produced the progeny that would be expected if the exceptional white were a normal pink. This analysis indicates that the inability of these exceptional adenine-dependent, methionine-independent cultures to produce the pink pigment was due to some mechanism which is restored to activity following hybridization.

The following hypothesis is invoked to explain the effect of outcrossing in restoring the pink color. Pink depends upon the presence of the two genes *ad* and *MET* plus some other substances (X, Y, Z, etc.). The substance X is an essential component of gene X which has no other components besides X. Continuous production of pink exhausts the supply of X and results in the "running out" of the character. The stock to which the outcross is made carries gene X with an intact supply of the X component for since the stock does not produce pink it does exhaust its supply of the X substance. The outcross automatically restores the X substance and reëstablishes the pink color. Other stocks may become white because Y or Z substances are exhausted. Mutations from pink to white are not the result of a drastic change in genotype but merely the result of the exhaustion of some gene component easily supplied by outcrossing to any normal stock.

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WIND-DRIVEN CURRENTS IN A BAROCLINIC OCEAN; WITH APPLICATION TO THE EQUATORIAL CURRENTS OF THE EASTERN PACIFIC*

BY H. U. SVERDRUP

SCRIPPS INSTITUTION OF OCEANOGRAPHY, UNIVERSITY OF CALIFORNIA, LA JOLLA, CALIFORNIA

1. Introduction.—Permanent ocean currents are computed from the observed distribution of density on the assumptions (1) that the horizontal pressure gradient is balanced by the Coriolis force (the deflecting force of the earth's rotation) and (2) that the horizontal velocities and the hori-

zontal pressure gradient vanish at a moderate depth below the sea surface. The second condition can be fulfilled only in a baroclinic system, that is, in a system in which the isosteric surfaces intersect the isobaric surfaces.

In the computation of currents acceleration and frictional forces are neglected. Experience indicates that the computations lead to nearly correct results, implying that accelerations and frictional forces are small, but since friction is not entirely lacking, energy must be supplied to the ocean in order to maintain the permanent currents and the corresponding permanent distribution of mass. This energy can be supplied by the effects of heating and cooling or by the stress which the prevailing winds exert on the sea surface. Of the sources the latter is generally considered to be the more important. We shall examine effects of the wind stress only, taking into account that the ocean waters in motion represent a baroclinic system.

Ekman[1] and Stockmann[2] have examined the currents which develop in a *homogeneous* ocean under the influence of a stress exerted on the free surface, and Fjeldstad[3] has solved a special problem dealing with baroclinic conditions. If the general problem for a baroclinic ocean could be solved, knowledge of the wind stress alone would enable us to compute the permanent ocean currents, provided the effects of heating and cooling were negligible. A treatment of this general problem would present great mathematical difficulties because it would require the introduction of lateral frictional stresses and complete boundary conditions. Here we shall deal with the special case of equatorial currents in a region where lateral stresses can be neglected, boundary conditions are relatively simple, wind systems are semipermanent, and where our results imply that effects of heating and cooling, if present, need not be considered explicitly.

The striking feature of the currents of the equatorial regions is that imbedded between the currents which flow toward the *west* under the influence of the prevailing trade winds equatorial counter currents flow toward the *east*. In the Pacific and Atlantic Oceans the counter current is particularly well developed in the eastern parts of the oceans where it is located north of the equator, its axis coinciding approximately with the location of the equatorial calm belt which is found further to the north in summer than in winter. In the Indian Ocean the counter current is found to the south of the equator, but in the northern winter only.

Our specific problem is to determine whether the equatorial currents, including the counter currents, can be accounted for on the basis of our knowledge of the wind stress only. This problem was first approached by Montgomery and Palmén,[4] but Stockmann[2] has shown that they did not treat it in a sufficiently general manner. Stockmann's theoretical results, however, are not applicable to the conditions in the ocean because he assumed homogeneous water, but a similar analysis for a baroclinic

system leads to a remarkable agreement between theoretical conclusions and observed conditions.

2. *Theory.*—The ocean waters are so nearly in hydrostatic equilibrium that at any depth the pressure, p, can be determined by a numerical integration of the hydrostatic equation:

$$dp = g\rho\, dz \tag{1}$$

provided that the density, ρ, is known from observations. In equation (1) and in the following equations the z-axis is positive downwards.

