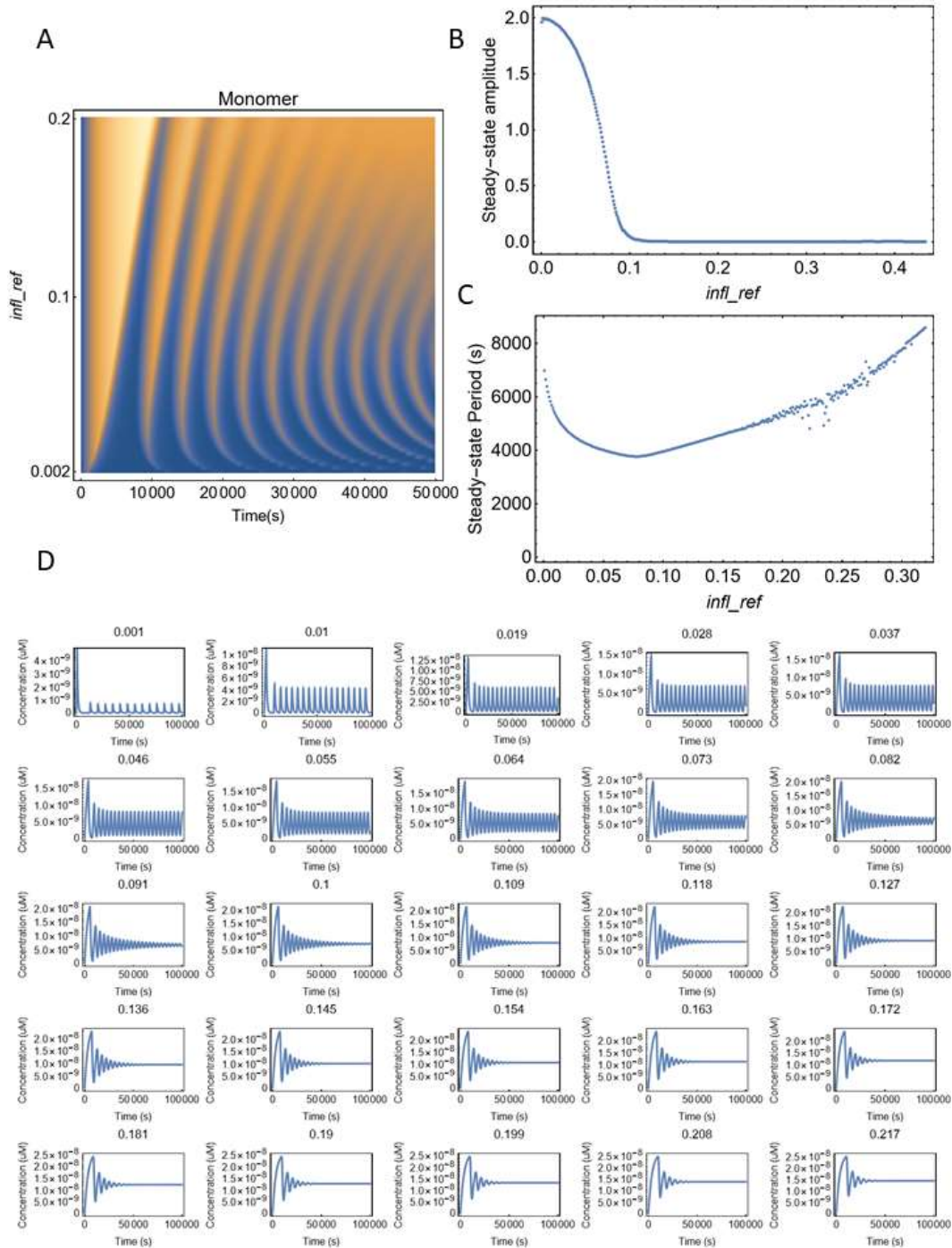
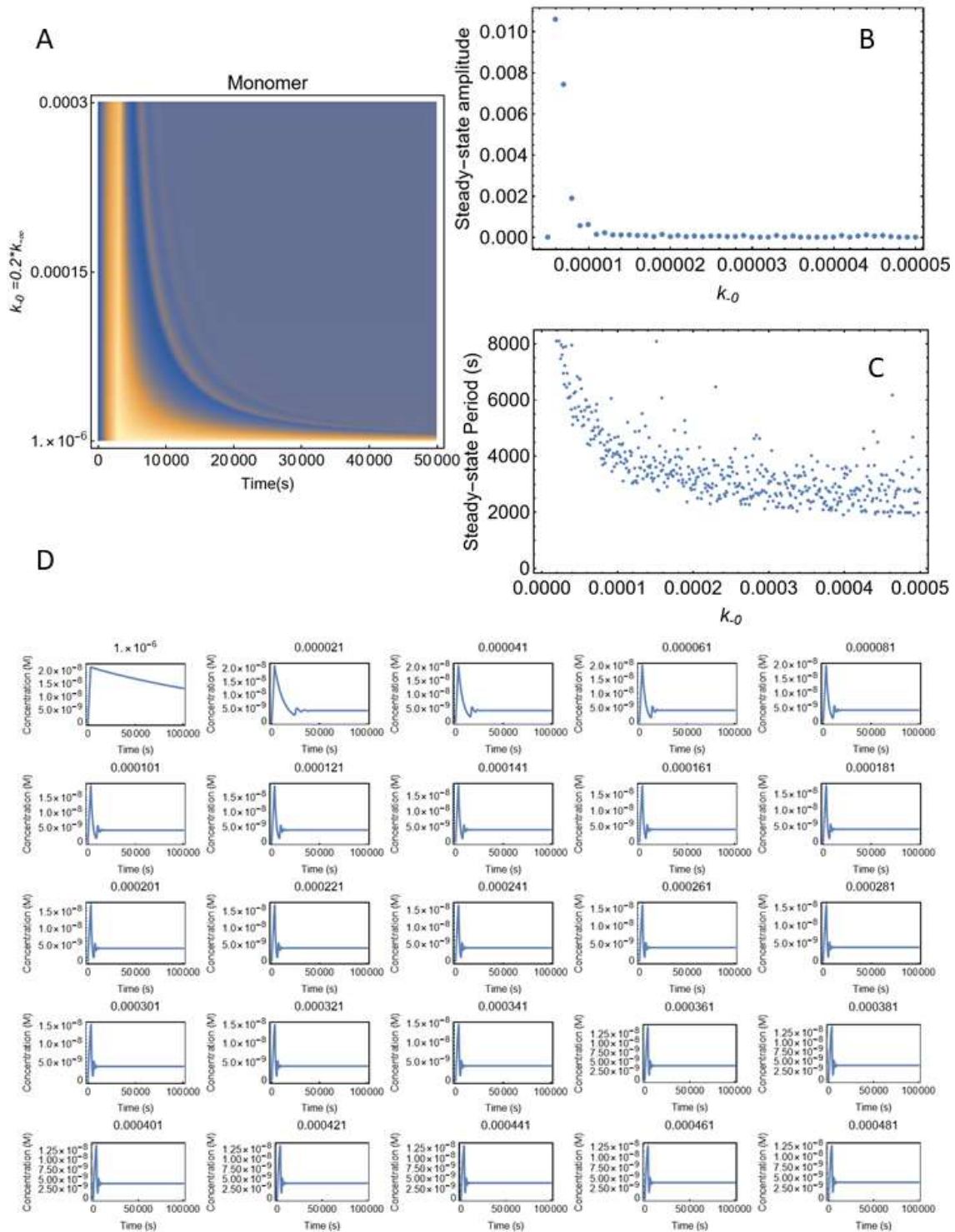


