

# Supplementary Materials for Geometric Framework for Statistical Analysis of Eye Tracking Heat Maps, with Application to a Tobacco Waterpipe Study

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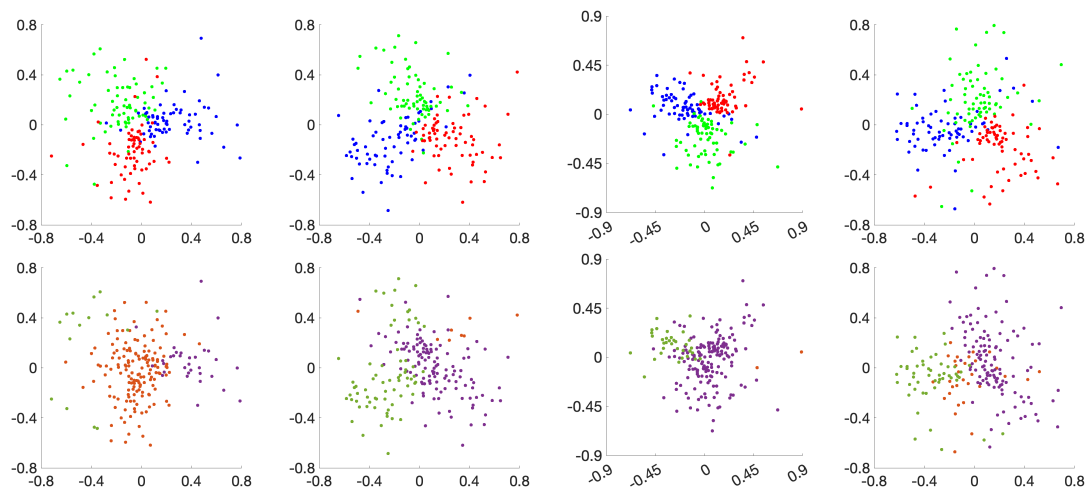
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## ARTICLE HISTORY

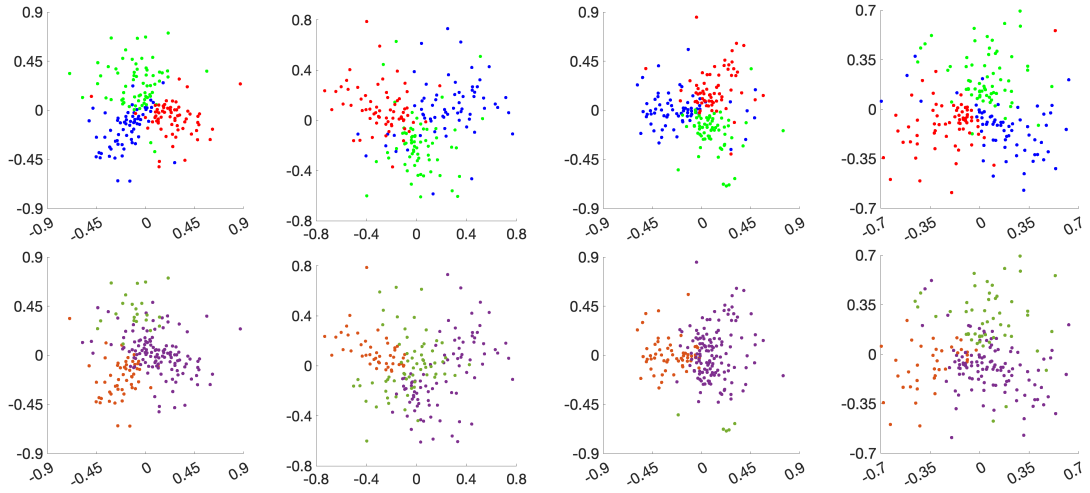
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### 1. Hierarchical Clustering of Heat Maps

