Supplementary material for the paper: Distribution-free Phase II triple EWMA control chart for joint monitoring the process location and scale parameters

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In the following lines, we compute the IC variance of the TL_j statistic.
The charting statistic TL_j is written as

$$TL_j = \frac{\lambda^3}{2}\sum_{i=1}^{j}(1-\lambda)^{j-i}(j-i+1)(j-i+2)L_j + (1-\lambda)^j\left[\lambda j(\lambda j+\lambda+2)+2\right].$$

Thus, we get

$$\begin{aligned} E\left(TL_j|\mathbf{X_m}, IC\right) &= \frac{\lambda^3}{2}\sum_{i=1}^{j}(1-\lambda)^{j-i}(j-i+1)(j-i+2)E\left(L_j|\mathbf{X_m}, IC\right) + \\ &\quad (1-\lambda)^j\left[\lambda j(\lambda j+\lambda+2)+2\right] \end{aligned}$$

and

$$Var\left(TL_j|\mathbf{X_m}, IC\right) = \frac{\lambda^6}{4}\sum_{i=1}^{j}(1-\lambda)^{2(j-i)}(j-i+1)^2(j-i+2)^2 Var\left(L_j|\mathbf{X_m}, IC\right).$$

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It is to be noted that

$$\begin{aligned}
& \frac{\lambda^3}{2}\sum_{i=1}^{j}(1-\lambda)^{j-i}(j-i+1)(j-i+2) \\
= & \frac{\lambda^3}{2}\sum_{u=1}^{j}(1-\lambda)^{u-1}u(u+1) = \frac{\lambda^3}{2}\left[2\sum_{u=1}^{j}u(1-\lambda)^{u-1} + (1-\lambda)\sum_{u=1}^{j}u(u-1)(1-\lambda)^{u-2}\right] \\
= & \frac{\lambda^3}{2}\left[2\left(\frac{1-(1-\lambda)^{j+1}}{\lambda^2} - \frac{(j+1)(1-\lambda)^j}{\lambda}\right) - (1-\lambda)\left(\frac{j(j+1)(1-\lambda)^{j-1}}{\lambda} + \right.\right. \\
& \left.\left.\frac{2(j+1)(1-\lambda)^j}{\lambda^2} - \frac{2(1-(1-\lambda)^{j+1}}{\lambda^3}\right)\right] \\
= & \lambda - \lambda(1-\lambda)^{j+1} - (j+1)\lambda^2(1-\lambda)^j - \frac{j(j+1)\lambda^2(1-\lambda)^j}{2} - (j+1)\lambda(1-\lambda)^{j+1} + \\
& 1-\lambda-(1-\lambda)^{j+2} \\
= & 1 - \frac{(1-\lambda)^j}{2}\left[2\lambda(1-\lambda) + 2(j+1)\lambda^2 + j(j+1)\lambda^2 + 2(j+1)\lambda(1-\lambda) + 2(1-\lambda)^2\right] \\
= & 1 - \frac{(1-\lambda)^j}{2}[\lambda j(\lambda j + \lambda + 2) + 2].
\end{aligned}$$

Thus,

$$\begin{aligned}
E\left(TL_j|\mathbf{X_m}, IC\right) = & \left[1 - \frac{(1-\lambda)^j}{2}[\lambda j(\lambda j + \lambda + 2) + 2]\right] E\left(L_j|\mathbf{X_m}, IC\right) + \\
& (1-\lambda)^j\left[\lambda j(\lambda j + \lambda + 2) + 2\right].
\end{aligned}$$

Taking $d = (1-\lambda)^2$, we have

$$\begin{aligned}
&\sum_{i=1}^{j}(1-\lambda)^{2(j-i)}(j-i+1)^2(j-i+2)^2 = \sum_{u=1}^{j} u^2(u+1)^2 d^{u-1} = \sum_{u=1}^{j}(u^4+2u^3+u^2)d^{u-1} \\
= \; & \sum_{u=1}^{j} u(u-1)(u-2)(u-3)d^{u-1} + 8\sum_{u=1}^{j} u(u-1)(u-2)d^{u-1} + 14\sum_{u=1}^{j} u(u-1)d^{u-1} + \\
& 4\sum_{u=1}^{j} ud^{u-1} \\
= \; & d^3\sum_{u=1}^{j} u(u-1)(u-2)(u-3)d^{u-4} + 8d^2\sum_{u=1}^{j} u(u-1)(u-2)d^{u-3} + 14d\sum_{u=1}^{j} u(u-1)d^{u-2} + \\
& 4\sum_{u=1}^{j} ud^{u-1} \\
= \; & d^3\left[-\frac{j(j^2-1)(j-2)d^{j-3}}{1-d} - \frac{4j(j^2-1)d^{j-2}}{(1-d)^2} - \frac{12j(j+1)d^{j-1}}{(1-d)^3} - \frac{24(j+1)d^j}{(1-d)^4} + \right. \\
& \left.\frac{24(1-d^{j+1})}{(1-d)^5}\right] + 8d^2\left[-\frac{j(j^2-1)d^{j-2}}{1-d} - \frac{3j(j+1)d^{j-1}}{(1-d)^2} - \frac{6(j+1)d^j}{(1-d)^3} + \right. \\
& \left.\frac{6(1-d^{j+1})}{(1-d)^4}\right] + 14d\left[-\frac{j(j+1)d^{j-1}}{1-d} - \frac{2(j+1)d^j}{(1-d)^2} + \frac{2(1-d^{j+1})}{(1-d)^3}\right] + \\
& 4\left[\frac{1-d^{j+1}}{(1-d)^2} - \frac{(j+1)d^j}{1-d}\right].
\end{aligned}$$

Therefore

$$\begin{aligned}
Var\left(TL_j|\mathbf{X_m}, IC\right) \; = \; & \left[\frac{d^3\lambda^6}{4}\left[-\frac{j(j^2-1)(j-2)d^{j-3}}{1-d} - \frac{4j(j^2-1)d^{j-2}}{(1-d)^2} - \frac{12j(j+1)d^{j-1}}{(1-d)^3} - \right.\right. \\
& \left.\frac{24(j+1)d^j}{(1-d)^4} + \frac{24\left(1-d^{j+1}\right)}{(1-d)^5}\right] + 2d^2\lambda^6\left[-\frac{j(j^2-1)d^{j-2}}{1-d} - \right. \\
& \left.\frac{3j(j+1)d^{j-1}}{(1-d)^2} - \frac{6(j+1)d^j}{(1-d)^3} + \frac{6\left(1-d^{j+1}\right)}{(1-d)^4}\right] + \\
& \frac{7d\lambda^6}{2}\left[-\frac{j(j+1)d^{j-1}}{1-d} - \frac{2(j+1)d^j}{(1-d)^2} + \frac{2(1-d^{j+1})}{(1-d)^3}\right] + \\
& \left.\lambda^6\left[\frac{1-d^{j+1}}{(1-d)^2} - \frac{(j+1)d^j}{1-d}\right]\right] Var\left(L_j|\mathbf{X_m}, IC\right).
\end{aligned}$$

From the above equations, we have that the IC variance of the TL_j statistic is

$$\begin{aligned}
Var(TL_j|IC) \; &= \; E\left[Var(TL_j|\mathbf{X_m}, IC)\right] + Var\left[E(TL_j|\mathbf{X_m}, IC)\right] \\
&= \; K_{T_j}\xi_1 + \left[1 - \frac{(1-\lambda)^j}{2}\left[\lambda j(\lambda j + \lambda + 2) + 2\right]\right]^2 \xi_2,
\end{aligned}$$

where

$$K_{T_j} = \left[\frac{d^3\lambda^6}{4}\left[-\frac{j(j^2-1)(j-2)d^{j-3}}{1-d} - \frac{4j(j^2-1)d^{j-2}}{(1-d)^2} - \frac{12j(j+1)d^{j-1}}{(1-d)^3} - \right.\right.$$
$$\left.\frac{24(j+1)d^j}{(1-d)^4} + \frac{24\left(1-d^{j+1}\right)}{(1-d)^5}\right] + 2d^2\lambda^6\left[-\frac{j(j^2-1)d^{j-2}}{1-d} - \right.$$
$$\left.\frac{3j(j+1)d^{j-1}}{(1-d)^2} - \frac{6(j+1)d^j}{(1-d)^3} + \frac{6\left(1-d^{j+1}\right)}{(1-d)^4}\right] +$$
$$\frac{7d\lambda^6}{2}\left[-\frac{j(j+1)d^{j-1}}{1-d} - \frac{2(j+1)d^j}{(1-d)^2} + \frac{2(1-d^{j+1})}{(1-d)^3}\right] +$$
$$\left.\lambda^6\left[\frac{1-d^{j+1}}{(1-d)^2} - \frac{(j+1)d^j}{1-d}\right]\right].$$

