# Supporting Information for

## Effect of Skull Morphology on Fox Snow Diving

**Jisoo Yuk, Anupam Pandey, Leena Park, William E. Bemis, and Sunghwan Jung**

**Corresponding Author: Sungwhan Jung**
**E-mail: sunnyjsh@cornell.edu**

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## Supporting Information Text

## A. 3D geometries of animal skulls and radius of curvature

The curvature plays a pivotal role in determining the force exerted by a curved object when it interacts with an interface. To compute the curvature of an asymmetric falling object, we derived an average 2D curvature by analyzing both side and dorsal views. Initially, we captured 2D front and side images of the 3D skull structures. These 2D images were subsequently transformed into Black and White (BW) images using MATLAB's image processing toolbox, and the coordinates of their boundaries were accurately defined.

To facilitate the curvature calculation for the drag force equation, we simplified the representation of the skull by modeling it as a combination of two basic spheroids. This was necessitated by the abrupt change in shape near the zygomatic arch. Thus, we divided the fox skull into two areas, around the snout and the part encompassing the zygomatic arch. We approximated each boundary, divided into two segments, by fitting an ellipse using the least squares method to calculate 2D curvature of dorsal and side view. Using the equation for the ellipse as, $\frac{x^2}{a^2} + \frac{z^2}{b^2} = 1$, the curvature at the lowest point of the ellipse (0,-b), is calculated as, $\kappa_m = b/a^2$. Subsequently, we obtained the final curvature by averaging the curvatures measured from both views. Specifically, $\kappa_{m1}$ and $\kappa_{m2}$ denote the average curvatures from the first spheroid near the snout and the second spheroid near the zygomatic arch, calculated by the equations: $\kappa_{m1} = (\kappa_{m1d} + \kappa_{m1s})/2$ and $\kappa_{m2} = (\kappa_{m2d} + \kappa_{m2s})/2$, where $\kappa_{m1d}$ & $\kappa_{m2d}$ are the curvatures from the dorsal view, and $\kappa_{m1s}$ & $\kappa_{m2s}$ are the curvatures from the side view.

## B. Condition of snow experiment

We conducted our dropping experiments exclusively during the winter season to ensure the availability of fresh snow. These experiments took place at Riley-Robb Hall, Cornell University, located in Ithaca. We sourced all pertinent weather information for Ithaca on the day of the experiment from the Timeanddate website (https://www.timeanddate.com/) (1). Please see Table S1.

## C. Experimental setup for physical experiment

### List of equipment.

- Water tank for water experiments
- 5 Gallon bucket for snow experiments
- Load cell, LC101-25 (Omegadyne Inc)
- Signal Conditioning Amplifier, 2360B (Micro-Measurements, Wendell, North Carolina, United States)
- Data acquisition card, USB-6001 (National Instruments, Austin, Texas, United States)
- 80/20 extrusion framing
- 80/20 linear bearing
- Rubber stopper
- High Speed camera (Photron, Tokyo, Japan)
- 50 mm Lens (Nikon, Tokyo, Japan)

### Procedure.

1. We create an experimental setup using 80/20 framing extrusions (as shown in Fig. S3) to allow objects to slide on top of the water tank or bucket.
2. We install a central 80/20 framing extrusion and insert 80/20 Linear Bearings to create a slider mechanism.
3. We attach a rubber stopper at the bottom of the central 80/20 framing extrusion to stop the slider at the end.
4. We mount a load cell on the slider to measure forces.
5. We connect a projectile to the load cell using a rod that is at least 50 cm long.
6. We connect the load cell to the Signal Conditioning Amplifier, 2360B (Micro-Measurements, Wendell, North Carolina, United States) following the instructions provided by the manufacturer of the load cell.
7. We connect the Signal Conditioning Amplifier to the Data Acquisition card, NI USB-6001.
8. We open the Analog Input Recorder in MATLAB and select the corresponding channel. Alternatively, you can use NI MAX software for this purpose.
9. We turn on the Signal Conditioning Amplifier and adjust the gain after verifying the input signal from the load cell. If a filter is required, select the desired filter size. In our diving experiments, we used a filter size of 1K.
10. We perform the calibration step to convert the load cell's signal into force measurements:
    (a) Attach a string to the rod connected to the load cell.
    (b) Suspend at least five different weights and record the corresponding voltage readings.
    (c) Plot the voltage values against each weight to obtain a calibration curve.