Neglecting lateral stresses the equations of horizontal motion can be written:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \lambda v + \frac{1}{\rho}\frac{\partial}{\partial z}\left(A\frac{\partial u}{\partial z}\right)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - \lambda u + \frac{1}{\rho}\frac{\partial}{\partial z}\left(A\frac{\partial v}{\partial z}\right) \tag{2}$$

where u and v are the horizontal velocity components in a rectilinear coordinate system, $\lambda = 2\omega \sin \varphi$ (ω the earth's angular velocity of rotation, φ the latitude, taken positive to the north of the equator), and A is the eddy viscosity.

We shall assume stationary conditions,

$$\frac{\partial u}{\partial t} = \frac{\partial v}{\partial t} = 0, \tag{3}$$

and shall neglect the non-linear terms, the field accelerations:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = 0, \tag{4}$$

thereby placing severe restrictions upon the possible lateral boundary conditions. Equations (2) reduce to:

$$\frac{\partial p}{\partial x} = \lambda \rho v + \frac{\partial}{\partial z}\left(A\frac{\partial u}{\partial z}\right)$$

$$\frac{\partial p}{\partial y} = -\lambda \rho u + \frac{\partial}{\partial z}\left(A\frac{\partial v}{\partial z}\right) \tag{5}$$

stating that the horizontal pressure gradient is balanced by the Coriolis force and frictional stresses exerted on horizontal surfaces. In *homogeneous* water the horizontal pressure gradient is independent of depth but in a *baroclinic* system varies with depth. In the ocean it generally vanishes at

a moderate depth, less than that to the bottom. We define a function P by the integrals

$$\frac{\partial P}{\partial x} = \int_0^d \frac{\partial p}{\partial x} dz, \qquad \frac{\partial P}{\partial y} = \int_0^d \frac{\partial p}{\partial y} dz \qquad (6)$$

where d is equal to or greater than the depth at which the horizontal pressure gradient becomes zero. The function P, which is closely related to the P-function introduced by Ekman,[5] can be computed from the observed vertical distribution of density at a single oceanographic station, using equation (1).

The horizontal velocity must vanish at or above the depth d. The integrals

$$M_x = \int_0^d \rho\, u\, dz, \qquad M_y = \int_0^d \rho\, v\, dz \qquad (7)$$

represent therefore the components of the *net* mass transport by the currents.

Integrating equations (5) from 0 to d, and introducing the horizontal boundary conditions:

$$\left(A\frac{\partial u}{\partial z}\right)_0 = -\tau_x, \qquad \left(A\frac{\partial u}{\partial z}\right)_d = 0$$

$$\left(A\frac{\partial v}{\partial z}\right)_0 = -\tau_y, \qquad \left(A\frac{\partial v}{\partial z}\right)_d = 0 \qquad (8)$$

where τ_x and τ_y are the components of the wind stress, we obtain:

$$\frac{\partial P}{\partial x} = \lambda M_y + \tau_x \qquad (9a)$$

$$\frac{\partial P}{\partial y} = -\lambda M_x + \tau_y \qquad (9b)$$

The terms in equations (9) are well known in oceanography. Omitting the stress components the equations give the mass transport related to the distribution of density, or assuming homogeneous water in hydrostatic equilibrium $(\partial P/\partial x = \partial P/\partial y = 0)$ they give the mass transport by pure wind currents. Equations (9) have been used by Defant[6] for computing the wind stress from oceanographic observations, including direct measurements of currents, but they have not been applied to other problems.

For application to other problems we add the equation:

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_y}{\partial y} = 0 \qquad (10)$$

which is obtained by integration of the equation of continuity, assuming

that the vertical velocity is zero at the free surface and at the depth d. The three equations (9a), (9b) and (10) can be considered as relating the three unknown quantities, P, M_x, and M_y, to the known wind stress. Consequently, the distribution of density, as described by the partial derivatives of P, and the mass transport by the corresponding currents can be expressed as functions of the stress.