Table S1: IC run-length characteristics of the TL control chart when $ARL_0 \approx 500$

		time-varying UCL				steady-state UCL			
m	n	$\lambda = 0.05$	0.10	0.25	0.50	0.05	0.10	0.25	0.50
100	5	500.37 (1560.69)	498.53 (1251.60)	501.05 (953.59)	501.90 (811.95)	500.82 (1340.46)	500.66 (1133.47)	500.84 (922.71)	500.66 (800.83)
		1,3,65,329,2185	1,26,140,445,2040	3,68,210,547,1882	11,88,243,576,1853	25,56,150,404,1873	22,70,184,472,1872	18,85,224,547,1831	17,93,246,576,1829
100	10	498.99 (1670.61)	499.52 (1335.75)	500.93 (1008.15)	500.14 (841.89)	499.66 (1404.68)	500.99 (1202.82)	499.35 (956.12)	500.75 (825.33)
		1,2,35,266,2198	1,13,110,408,2110	1,54,188,523,1971	9,77,232,584,1871	20,42,122,368,1991	19,58,165,451,1923	16,72,206,527,1908	15,83,237,585,1866
100	15	501.09 (1797.40)	500.76 (1435.23)	500.17 (1072.17)	499.40 (885.08)	499.53 (1520.16)	499.84 (1242.44)	499.56 (1010.59)	499.59 (868.43)
		1,1,15,190,2289	1,6,74,364,2207	1,39,159,504,2044	7,65,209,561,1927	13,26,82,319,2110	16,44,138,427,2054	13,59,181,515,1979	12,72,215,562,1904
300	5	500.44 (987.63)	499.83 (789.49)	499.91 (673.52)	500.77 (618.65)	500.30 (795.96)	499.55 (700.84)	499.59 (653.84)	500.01 (606.75)
		1,27,186,550,1947	1,71,243,609,1817	8,101,284,640,1697	17,117,304,662,1622	38,105,252,566,1743	30,113,275,611,1698	24,116,291,633,1658	23,123,307,661,1599
300	10	499.96 (998.59)	500.08 (826.42)	499.91 (661.54)	499.87 (606.27)	499.71 (804.93)	500.55 (722.27)	499.86 (633.49)	499.62 (594.98)
		1,20,175,559,2015	1,64,233,605,1870	6,97,277,643,1720	16,113,295,654,1668	37,101,247,568,1779	29,107,269,607,1734	25,112,287,640,1698	22,119,298,652,1652
300	15	500.75 (1060.92)	500.06 (848.28)	500.98 (685.67)	500.69 (624.47)	499.96 (841.92)	500.97 (754.95)	500.48 (655.11)	500.21 (614.82)
		1,14,152,528,2100	1,56,224,591,1921	5,94,274,633,1731	15,108,298,655,1688	35,93,236,561,1772	29,102,262,598,1747	23,110,283,631,1682	22,115,301,649,1678

Table S2: Performance of the TL, DL and EL control charts under the $N(\theta, \delta)$ distribution with a time-varying UCL when $m = 100$, $n = 5$ and $ARL_0 \approx 500$

	TL				DL		EL	
	$\lambda = 0.05$	0.10	0.25	0.50	0.05	0.25	0.05	0.25
θ	$L = 0.648$	1.236	2.140	3.020	0.936	2.472	1.894	3.497
				$\delta = 1$				
0	500.37	498.53	501.05	501.90	501.24	500.06	498.15	501.96
	(1560.69)	(1251.60)	(953.59)	(811.95)	(1462.29)	(910.02)	(1211.95)	(824.66)
0.1	403.85	407.54	425.40	433.63	**394.76**	424.55	414.23	435.15
	(1394.06)	(1062.86)	(847.90)	(727.79)	(1236.13)	(789.98)	(1076.23)	(718.18)
0.25	**137.54**	162.34	196.58	215.86	143.83	196.26	164.06	215.78
	(664.63)	(562.25)	(479.48)	(399.16)	(615.63)	(431.52)	(543.13)	(396.93)
0.5	**12.46**	18.20	27.66	38.03	13.14	29.24	18.48	40.49
	(32.33)	(38.93)	(57.81)	(72.38)	(32.29)	(60.92)	(54.67)	(92.94)
1	**1.74**	2.32	2.97	3.56	1.82	3.00	2.34	3.84
	(1.73)	(2.42)	(2.75)	(3.35)	(1.64)	(2.71)	(2.03)	(3.78)
1.5	**1.09**	1.16	1.31	1.46	1.11	1.33	1.23	1.53
	(0.36)	(0.50)	(0.71)	(0.83)	(0.38)	(0.71)	(0.53)	(0.86)
2	**1.01**	1.02	1.04	1.08	**1.01**	1.05	1.03	1.10
	(0.09)	(0.14)	(0.22)	(0.29)	(0.11)	(0.24)	(0.19)	(0.33)
				$\delta = 1.1$				
0	**130.64**	161.14	192.67	212.19	136.94	198.78	156.47	210.27
	(491.64)	(445.34)	(353.80)	(343.28)	(475.71)	(370.14)	(413.91)	(331.50)
0.1	**111.26**	135.20	168.83	190.07	115.63	172.38	133.58	188.18
	(459.40)	(355.42)	(341.24)	(314.48)	(417.28)	(318.04)	(358.78)	(307.66)
0.25	**49.42**	65.33	89.68	108.36	52.68	93.67	63.82	109.62
	(185.20)	(163.33)	(187.38)	(192.95)	(180.87)	(186.96)	(151.52)	(201.96)
0.5	**9.87**	14.10	20.05	26.81	10.32	21.52	13.53	27.12
	(17.77)	(24.64)	(33.52)	(44.84)	(19.38)	(35.88)	(24.13)	(40.32)
1	**1.85**	2.39	3.13	3.63	1.93	3.13	2.40	3.85
	(1.90)	(2.43)	(2.95)	(3.39)	(1.79)	(2.88)	(2.11)	(3.65)
1.5	**1.12**	1.21	1.38	1.55	1.15	1.42	1.28	1.63
	(0.42)	(0.59)	(0.82)	(0.92)	(0.45)	(0.80)	(0.60)	(0.96)
2	**1.01**	1.03	1.07	1.11	1.02	1.08	1.05	1.14
	(0.13)	(0.18)	(0.28)	(0.36)	(0.15)	(0.30)	(0.23)	(0.40)
				$\delta = 1.25$				
0	**26.28**	36.91	52.79	67.75	27.51	56.06	35.37	67.08
	(61.09)	(68.80)	(88.94)	(102.19)	(59.72)	(92.87)	(68.45)	(98.09)
0.1	**24.28**	34.22	47.98	61.77	25.36	50.67	32.34	61.79
	(57.52)	(66.44)	(78.49)	(92.10)	(53.23)	(81.35)	(58.85)	(88.72)
0.25	**16.73**	23.24	32.49	42.70	17.08	34.67	21.77	43.10
	(34.26)	(40.50)	(49.64)	(62.85)	(34.88)	(54.59)	(38.00)	(60.94)
0.5	**6.97**	9.85	13.05	16.66	6.99	13.38	8.88	16.87
	(10.97)	(13.59)	(17.66)	(22.49)	(10.56)	(18.17)	(12.96)	(22.06)
1	**1.95**	2.45	3.19	3.63	1.98	3.17	2.42	3.81
	(1.98)	(2.48)	(3.02)	(3.33)	(1.84)	(2.95)	(2.14)	(3.50)
1.5	**1.16**	1.28	1.50	1.66	1.20	1.52	1.35	1.74
	(0.50)	(0.73)	(0.92)	(1.06)	(0.53)	90.93)	(0.70)	(1.09)
2	**1.03**	1.05	1.11	1.17	1.04	1.13	1.09	1.20
	(0.18)	(0.26)	(0.37)	(0.45)	(0.21)	(0.38)	(0.31)	(0.49)

Table S2: continued

	TL				DL		EL	
	$\lambda = 0.05$	0.10	0.25	0.50	0.05	0.25	0.05	0.25
θ	$L = 0.648$	1.236	2.140	3.020	0.936	2.472	1.894	3.497
				$\delta = 1.5$				
0	**7.30**	10.07	13.56	17.74	**7.30**	14.11	8.82	18.49
	(10.99)	(13.05)	(15.44)	(21.01)	(10.52)	(17.07)	(11.54)	(22.44)
0.1	**7.04**	9.74	13.18	17.36	7.05	13.63	11.30	21.61
	(10.44)	(12.22)	(15.02)	(20.53)	(9.94)	(16.14)	(11.30)	(21.61)
0.25	**6.14**	8.34	11.39	14.42	6.21	11.66	7.46	14.90
	(8.88)	(10.44)	(12.83)	(16.67)	(8.58)	(13.57)	(9.30)	(17.21)
0.5	4.32	5.86	7.67	9.12	**4.29**	7.65	5.17	9.29
	(5.99)	(7.04)	(8.42)	(10.10)	(5.51)	(8.51)	(5.96)	(10.22)
1	**1.92**	2.38	3.04	3.50	1.93	3.02	2.35	3.81
	(1.88)	(2.38)	(2.81)	(3.15)	(1.71)	(2.69)	(2.02)	(3.50)
1.5	**1.24**	1.39	1.63	1.82	1.27	1.65	1.45	1.74
	(0.67)	(0.92)	(1.15)	(1.22)	(0.67)	(1.08)	(0.83)	(1.09)
2	**1.06**	1.10	1.18	1.27	1.07	1.20	1.14	1.32
	(0.27)	(0.38)	(0.51)	(0.60)	(0.30)	(0.50)	(0.40)	(0.63)
				$\delta = 2$				
0	**2.53**	3.30	4.37	5.22	2.58	4.36	3.06	5.47
	(2.99)	(3.66)	(4.16)	(4.89)	(2.82)	(4.11)	(2.90)	(4.96)
0.1	**2.52**	3.29	4.36	5.20	2.54	4.35	3.05	5.46
	(2.94)	(3.59)	(4.15)	(4.78)	(2.71)	(4.09)	(2.90)	(4.97)
0.25	**2.44**	3.23	4.23	5.01	2.45	4.20	3.01	5.20
	(2.81)	(3.51)	(4.00)	(4.62)	(2.54)	(3.91)	(2.83)	(4.75)
0.5	**2.21**	2.87	3.71	4.36	2.23	3.69	2.72	4.51
	(2.43)	(3.09)	(3.50)	(3.91)	(2.27)	(3.36)	(2.46)	(3.99)
1	**1.64**	1.97	2.49	2.85	1.67	2.49	1.98	2.95
	(1.43)	(1.80)	(2.14)	(2.29)	(1.31)	(2.03)	(1.47)	(2.35)
1.5	**1.29**	1.45	1.71	1.92	1.31	1.73	1.50	1.99
	(0.78)	(1.01)	(1.24)	(1.32)	(0.74)	(1.17)	(0.88)	(1.31)
2	**1.11**	1.18	1.33	1.45	1.13	1.35	1.24	1.50
	(0.41)	(0.54)	(0.73)	(0.81)	(0.42)	(0.71)	(0.55)	(0.82)

