

Supplementary Materials for “Accommodating time-varying
heterogeneity in risk estimation under the Cox model:
a transfer learning approach”

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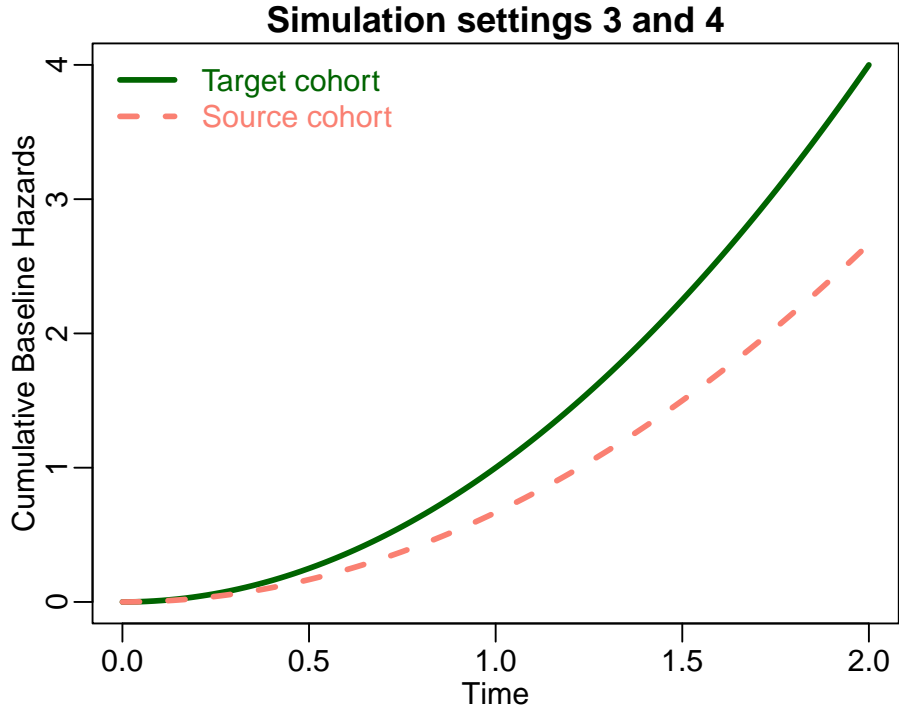


Figure S1: True cumulative baseline hazards for the simulation Settings 3 and 4.

