Carbon Economics and Assimilation in Dynamic Light Environments Calculations

1. Construction Cost Equations:

(1.1) $$Alkalinity\ \left(\frac{mEq}{g}\right) = \frac{HCl\ (Mol) - NaOH\ (Mol)}{2} / Ash\ (g)$$

(1.2) $$Mineral\ Content\ \left(\frac{mg}{g}\right) = Ash\ \left(\frac{mg}{g}\right) - Alkalinity\ \left(\frac{mEq}{g}\right) * 30 + Nitrate\ \left(\frac{mg}{g}\right)$$

(1.3) $$Construction\ Cost\left(\frac{g\ glucose}{g\ tissue}\right) = \left(-1.041 + 5.077 * \left(\frac{C\left(\frac{mg}{g}\right)}{1000}\right)\right) * \left(1 - \left(\frac{Mineral\left(\frac{mg}{g}\right)}{1000}\right)\right) + \left(5.325 * \left(\frac{N\left(\frac{mg}{g}\right) - Nitrate\left(\frac{mg}{g}\right)}{1000}\right)\right)$$

2. Assimilation Equations:

(2.1) $$Assimilation\ (\mu mol\ m^{-2}s^{-1}) = \frac{\left(phi*PAR + A_{sat} - \left(\sqrt{((phi*PAR + A_{sat})^2 - 4*theta*phi*PAR*A_{sat}}\right)\right)}{2*theta}$$

3. Converting assimilation from μmol CO_2 m^{-2} to grams glucose per gram tissue:

(3.1) $$\frac{glucose(g)}{tissue(g)} = \left(\frac{(\mu mol\ m^{-2}) * SLA\left(\frac{cm^2}{g}\right)}{6000000000}\right) * 180$$

4. Carbon Economic Trait Equations:

(4.1) $$Payback\ time\ (days) = \frac{Construction\ Cost}{\left(\left(Assimilation\left(\frac{gglucose}{g\ s^{-1}}\right) + Rd\left(\frac{gglucose}{g\ s^{-1}}\right)\right) * 3600\right) * 12}$$

(4.2) $$Return\ on\ Investment\ (g\ glucose) = \left(\left(\left(Assimilation\left(\frac{gglucose}{g\ s^{-1}}\right) + Rd\left(\frac{gglucose}{g\ s^{-1}}\right)\right) * 3600\right) * 12\right) * (Leaf\ lifespan - payback\ time)$$

5. Daily Integrated PPFD when PAR is constant (12-hour light period):

(5.1) $$Integrated\ PPFD\ (mol\ m^{-2}d^{-1}) = \frac{PAR*43200}{10^6}$$

6. Light Levels Across the Day (all trig functions using radians):

(6.1) $$Solar\ declination\ angle = \left(-23.5 * \frac{\cos(6.28*day\ (julian\ days)+10)}{365}\right) * \left(\frac{pi}{180}\right)$$

(6.2) $$Hour\ angle = 0.262 * (time\ (hours) - 12)$$

(6.3) $$Solar\ elevation\ angle = \text{asin}(sin(latitude) * sin(solar\ declination\ angle) + cos(latitude) * cos(solar\ declination\ angle) * cos(hour\ angle))$$

(6.4) $$Direct\ Light = 2600 * 0.75^{\left(\frac{1}{\sin(solar\ elevation\ angle)}\right)} * \sin(solar\ elevation\ angle)$$

(6.5) $$Diffuse\ Light = 0.3 * \left(1 - 0.75^{\left(\frac{1}{\sin(solar\ elevation\ angle)}\right)}\right) * 2600 * \sin(solar\ elevation\ angle)$$

(6.6) $$Light\ during\ sunfleck = (Diffuse\ Light)exp^{-0.78*LAI} + Direct\ Light$$

(6.7) $$Light\ during\ shadefleck = (Diffuse\ Light)exp^{-0.78*LAI}$$

7. Sun and Shadefleck equations:

(7.1) $$Sunfleck\ length = shadefleck\ length / \frac{1-exp\left(-\frac{LAI}{\sin(solarelevationangle)}\right)}{exp\left(-\frac{LAI}{\sin(solarelevationangle)}\right)}$$

(7.2) $$Shadefleck\ length = sunfleck\ length * \frac{1-exp\left(-\frac{LAI}{\sin(solarelevationangle)}\right)}{exp\left(-\frac{LAI}{\sin(solarelevationangle)}\right)}$$

8. Light induction variables equations:

(8.1) $$Af^* = (A_{sat} + Rd) * (\frac{0.65*400}{C_{i-sat}})$$

(8.2) $$A^* = (A + Rd) * (\frac{0.65*400}{C_i})$$

(8.3) $$tau = -1/slope\ of\ \ln(Af^* - A^*)\ vs\ Time\ from\ minutes\ 1-10\ of\ induction$$