Table S3: Performance of the TL, DL and EL control charts under the $N(\theta, \delta)$ distribution with a steady-state UCL when $m = 100$, $n = 5$ and $ARL_0 \approx 500$

	TL				DL		EL	
	$\lambda = 0.05$	0.10	0.25	0.50	0.05	0.25	0.05	0.25
θ	$L = 0.500$	1.161	2.114	3.011	0.816	2.455	1.838	3.484
				$\delta = 1$				
0	500.82	500.66	500.84	500.66	499.68	502.01	503.36	496.75
	(1340.46)	(1133.47)	(922.71)	(800.83)	(1281.16)	(891.05)	(1125.90)	(798.89)
0.1	411.34	418.96	429.48	433.80	408.40	428.43	420.83	430.92
	(1154.74)	(993.92)	(809.00)	(716.85)	(1102.78)	(766.79)	(996.09)	(703.62)
0.25	160.96	175.38	203.11	217.92	161.52	201.58	177.03	216.35
	(546.46)	(487.54)	(472.56)	(392.82)	(525.63)	(419.62)	(499.36)	(393.44)
0.5	30.26	29.41	31.98	39.91	25.26	32.97	25.51	41.02
	(34.69)	(39.92)	(57.15)	(67.64)	(32.85)	(61.05)	(53.07)	(70.21)
1	12.76	9.76	5.99	4.58	7.59	4.72	4.30	4.48
	(2.61)	(2.48)	(2.42)	(3.21)	(2.43)	(2.65)	(2.66)	(3.71)
1.5	9.08	6.66	3.69	2.25	4.72	2.54	2.05	1.86
	(1.18)	(1.06)	(0.85)	(0.85)	(1.01)	(0.82)	(0.95)	(0.99)
2	7.50	5.41	2.89	1.60	3.61	1.88	1.36	1.23
	(0.67)	(0.58)	(0.53)	(0.55)	(0.61)	(0.51)	(0.53)	(0.46)
				$\delta = 1.1$				
0	155.31	175.28	197.67	214.89	157.69	203.86	169.05	210.59
	(426.78)	(406.05)	(344.89)	(340.26)	(403.18)	(365.31)	(373.09)	(325.24)
0.1	137.02	151.85	175.39	192.94	134.97	177.33	146.48	188.77
	(397.90)	(326.00)	(327.69)	(313.46)	(339.94)	(310.89)	(330.95)	(303.82)
0.25	71.35	79.59	94.79	110.75	69.66	97.90	78.94	110.66
	(146.81)	(147.08)	(169.42)	(191.95)	(149.88)	(171.07)	(177.03)	(198.49)
0.5	26.37	24.64	24.57	28.58	21.12	24.71	19.21	29.46
	(17.06)	(22.85)	(34.47)	(43.54)	(20.13)	(36.18)	(22.58)	(47.72)
1	12.85	9.85	6.07	4.61	7.65	4.79	4.37	4.51
	(2.68)	(2.54)	(2.56)	(3.28)	(2.50)	(2.70)	(2.74)	(3.70)
1.5	9.27	6.81	3.80	2.35	4.86	2.63	2.16	1.96
	(1.25)	(1.14)	(0.92)	(0.93)	(1.08)	(0.91)	(1.03)	(1.08)
2	7.66	5.54	2.98	1.68	3.73	1.94	1.44	1.29
	(0.73)	(0.65)	(0.57)	(0.58)	(0.66)	(0.55)	(0.58)	(0.52)
				$\delta = 1.25$				
0	46.90	49.98	58.06	67.76	43.18	60.10	45.36	68.50
	(54.30)	(64.56)	(85.27)	(89.59)	(57.10)	(91.02)	(68.43)	(97.09)
0.1	44.43	46.51	53.53	64.09	40.47	55.11	41.90	63.56
	(51.99)	(57.23)	(78.00)	(89.81)	(55.39)	(81.31)	(60.75)	(88.66)
0.25	35.19	34.61	37.39	44.88	30.09	37.90	29.67	44.42
	(31.25)	(36.03)	(47.81)	(62.72)	(32.25)	(49.42)	(38.89)	(62.88)
0.5	22.14	19.32	16.85	18.32	16.49	16.22	13.77	18.26
	(10.93)	(11.90)	(16.46)	(23.09)	(10.84)	(18.03)	(12.98)	(23.39)
1	12.85	9.86	6.10	4.64	7.67	4.82	4.37	4.42
	(2.69)	(2.58)	(2.61)	(3.16)	(2.54)	(2.73)	(2.73)	(3.49)
1.5	9.52	7.02	3.94	2.48	5.06	2.75	2.31	2.10
	(1.34)	(1.22)	(1.02)	(1.06)	(1.18)	(1.02)	(1.14)	(1.22)
2	7.91	5.73	3.10	1.79	3.90	2.02	1.55	1.39
	(0.82)	(0.74)	(0.62)	(0.63)	(0.73)	(0.60)	(0.65)	(0.61)

Table S3: continued

	TL				DL		EL	
	$\lambda = 0.05$	0.10	0.25	0.50	0.05	0.25	0.05	0.25
θ	$L = 0.500$	1.161	2.114	3.011	0.816	2.455	1.838	3.484
				$\delta = 1.5$				
0	22.82	19.92	17.55	19.82	17.10	17.09	14.38	19.88
	(9.60)	(10.96)	(15.17)	(21.44)	(10.19)	(16.86)	(12.39)	(22.15)
0.1	22.54	19.69	17.31	19.28	16.83	16.93	14.07	19.30
	(9.35)	(10.81)	(15.40)	(21.29)	(9.99)	(17.46)	(12.07)	(21.53)
0.25	21.23	18.16	15.32	16.42	15.45	14.04	12.45	16.61
	(8.51)	(9.62)	(13.09)	(17.81)	(8.88)	(12.90)	(10.54)	(18.10)
0.5	17.86	14.61	10.91	10.56	12.17	9.83	8.81	10.40
	(5.62)	(6.14)	(7.85)	(10.19)	(5.85)	(8.35)	(7.07)	(10.57)
1	12.63	9.66	5.91	4.41	7.48	4.62	4.19	4.16
	(2.59)	(2.41)	(2.35)	(2.82)	(2.39)	(2.48)	(2.56)	(3.08)
1.5	9.83	7.29	4.13	2.63	5.30	2.92	2.47	2.26
	(1.46)	(1.33)	(1.14)	(1.19)	(1.30)	(1.15)	(1.27)	(1.35)
2	8.27	6.01	3.28	1.93	4.16	2.17	1.71	1.52
	(0.95)	(0.86)	(0.71)	(0.70)	(0.84)	(0.70)	(0.76)	(0.74)
				$\delta = 2$				
0	14.80	11.51	7.52	6.29	9.29	6.34	5.84	6.20
	(3.24)	(3.10)	(3.36)	(4.44)	(3.15)	(3.95)	(3.65)	(4.78)
0.1	14.76	11.49	7.50	6.22	9.27	6.31	5.81	6.17
	(3.23)	(3.09)	(3.34)	(4.30)	(3.15)	(3.85)	(3.59)	(4.76)
0.25	14.54	11.30	7.32	6.05	9.09	6.13	5.65	5.92
	(3.17)	(3.01)	(3.20)	(4.19)	(3.08)	(3.69)	(3.49)	(4.51)
0.5	13.81	10.67	6.79	5.39	8.47	5.50	5.07	5.21
	(2.88)	(2.74)	(2.89)	(3.63)	(2.78)	(3.14)	(3.07)	(3.86)
1	11.81	8.96	5.36	3.80	6.83	4.06	3.63	3.50
	(2.11)	(1.97)	(1.86)	(2.15)	(1.96)	(1.96)	(2.06)	(2.41)
1.5	10.04	7.48	4.27	2.76	5.46	3.03	2.59	2.41
	(1.50)	(1.38)	(1.21)	(1.27)	(1.35)	(1.23)	(1.35)	(1.50)
2	8.77	6.42	3.55	2.16	4.52	2.39	1.96	1.76
	(1.11)	(1.00)	(0.85)	(0.84)	(0.99)	(0.85)	(0.93)	(0.94)

Table S4: Performance of the TL, DL and EL control charts under the $L(\theta, \delta/\sqrt{2})$ distribution with a time-varying UCL when $m = 100$, $n = 5$ and $ARL_0 \approx 500$