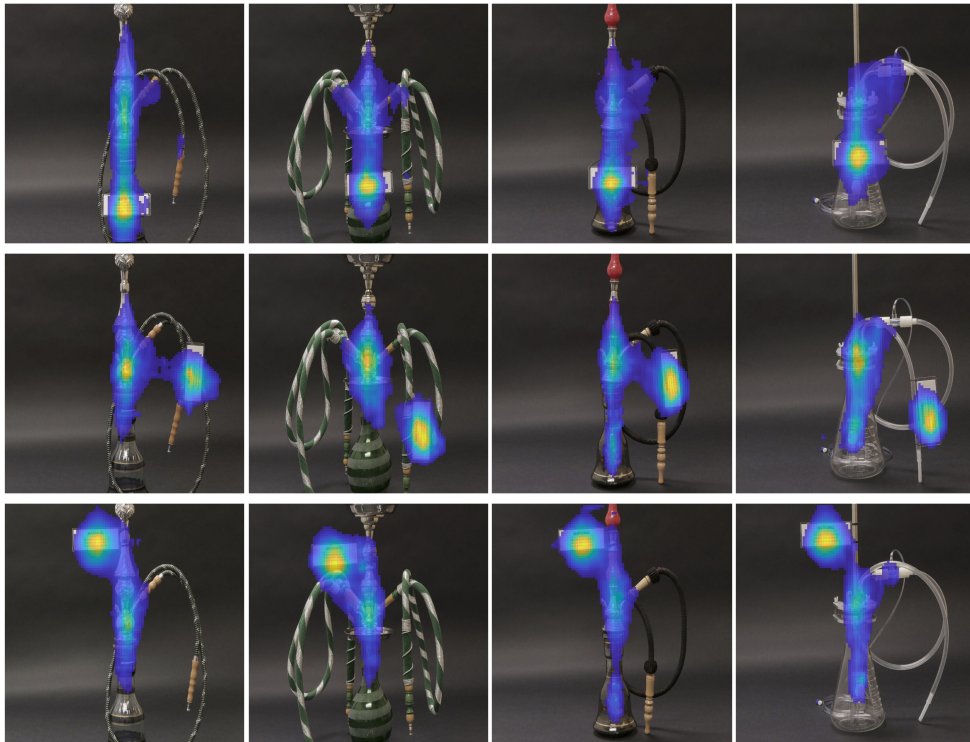
Figures 1 and 2 present the heat map data of the participants after multidimensional scaling, for waterpipes 1-4 (left to right), with colors indicating the placement of the *text* and *lung+text* warning labels (water bowl=blue, hose=green and stem=red), respectively. These figures complement the results presented in Section 3.2 in the main article; the patterns observed here are very similar to those seen in Figure 5 in the main article.



**Figure 1.** Heat map data after MDS, for waterpipes 1-4 (left to right), colored according to the placement (water bowl=blue, hose=green, stem=red) of the text label (top row), and the partitions estimated via hierarchical clustering (bottom row).



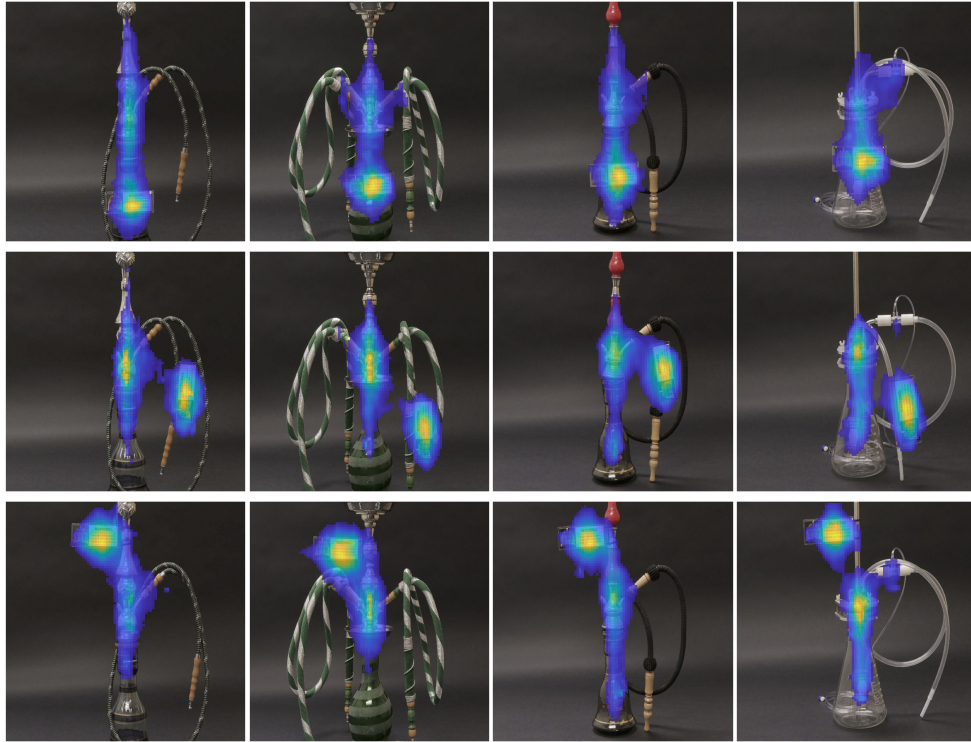
**Figure 2.** Heat map data after MDS, for waterpipes 1-4 (left to right), colored according to the placement (water bowl=blue, hose=green, stem=red) of the lung+text label (top row), and the partitions estimated via hierarchical clustering (bottom row).