    Once the calibration curve is obtained, we place the testing projectile on the rod.
11. To capture synchronized video footage of the entering motion:
    (a) Position a high-speed camera in front of the water tank.
    (b) Set the frame rate of the camera to 4,000 frames per second (or other frame rates).

    (c) Connect the high-speed camera to a trigger and the Data Acquisition card for synchronization.
    (d) Push the trigger to start recording, and the triggering time is recorded by the Data Acquisition card.
12. We drop the object into the water tank while simultaneously recording force data and high-speed imaging.
13. Using the calibration curve, we convert the voltage signals obtained during the dropping experiments into corresponding force data.

**Table S1. List of snow and weather conditions during the skull dropping experiments.**

| Snow experiment condition | | | |
|---|---|---|---|
| Tested model | Arctic fox, original | Arctic fox, original | Arctic fox, flattened |
| Experimental date | Feb 3rd, 2021 | Feb 4th, 2022 | Feb 4th, 2022 |
| Highest temperature | -3.9 | -7.8 | -7.8 |
| Lowest temperature | -6.1 | -11.1 | -11.1 |
| Forecast | Light snow, Ice fog | Light snow, Ice fog | Light snow, Ice fog |
| Humidity | 81% | 87% | 87% |
| Snow density | 105.7 | 86.5 | 97.5 |