	TL				DL		EL	
	$\lambda = 0.05$	0.10	0.25	0.50	0.05	0.25	0.05	0.25
θ	$L = 0.648$	1.236	2.140	3.020	0.936	2.472	1.894	3.497
				$\delta = 1$				
0	495.70	503.08	501.08	499.80	503.05	498.80	496.69	495.19
	(1559.88)	(1261.44)	(949.60)	(784.55)	(1472.20)	(881.88)	(1208.44)	(786.66)
0.1	**389.47**	395.71	421.77	432.30	394.81	422.36	401.94	428.98
	(1348.10)	(1061.51)	(844.30)	(743.13)	(1272.15)	(798.38)	(1054.16)	(722.28)
0.25	**97.72**	127.80	168.15	204.44	110.12	180.25	134.47	212.93
	(454.88)	(437.60)	(391.91)	(418.71)	(471.10)	(424.72)	(425.63)	(423.13)
0.5	**7.38**	11.13	17.25	26.59	7.78	19.32	11.22	29.19
	(19.16)	(24.91)	(38.18)	(64.01)	(20.12)	(46.88)	(27.06)	(73.45)
1	**1.29**	1.51	1.89	2.21	1.34	1.94	1.59	2.38
	(0.97)	(1.23)	(1.70)	(1.87)	(0.95)	(1.62)	(1.16)	(1.99)
1.5	**1.02**	1.05	1.11	1.18	1.04	1.13	1.09	1.23
	(0.17)	(0.25)	(0.39)	(0.49)	(0.20)	(0.40)	(0.31)	(0.53)
2	**1.00**	1.01	1.02	1.03	**1.00**	1.02	1.01	1.04
	(0.05)	(0.08)	(0.13)	(0.18)	(0.06)	(0.14)	(0.11)	(0.22)
				$\delta = 1.1$				
0	**203.42**	225.31	252.57	270.10	208.00	254.11	219.40	268.12
	(781.80)	(609.39)	(470.45)	(439.88)	(713.52)	(446.09)	(584.02)	(443.43)
0.1	**163.17**	186.41	215.08	236.19	168.17	218.80	180.31	232.99
	(679.10)	(548.43)	(416.61)	(412.31)	(642.58)	(406.74)	(480.03)	(394.93)
0.25	**50.34**	69.62	98.88	123.04	55.59	104.83	72.33	125.86
	(175.18)	(193.17)	(222.20)	(239.02)	(197.35)	(233.53)	(211.47)	(242.88)
0.5	**6.13**	9.08	14.36	20.70	6.85	15.68	9.44	22.27
	(11.66)	(17.52)	(28.03)	(43.19)	(15.70)	(33.45)	(20.05)	(46.20)
1	**1.33**	1.56	1.96	2.27	1.37	2.00	1.64	2.43
	(1.04)	(1.39)	(1.73)	(1.86)	(1.00)	(1.69)	(1.21)	(2.01)
1.5	**1.03**	1.06	1.14	1.22	1.05	1.16	1.10	1.27
	(0.21)	(0.30)	(0.44)	(0.54)	(0.24)	(0.46)	(0.34)	(0.58)
2	**1.00**	1.01	1.02	1.04	1.01	1.03	1.02	1.06
	(0.06)	(0.09)	(0.16)	(0.22)	(0.08)	(0.17)	(0.14)	(0.24)
				$\delta = 1.25$				
0	**53.38**	70.63	93.62	113.05	56.30	96.78	68.02	113.08
	(180.55)	(172.63)	(160.75)	(170.24)	(175.14)	(158.74)	(155.17)	(171.34)
0.1	**46.34**	62.02	83.98	101.88	48.73	88.07	59.42	101.64
	(163.47)	(168.18)	(161.53)	(162.31)	(151.82)	(165.33)	(131.03)	(157.21)
0.25	**22.50**	31.67	44.96	59.84	24.08	48.50	30.77	62.38
	(63.34)	(74.14)	(84.80)	(105.96)	(69.63)	(89.83)	(75.05)	(115.49)
0.5	**5.34**	7.66	10.82	14.92	5.49	11.44	7.14	15.69
	(9.61)	(14.48)	(17.37)	(26.47)	(10.05)	(19.34)	(13.29)	(26.72)
1	**1.36**	1.63	2.03	2.35	1.40	2.07	1.68	2.49
	(1.07)	(1.48)	(1.84)	(1.98)	(1.05)	(1.75)	(1.25)	(2.04)
1.5	**1.04**	1.08	1.18	1.28	1.06	1.20	1.13	1.34
	(0.25)	(0.35)	(0.52)	(0.62)	(0.27)	(0.52)	(0.39)	(0.68)
2	**1.01**	1.01	1.04	1.07	**1.01**	1.05	1.03	1.09
	(0.08)	(0.12)	(0.20)	(0.27)	(0.10)	(0.23)	(0.17)	(0.31)

Table S4: continued

	TL				DL		EL	
	$\lambda = 0.05$	0.10	0.25	0.50	0.05	0.25	0.05	0.25
θ	$L = 0.648$	1.236	2.140	3.020	0.936	2.472	1.894	3.497
				$\delta = 1.5$				
0	**13.48**	19.03	26.84	35.29	13.85	28.11	17.86	36.44
	(25.82)	(30.39)	(39.11)	(47.81)	(26.11)	(39.83)	(30.21)	(49.53)
0.1	**12.49**	17.69	24.78	33.24	12.86	26.23	16.66	34.23
	(22.66)	(28.46)	(36.33)	(46.80)	(23.70)	(38.25)	(29.55)	(47.87)
0.25	**8.87**	12.57	17.49	23.28	9.03	18.49	11.50	24.09
	(15.31)	(19.19)	(25.11)	(33.01)	(15.59)	(27.52)	(17.89)	(33.69)
0.5	**3.89**	5.42	7.38	9.46	3.95	7.57	5.06	10.01
	(5.98)	(7.28)	(9.19)	(12.27)	(5.69)	(9.83)	(6.70)	(13.15)
1	**1.39**	1.66	2.07	2.39	1.43	2.10	1.72	2.52
	(1.08)	(1.50)	(1.84)	(1.98)	(1.06)	(1.75)	(1.27)	(2.02)
1.5	**1.07**	1.13	1.24	1.35	1.09	1.27	1.17	1.42
	(0.31)	(0.45)	(0.63)	(0.72)	(0.34)	(0.63)	(0.46)	(0.77)
2	**1.01**	1.02	1.06	1.11	1.02	1.07	1.05	1.13
	(0.13)	(0.19)	(0.26)	(0.35)	(0.15)	(0.29)	(0.23)	(0.38)
				$\delta = 2$				
0	4.18	5.67	7.49	9.47	**4.12**	7.61	5.08	9.96
	(5.61)	(6.79)	(8.00)	(10.42)	(5.13)	(8.26)	(5.68)	(10.76)
0.1	4.04	5.48	7.29	9.10	**4.01**	7.36	4.94	9.60
	(5.47)	(6.52)	(7.63)	(9.80)	(4.98)	(7.88)	(5.43)	(10.14)
0.25	3.54	4.80	6.25	7.77	**3.52**	6.29	4.34	8.12
	(4.55)	(5.58)	(6.33)	(8.10)	(4.22)	(6.44)	(4.60)	(8.27)
0.5	2.55	3.30	4.31	5.23	**2.54**	4.33	3.09	5.51
	(3.16)	(3.79)	(4.33)	(5.27)	(2.85)	(4.34)	(3.13)	(5.42)
1	**1.37**	1.60	1.98	2.30	1.41	2.02	1.66	2.43
	(1.04)	(1.35)	(1.62)	(1.83)	(0.98)	(1.56)	(1.15)	(1.92)
1.5	**1.10**	1.17	1.32	1.46	1.12	1.35	1.24	1.52
	(0.38)	(0.54)	(0.75)	(0.85)	(0.40)	(0.72)	(0.56)	(0.88)
2	**1.03**	1.05	1.11	1.18	1.04	1.13	1.08	1.21
	(0.18)	(0.26)	(0.38)	(0.46)	(0.21)	(0.40)	(0.30)	(0.50)

Table S5: Performance of the TL, DL and EL control charts under the $L(\theta, \delta/\sqrt{2})$ distribution with a steady-state UCL when $m = 100$, $n = 5$ and $ARL_0 \approx 500$