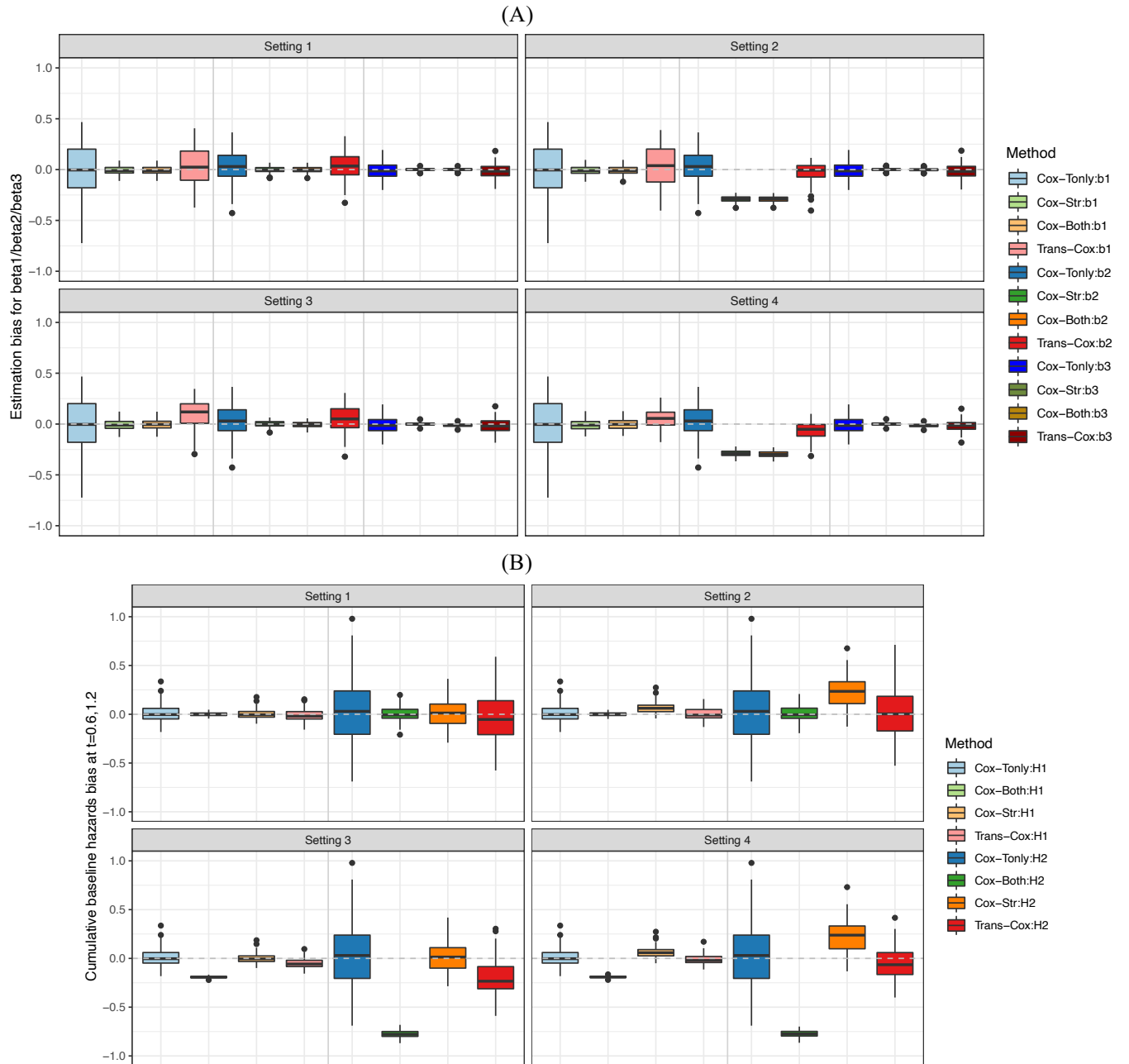


Figure S2: Estimation bias for coefficient β_1 , β_2 , and β_3 (panel A) and cumulative baseline hazards at time 0.6 and 1.2 (panel B) in simulation studies when the third covariate follows $N(0, 1)$ in the target cohort and $N(0.5, 1)$ in the source cohort. The sample size is 250 for the target cohort and 6400 for the source cohort. The dotted gray line shows the place where bias equals zero. Results are summarized over 100 Monte Carlo simulations.

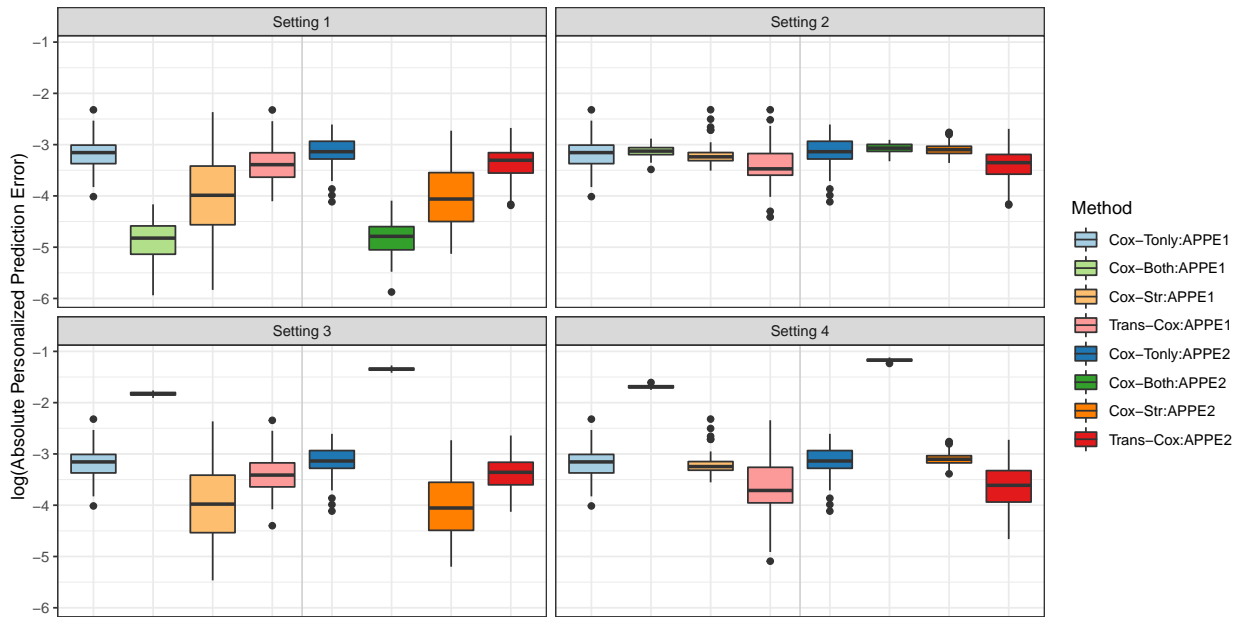


Figure S3: Boxplots for absolute personalized prediction error of Trans-Cox and other existing methods in different simulation settings when the third covariate follows $N(0, 1)$ in the target cohort and $N(0.5, 1)$ in the source cohort. The sample size is set as 250 for the target cohort and 6400 for the source cohort. The results are summarized over 100 Monte Carlo datasets.