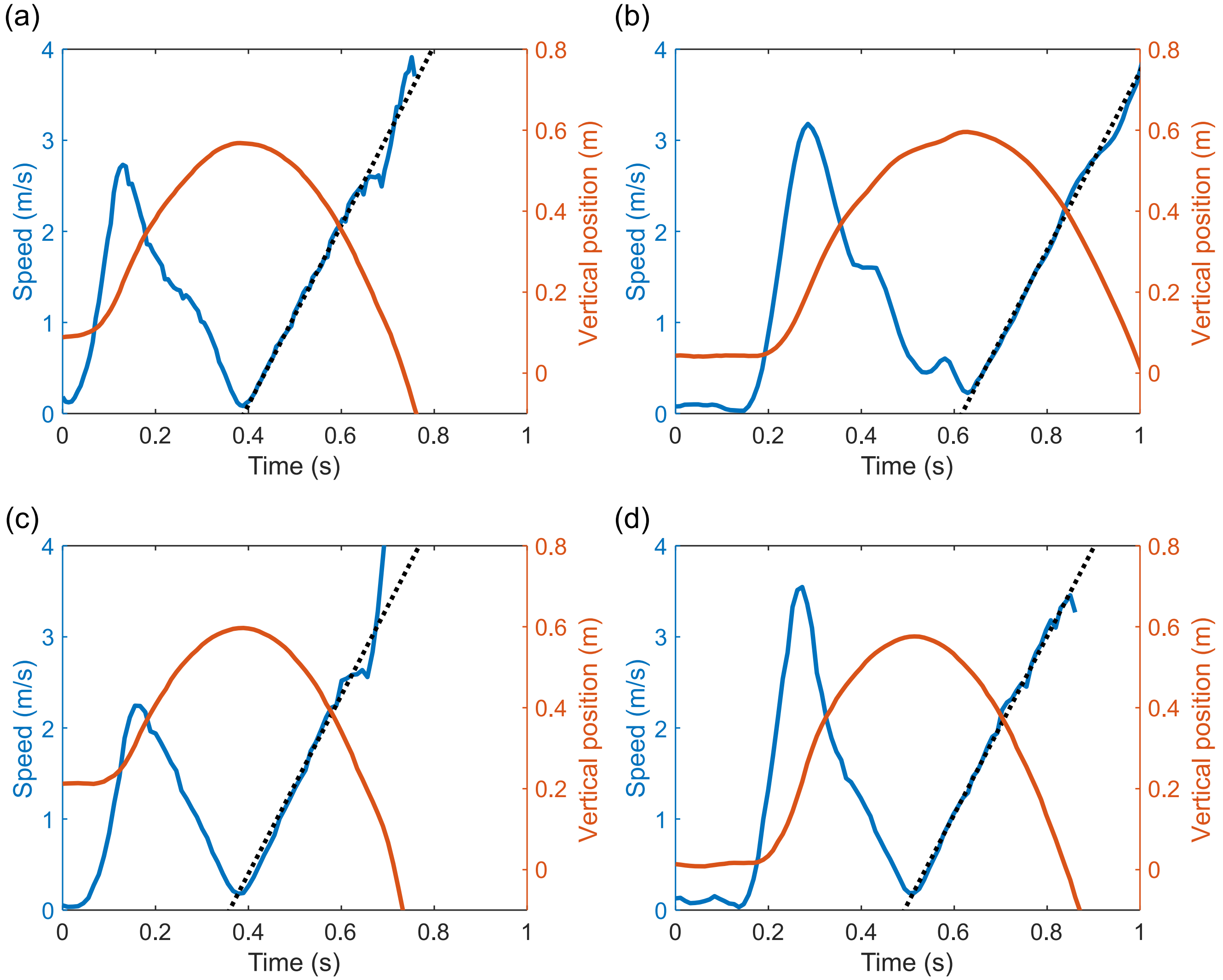

**Fig. S1.** Diving profiles of arctic foxes and red foxes. The vertical velocity is displayed on the left y-axis, while the height of the snout is shown on the right y-axis. (a) and (b) show the diving profiles of the arctic foxes. The corresponding footage IDs are Creative # 1150716601 for (a) and Creative # 1150681603 for (b). (c) and (d) show the diving profiles of the red foxes. The corresponding footage IDs are Creative # 564830149 for (c) and Creative # 564829945 for (d). The results reveal that both species exhibit similar maximum diving heights and speeds.