In applying equations (9) and (10) to equatorial currents we place the positive x-axis toward the east and the positive y-axis toward the north, and let $y = 0$ at the equator ($\varphi = 0$). Since

$$dy = R \, d\varphi \tag{11}$$

where R is the radius of the earth:

$$\frac{\partial \lambda}{\partial x} = 0, \qquad \frac{\partial \lambda}{\partial y} = \frac{2\omega \cos \varphi}{R}, \qquad \frac{\partial^2 \lambda}{\partial y^2} = -\frac{2\omega \sin \varphi}{R^2} \tag{12}$$

Differentiating equation (9a) with respect to y and (9b) with respect to x, subtracting and taking equations (10) and (12) into account, we obtain

$$M_y \frac{\partial \lambda}{\partial y} + \left(\frac{\partial \tau_x}{\partial y} - \frac{\partial \tau_y}{\partial x} \right) = 0 \tag{13}$$

In the trade-wind belt of the eastern Pacific the term $\partial \tau_y / \partial x$ is so small that with good approximation:

$$M_y = \frac{\partial \tau_x}{\partial y} \Big/ \frac{\partial \lambda}{\partial y} = -\frac{\partial \tau_x}{\partial y} \frac{R}{2\omega \cos \varphi} \tag{14}$$

Introducing equation (14) in (9a):

$$\frac{\partial P}{\partial x} = -\frac{\partial \tau_x}{\partial y} R \tan \varphi + \tau_x \tag{15}$$

or, writing differences on the left-hand side:

$$\frac{\Delta P}{\Delta x} = -\frac{\overline{\partial \tau_x}}{\partial y} R \tan \varphi + \bar{\tau}_x \tag{16}$$

where averages over the distance Δx are indicated by bars.

From equations (10) and (14) follows

$$\frac{\partial M_x}{\partial x} = \frac{1}{2\omega \cos \varphi} \left(\frac{\partial \tau_x}{\partial y} \tan \varphi + \frac{\partial^2 \tau_x}{\partial y^2} R \right) \tag{17}$$

When integrating equation (17) from 0 to Δx we shall assume a north-south vertical boundary at $x = 0$ at which the kinematic boundary condition $u_0 = 0$ must be satisfied in the form $M_0 = 0$. We obtain:

$$M_x = \frac{\Delta x}{2\omega \cos \varphi} \left(\overline{\frac{\partial \tau_x}{\partial y}} \tan \varphi + \overline{\frac{\partial^2 \tau_x}{\partial y^2}} R \right) \tag{18}$$

Equation (18) cannot hold at a second north-south boundary at, say, $x = L$, at which the condition $M_L = 0$ must be satisfied. This inadequacy of our solution is due to the neglect of the field accelerations (eq. 4). Attempts will be made to find more general solutions and to study other special cases.

Substituting equation (18) in (9b):

$$\frac{\partial P}{\partial y} = -\Delta x \tan \varphi. \left(\overline{\frac{\partial \tau_x}{\partial y}} \tan \varphi + \overline{\frac{\partial^2 \tau_x}{\partial y^2}} R \right) + \tau_y \tag{19}$$

Equations (15) or (16) and (19), together with (14) and (18), represent in our special case the relationships of the distribution of mass and the corresponding mass transport to the wind stress. The validity of our results can be tested where suitable observations are available.

3. *Discussion.*—The available oceanographic observations comprise (1) a line of 8 stations between latitudes 22°N and 10°S, longitudes 137°W and 162°W, occupied by the *Carnegie* between October 21 and November 4, 1929 (Fleming[7]); (2) a line of 12 stations between latitudes 6°N and 9°S, longitudes 80°W and 108°W, occupied by the *Carnegie* between October 26 and November 21, 1928; and (3) a line of 8 stations between latitudes 9°N and 21°N, longitudes 87°W and 109°W, occupied by the Bushnell between March 18 and March 24, 1939 (Sverdrup[8]). From the observations at each of these stations the value of the function P was computed by integrating to a depth of 1000 meters. From all data the ratio $\Delta P/\Delta x$ was found, and from the *Carnegie* section in mid-ocean $\partial P/\partial y$ was derived.