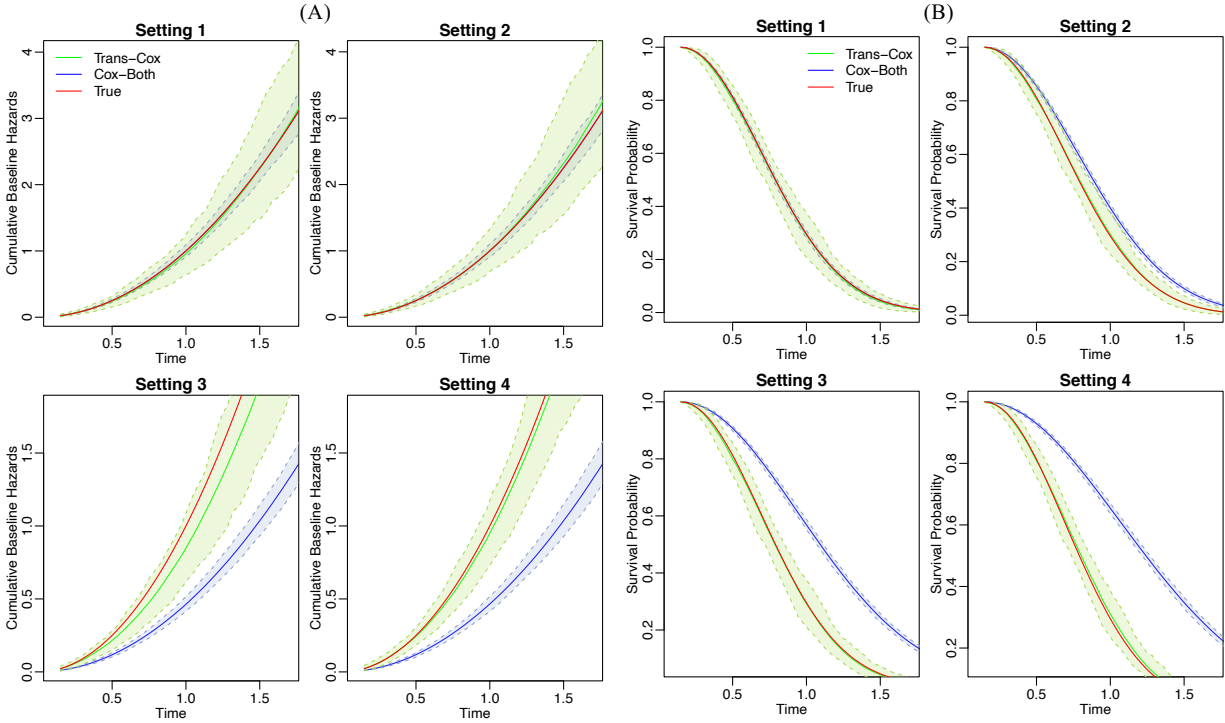


Figure S4: The cumulative baseline hazard (panel A) and the survival curves (panel B) for the proposed method (Trans-Cox, green) and the Cox proportional hazard model using the target/source combined data (Cox-B, blue) in comparison to the true results (red curves). In this simulation study, the third covariate follows $N(0, 1)$ in the target cohort and $N(0.5, 1)$ in the source cohort. The sample size is set as 250 for the target cohort and 6400 for the source cohort. For survival curves, the covariates are fixed at $\mathbf{X} = (0.5, 1, 0.1, 0.5, 0.5)$. The shaded area is the 95% confidence band area from bootstrap with 100 bootstrap iterations. Results are summarized over 100 Monte Carlo experiments.