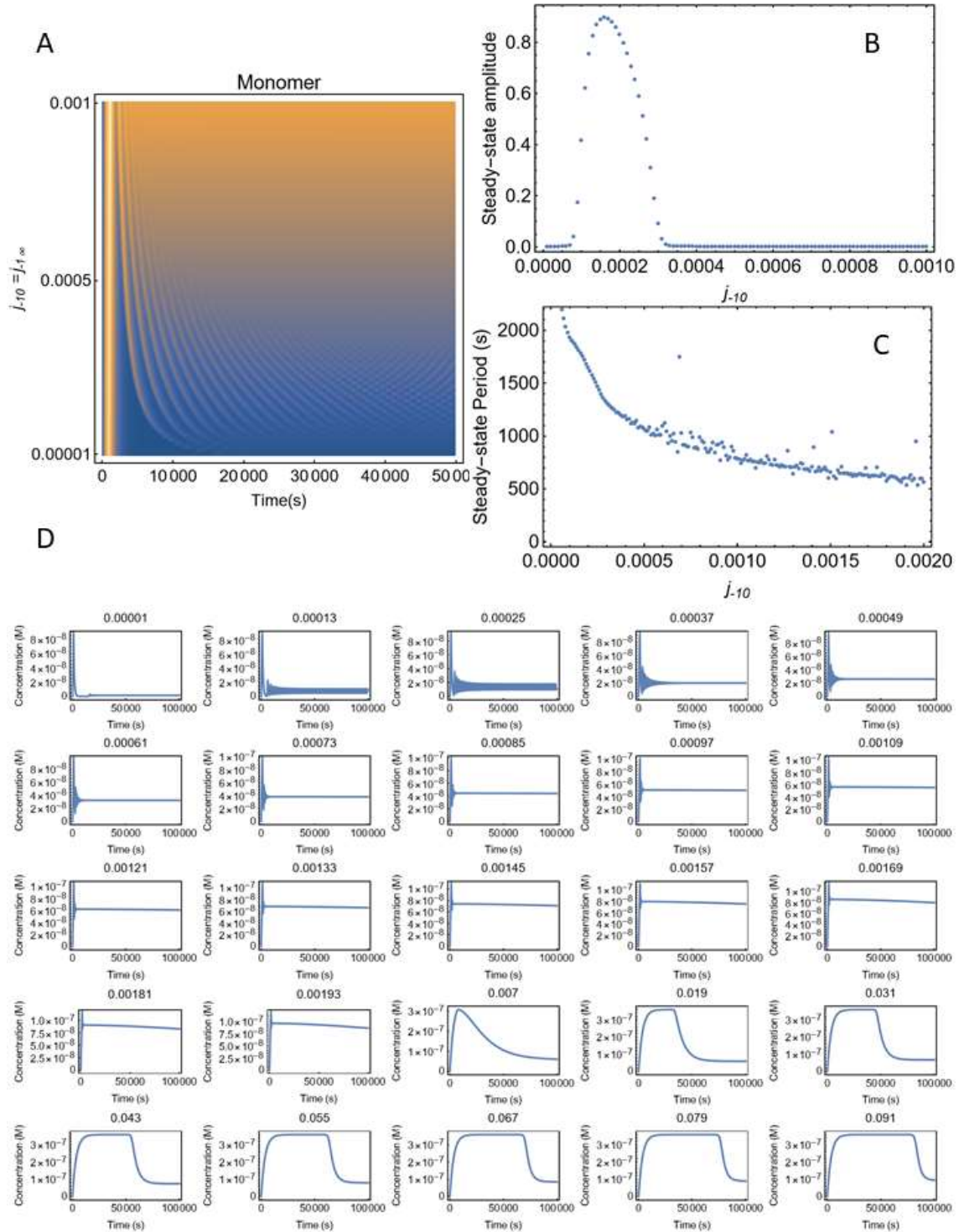
**Supplementary Figure 1.** The oscillatory potential of the *steep* parameter. (a) The contour map of the Aβ monomer with respect to the *steep* parameter along the y-axis and the time along x-axis. Golden color implies peak of monomer concentration (z-axis) and the blue color implies valley of Aβ monomer concentration. (b) The normalized steady-state amplitude of oscillation peaks to maximum value of 2 at the *steep* value 20. (c) The steady-state period of oscillation has a regular trend beyond the *steep* value 20. (d) the time-course of Aβ monomer concentrations for different *steep* values (mentioned on top of each panel).