	TL				DL		EL	
	$\lambda = 0.05$	0.10	0.25	0.50	0.05	0.25	0.05	0.25
θ	$L = 0.500$	1.161	2.114	3.011	0.816	2.455	1.838	3.484
				$\delta = 1$				
0	500.71	504.59	505.68	498.95	494.20	502.62	497.27	490.52
	(1341.24)	(1134.75)	(918.42)	(774.37)	(1239.40)	(855.57)	(1080.90)	(772.17)
0.1	391.15	400.83	422.93	432.30	391.42	423.76	405.00	425.77
	(1117.84)	(968.21)	(803.48)	(728.22)	(1066.24)	(775.58)	(967.63)	(709.15)
0.25	117.84	140.20	172.72	206.55	121.67	183.70	141.73	212.27
	(351.16)	(381.92)	(383.53)	(411.18)	(364.96)	(409.58)	(373.95)	(416.10)
0.5	23.26	21.08	20.96	28.24	17.88	21.93	16.51	30.54
	(21.86)	(23.40)	(37.96)	(64.84)	(23.73)	(46.48)	(27.99)	(74.26)
1	10.91	8.15	4.71	3.18	6.11	3.44	3.02	2.91
	(1.86)	(1.69)	(1.46)	(1.68)	(1.69)	(1.57)	(1.66)	(2.08)
1.5	8.31	6.04	3.29	1.95	4.18	2.20	1.67	1.50
	(0.93)	(0.82)	(0.62)	(0.61)	(0.78)	(0.56)	(0.71)	(0.69)
2	7.25	5.22	2.77	1.48	3.41	1.82	1.24	1.15
	(0.57)	(0.48)	(0.48)	(0.52)	(0.53)	(0.45)	(0.45)	(0.37)
				$\delta = 1.1$				
0	224.24	236.16	259.86	266.31	222.70	259.99	234.48	268.54
	(667.12)	(534.58)	(455.97)	(434.54)	(603.52)	(435.80)	(537.20)	(440.09)
0.1	181.54	200.60	221.20	232.78	182.94	224.46	193.87	233.25
	(544.45)	(509.96)	(404.85)	(402.96)	(518.76)	(399.66)	(449.11)	(392.70)
0.25	73.40	85.13	103.92	121.40	72.40	108.23	82.81	126.52
	(154.22)	(203.39)	(216.88)	(234.18)	(163.69)	(227.80)	(192.71)	(126.52)
0.5	21.93	19.45	18.08	20.55	16.41	18.26	14.46	23.47
	(14.82)	(19.51)	(28.64)	(42.53)	(16.20)	(33.64)	(21.99)	(46.16)
1	11.02	8.25	4.80	3.24	6.19	3.51	3.08	2.96
	(1.90)	(1.73)	(1.59)	(1.74)	(1.73)	(1.64)	(1.73)	(2.08)
1.5	8.43	6.13	3.35	1.99	4.26	2.24	1.73	1.50
	(0.97)	(0.86)	(0.66)	(0.65)	(0.81)	(0.60)	(0.75)	(0.69)
2	7.34	5.29	2.82	1.53	3.48	1.85	1.29	1.17
	(0.60)	(0.52)	(0.49)	(0.54)	(0.56)	(0.45)	(0.48)	(0.40)
				$\delta = 1.25$				
0	76.65	85.93	100.28	116.14	74.13	102.62	81.27	113.08
	(156.97)	(157.29)	(155.54)	(170.58)	(143.79)	(158.68)	(149.91)	(171.34)
0.1	68.80	75.93	90.18	104.72	66.01	92.88	70.75	102.79
	(149.85)	(152.21)	(159.05)	(160.82)	(136.99)	(161.64)	(126.61)	(154.89)
0.25	42.12	44.12	50.65	61.95	38.06	52.49	39.42	63.79
	(61.56)	(70.95)	(85.05)	(105.35)	(63.45)	(89.82)	(73.35)	(114.41)
0.5	20.09	17.17	14.43	16.33	14.45	14.00	11.76	16.88
	(10.55)	(13.61)	(18.25)	(26.99)	(11.34)	(20.94)	(15.62)	(26.83)
1	11.17	8.37	4.86	3.33	6.28	3.59	3.15	3.03
	(2.00)	(1.83)	(1.64)	(1.85)	(1.78)	(1.68)	(1.77)	(2.09)
1.5	8.59	6.26	3.43	2.05	4.37	2.31	1.80	1.63
	(1.02)	(0.90)	(0.71)	(0.69)	(0.87)	(0.66)	(0.80)	(0.81)
2	7.47	5.39	2.88	1.59	3.58	1.89	1.35	1.22
	(0.66)	(0.57)	(0.51)	(0.55)	(0.60)	(0.48)	(0.52)	(0.45)

Table S5: continued

	TL				DL		EL	
	$\lambda = 0.05$	0.10	0.25	0.50	0.05	0.25	0.05	0.25
θ	$L = 0.500$	1.161	2.114	3.011	0.816	2.455	1.838	3.484
				$\delta = 1.5$				
0	31.24	30.34	31.50	34.97	26.10	31.77	24.92	37.97
	(23.21)	(29.14)	(38.51)	(47.29)	(25.71)	(39.66)	(30.04)	(48.51)
0.10	29.97	28.80	29.52	35.29	24.77	29.69	23.60	35.72
	(20.48)	(27.09)	(36.21)	(46.56)	(22.53)	(37.88)	(28.91)	(47.67)
0.25	25.15	22.97	21.56	25.11	19.64	21.57	17.33	25.38
	(14.82)	(19.06)	(25.35)	(32.74)	(16.46)	(28.18)	(18.97)	(33.14)
0.50	17.58	14.37	10.83	10.83	11.93	9.83	8.71	10.96
	(6.05)	(6.62)	(9.18)	(12.45)	(6.22)	(9.79)	(7.69)	(12.93)
1.00	11.22	8.42	4.90	3.36	6.32	3.63	3.21	3.08
	(1.98)	(1.81)	(1.58)	(1.85)	(1.77)	(1.68)	(1.78)	(2.12)
1.50	8.81	6.44	3.54	2.14	4.53	2.41	1.92	1.72
	(1.09)	(0.98)	(0.78)	(0.77)	(0.94)	(0.76)	(0.87)	(0.88)
2.00	7.67	5.53	2.98	1.68	3.73	1.95	1.45	1.30
	(0.74)	(0.64)	(0.55)	(0.58)	(0.66)	(0.53)	(0.59)	(0.52)
				$\delta = 2$				
0	17.95	14.66	10.93	10.84	12.23	9.81	8.90	11.00
	(5.33)	(5.72)	(7.35)	(10.31)	(5.46)	(7.69)	(6.51)	(10.81)
0.10	17.75	14.45	10.70	10.43	12.03	9.60	8.67	10.65
	(5.20)	(5.52)	(7.01)	(9.69)	(5.31)	(7.57)	(6.25)	(10.23)
0.25	16.84	13.51	9.60	9.06	11.13	8.52	7.67	9.07
	(4.52)	(4.63)	(5.70)	(8.00)	(4.47)	(6.26)	(5.21)	(8.29)
0.50	14.68	11.48	7.51	6.33	9.20	6.29	5.76	6.24
	(3.43)	(3.44)	(3.80)	(5.08)	(3.39)	(4.16)	(3.87)	(5.33)
1.00	11.08	8.29	4.81	3.25	6.20	3.53	3.11	2.96
	(1.83)	(1.67)	(1.48)	(1.70)	(1.64)	(1.56)	(1.63)	(1.96)
1.50	9.10	6.67	3.70	2.27	4.74	2.53	2.07	1.87
	(1.16)	(1.06)	(0.87)	(0.86)	((1.02)	(0.85)	(0.94)	(1.00)
2.00	8.01	5.80	3.14	1.83	3.97	2.07	1.60	1.42
	(0.84)	(0.75)	(0.62)	(0.63)	(0.74)	(0.60)	(0.68)	(0.64)

Table S6: Performance of the TL, DL and EL control charts under the $SE(\theta, \delta)$ distribution with a time-varying UCL when $m = 100$, $n = 5$ and $ARL_0 \approx 500$

	TL				DL		EL	
	$\lambda = 0.05$	0.10	0.25	0.50	0.05	0.25	0.05	0.25
θ	$L = 0.648$	1.236	2.140	3.020	0.936	2.472	1.894	3.497
				$\delta = 1$				
0	495.70	503.08	501.75	499.80	504.04	498.80	496.69	495.19
	(1559.88)	(1261.44)	(949.60)	(784.55)	(1472.20)	(881.88)	(1208.44)	(786.66)
0.1	1491.40	1331.85	1096.00	**956.14**	1452.33	1052.89	1313.79	966.76
	(3582.55)	(2977.57)	(2180.56)	(1699.59)	(3385.46)	(2041.15)	(2880.36)	(1698.97)
0.25	**122.53**	127.19	157.47	220.09	125.98	168.08	139.46	248.48
	(969.56)	(775.96)	(630.13)	(545.39)	(925.01)	(558.01)	(792.95)	(571.66)
0.5	**2.41**	4.50	8.05	17.56	2.96	9.57	5.56	22.66
	(3.21)	(5.13)	(9.19)	(30.07)	(3.46)	(12.68)	(5.84)	(39.41)
1	**1.05**	1.35	1.97	2.72	1.17	2.13	1.66	3.29
	(0.22)	(0.73)	(1.46)	(2.42)	(0.42)	(1.56)	(1.05)	(3.31)
1.5	**1.00**	1.01	1.04	1.12	**1.00**	1.06	1.02	1.19
	(0.02)	(0.08)	(0.23)	(0.39)	(0.04)	(0.27)	(0.16)	(0.50)
2	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
	(0.00)	(0.00)	(0.02)	(0.04)	(0.00)	(0.03)	(0.00)	(0.06)
				$\delta = 1.1$				
0	**293.30**	314.82	333.81	345.25	297.23	336.68	304.80	341.05
	(1070.50)	(878.66)	(635.25)	(558.85)	(986.05)	(618.03)	(785.74)	(558.43)
0.1	580.45	540.35	492.89	457.36	570.53	479.14	517.00	**440.74**
	(2047.64)	(1651.33)	(1221.63)	(898.94)	(1900.96)	(1110.37)	(1510.34)	(858.43)
0.25	**45.70**	55.75	75.93	117.03	47.69	87.49	59.64	129.74
	(482.62)	(407.55)	(250.03)	(256.55)	(440.50)	(275.41)	(381.97)	(277.14)
0.5	**2.24**	4.03	6.94	13.58	2.71	8.07	4.89	16.87
	(2.79)	(4.35)	(7.19)	(21.02)	(2.95)	(10.50)	(4.83)	(24.92)
1	**1.04**	1.27	1.77	2.32	1.13	1.89	1.53	2.75
	(0.21)	(0.64)	(1.27)	(1.90)	(0.38)	(1.34)	(0.92)	(2.52)
1.5	**1.00**	**1.00**	1.03	1.09	**1.00**	1.05	1.02	1.15
	(0.01)	(0.07)	(0.18)	(0.33)	(0.03)	(0.24)	(0.13)	(0.43)
2	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
	(0.00)	(0.00)	(0.01)	(0.03)	(0.00)	(0.04)	(0.00)	(0.05)
				$\delta = 1.25$				
0	**87.91**	113.66	141.85	161.96	93.45	147.82	111.06	159.25
	(341.97)	(317.59)	(288.16)	(274.94)	(311.39)	(293.50)	(285.38)	(266.44)
0.1	**99.83**	111.15	134.15	149.39	102.73	137.00	111.45	150.23
	(566.60)	(403.28)	(321.89)	(282.66)	(531.49)	(309.21)	(415.69)	(287.16)
0.25	**13.48**	19.69	31.67	49.71	14.55	35.74	20.89	54.12
	(89.03)	(60.72)	(75.76)	(95.91)	(84.55)	(81.57)	(68.76)	(108.86)
0.5	**1.97**	3.36	5.52	9.09	2.35	6.09	3.98	10.82
	(2.14)	(3.45)	(5.34)	(12.52)	(2.30)	(6.61)	(3.65)	(13.73)
1	**1.03**	1.19	1.56	1.95	1.09	1.65	1.38	2.23
	(0.17)	(0.53)	(1.04)	(1.43)	(0.32)	(1.09)	(0.75)	(1.74)
1.5	**1.00**	**1.00**	1.02	1.07	**1.00**	1.03	1.01	1.11
	(0.00)	(0.06)	(0.14)	(0.29)	(0.03)	(0.19)	(0.11)	(0.36)
2	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
	(0.00)	(0.00)	(0.00)	(0.03)	(0.00)	(0.03)	(0.00)	(0.05)