(8.4) $$Induction\ state\ (\%) = \left(\frac{A-Rd}{A_{sat}-Rd}\right) * 100$$

9. Assimilation in dynamic light equations:

(9.1) $$Af = \frac{\left(phi*PAR+A_{sat}-\left(\sqrt{((phi*PAR+A_{sat})^2-4*theta*phi*PAR*A_{sat}}\right)\right)}{(2*theta)}$$

(9.2) *Ai* = *A* calculated for the proceeding interval during a sunfleck, *A* calculated for the proceeding interval using tau when light is decreasing (estimated at 5x induction tau) during a shadefleck, or is 0 at first light.

(9.3) $$A = Af - (Af - Ai) * exp^{-\frac{1}{tau}}$$

(9.4) $$Aint\ (integrated\ CO_2\ assimilation - 1\ \min intervals) = Af * 60 - (Af - Ai) * tau + (Af - Ai) * tau * exp^{(-\frac{1}{tau})}$$

Abbreviations and Variables:

A – Instantaneous assimilation rate
*A** -- Assimilation rate corrected for changes in C_i
Af – Potential maximum assimilation rate
*Af** -- Potential maximum assimilation rate corrected for changes in C_i
Ai – Initial assimilation rate prior to induction
Aint – Integrated assimilation
A_{sat} – Light saturated assimilation rate
C_i – Intercellular CO_2 concentration
LAI – Leaf area index
PAR – Photosynthetically Active Radiation
phi – quantum yield of photosynthetic light response
PPFD – Photosynthetic Photon Flux Density
SLA – Specific Leaf Area
tau – relaxation time for Rubisco activation
theta – curvature of photosynthetic light response
Rd – Respiration

References:

Campbell GS, Norman JM. **1998**. *An Introduction to Environmental Biophysics*. New York: Springer Verlag.
[Equations:6.4, 6.5]

Lobo F de A, de Barros MP, Dalmagro HJ, Dalmolin ÂC, Pereira WE, de Souza ÉC, Vourlitis GL, Rodríguez Ortíz CE. **2013**. Fitting net photosynthetic light-response curves with Microsoft Excel-a critical look at the models. *Photosynthetica* **51**: 445–456.
[Equation: 2.1]

Mott KA, Woodrow IE. **2000**. Modelling the role of Rubisco activase in limiting non-steady-state photosynthesis. *Journal of Experimental Botany* **51**: 399–406.
[Equations: 9.3, 9.4]

Poorter H, Villar R. 1997. The Fate of Acquired Carbon in Plants: Chemical Composition and Construction Costs. In: Bazzaz FA, Grace J, eds. Plant Resource Allocation. New York: Academic Press, 39–72.
[Equations: 1.1, 1.2, 1.3]

Poorter H. **1994**. Construction costs and payback time of biomass: a whole plant perspective. In: J. Roy and E. Garnier, ed. A Whole Plant Perspective on Carbon-Nitrogen Interactions. The Netherlands: SPB Academic Publishing, 111–127.
[Equation: 1.3]

Poorter H, Pepin S, Rijkers T, De Jong Y, Evans JR, Kö Rner C. **2006**. Construction costs, chemical composition and payback time of high-and low-irradiance leaves. *Journal of Experimental Botany* **57**: 355–371.
[Equations: 3.1, 4.1]

Salter WT, Merchant AM, Richards RA, Trethowan R, Buckley TN. **2019**. Rate of photosynthetic induction in fluctuating light varies widely among genotypes of wheat. *Journal of Experimental Botany* **70**: 2787–2796.
[Equations: 6.4, 6.5, 6.6, 6.7, 7.1, 7.2]

Taylor SH, Long SP. **2017**. Slow induction of photosynthesis on shade to sun transitions in wheat may cost at least 21% of productivity. *Philosophical Transactions of the Royal Society B* **372**: 20160543.
[Equations: 8.1, 9.1, 9.2]

Woodrow IE, Mott KA. **1989**. Rate Limitation of Non-steady-state Photosynthesis by Ribulose-1,5-bisphosphate Carboxylase in Spinach. *Australian Journal of Plant Physiology* **16**: 487–500.
[Equations: 8.1, 8.2, 8.3, 9.2]

Zhu X-G, Ort DR, Whitmarsh J, Long SP. **2004**. The slow reversibility of photosystem II thermal energy dissipation on transfer from high to low light may cause large losses in carbon gain by crop canopies: a theoretical analysis. *Journal of Experimental Botany* **55**: 1167–1175.
[Equations: 6.1, 6.2, 6.3]