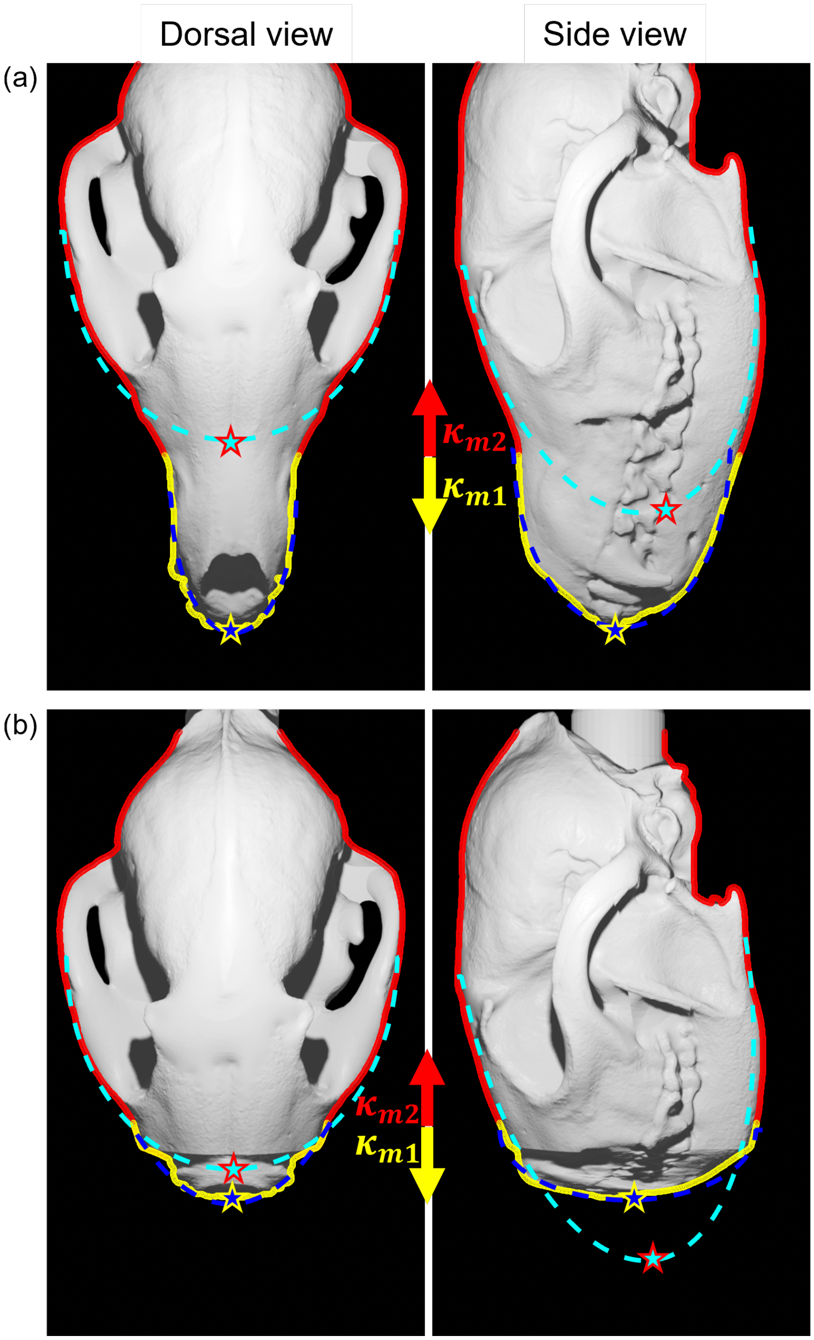

**Fig. S2.** (a) presents the original arctic fox, while (b) shows the flattened snout model of the arctic fox, with their dorsal and side views provided. The best fitting ellipsoids are obtained using the MATLAB image processing toolbox and the least square method. The boundary of the snout is depicted with yellow lines, while the boundary of the rear part is depicted with red lines. The optimal ellipsoid that best matches the snout part is depicted with dark blue lines, while the optimal ellipsoid for the rear part is indicated by cyan lines. From these fitted curves, we obtained the curvature values at the lowest point along the z-axis, which were then used to estimate the force. In the figure, the star symbols denote the lowest points of each ellipse. For the force calculation, we utilized the first curvature ($\kappa_{m1}$) when the snout penetrated the snow, while the second curvature ($\kappa_{m2}$) was applied when the zygomatic arch and rear part penetrated the snow.

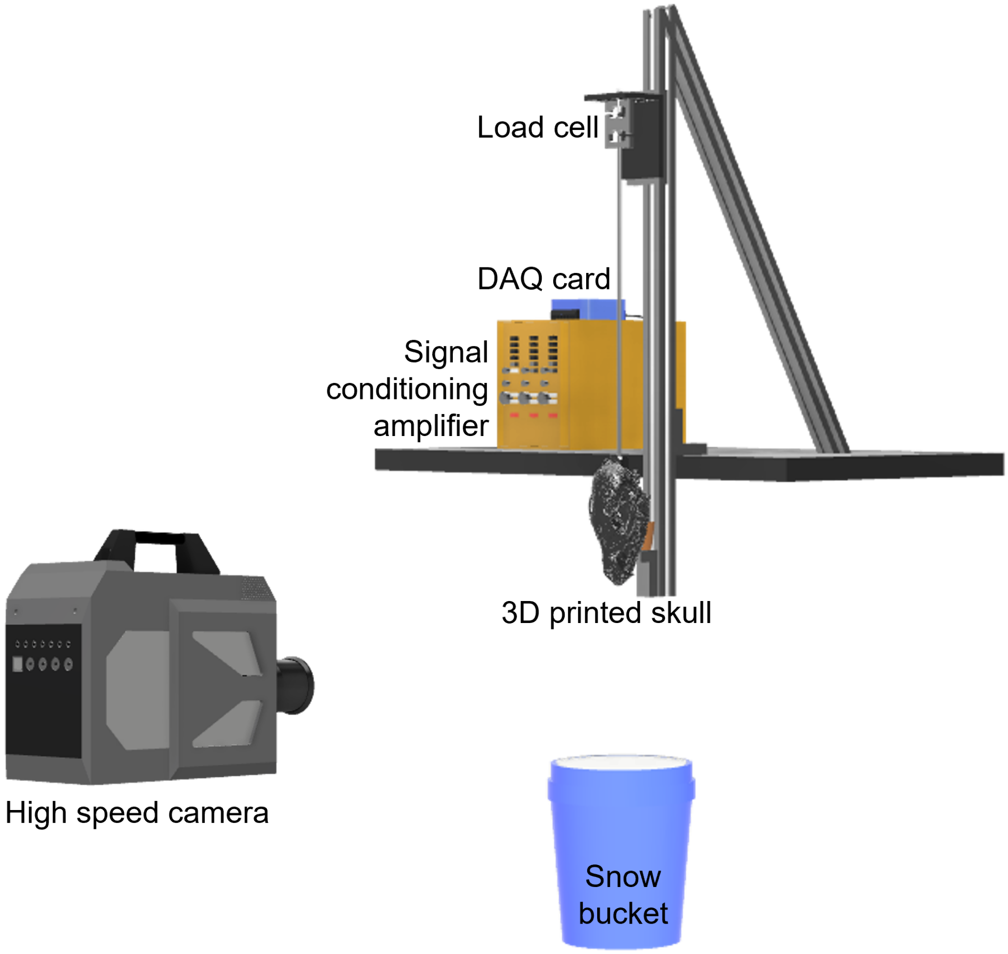

**Fig. S3.** Experimental setup for diving into the snow. The 3D printed fox skull is attached to a load cell with a rod. The load cell is connected to a DAQ card through a signal conditioning amplifier. The skull slides along an 80/20 extrusion framing and enters into the snow. The force at the moment of diving is measured by the load cell, and the scene is captured by a high speed camera.