Table S6: continued

	TL				DL		EL	
	$\lambda = 0.05$	0.10	0.25	0.50	0.05	0.25	0.05	0.25
θ	$L = 0.648$	1.236	2.140	3.020	0.936	2.472	1.894	3.497
				$\delta = 1.5$				
0	**17.13**	24.39	34.23	44.01	18.18	35.93	22.95	44.60
	(36.32)	(45.77)	(57.81)	(69.75)	(40.54)	(59.27)	(44.31)	(68.86)
0.1	**13.51**	19.44	27.27	35.60	14.14	28.97	18.37	35.75
	(31.73)	(39.94)	(48.15)	(58.19)	(33.47)	(51.25)	(38.08)	(56.60)
0.25	**5.32**	7.89	11.53	16.61	5.55	12.37	7.74	17.51
	(9.23)	(12.18)	(16.44)	(24.87)	(9.01)	(17.94)	(10.92)	(25.17)
0.5	**1.64**	2.52	3.83	5.33	1.90	4.06	2.90	6.08
	(1.50)	(2.36)	(3.53)	(5.57)	(1.59)	(3.67)	(2.42)	(6.50)
1	**1.02**	1.11	1.33	1.60	1.06	1.41	1.23	1.77
	(0.13)	(0.39)	(0.75)	(1.04)	(0.25)	(0.80)	(0.56)	(1.21)
1.5	**1.00**	**1.00**	1.01	1.04	**1.00**	1.02	1.01	1.07
	(0.00)	(0.04)	(0.11)	(0.21)	(0.02)	(0.13)	(0.08)	(0.27)
2	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
	(0.00)	(0.00)	(0.00)	(0.03)	(0.00)	(0.02)	(0.00)	(0.03)
				$\delta = 2$				
0	**4.02**	5.50	7.17	8.74	**4.02**	7.26	4.93	8.98
	(5.72)	(6.89)	(7.92)	(9.96)	(5.31)	(8.22)	(5.83)	(10.08)
0.1	**3.42**	4.70	6.18	7.45	3.47	6.19	4.34	7.70
	(4.58)	(5.70)	(6.63)	(8.25)	(4.32)	(6.76)	(4.78)	(8.54)
0.25	**2.30**	3.10	4.19	5.06	2.40	4.26	3.11	5.36
	(2.71)	(3.35)	(4.02)	(4.89)	(2.52)	(4.16)	(2.95)	(5.24)
0.5	**1.31**	1.71	2.32	2.81	1.44	2.39	1.91	3.07
	(0.83)	(1.32)	(1.84)	(2.24)	(0.92)	(1.86)	(1.38)	(2.52)
1	**1.01**	1.05	1.15	1.30	1.03	1.20	1.11	1.38
	(0.08)	(0.24)	(0.47)	(0.65)	(0.16)	(0.54)	(0.37)	(0.72)
1.5	**1.00**	**1.00**	**1.00**	1.02	**1.00**	1.01	**1.00**	1.03
	(0.00)	(0.03)	(0.07)	(0.13)	(0.00)	(0.08)	(0.06)	(0.18)
2	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
	(0.00)	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)	(0.00)	(0.02)

Table S7: Performance of the TL, DL and EL control charts under the $SE(\theta, \delta)$ distribution with a steady-state UCL when $m = 100$, $n = 5$ and $ARL_0 \approx 500$

	TL				DL		EL	
	$\lambda = 0.05$	0.10	0.25	0.50	0.05	0.25	0.05	0.25
θ	$L = 0.500$	1.161	2.114	3.011	0.816	2.455	1.838	3.484
				$\delta = 1$				
0	500.70	504.59	505.68	498.95	494.20	502.62	497.25	490.52
	(1341.24)	(1134.75)	(918.42)	(774.37)	(1239.40)	(855.57)	(1080.90)	(772.17)
0.1	1470.31	1291.33	1084.89	947.92	1401.46	1035.30	1271.01	952.93
	(3290.25)	(2795.54)	(2138.20)	(1670.71)	(3115.98)	(1980.78)	(2707.17)	(1675.27)
0.25	139.70	134.96	160.11	220.39	133.44	169.94	146.41	248.34
	(890.44)	(725.33)	(613.83)	(533.66)	(825.65)	(545.69)	(755.77)	(566.32)
0.5	17.84	14.44	11.07	18.49	11.99	11.44	9.59	23.54
	(3.78)	(4.31)	(8.57)	(29.79)	(4.00)	(13.18)	(6.26)	(39.08)
1	11.74	8.84	5.16	3.72	6.71	3.85	3.53	3.93
	(1.91)	(1.68)	(1.38)	(2.20)	(1.61)	(1.45)	(1.63)	(3.37)
1.5	8.68	6.33	3.43	2.09	4.43	2.29	1.82	1.56
	(1.00)	(0.84)	(0.60)	(0.46)	(0.77)	(0.53)	(0.64)	(0.69)
2	7.43	5.33	2.94	1.68	3.56	1.98	1.21	1.07
	(0.60)	(0.51)	(0.33)	(0.47)	(0.55)	(0.22)	(0.41)	(0.26)
				$\delta = 1.1$				
0	311.56	320.11	339.04	347.11	309.22	340.27	311.74	340.41
	(913.97)	(774.84)	(606.04)	(551.12)	(865.84)	(597.52)	(700.06)	(545.87)
0.1	579.61	536.45	491.66	457.68	559.69	481.53	515.62	440.41
	(1837.64)	(1530.07)	(1180.69)	(882.83)	(1715.71)	(1081.52)	(1433.47)	(856.63)
0.25	62.69	66.05	80.26	118.71	60.60	91.08	67.49	129.35
	(399.68)	(358.01)	(244.49)	(255.04)	(402.77)	(274.04)	(339.90)	(267.82)
0.5	17.08	13.71	10.04	14.50	11.29	9.88	8.63	17.73
	(3.52)	(3.96)	(6.67)	(20.64)	(3.58)	(10.13)	(5.44)	(24.60)
1	11.24	8.42	4.86	3.32	6.31	3.56	3.21	3.34
	(1.78)	(1.56)	(1.26)	(1.69)	(1.49)	(1.29)	(1.47)	(2.53)
1.5	8.50	6.19	3.34	2.09	4.30	2.22	1.73	1.48
	(0.94)	(0.80)	(0.55)	(0.46)	(0.71)	(0.47)	(0.61)	(0.64)
2	7.36	5.27	2.91	1.63	3.49	1.96	1.17	1.06
	(0.57)	(0.47)	(0.34)	(0.48)	(0.54)	(0.24)	(0.38)	(0.24)
				$\delta = 1.25$				
0	111.82	128.11	147.73	164.26	112.18	152.05	123.67	159.68
	(280.47)	(278.03)	(271.56)	(272.41)	(262.57)	(282.44)	(271.15)	(258.58)
0.1	121.24	125.34	140.97	151.26	119.60	141.83	121.21	150.15
	(512.93)	(387.15)	(333.76)	(279.61)	(484.49)	(303.99)	(393.78)	(277.90)
0.25	30.82	30.62	35.74	51.71	26.26	38.77	27.98	55.36
	(45.81)	(59.77)	(75.50)	(97.31)	(62.86)	(81.86)	(65.70)	(108.99)
0.5	15.88	12.55	8.61	10.03	10.19	7.88	7.20	11.71
	(3.18)	(3.26)	(4.53)	(12.27)	(3.12)	(6.30)	(4.25)	(13.60)
1	10.63	7.92	4.50	2.96	5.85	3.22	2.82	2.76
	(1.60)	(1.40)	(1.12)	(1.27)	(1.32)	(1.12)	(1.26)	(1.78)
1.5	8.27	6.00	3.24	1.96	4.16	2.15	1.62	1.38
	(0.87)	(0.74)	(0.50)	(0.43)	(0.67)	(0.41)	(0.59)	(0.56)
2	7.26	5.20	2.87	1.57	3.41	1.94	1.13	1.04
	(0.52)	(0.42)	(0.37)	(0.50)	(0.52)	(0.27)	(0.34)	(0.20)