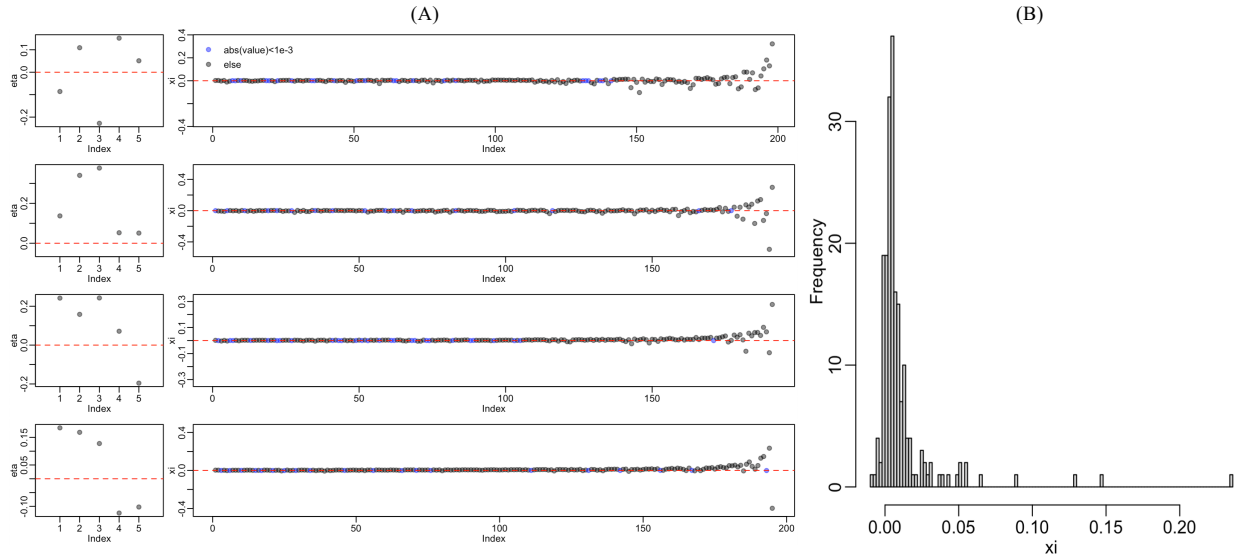


Figure S5: Scatter plot of the estimated values for η and ξ in the simulation study with sample size 250 in the target cohort and 6400 in the source cohort (A) when the third covariate follows $N(0, 1)$ in the target cohort and $N(0.5, 1)$ in the source cohort. From top to bottom, the panels correspond to Settings 1 to 4. Histogram plot to show the distribution of ξ in all four settings in this simulation study(B).

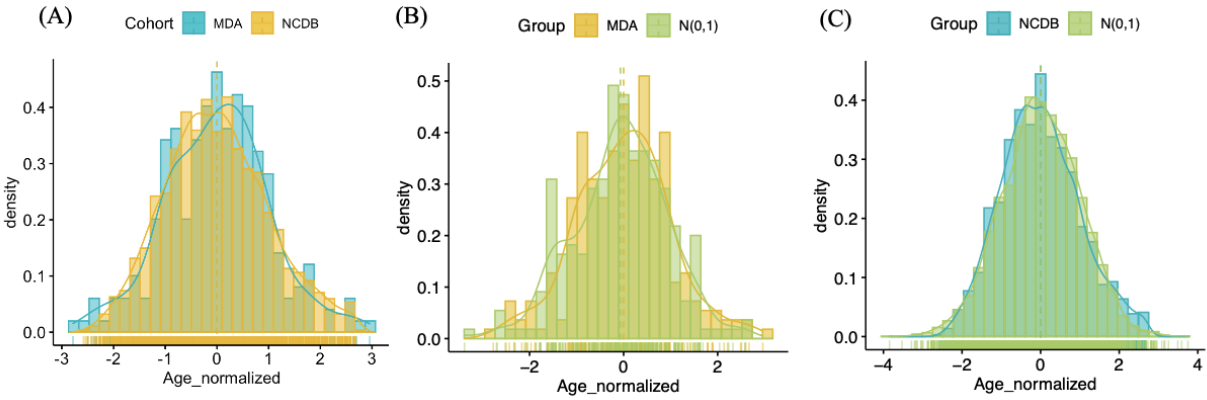


Figure S6: The histogram of normalized age (A), the comparison of normalized age in MDA versus standard normal distribution (B), and the comparison of normalized age in NCDB versus standard normal distribution (C) in the real data analysis.

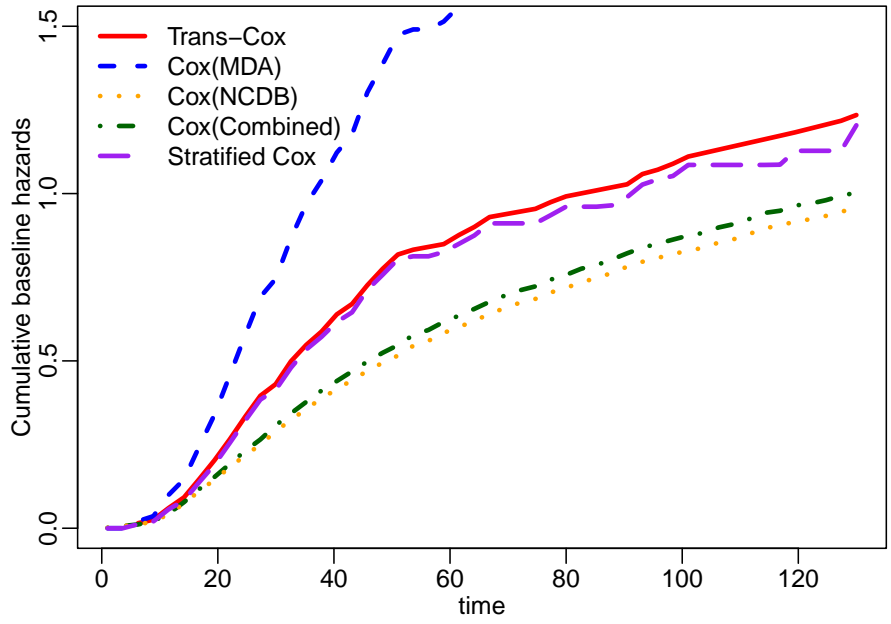


Figure S7: Cumulative baseline hazards estimated by Trans-Cox, Cox models with the target cohort only (MDA), with the source cohort only (NCDB), and with the combination of MDA and NCDB cohorts (Combined).