Wind observations comprise (1) monthly wind roses for 5-degree squares published in the Pilot Charts of the North and South Pacific, giving the percentage of winds from different directions and the corresponding average wind force (on the Beaufort scale) and (2) compilations of frequencies of winds of different forces in the "Atlas of Climatological Charts of the Oceans."[9] From the wind data the average wind stresses in October and November were computed, using the relationship

$$\tau = \gamma^2 \rho' U^2$$

where γ^2 is the resistance coefficient, ρ' the density of the air, and U the wind speed as estimated at a height of about 10 meters. At wind force 3 Beaufort or less the sea surface was assumed to be hydrodynamically smooth, with a resistance coefficient of about 0.8×10^{-3}, decreasing somewhat with increasing wind speed.. At wind force 4 Beaufort and higher a constant value, $\gamma^2 = 2.6 \times 10^{-3}$, was used, corresponding to a hydrodynamically rough surface (Rossby[10]). The manner in which all computa-

tions were carried out will be described elsewhere by the author and R. O. Reid, who has prepared the figures in this paper.
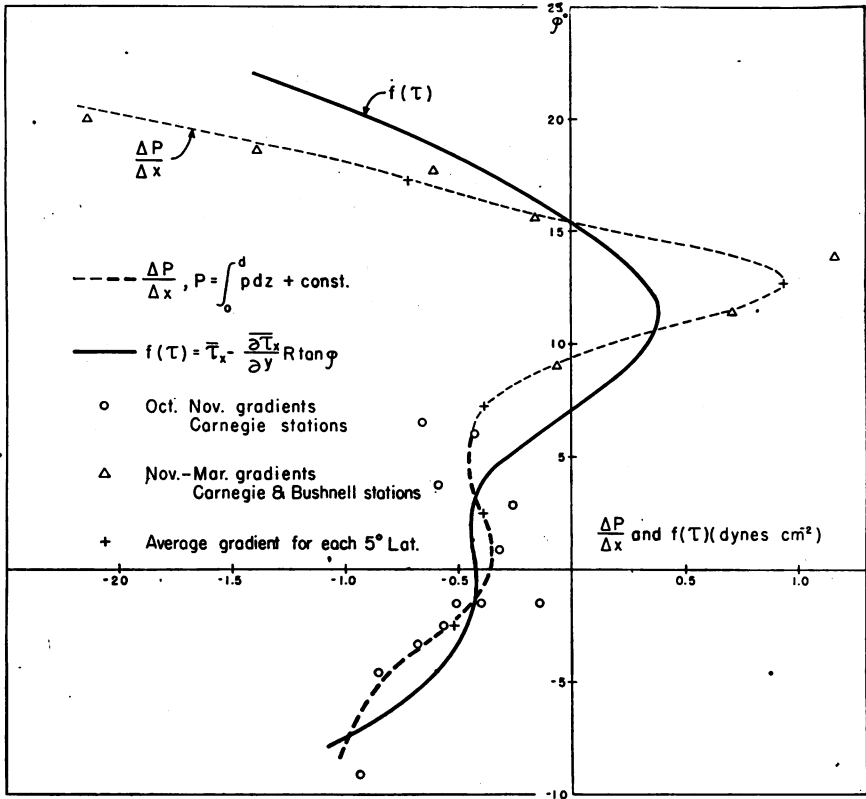

FIGURE 1

In figure 1 the terms of equation (16) are shown as functions of latitude. The curve that represents the left-hand term is heavily dashed to the south of latitude 6°N where the oceanographic observations upon which it is based were all taken in October–November, although in different years. To the north of 6°N the curve is shown by light dashes because observations off the American west coast in March have been combined with observations in mid-ocean in October. The right-hand term, the stress function, is shown by a full-drawn curve and is based on climatological wind data for the months October–November. The agreement between the curves is very good, considering that results of average wind conditions are compared with results derived from a few oceanographic stations which have been occupied in different seasons.