Table S7: continued

	TL				DL		EL	
	$\lambda = 0.05$	0.10	0.25	0.50	0.05	0.25	0.05	0.25
θ	$L = 0.500$	1.161	2.114	3.011	0.816	2.455	1.838	3.484
				$\delta = 1.5$				
0	35.67	36.20	39.05	46.09	30.99	39.71	31.00	45.92
	(35.78)	(47.42)	(57.54)	(70.20)	(38.46)	(60.13)	(46.07)	(68.14)
0.1	31.36	30.54	31.80	37.40	26.15	32.26	25.41	37.08
	(45.13)	(42.54)	(49.57)	(57.29)	(45.21)	(51.39)	(40.25)	(56.22)
0.25	20.31	17.35	14.95	17.99	14.63	14.73	12.32	18.70
	(8.58)	(11.22)	(16.38)	(24.70)	(9.30)	(17.99)	(12.31)	(25.52)
0.5	14.09	10.91	6.93	6.26	8.65	5.80	5.44	6.82
	(2.67)	(2.55)	(2.77)	(5.40)	(2.50)	(3.43)	(3.05)	(6.47)
1	9.88	7.29	4.08	2.57	5.28	2.84	2.41	2.25
	(1.38)	(1.20)	(0.95)	(0.94)	(1.12)	(0.91)	(1.02)	(1.29)
1.5	7.97	5.75	3.12	1.87	3.94	2.07	1.49	1.28
	(0.77)	(0.67)	(0.42)	(0.43)	(0.61)	(0.33)	(0.55)	(0.49)
2	7.13	5.11	2.79	1.47	3.29	1.89	1.09	1.03
	(0.46)	(0.34)	(0.42)	(0.50)	(0.46)	(0.32)	(0.29)	(0.16)
				$\delta = 2$				
0	17.39	14.15	10.49	9.99	11.78	9.44	8.44	9.84
	(5.59)	(5.89)	(7.51)	(9.84)	(5.82)	(8.17)	(6.74)	(9.88)
0.1	16.36	13.14	9.33	8.60	10.82	8.20	7.46	8.48
	(4.76)	(4.96)	(6.04)	(8.15)	(4.92)	(6.63)	(5.72)	(8.31)
0.25	14.30	11.14	7.25	6.05	8.89	6.02	5.55	6.08
	(3.30)	(3.23)	(3.56)	(4.69)	(3.18)	(3.93)	(3.62)	(5.22)
0.5	11.70	8.83	5.22	3.73	6.72	3.94	3.58	3.64
	(1.95)	(1.79)	(1.60)	(2.09)	(1.77)	(1.76)	(1.90)	(2.54)
1	8.92	6.52	3.58	2.18	4.61	2.42	1.95	1.74
	(1.08)	(0.94)	(0.73)	(0.67)	(0.89)	(0.68)	(0.81)	(0.85)
1.5	7.56	5.43	2.98	1.71	3.67	1.97	1.32	1.16
	(0.64)	(0.56)	(0.37)	(0.48)	(0.58)	(0.31)	(0.49)	(0.38)
2	6.97	5.03	2.64	1.33	3.15	1.79	1.05	1.01
	(0.40)	(0.25)	(0.48)	(0.47)	(0.36)	(0.41)	(0.22)	(0.11)

Table S8: Performance of the TL, DL and EL control charts under the $Gumbel(\theta, \delta)$ distribution with a time-varying UCL when $m = 100$, $n = 5$ and $ARL_0 \approx 500$

	TL				DL		EL	
	$\lambda = 0.05$	0.10	0.25	0.50	0.05	0.25	0.05	0.25
θ	$L = 0.648$	1.236	2.140	3.020	0.936	2.472	1.894	3.497
				$\delta = 1$				
0	495.70	503.08	501.75	499.80	503.05	498.80	496.69	495.19
	(1559.88)	(1261.44)	(949.60)	(784.55)	(1472.20)	(881.88)	(1208.44)	(786.66)
0.1	608.35	592.73	579.07	571.63	601.78	569.12	582.97	**563.33**
	(1874.52)	(1486.65)	(1127.43)	(951.23)	(1722.57)	(1046.90)	(1382.66)	(896.68)
0.25	**368.19**	386.73	410.79	433.69	382.66	415.65	394.90	441.18
	(1425.38)	(1181.14)	(927.55)	(808.35)	(1368.41)	(898.69)	(1156.90)	(816.88)
0.5	**39.60**	55.81	85.99	122.86	45.11	95.69	61.32	134.65
	(283.77)	(296.07)	(258.39)	(260.63)	(303.67)	(252.55)	(289.56)	(299.84)
1	**2.58**	3.98	6.04	9.13	2.84	6.54	4.21	10.36
	(3.40)	(4.69)	(6.92)	(14.10)	(3.33)	(8.08)	(4.59)	(16.14)
1.5	**1.17**	1.43	1.91	2.34	1.26	1.99	1.62	2.61
	(0.59)	(0.99)	(1.48)	(1.86)	(0.67)	(1.50)	(1.05)	(2.17)
2	**1.01**	1.04	1.12	1.24	1.02	1.16	1.09	1.33
	(0.11)	(0.23)	(0.42)	(0.57)	(0.15)	(0.46)	(0.33)	(0.65)
				$\delta = 1.1$				
0	**145.30**	170.89	205.44	226.06	153.80	208.71	171.55	225.10
	(560.06)	(448.18)	(375.70)	(362.58)	(528.41)	(362.55)	(457.94)	(353.50)
0.1	**181.27**	204.24	237.14	255.64	188.14	241.96	200.86	253.81
	(715.99)	(574.64)	(451.66)	(418.83)	(678.23)	(443.78)	(510.67)	(412.22)
0.25	**135.28**	155.95	192.49	212.26	138.84	199.44	160.49	213.75
	(613.78)	(460.25)	(430.16)	(378.96)	(529.88)	(432.50)	(473.32)	(380.64)
0.5	**22.43**	36.71	55.97	77.67	22.47	61.37	38.45	81.85
	(105.51)	(106.70)	(146.92)	(157.56)	(98.80)	(144.04)	(117.96)	(168.88)
1	**2.65**	3.91	5.65	7.94	2.88	5.94	4.07	8.87
	(3.44)	(4.49)	(6.18)	(10.66)	(3.43)	(6.83)	(4.50)	(12.15)
1.5	**1.21**	1.48	1.96	2.36	1.30	2.01	1.65	2.58
	(0.70)	(1.08)	(1.55)	(1.90)	(0.75)	(1.54)	(1.10)	(2.10)
2	**1.01**	1.06	1.16	1.29	1.03	1.19	1.12	1.37
	(0.14)	(0.28)	(0.49)	(0.64)	(0.20)	(0.51)	(0.38)	(0.71)
				$\delta = 1.25$				
0	**31.44**	43.70	62.13	77.47	33.71	65.48	42.57	79.15
	(76.08)	(81.55)	(100.43)	(109.33)	(76.14)	(101.79)	(82.72)	(113.45)
0.1	**35.27**	49.35	69.84	86.15	37.95	72.65	47.74	87.52
	(94.98)	(102.86)	(131.15)	(128.82)	(96.47)	(118.96)	(97.06)	(137.25)
0.25	**31.19**	43.49	61.21	77.40	32.97	64.22	41.88	78.30
	(86.22)	(98.53)	(115.94)	(128.01)	(88.99)	(111.94)	(93.86)	(131.31)
0.5	**13.52**	19.51	28.50	38.21	14.20	30.34	18.90	39.40
	(32.89)	(40.09)	(53.06)	(66.71)	(33.35)	(56.17)	(38.47)	(66.69)
1	**2.65**	3.73	5.18	6.73	2.79	5.25	3.73	7.26
	(3.39)	(4.38)	(5.52)	(8.12)	(3.30)	(5.65)	(4.02)	(8.80)
1.5	**1.28**	1.56	2.00	2.38	1.35	2.05	1.69	2.56
	(0.85)	(1.22)	(1.62)	(1.91)	(0.84)	(1.57)	(1.16)	(2.06)
2	**1.03**	1.09	1.21	1.35	1.05	1.24	1.16	1.43
	(0.22)	(0.37)	(0.58)	(0.71)	(0.27)	(0.59)	(0.45)	(0.78)

Table S8: continued

	TL				DL		EL	
	$\lambda = 0.05$	0.10	0.25	0.50	0.05	0.25	0.05	0.25
θ	$L = 0.648$	1.236	2.140	3.020	0.936	2.472	1.894	3.497
				$\delta = 1.5$				
0	**8.62**	12.06	16.44	21.73	8.67	17.13	10.92	22.45
	(13.64)	(16.07)	(20.60)	(27.15)	(13.39)	(21.56)	(15.23)	(27.96)
0.1	**8.93**	12.40	17.15	22.62	9.06	17.95	11.31	23.41
	(14.45)	(17.15)	(22.71)	(29.14)	(14.38)	(23.64)	(16.35)	(30.48)
0.25	**8.44**	11.89	16.25	21.45	8.51	17.02	10.77	22.01
	(13.64)	(16.98)	(21.54)	(28.51)	(13.59)	(22.72)	(15.60)	(29.10)
0.5	**6.10**	8.47	11.53	14.85	6.11	11.95	7.73	15.24
	(9.83)	(11.88)	(14.62)	(19.41)	(9.41)	(15.87)	(10.77)	(19.95)
1	**2.43**	3.22	4.31	5.09	2.50	4.31	3.17	5.44
	(2.92)	(3.60)	(4.36)	(5.05)	(2.77)	(4.35)	(3.19)	(5.65)
1.5	**1.36**	1.63	2.05	2.36	1.41	2.07	1.71	2.49
	(1.00)	(1.34)	(1.69)	(1.86)	(0.98)	(1.60)	(1.20)	(1.96)
2	**1.07**	1.15	1.29	1.44	1.10	1.33	1.21	1.53
	(0.35)	(0.51)	(0.72)	(0.82)	(0.39)	(0.71)	(0.54)	(0.89)
				$\delta = 2$				
0	2.90	3.87	5.02	6.04	**2.87**	5.02	3.56	6.33
	(3.59)	(4.44)	(4.96)	(5.96)	(3.28)	(4.92)	(3.59)	(6.15)
0.1	2.92	3.87	5.05	6.07	**2.89**	5.05	3.57	6.36
	(3.62)	(4.46)	(5.04)	(6.08)	(3.31)	(5.06)	(3.66)	(6.29)
0.25	**2.77**	3.73	4.88	5.87	2.84	4.90	3.51	6.15
	(3.46)	(4.33)	(4.88)	(5.80)	(3.24)	(4.92)	(3.61)	(6.03)
0.5	**2.55**	3.35	4.35	5.17	2.56	4.34	3.16	5.51
	(3.06)	(3.84)	(4.36)	(4.97)	(2.85)	(4.31)	(3.12)	(5.40)
1	**1.87**	2.32	2.99	3.42	1.89	2.97	2.28	3.57
	(1.89)	(2.33)	(2.79)	(3.06)	(1.71)	(2.67)	(1.92)	(3.10)
1.5	**1.39**	1.62	1.96	2.22	1.43	1.96	1.66	2.29
	(1.01)	(1.31)	(1.56)	(1.66)	(0.96)	(1.47)	(1.10)	(1.67)
2	**1.13**	1.23	1.42	1.56	1.16	1.44	1.30	1.63
	(0.48)	(0.68)	(0.88)	(0.96)	(0.51)	(0.84)	(0.66)	(0.99)