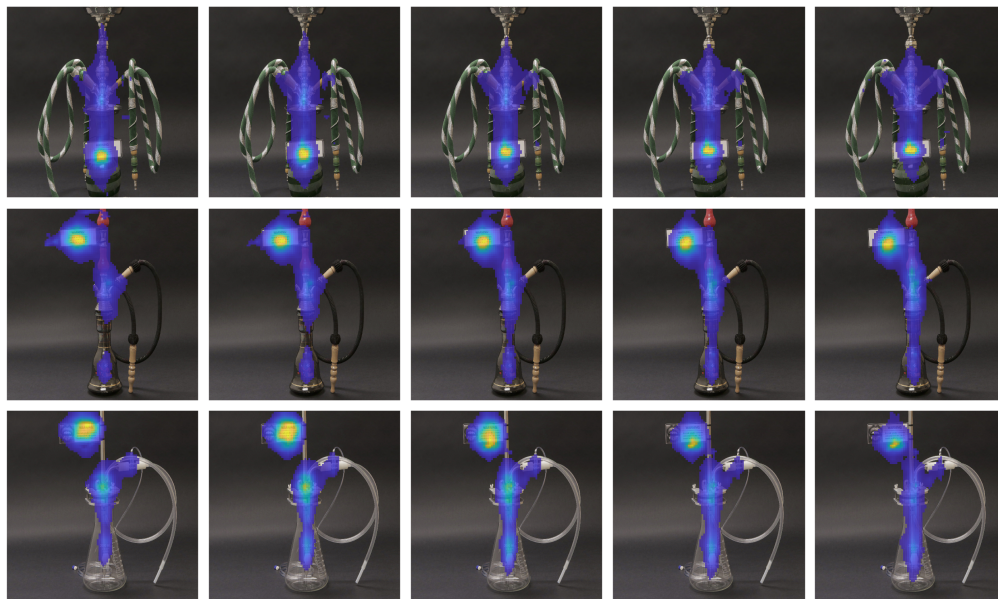
**Figure 3.** Karcher means of participants' heat maps for text only label. Waterpipes 1-4 are shown along the rows from left to right, respectively. Different label placements are shown along the columns with water bowl, hose and stem from top to bottom, respectively.

## 2. Sample Averaging of Heat Maps

Figures 3 and 4 display heat map Karcher means for the text and lung+text labels for waterpipes 1-4 (left to right), and water bowl, hose and stem label placements (top to bottom), respectively. These figures complement the results presented in Section 3.3



**Figure 4.** Karcher means of participants' heat maps for pipe images with lung+text label. Waterpipes 1-4 are shown along the rows from left to right, respectively. Label placements are shown along the columns with water bowl, hose and stem from top to bottom, respectively.



**Figure 5.** Second mode of variation for three cases: (i) waterpipe 2, text, water bowl (top), (ii) waterpipe 3, text, stem (middle), and (iii) waterpipe 4, mouth+text, stem (bottom).

in the main article; the patterns observed here are very similar to those seen in Figure 6 in the main article.

### 3. Assessment of Variability Among Heat Maps

Figure 5 provides a visual assessment of the second mode of variability in the heat map data for three different cases: (i) waterpipe 2, text label, water bowl placement (top row), (ii) waterpipe 3, text label, stem placement (middle row), and (iii) waterpipe 4, mouth+text label, stem placement (bottom row). The displayed paths were generated using Equation (8) and complement the results described in Section 3.4 in the main article. There appears to be much less variation along these second modes of variability as compared to the principal modes displayed in Figure 7 in the main article.