In figure 2 the P function and the terms of equation (19) are plotted against latitude. The P function is based on the *Carnegie* observations
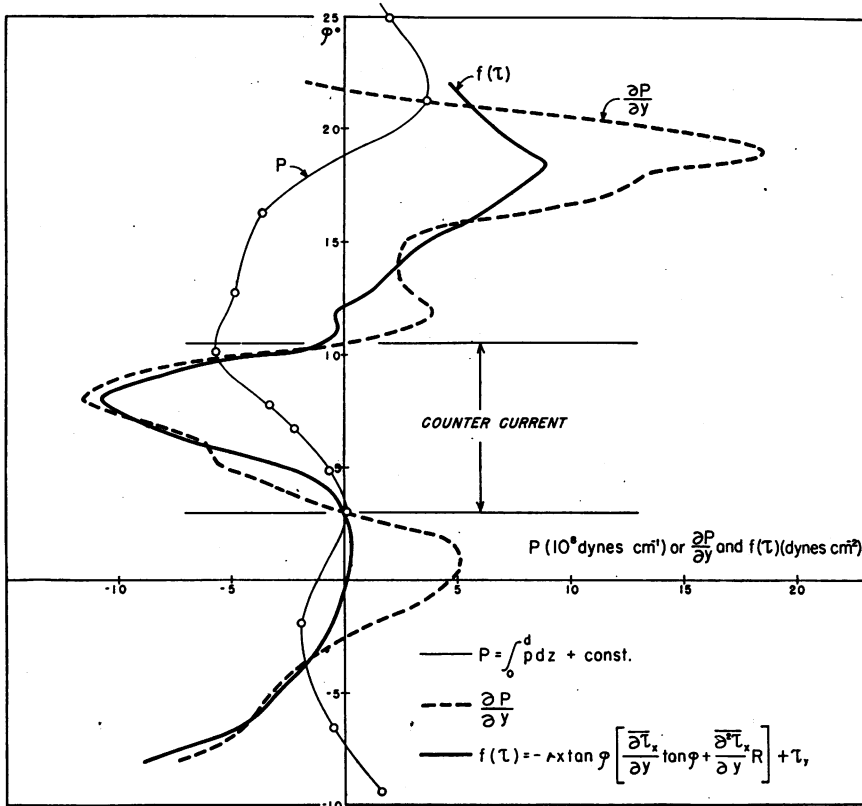

P (10^8 dynes cm^{-1}) or $\frac{\partial P}{\partial y}$ and $f(\tau)$(dynes cm^{-2})

$$P = \int_0^d p\,dz + const.$$

$$\frac{\partial P}{\partial y}$$

$$f(\tau) = -\Lambda x \tan \varphi \left[\frac{\overline{\partial \tau_x}}{\partial y} \tan\varphi + \frac{\overline{\partial^2 \tau_x}}{\partial y}R \right] + \tau,$$

COUNTER CURRENT

FIGURE 2

in mid-ocean in October–November, 1928, and the stress function on the average wind conditions in October–November over the ocean from the American west coast to the *Carnegie* section. A good agreement is obtained between the results based on a single oceanographic section and those derived from climatological wind charts.

4. *Conclusions.*—The distribution of density and the mass transport by the accompanying currents of the eastern equatorial Pacific depend entirely upon the average stress exerted on the sea surface by the prevailing winds. This conclusion is probably valid for the equatorial currents of all oceans but it has been demonstrated only for a case in which the non-linear terms in the equation of motion can be neglected.

It appears possible that the analysis of the relationship between wind stress and prevailing currents, assuming baroclinic conditions, can be

extended to other cases and can be developed into a powerful tool for examining permanent currents as well as changes produced by changing winds.   Efforts in this direction are being continued.

* Contributions from the Scripps Institution of Oceanography, New Series, No. 324.

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THE PROBLEMS OF CONGRUENT NUMBERS AND CONCORDANT FORMS

By E. T. Bell

CALIFORNIA INSTITUTE OF TECHNOLOGY, PASADENA

1. Four Related Problems.—All letters in formulas denote rational integers, and solution means the *complete* solution in such integers.   The problem of solving the simultaneous diophantine equations

$$rX^2 + mY^2 = rZ^2, \quad sX^2 + nY^2 = sW^2$$

includes as special cases two classical problems.

Problem 1.—If $r = s = Y^2 = 1$, $n = -m$, where m is a given constant, the problem is that of congruent numbers.   It goes back to Diophantus in the third century, the Arabs of the tenth and eleventh centuries, and Leonardo of Pisa (Fibonacci) in the early thirteenth century.   For m arbitrarily assigned it is still unsolved.

Problem 2.—For $r = s = 1$ the problem is Euler's (1780) of concordant forms, also unsolved.

Many special cases of these two have been investigated.   Thus Fermat proved by his method of descent that if $m = n = -1$ in Problem 2, there are no integers X, Y, Z, W all different from zero satisfying the equations. From this his theorem for fourth powers follows.   Modern work originating in these problems has been concerned with cubics and quartics having