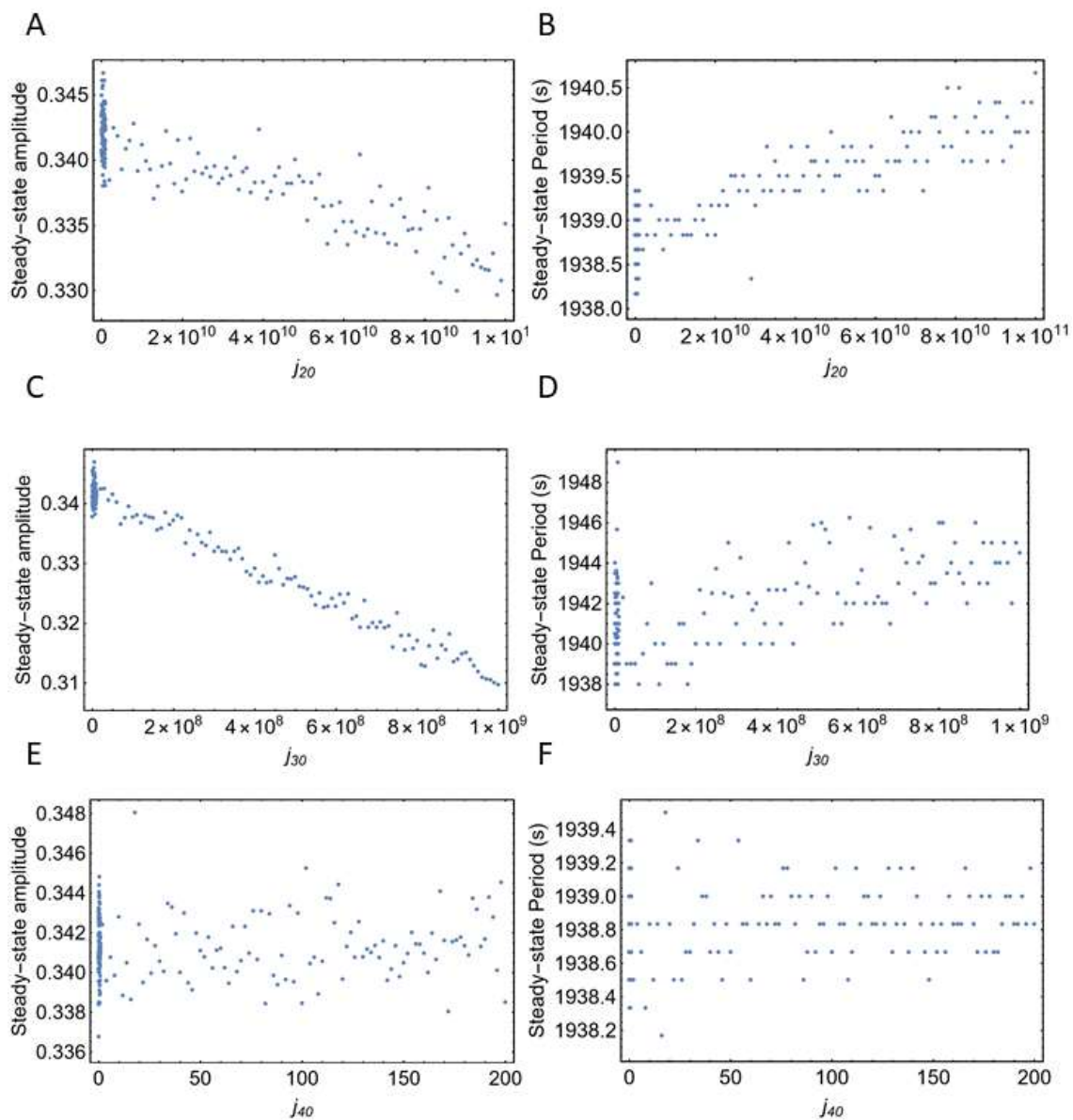
**Supplementary Figure 2.** The oscillatory potential of the *infl\_ref* parameter. (a) The contour map of the A $\beta$  monomer with respect to the *infl\_ref* parameter along the y-axis and the time along x-axis. Golden color implies peak of monomer concentration (z-axis) and the blue color implies valley of A $\beta$  monomer concentration. (b) The normalized steady-state amplitude of oscillation disappears at the *infl\_ref* value 0.1. (c) The steady-state period of oscillation has a regular trend before the *infl\_ref* value 0.1. (d) the time-course of A $\beta$  monomer concentrations for different *infl\_ref* values (mentioned on top of each panel).