Table S9: Performance of the TL, DL and EL control charts under the $Gumbel(\theta, \delta)$ distribution with a steady-state UCL when $m = 100$, $n = 5$ and $ARL_0 \approx 500$

	TL				DL		EL	
	$\lambda = 0.05$	0.10	0.25	0.50	0.05	0.25	0.05	0.25
θ	$L = 0.500$	1.161	2.114	3.011	0.816	2.455	1.838	3.484
				$\delta = 1$				
0	500.71	504.59	505.68	498.95	494.20	502.62	497.25	490.52
	(1341.24)	(1134.75)	(918.42)	(774.37)	(1239.40)	(855.57)	(1080.90)	(772.17)
0.1	582.79	577.89	578.83	570.12	577.13	569.41	579.31	558.79
	(1550.00)	(1317.86)	(1078.26)	(934.00)	(1462.33)	(1022.89)	(1280.24)	(883.30)
0.25	381.06	387.31	408.44	432.99	381.46	418.21	393.47	437.25
	(1256.30)	(1065.08)	(889.25)	(792.95)	(1195.02)	(878.51)	(1050.94)	(802.09)
0.5	60.98	68.85	91.11	125.05	58.94	98.92	71.04	135.57
	(264.26)	(281.18)	(258.14)	(258.72)	(273.44)	(235.63)	(281.70)	(296.37)
1	16.18	12.93	9.27	10.37	10.58	8.47	7.55	11.45
	(4.05)	(4.27)	(6.34)	(13.89)	(4.07)	(7.97)	(5.41)	(16.63)
1.5	11.13	8.35	4.84	3.31	6.26	3.54	3.15	3.14
	(1.80)	(1.63)	(1.38)	(1.72)	(1.57)	(1.43)	(1.56)	(2.22)
2	8.81	6.43	3.52	2.13	4.52	2.38	1.88	1.66
	(1.05)	(0.90)	(0.69)	(0.61)	(0.84)	(0.63)	(0.77)	(0.79)
				$\delta = 1.1$				
0	173.02	187.79	211.54	228.41	171.95	214.78	184.10	225.39
	(495.93)	(416.98)	(357.49)	(352.88)	(452.24)	(353.60)	(403.79)	(346.33)
0.1	199.49	217.17	242.61	257.69	204.08	246.67	211.92	253.37
	(590.21)	(516.00)	(426.71)	(410.01)	(593.23)	(425.62)	(456.75)	(403.57)
0.25	154.17	171.45	196.52	214.43	154.41	204.20	168.34	213.75
	(486.55)	(431.43)	(407.94)	(370.80)	(442.04)	(421.58)	(418.91)	(375.84)
0.5	45.17	48.84	60.79	79.96	42.08	65.29	46.87	83.15
	(85.93)	(100.10)	(139.02)	(157.95)	(105.78)	(141.72)	(107.04)	(167.49)
1	15.95	12.70	8.95	9.17	10.35	8.05	7.21	9.75
	(4.14)	(4.34)	(5.88)	(11.14)	(4.10)	(7.03)	(5.26)	(12.04)
1.5	11.11	8.34	4.84	3.32	6.25	3.55	3.15	3.11
	(1.81)	(1.66)	(1.42)	(1.76)	(1.60)	(1.47)	(1.60)	(2.15)
2	8.87	6.48	3.55	2.16	4.56	2.41	1.92	1.70
	(1.07)	(0.93)	(0.71)	(0.66)	(0.88)	(0.67)	(0.81)	(0.83)
				$\delta = 1.25$				
0	53.22	58.07	68.40	80.27	50.17	70.67	53.67	81.07
	(64.51)	(80.33)	(97.94)	(108.63)	(69.74)	(102.09)	(82.34)	(115.14)
0.1	57.95	63.54	75.91	88.94	54.74	77.56	58.91	88.98
	(84.19)	(95.27)	(126.18)	(127.96)	(85.77)	(118.32)	(93.47)	(134.65)
0.25	52.07	56.74	66.97	79.68	48.72	68.54	52.29	79.82
	(79.87)	(92.56)	(114.68)	(126.99)	(84.42)	(11.93)	(93.74)	(130.59)
0.5	31.24	30.73	32.83	40.12	26.24	33.57	25.99	40.67
	(32.76)	(42.74)	(52.84)	(66.58)	(34.71)	(55.89)	(41.50)	(66.26)
1	15.45	12.23	8.38	7.91	9.92	7.31	6.71	8.07
	(4.05)	(4.12)	(5.01)	(8.18)	(3.97)	(5.80)	(4.95)	(8.80)
1.5	11.09	8.33	4.84	3.30	6.24	3.56	3.15	3.08
	(1.85)	(1.69)	(1.47)	(1.74)	(1.65)	(1.52)	(1.65)	(2.12)
2	8.96	6.56	3.62	2.21	4.63	2.46	1.97	1.77
	(1.11)	(0.98)	(0.77)	(0.72)	(0.92)	(0.73)	(0.86)	(0.89)

Table S9: continued

	TL				DL		EL	
	$\lambda = 0.05$	0.10	0.25	0.50	0.05	0.25	0.05	0.25
θ	$L = 0.500$	1.161	2.114	3.011	0.816	2.455	1.838	3.484
				$\delta = 1.5$				
0	24.70	22.23	20.59	23.56	19.09	20.12	16.70	23.90
	(12.20)	(14.58)	(20.27)	(27.00)	(13.08)	(21.46)	(15.98)	(27.68)
0.1	25.10	22.72	21.27	24.42	19.51	21.01	17.23	24.77
	(13.13)	(15.78)	(22.12)	(29.08)	(14.09)	(23.54)	(17.00)	(30.29)
0.25	24.41	22.02	20.23	23.14	18.84	19.93	16.49	23.27
	(12.66)	(16.22)	(20.95)	(28.35)	(14.28)	(22.39)	(16.56)	(28.86)
0.5	20.95	18.00	15.20	16.29	15.27	14.49	12.41	16.43
	(9.22)	(11.06)	(15.56)	(19.23)	(9.87)	(16.19)	(12.39)	(19.83)
1	14.45	11.30	7.37	6.19	9.03	6.17	5.66	6.20
	(3.54)	(3.51)	(3.76)	(4.93)	(3.41)	(4.20)	(3.88)	(5.62)
1.5	11.02	8.26	4.80	3.27	6.18	3.54	3.12	2.98
	(1.86)	(1.70)	(1.49)	(1.72)	(1.67)	(1.55)	(1.68)	(2.01)
2	9.08	6.67	3.70	2.27	4.73	2.54	2.06	1.86
	(1.17)	(1.06)	(0.85)	(0.83)	(1.01)	(0.82)	(0.95)	(0.98)
				$\delta = 2$				
0	15.49	12.26	8.24	7.23	9.92	7.04	6.46	7.21
	(3.70)	(3.67)	(4.32)	(5.84)	(3.67)	(4.73)	(4.22)	(6.14)
0.1	15.49	12.24	8.26	7.25	9.93	7.05	6.45	7.23
	(3.74)	(3.83)	(4.39)	(5.91)	(3.72)	(4.84)	(4.25)	(6.28)
0.25	15.33	12.09	8.13	7.06	9.79	6.91	6.35	6.97
	(3.74)	(3.80)	(4.34)	(5.76)	(3.72)	(4.77)	(4.26)	(5.84)
0.5	14.64	11.48	7.50	6.36	9.18	6.31	5.76	6.28
	(3.43)	(3.47)	(3.75)	(5.03)	(3.36)	(4.18)	(3.77)	(5.40)
1	12.62	9.64	5.92	4.46	7.48	4.63	4.21	4.19
	(2.52)	(2.40)	(2.38)	(2.97)	(2.38)	(2.53)	(2.57)	(3.16)
1.5	10.69	7.99	4.63	3.09	5.93	3.37	2.93	2.74
	(1.76)	(1.62)	(1.42)	(1.57)	(1.59)	(1.44)	(1.56)	(1.74)
2	9.24	6.80	3.80	2.34	4.85	2.62	2.16	1.97
	(1.26)	(1.14)	(0.95)	(0.95)	(1.10)	(0.94)	(1.05)	(1.10)