Supporting Appendix 1: Cylinder Thinning Model

Here, we give a detailed derivation of our analytic model for the thinning dynamics of the intracellular bridge during the last stage of cytokinesis. This model is a simple extension of the model first used by Entov and Hinch (1) to describe elastic effects in the thinning of a polymeric liquid bridge. We begin by approximating the intercellular bridge as a perfect cylinder of radius $a(t)$. The cortical stretch modulus, S_c, gives rise to a Laplace pressure across the cortex, which is proportional to the mean curvature. For a sphere of radius R, the Laplace pressure is

$$P = \frac{2}{R} S_c \,. \qquad \textbf{[1]}$$

For a cylinder, the Laplace pressure is

$$P = \frac{S_c}{a} \,. \qquad \textbf{[2]}$$

If we approximate the two daughter cells as perfect spheres and the bridge as a perfect cylinder, we find the Laplace pressure P is larger inside a 1-μm radius bridge than the 4-μm radius daughter cells, thus the Laplace pressure tends to squeeze materials from the bridge into the two daughter cells. If we associate a viscosity μ with flow of the material from the bridge into the daughter cells, then dimensional analysis gives a characteristic velocity U_* for the outflow

$$U_* = \frac{S_c}{\mu} \,. \qquad \textbf{[3]}$$

In the rest of the appendix, we derive simple, analytic expressions, which describe bridge-thinning dynamics precisely, provided the geometry of the furrow remains cylindrical. We consider two extreme scenarios. First, we examine the thinning dynamics driven by a contractile radial stress exerted by myosin II, which acts against compressive stresses from the two daughter cells. This thinning dynamics is a combination of a linear decrease and an exponential decay, with the exponential decay preceding the linear decrease. Second, we examine the thinning dynamics associated with elastic stresses associated with materials inside the intercellular bridge. Here, a dominance of elastic stresses results in an exponential decay. In both cases, we retain the stretch modulus S_c in the analysis.

For the first scenario, we begin with volume conservation. As the cylinder thins and lengthens, volume conservation requires that the decrease in volume due to radial thinning in a volume element with length Δz in the cylinder, $\partial / \partial t(\pi a^2 \Delta z)$, is balanced by volume flux, $-[(\pi a^2 U_z)|_z^{z+\Delta z}]$, out of the two ends of the same volume element. Here U_z denotes the axial velocity inside the cylinder. Simplifying the expression and taking the limit of Δz going to 0, we obtain

$$\frac{\partial a}{\partial t} = -\frac{1}{2}\frac{\partial U_z}{\partial z}\, a(t) = -\frac{1}{2}\, e(t)\, a(t), \qquad \textbf{[4]}$$

where the axial strain rate $e(t)$ is unknown. In addition to volume conservation, the viscous flow in the thinning bridge must satisfy boundary conditions on the cylinder surface and at the two ends of the cylinder. These are the conditions that the fluid stresses should be continuous across a fluid interface. The full expression can be found in any textbook on fluid mechanics (2). Because of the extreme simplicity of the bridge geometry, the boundary conditions reduce to radial stress balance across the cylinder and axial stress balance across the ends of the cylinder. The radial stress balance has the form

$$\sigma_{rr} + \frac{S_c}{a} = P - 2\,\mu\,\frac{\partial U_r}{\partial r} = P + \mu e(t), \qquad \textbf{[5]}$$

where σ_{rr} is the actively generated radial stress on the bridge surface, P is the fluid pressure and U_r is the radial velocity. We also assume σ_{rr} is entirely contractile, corresponding to a radial force exerted inwards. In arriving at the right hand side of **5**, we made use of the fact that the velocity field should be incompressible, so that

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rU_r\right) + \frac{\partial U_z}{\partial z} = 0\,, \qquad \textbf{[6]}$$

which relates the radial derivative of U_r to the axial derivative of U_z. In our problem, **6** is simply

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rU_r\right) + e(t) = 0\,. \qquad \textbf{[7]}$$

Integrating **7** with respect to r, we find

$$U_r = -\frac{e(t)r}{2} - \frac{c_0}{r}\,. \qquad \textbf{[8]}$$

Because the centerline radial velocity U_r $(r = 0)$ must be bounded, $c_0 = 0$. This finding means $\partial(U_r)/\partial r = -e(t)/2$, hence the right hand side of Eq. **5**.