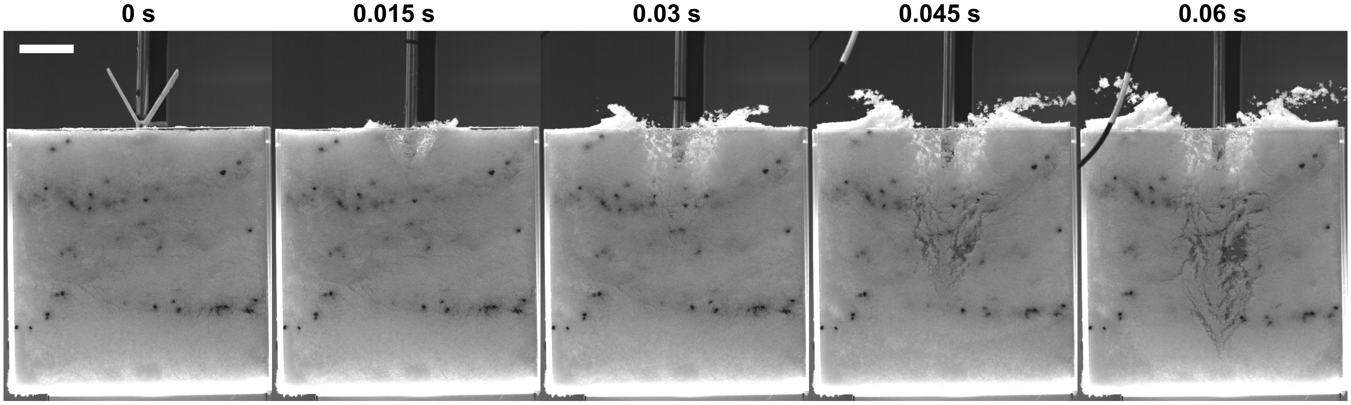

**Fig. S4.** Image sequences of a pressure propagation experiment, in which a 60-degree wedge is dropped into a glass container. At 0 s, the wedge just touches the snow surface, and at 0.015 s, it is fully submerged. At 0.03 s, 0.045 s, 0.06 s, the wedge has deeper penetrated 2, 3, 4 times of the skull, respectively. (Scale bar = 5 cm)

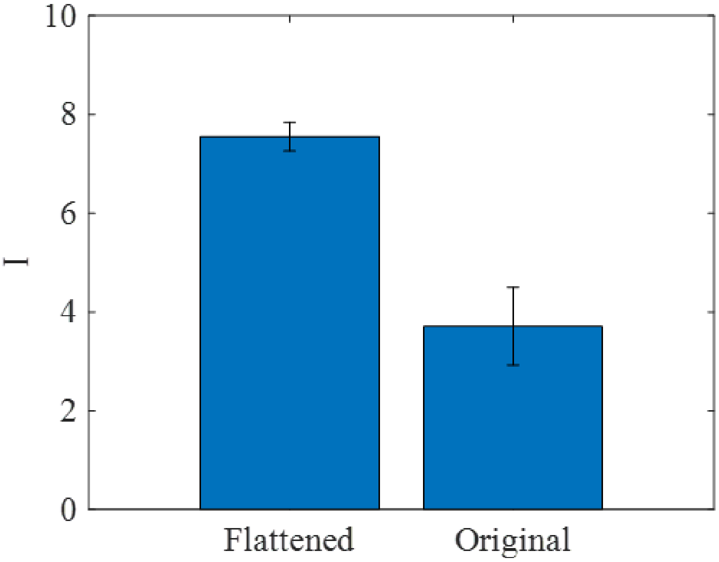

**Fig. S5.** The amount of time-averaged impulse that occurs when a flattened snout and an original snout are dropped in the snow. It was calculated by integrating the force until the moment the skull was completely submerged in the snow, that is, $\bar{T} = 1$, and dividing it by the total time. The flattened snout produced the impulse twice as high as the original snout.

**Movie S1. The dropping test with the skull of the Arctic fox into snow.**

**Movie S2. The dropping test with the skull of the flattened Arctic fox into snow.**

**Movie S3. The pressure propagation experiment in snow.**

## References

1. timeanddate, Past weather in ithaca, new york, usa, https://www.timeanddate.com/weather/@5122432/historic (accessed: 11.29.2023).