days is taken out of 365 days of the year. While it is not required in the equation for estimating  $v_{ij}$ , the length of time T should be constant for all individuals in the sample, for convenience in computing variances. The most suitable value for T is six weeks: not so long as to make the information unreliable, but long enough for changes in health to occur. To adequately cover the health status of a population during the year, the observation periods should be distributed evenly over the 52 weeks. With a sample size of N = 47m, the first group of m individuals should be observed from the first week to the sixth week, the second group of m individuals from the second week to the seventh week, and so on to the 47th group, observed from the 47th week to the 52nd, or the last week of the year.

Data collection should include an initial interview with each member in the sample, a daily diary of health events covering the six weeks for each sample member, and a final interview at the end of the diary period to review the diary and collect additional health status data similar to that obtained at the first contact.

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