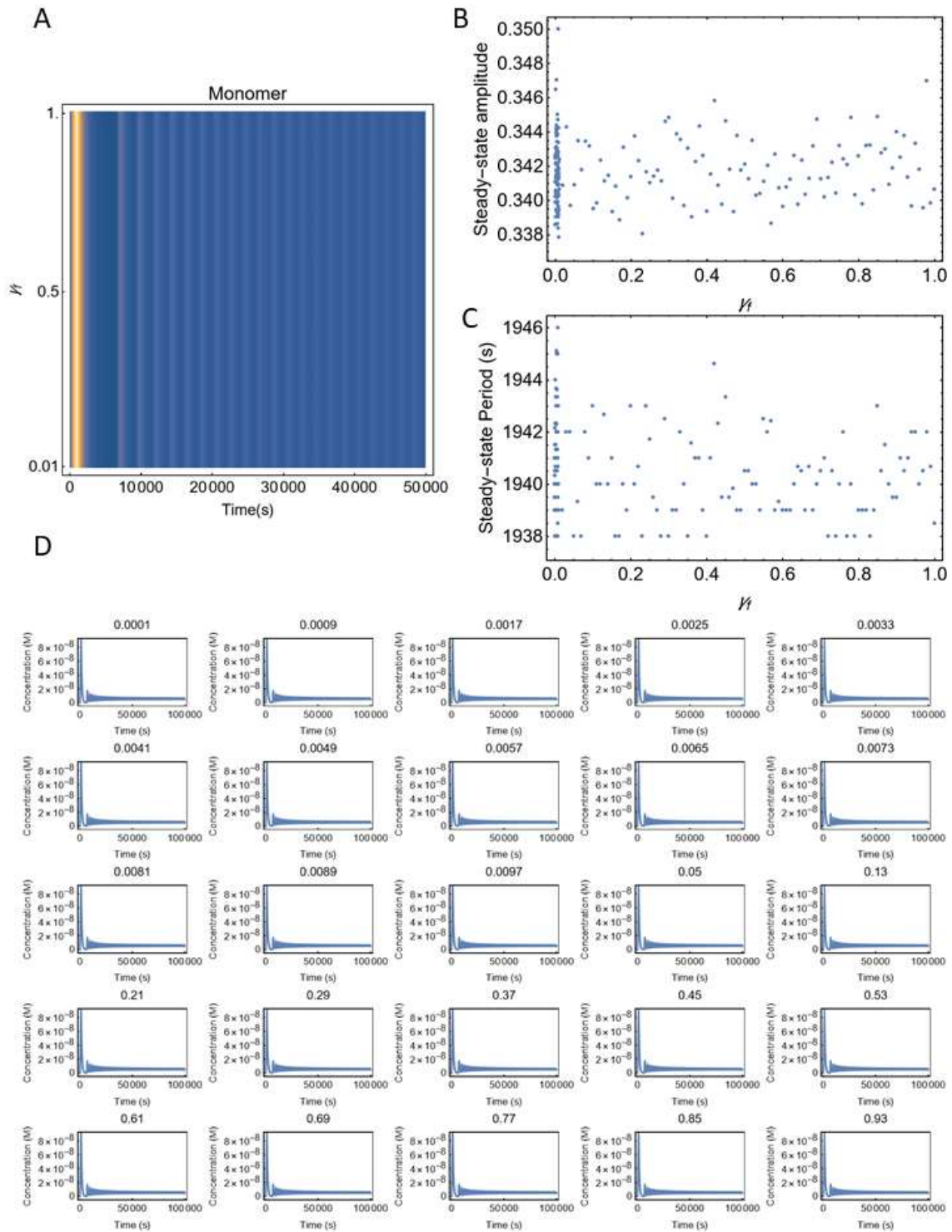
**Supplementary Figure 3.** The  $k_0$  parameter does not affect the oscillatory behavior. (a) The contour map of the A $\beta$  monomer with respect to  $k_0$  parameter along the y-axis and the time along x-axis. Golden color implies peak of monomer concentration (z-axis) and the blue color implies valley of A $\beta$  monomer concentration. (b) The normalized steady-state amplitude of oscillation is insignificant. (c) The steady-state period of oscillation shows high scatter. (d) the time-course of A $\beta$  monomer concentrations for different  $k_0$  values (mentioned on top of each panel) does not show steady oscillation.