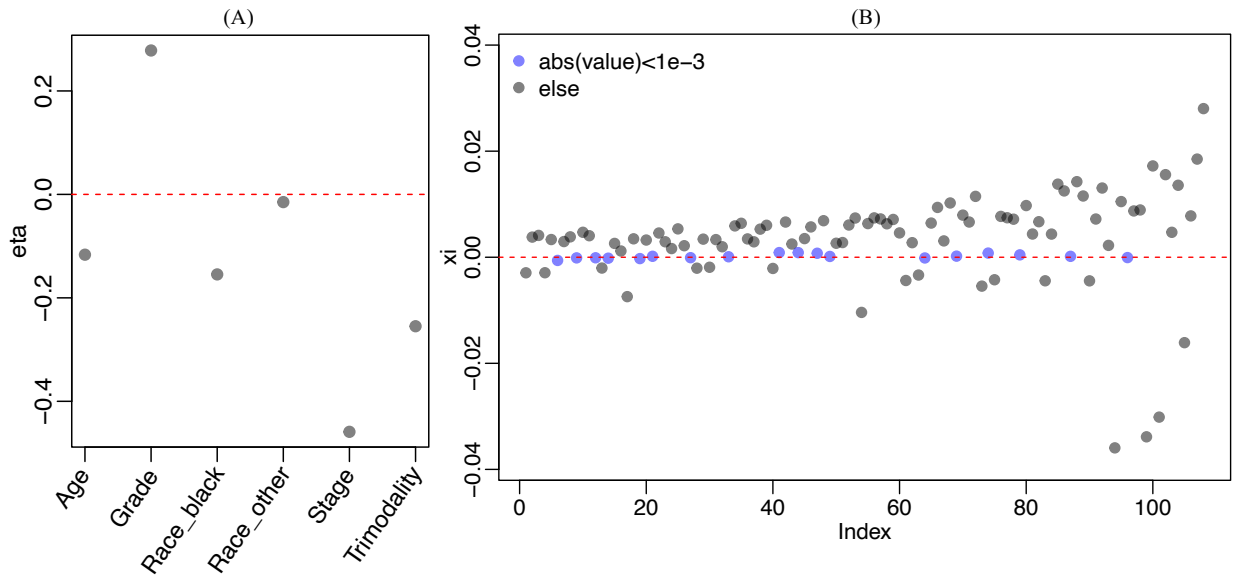


Figure S8: Scatterplot of the estimated values for η (A) and ξ (B) in the real data analysis.

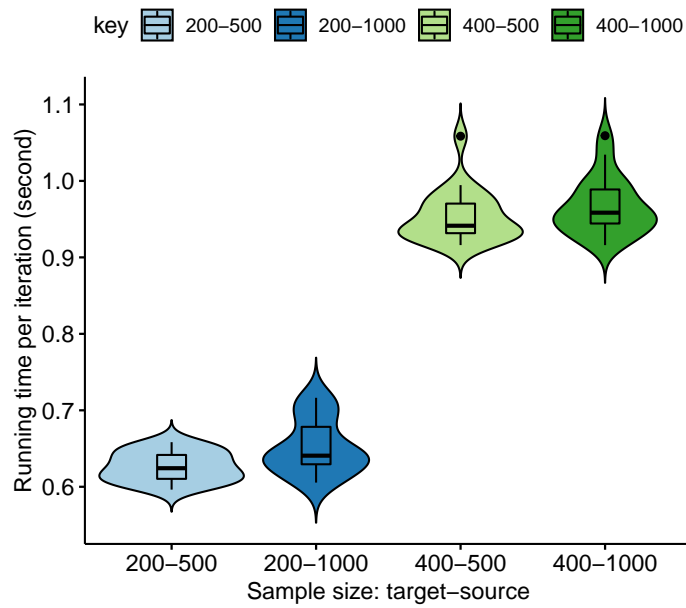


Figure S9: The computational cost evaluated using the simulation datasets with different sample sizes. The x axis shows the samples sizes in the target and source cohorts, e.g., 200-500 means 200 subjects in the target cohort and 500 subjects in the source cohort.