At the end of the cylindrical bridge, where the bridge joins onto a daughter cell, the axial stress balance takes the form

$$-\,\sigma_{zz} = -\,P + 2\mu e(t), \qquad \textbf{[9]}$$

where σ_{zz} is the compressive axial stress exerted by the daughter cell at the ends of the cylinder. If we take **9** as an equation for the pressure and substitute it into Eq. **5**, we then obtain the following expression for the axial strain rate $e(t)$

$$e(t) = \frac{S_c}{3\mu a(t)} + \frac{\sigma_{rr} - \sigma_{zz}}{3\mu}. \qquad \textbf{[10]}$$

Substituting **10** into **4** yields

$$\frac{\partial a}{\partial t} = -\frac{S_c}{6\mu} - \frac{(\sigma_{rr} - \sigma_{zz})}{6\mu} a(t). \qquad \textbf{[11]}$$

Note if $\sigma_{rr} = \sigma_{zz} = 0$, then $a(t) = a_0 - (S_c/6\mu)\, t$. The radius thins linearly over time with a characteristic velocity proportional to U_*, consistent with the earlier dimensional analysis.

Now suppose both of the applied stresses are constant over the time, **11** can be integrated to yield

$$a(t) = a_0 e^{-(\Delta\sigma t)/6\mu} - \frac{S_c}{\Delta\sigma}\,(1 - e^{-(\Delta\sigma t)/(6\mu)}), \qquad \textbf{[12]}$$

where $\Delta\sigma = \sigma_{rr} - \sigma_{zz} > 0$ and we have used the initial condition $a(t = 0) = a_0$. Eq. **12** describes a thinning dynamics which is a combination of linear decrease and exponential decay. The linear decrease component can be seen most easily by considering the situation where $\Delta\sigma/(6\mu)$ is small. In that case, Taylor expansion of **12** shows that the bridge thinning dynamics reduces to a linear thinning at leading order, as

$$a(t) = a_0 - \frac{S_c}{6\mu}\, t + \frac{S_c \Delta\sigma}{72\mu^2} t^2 + \dots. \qquad \textbf{[13]}$$

Here, the effect of the applied stress enters only as a correction to the linear dynamics. In short, depending on the magnitude of $\Delta\sigma$, the thinning dynamics changes continuously from a linear decrease to an exponential thinning. One trend, however, is robust and does not depend on the magnitude of $\Delta\sigma$: the dynamics always looks more exponential at the beginning, when $a(t)$ is large. As $a(t)$ decreases, the linear decrease becomes more obvious.

We now turn to the second scenario: the buildup of elastic stresses inside the thinning bridge. We begin with the simplest possible model of elastic effects. The material in the bridge is assumed to be isotropic and that the deformation lies within the linear elasticity regime. More precisely, we assume that the deformation is purely an axial stretch, denoted by A_z, and behaves as

$$\frac{\partial A_z}{\partial t} = 2e(t)A_z(t) - \frac{A_z(t)}{\tau}, \qquad \textbf{[14]}$$

where τ is an elastic relaxation time. Using Eq. **4** to substitute for $e(t)$ in **14** and after rearrangement, we find **14** can be rewritten as

$$\frac{\partial}{\partial t}\left[\ln(A_z)\right] = -4\frac{\partial}{\partial t}[\ln(a(t))] - \frac{1}{\tau}. \qquad \textbf{[15]}$$

Upon integrating **15** with respect to t and using the initial condition $A_z(t = 0) = 1$, we find

$$A_z = \left(\frac{a_0}{a(t)}\right)^4 e^{-t/\tau}. \qquad \textbf{[16]}$$

To get $a(t)$, we assume in addition that the elastic stress is sufficiently large so that the viscous stress associated with flow inside the bridge is negligible in comparison. In other words, the elastic stress completely balances the Laplace pressure due to the surface stretch modulus

$$\frac{S_c}{a(t)} = gA_z(t), \qquad \textbf{[17]}$$

where g is the elastic modulus. Substituting **16** into **17** yields

$$a(t) = a_0\left(\frac{a_0}{S_c}\, g\, e^{-t/\tau}\right)^{1/3}. \qquad \textbf{[18]}$$

The bridge radius decays as a near exponential with the characteristic decay time given by the relaxation time of the elastic stresses. Note in arriving at **18** we have simplified the dynamics considerably. A more complete treatment, which considers the dynamics at earlier times where viscous stresses are comparable with elastic stresses, can be found in Entov and Hinch (1). They found that when both effects are considered, the thinning dynamics first appears linear and, then, approaches an exponential decay as $a(t)$ becomes small.

Eqs. **12** and **18** were used to compare theoretical furrow thinning dynamics to observed furrow-thinning dynamics for wild-type and mutant cytokineses in Fig. 4.

1. Entov, V. M. & Hinch, E. J. (1997) *J. Non-Newtonian Fluid Mech.* **72,** 31-53.
2. Batchelor, G. K. (1967) *An Introduction to Fluid Mechanics* (Cambridge Univ. Press, Cambridge, U.K.).