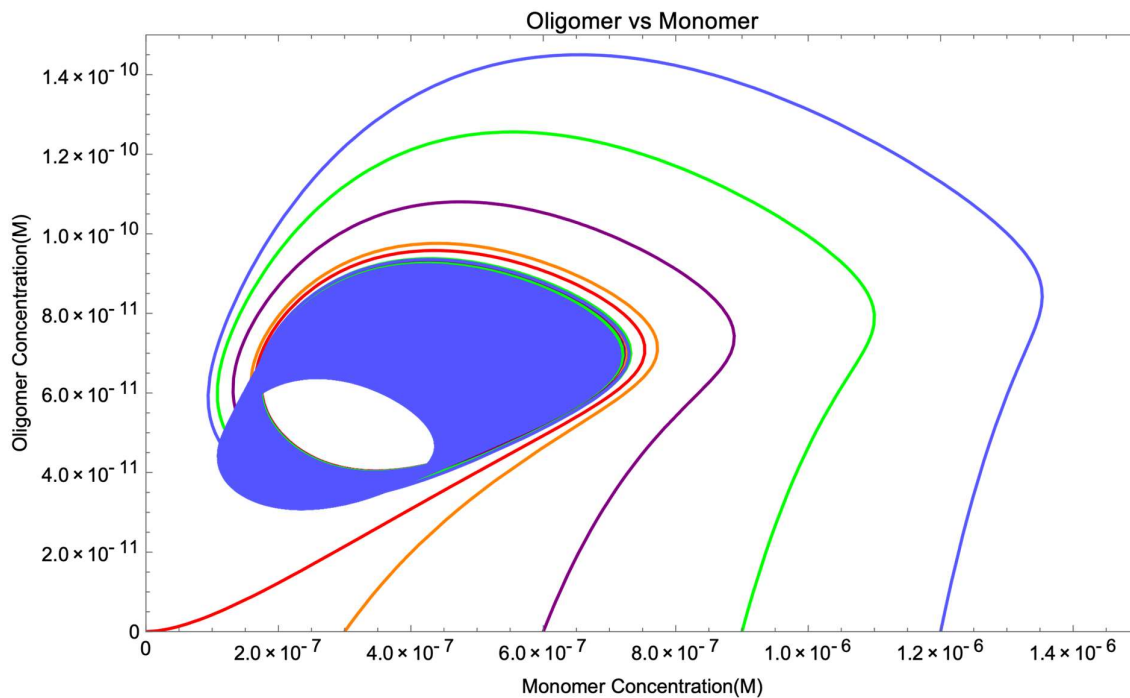
**Supplementary Figure 4.** The  $j_{-1}$  parameter has minor effect on the oscillatory behavior. (a) The contour map of the A $\beta$  monomer with respect to  $j_{-1}$  parameter along the y-axis and the time along x-axis. Golden color implies the peak of monomer concentration (z-axis) and the blue color implies the valley of A $\beta$  monomer concentration. (b) The normalized steady-state amplitude of oscillation is significant between 0.0001 and 0.0003. (c) The steady-state period of oscillation shows a regular trend till 0.0003. (d) the time-course of A $\beta$  monomer concentrations for different  $j_{-1}$  values (mentioned on top of each panel) shows steady oscillation in a narrow range.