Table S1: Simulation results when the third covariate follows $Uniform(0, 1)$ in the target cohort and $Beta(1, 2)$ in the source cohort. The sample size is 250 for the target cohort and 6400 for the source cohort.

| | | Trans-Cox | | | Cox-Tonly | | | Cox-Both | | | Cox-Str | | |
|------------|-------------|-----------------|------------------|-----------------|--------------|-------------|-------------|-------------|--------------|-------------|--------------|--------------|-----------|
| | | $\beta_1 = 0.5$ | $\beta_2 = -0.5$ | $\beta_3 = 0.2$ | β_1 | β_2 | β_3 | β_1 | β_2 | β_3 | β_1 | β_2 | β_3 |
| Bias (SD) | S1 | 14.4(169.5) | 17.8(117.2) | 26(154.9) | -27.1(265.6) | 4.2(144) | -3.6(240.7) | 0.4(42.4) | -1.2(26.3) | 4.4(57) | 0.4(42.5) | -1.2(26.2) | 4.8(58) |
| | S2 | 13.3(170.3) | -50.2(88.6) | 18.3(157.8) | -27.1(265.6) | 4.2(144) | -3.6(240.7) | 0.7(42.5) | -291.4(25.5) | 20(57.2) | 0.5(43) | -290.5(25.4) | 5.9(58.3) |
| | S3 | 3.3(50.4) | 5.9(51.6) | 14.4(64.3) | -27.1(265.6) | 4.2(144) | -3.6(240.7) | 3.7(43) | -6.7(28.5) | 74.8(61.2) | -2.4(44.7) | -1.2(28.7) | 9(62.1) |
| | S4 | 54.2(93.6) | -82.3(82) | 52(89) | -27.1(265.6) | 4.2(144) | -3.6(240.7) | 4.5(43.2) | -296.8(27.9) | 83.9(63.9) | -2.8(44.9) | -287.8(28) | 9(64.9) |
| MSE | | β_1 | β_2 | β_3 | β_1 | β_2 | β_3 | β_1 | β_2 | β_3 | β_1 | β_2 | β_3 |
| | S1 | 28.9 | 14.1 | 24.6 | 71.3 | 20.8 | 57.9 | 1.8 | 0.692 | 3.27 | 1.81 | 0.69 | 3.39 |
| | S2 | 29.2 | 10.4 | 25.2 | 71.3 | 20.8 | 57.9 | 1.8 | 85.6 | 3.67 | 1.85 | 85 | 3.43 |
| | S3 | 2.55 | 2.7 | 4.34 | 71.3 | 20.8 | 57.9 | 1.87 | 0.856 | 9.35 | 2 | 0.826 | 3.94 |
| S4 | 11.7 | 13.5 | 10.6 | 71.3 | 20.8 | 57.9 | 1.89 | 88.8 | 11.1 | 2.03 | 83.6 | 4.3 | |
| Bias (SD) | | H(0.6)=0.36 | | H(1.2)=1.44 | | H(0.6) | | H(1.2) | | H(0.6) | | H(1.2) | |
| | S1 | -16.6(57.8) | | -62.6(233.9) | | 16.8(105.6) | | 88.6(434.5) | | -1.3(19.4) | | -6.8(75) | |
| | S2 | 3.2(63.7) | | 12.7(265.8) | | 16.8(105.6) | | 88.6(434.5) | | -2.9(19.7) | | -12.9(73.1) | |
| | S3 | -1.5(44.9) | | -12.5(161.8) | | 16.8(105.6) | | 88.6(434.5) | | -196.8(9.5) | | -796.7(35.1) | |
| S4 | -13.9(45.4) | | -75.6(171.5) | | 16.8(105.6) | | 88.6(434.5) | | -195.9(9.7) | | -797.2(35.9) | | |
| Bias (SD) | | RMST(0.6)=510 | | RMST(1.2)=706 | | RMST(0.6) | | RMST(1.2) | | RMST(0.6) | | RMST(1.2) | |
| | S1 | -0.4(11) | | -0.2(30.9) | | -0.4(12.4) | | -1.6(37.4) | | 0(2.3) | | -0.3(6.8) | |
| | S2 | 1.3(10.8) | | 6.3(29.9) | | -0.4(12.4) | | -1.6(37.4) | | 20.2(2) | | 79.9(6.9) | |
| | S3 | -0.7(10.1) | | -0.8(24.1) | | -0.4(12.4) | | -1.6(37.4) | | 45.5(1.6) | | 202.9(6.1) | |
| S4 | 3.5(9.8) | | 15.4(25.6) | | -0.4(12.4) | | -1.6(37.4) | | 55.9(1.3) | | 261.8(5.6) | | |
| Error (SD) | | APP(0.6) | | APP(1.2) | | APP(0.6) | | APP(1.2) | | APP(0.6) | | APP(1.2) | |
| | S1 | 34(13.9) | | 32.6(10.9) | | 42.6(13.9) | | 41.7(11.6) | | 8.8(3.2) | | 8.5(2.6) | |
| | S2 | 33.8(14.2) | | 32.3(11.5) | | 42.6(13.9) | | 41.7(11.6) | | 46.5(4.9) | | 43.9(3.6) | |
| | S3 | 26.3(17.3) | | 23.9(14.2) | | 42.6(13.9) | | 41.7(11.6) | | 169.5(5.7) | | 257(7.5) | |
| S4 | 30.6(15.4) | | 28.7(12.7) | | 42.6(13.9) | | 41.7(11.6) | | 194.7(6.5) | | 308.1(8.7) | | |