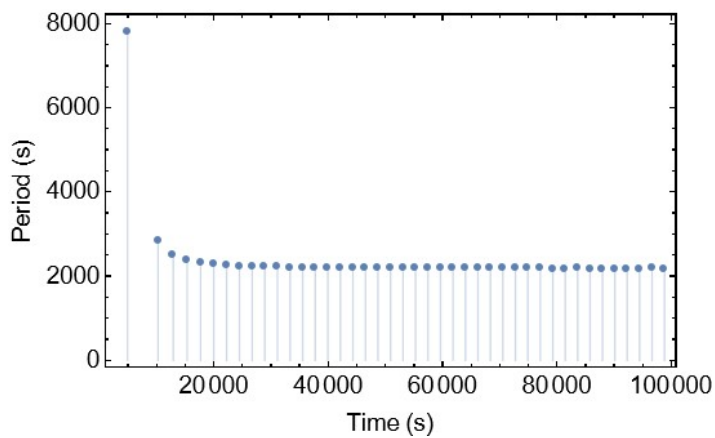
**Supplementary Figure 5.** The  $j_2$ ,  $j_3$ ,  $j_4$  parameters do not affect the oscillatory behavior. (a) The normalized steady-state amplitude of oscillation of the A $\beta$  monomer with respect to the  $j_2$  parameter shows a high scatter. (b) The steady-state period similarly shows a high scatter for the  $j_2$  parameter. A similar absence of steady oscillation is observed with respect to  $j_3$  for amplitude (c) and period (d), and the same behavior is observed with respect to  $j_4$  for amplitude (e) and (f) period.



**Supplementary Figure 6.** The  $\gamma_f$  parameter does not affect the oscillatory behavior. (a) The contour map of the A $\beta$  monomer with respect to  $\gamma_f$  parameter along the y-axis and the time along x-axis. The golden color implies the peak of monomer concentration (z-axis) and the blue color implies the valley of A $\beta$  monomer concentration. (b) The normalized steady-state amplitude of oscillation is insignificant. (c) The steady-state period of oscillation shows high scatter. (d) the time courses of A $\beta$  monomer concentrations for different  $\gamma_f$  values (mentioned on top of each panel) do not show steady oscillation.

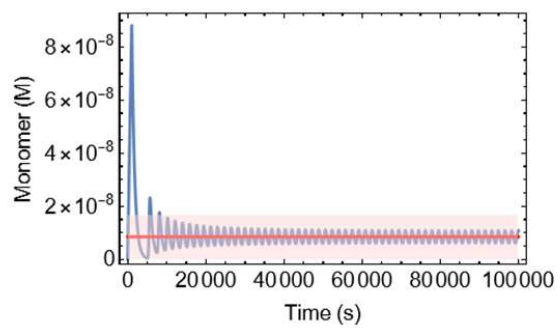


**Supplementary Figure 7.** The convergence of the oscillatory behaviors and the limit cycle of open coupled systems with different A $\beta$  monomer concentrations (0  $\rightarrow$  red; 0.3  $\mu$ M  $\rightarrow$  orange, 0.6  $\mu$ M  $\rightarrow$  purple, 0.9  $\mu$ M  $\rightarrow$  light green, 1.2  $\mu$ M  $\rightarrow$  blue).

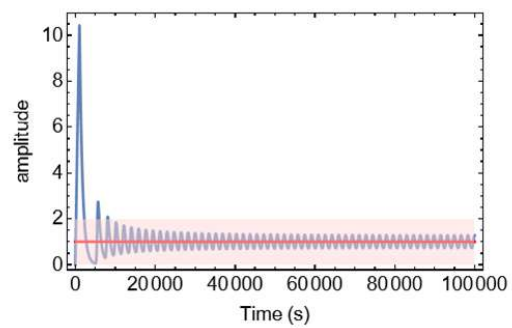


**Supplementary Figure 8.** The period of oscillation varies during the time course of simulations. In the case of steady oscillation, the period reaches an asymptotic value within the time course of the simulation.

A



B



**Supplementary Figure 9.** The amplitude of oscillation is shown for a representative case of steady oscillation. (a) The baseline is calculated as the midpoint of the steady peaks and valleys of the oscillation after the initial period of stabilization, such that the baseline will assume the value of 1 (pink box). (b) The resultant normalized amplitudes can vary between the values of 0 and 2 (pink box).



**Supplementary Table 1:** Results of sensitivity analysis for the open uncoupled model. The numerical values represent the scaled sensitivity of the corresponding state variable ( $a_m$ ,  $a_o$ ,  $a_{fp}$ ,  $a_f$ ) vs different model parameters. \*