Bias, bias $\times 10^3$; SD, standard deviation $\times 10^3$; MSE, mean squared error $\times 10^3$; RMST, restricted mean survival time $\times 10^3$; APP, absolute personalized risk prediction $\times 10^3$.

S1 - S4: Setting 1 to Setting 4.

Table S2: Simulation results when the third covariate follows $N(0, 1)$ in the target cohort and $N(0.5, 1)$ in the source cohort. The sample size is 250 for the target cohort and 6400 for the source cohort.

| | | Trans-Cox | | | Cox-Tonly | | | Cox-Both | | | Cox-Str | | |
|------------|-------------|-----------------|------------------|-----------------|--------------|-------------|-------------|-------------|--------------|--------------|--------------|--------------|-----------|
| | | $\beta_1 = 0.5$ | $\beta_2 = -0.5$ | $\beta_3 = 0.2$ | β_1 | β_2 | β_3 | β_1 | β_2 | β_3 | β_1 | β_2 | β_3 |
| Bias (SD) | S1 | 29.6(186.3) | 36(135.4) | -11.9(67.8) | -13.1(280.7) | 33.1(152.2) | -10.4(71.7) | -7.7(46.9) | 0.3(29.2) | 0.4(13.8) | -7.8(47) | 0.2(29.1) | 0.4(13.9) |
| | S2 | 30.7(198.1) | -25.2(95) | -13.4(69) | -13.1(280.7) | 33.1(152.2) | -10.4(71.7) | -7.9(47.2) | -289.5(28.4) | -2.1(14.2) | -8.3(47.2) | -288.5(28.3) | 0.4(14.1) |
| | S3 | 98.7(141.8) | 53.1(128.9) | -15.8(67.4) | -13.1(280.7) | 33.1(152.2) | -10.4(71.7) | -2.8(52.8) | -5.3(28.7) | -12.8(15.4) | -8.7(53.1) | 0.8(29.1) | 0.4(15.2) |
| | S4 | 58.2(92.6) | -67.1(84.4) | -15.2(53.7) | -13.1(280.7) | 33.1(152.2) | -10.4(71.7) | -0.6(52.1) | -296(29) | -15.1(15.2) | -7.7(52.2) | -286.6(29.4) | 0.1(15.1) |
| MSE | | β_1 | β_2 | β_3 | β_1 | β_2 | β_3 | β_1 | β_2 | β_3 | β_1 | β_2 | β_3 |
| | S1 | 35.6 | 19.6 | 4.74 | 78.9 | 24.3 | 5.24 | 2.26 | 0.855 | 0.192 | 2.27 | 0.847 | 0.193 |
| | S2 | 40.2 | 9.67 | 4.94 | 78.9 | 24.3 | 5.24 | 2.29 | 84.6 | 0.206 | 2.3 | 84.1 | 0.2 |
| | S3 | 29.8 | 19.4 | 4.8 | 78.9 | 24.3 | 5.24 | 2.79 | 0.852 | 0.399 | 2.89 | 0.847 | 0.231 |
| S4 | 12 | 11.6 | 3.11 | 78.9 | 24.3 | 5.24 | 2.72 | 88.4 | 0.458 | 2.78 | 83 | 0.227 | |
| Bias (SD) | | H(0.6)=0.36 | | H(1.2)=1.44 | | H(0.6) | | H(1.2) | | H(0.6) | | H(1.2) | |
| | S1 | -12.9(64.2) | | -34(245.3) | | 12.8(101) | | 82.2(412.7) | | 0.5(19.7) | | 1(45.8) | |
| | S2 | 2.2(67.3) | | 20(255.5) | | 12.8(101) | | 82.2(412.7) | | 1.6(20) | | 64.8(53.6) | |
| | S3 | -51.9(51.1) | | -202.2(181.3) | | 12.8(101) | | 82.2(412.7) | | -191.4(10.9) | | -774.8(37.7) | |
| S4 | -12.9(50.8) | | -63.2(169) | | 12.8(101) | | 82.2(412.7) | | -190.3(10.8) | | -773.8(36.9) | | |
| Bias (SD) | | RMST(0.6)=510 | | RMST(1.2)=706 | | RMST(0.6) | | RMST(1.2) | | RMST(0.6) | | RMST(1.2) | |
| | S1 | -3.3(11.2) | | -5(29.6) | | -2.3(11.5) | | -3.2(31) | | -0.2(2.4) | | -0.4(6.2) | |
| | S2 | -1.4(10.5) | | 2.1(26.3) | | -2.3(11.5) | | -3.2(31) | | 19.8(2) | | 78.5(6.2) | |
| | S3 | -3.7(11.1) | | -5.8(29.1) | | -2.3(11.5) | | -3.2(31) | | 44.5(1.7) | | 198(5.9) | |
| S4 | 0(10.2) | | 8.5(26) | | -2.3(11.5) | | -3.2(31) | | 55.1(1.4) | | 257.2(5.5) | | |
| Error (SD) | | APP(0.6) | | APP(1.2) | | APP(0.6) | | APP(1.2) | | APP(0.6) | | APP(1.2) | |
| | S1 | 35.8(13.2) | | 36.8(11) | | 42.6(13.3) | | 44.9(12.4) | | 8.2(2.9) | | 8.3(2.6) | |
| | S2 | 35.4(13.7) | | 36(11.4) | | 42.6(13.3) | | 44.9(12.4) | | 44.1(4.6) | | 46.7(4.4) | |
| | S3 | 35(13.1) | | 35.8(11.1) | | 42.6(13.3) | | 44.9(12.4) | | 160.9(5.5) | | 260(6.8) | |
| S4 | 29.5(15.4) | | 29.2(12.2) | | 42.6(13.3) | | 44.9(12.4) | | 184.6(5.8) | | 310.3(7.3) | | |