|            | $a_m$      | $a_o$      | $a_{fp}$   | $a_f$      |
|------------|------------|------------|------------|------------|
| $k_+$      | 0.955954   | 0.906956   | 10.3241    | 21.2092    |
| $k_-$      | -0.989641  | -0.828797  | -3.42233   | -4.43007   |
| $\gamma_o$ | -0.0009986 | -0.0150183 | -0.0175357 | -0.0183818 |
| $\gamma_f$ | 1.49E-06   | -0.0004099 | -0.0001038 | -0.0823782 |
| $j_1$      | -0.0032097 | 0.997391   | 0.991077   | 0.988581   |
| $j_{-1}$   | 0.00316319 | -0.987164  | -0.973237  | -0.96496   |
| $j_2$      | -0.0021791 | 0.00330252 | 0.998044   | 0.997033   |
| $j_3$      | -0.0021791 | 0.00330253 | -0.0019536 | 0.997023   |
| $j_4$      | -1.60E-05  | 0.00503353 | 0.00168495 | 0.00086866 |

\*  $a_m$ : monomeric A $\beta$  ;  $a_o$ : oligomeric A $\beta$ ;  $a_{fp}$ : fibrillar A $\beta$  (particle number concentration);  $a_f$ : fibrillar A $\beta$  (mass concentration); For the meaning of parameters (first column), see the text.

**Supplementary Table 2:** Results of sensitivity analysis for the open coupled model. The numerical values represent the scaled sensitivity of the corresponding state variable ( $a_m$ ,  $a_o$ ,  $a_{fp}$ ,  $a_f$ ,  $infl$ ) vs different model parameters. \*

|                | $a_m$    | $a_o$    | $a_{fp}$ | $a_f$    | $infl$    |
|----------------|----------|----------|----------|----------|-----------|
| $k_{+0}$       | 0.02883  | 0.01455  | 0.1782   | 0.40603  | 0.014586  |
| $k_{+\infty}$  | 0        | 0        | 0        | 0        | 0         |
| $k_{-0}$       | -0.02333 | -0.02279 | -0.1261  | 228.818  | -0.002    |
| $k_{-\infty}$  | -0.01292 | -0.03962 | -0.12496 | 219.167  | -0.01971  |
| $\gamma_o$     | 0.04333  | -0.01194 | 0.11708  | 120.257  | -0.00101  |
| $\gamma_f$     | -0.00048 | -0.00039 | 0.00245  | 4.45198  | 1.06E-05  |
| $j_{10}$       | -0.48191 | -0.00699 | -1.35344 | 196.133  | 0.010828  |
| $j_{1\infty}$  | -0.74636 | 0.00579  | -1.86037 | 120.944  | 0.016777  |
| $j_{-10}$      | 0.4161   | -0.02907 | 1.05527  | 216.875  | -0.00936  |
| $j_{-1\infty}$ | 0.67887  | -0.03204 | 1.6867   | 184.672  | -0.01526  |
| $j_{20}$       | -0.00093 | -0.00076 | 0.39553  | 8.57599  | 2.04E-05  |
| $j_{2\infty}$  | -0.00092 | -0.00076 | 0.61228  | 8.54691  | 2.04E-05  |
| $j_{30}$       | -0.00175 | -0.00144 | 0.00759  | 16.2318  | 3.83E-05  |
| $j_{3\infty}$  | 0.00037  | 0.00031  | -0.00226 | -3.48981 | -8.29E-06 |
| $j_{40}$       | -0.00037 | -0.00031 | 0.00174  | 3.45343  | 8.24E-06  |
| $j_{4\infty}$  | -0.00077 | -0.00063 | 0.00353  | 7.11175  | 1.69E-05  |
| $\delta_o$     | -1.21779 | -0.99223 | -4.22203 | 227.194  | 0.027417  |
| $\delta_f$     | -0.00016 | -0.00013 | 3.36E-05 | 0.42849  | 3.56E-06  |
| $k_{infl}$     | 1.16939  | 0.95231  | 4.00556  | 235.928  | -0.02631  |
| $infl_{ref}$   | 1.17927  | 0.96042  | 4.09029  | 144.291  | 0.973439  |
| $steep$        | -0.01402 | -0.01143 | -0.06082 | 27.1782  | -0.00896  |

\*  $a_m$ : monomeric A $\beta$  ;  $a_o$ : oligomeric A $\beta$ ;  $a_{fp}$ : fibrillar A $\beta$  (particle number concentration);  $a_f$ : fibrillar A $\beta$  (mass concentration);  $infl$ : inflammation level; For the meaning of parameters (first column), see the text.