Bias, bias $\times 10^3$; SD, standard deviation $\times 10^3$; MSE, mean squared error $\times 10^3$; RMST, restricted mean survival time $\times 10^3$; APP, absolute personalized risk prediction $\times 10^3$.

S1 - S4: Setting 1 to Setting 4.

Table S3: Simulation results to evaluate the bootstrap procedure for inference using sample size of source cohort $N_s = 500$.

| | | β_1 | | | β_2 | | | |
|-----------|----|-----------|-----------------|-------|------------|-----------------|-------|-----|
| | | Bias (SD) | SE | Cov | Bias (SD) | SE | Cov | |
| $N = 200$ | S1 | 290(1686) | 1445 | 95.55 | 165(1327) | 1069 | 94.13 | |
| | S2 | 226(1702) | 1462 | 95.38 | -419(1346) | 1136 | 90.56 | |
| | S3 | 290(1525) | 1377 | 93.97 | 59(1161) | 971.7 | 94.37 | |
| | S4 | 315(1515) | 1381 | 92.21 | -825(1195) | 979.1 | 89.98 | |
| | | | $H(0.6) = 0.36$ | | | $H(1.2) = 1.44$ | | |
| | | | Bias (SD) | SE | Cov | Bias (SD) | SE | Cov |
| | S1 | -36(611) | 601.1 | 94.84 | -45(1949) | 1824 | 95.45 | |
| | S2 | 102(631) | 634.5 | 94.18 | 477(2036) | 1916 | 93.27 | |
| | S3 | -21(592) | 578.5 | 95.38 | -52(1871) | 1770 | 95.68 | |
| | S4 | 168(631) | 614.7 | 93.83 | 629(1965) | 1844 | 93.23 | |
| | | β_1 | | | β_2 | | | |
| | | Bias (SD) | SE | Cov | Bias (SD) | SE | Cov | |
| $N = 400$ | S1 | 529(1281) | 1638 | 94.29 | 126(910) | 1220 | 94.99 | |
| | S2 | 465(1311) | 1296 | 93.86 | -354(980) | 900.5 | 92.25 | |
| | S3 | 522(1316) | 1354 | 91.84 | 111(850) | 922 | 94.96 | |
| | S4 | 642(1325) | 1253 | 89.35 | -471(920) | 837.9 | 91.46 | |
| | | | $H(0.6) = 0.36$ | | | $H(1.2) = 1.44$ | | |
| | | | Bias (SD) | SE | Cov | Bias (SD) | SE | Cov |
| | S1 | -97(435) | 3050 | 94.39 | -374(1452) | 4623 | 95.8 | |
| | S2 | 16(452) | 590 | 94.77 | 29(1495) | 1662 | 96.68 | |
| | S3 | -95(440) | 1067 | 92.65 | -392(1449) | 2224 | 93.35 | |
| | S4 | 9(467) | 452.5 | 94.37 | -38(1522) | 1490 | 95.88 | |

Bias, $(\hat{\phi} - \phi_0) \times 10^4$; SD, standard deviation $\times 10^4$; SE, standard error estimated using bootstrap-based variance estimation $\times 10^4$; Cov (%), coverage rates of 95% confidence interval.