nature portfolio

Peer Review File

Probing spin hydrodynamics on a superconducting quantum simulator

Editorial Note: This manuscript has been previously reviewed at another journal that is not operating a transparent peer review scheme. This document only contains reviewer comments and rebuttal letters for versions considered at *Nature Communications*.

Reviewers' Comments:

Reviewer #1:
Remarks to the Author:
I would like to thank the authors for their thorough response to my earlier comments. I find that all my comments have been adequately addressed, and believe that the revised manuscript has been significantly improved. I would therefore like to recommend the manuscript for publication in Nature Communications.

Minor note:

Ref. 43 has now been peer-reviewed and published: https://doi.org/10.1038/s41586-024-07325-z

Reviewer #2:
Remarks to the Author:
Yun-Hao Shi et al. perform a quantum simulation of emergent hydrodynamics in a coupled spin ladder of 24 qubits using their superconducting quantum information processor. In this work a novel experimental scheme is applied to measure infinite-temperature correlation functions that is based on typicality arguments. The authors apply this technique to ergodic systems, many-body localized systems, and systems subjected to a tilt. The autocorrelation function of these distinct systems is then analyzed.

Here are a couple of comments and questions:

1) In order to observe hydrodynamics, the total magnetization needs to be conserved. How does the magnetization evolve over the experimental time scales? In general, there could be leakage to higher occupation states of the resonator or just decay. It would be important to characterize the magnetization as a function of time. Such a plot should be added to the manuscript or supplement.
2) The authors have slightly rephrased the discussion on page 5 first paragraph and stated that for small t_R, the values of the observables are incompatible with infinite temperature autocorrelation functions. Could it still be that the prepared state is a close to an infinite temperature state (in the sense of ETH), but that time scales are too short

to get rid of coherence?
3) Any finite-time, finite-size system may exhibit intermediate regimes, in which the decay of correlation functions may look like a powerlaw of some sort. In the case of the system with a tilt, the observed power laws might be crossover effects; see for example the cold atom experiment: Phys. Rev. X 10, 011042 (2020), where a tilted Fermi-Hubbard model (2D) is analyzed (I think this work should be cited). There the authors find a crossover from diffusive (z=2) to subdiffusive (z=4) dynamics. As discussed in detail there, this is a finite size effect. For a thermodynamically large system, any tilt would do to observe z=4. Assuming that the tilted two-leg ladder is ergodic, similarly to the 2D Hubbard model, one would expect that at late times and for large systems z=4 is attained. I believe that the manuscript would benefit from including a discussion of that type.
4) Minor comments: (i) Usually in the recent literature z is the exponent relating space time in such a way that z=2 is diffusion and z=1 is ballistic. This corresponds to 1/z as used in this manuscript. To avoid confusion, I would suggest to go with the conventional definition of z or replace it by some other label. (ii) y-label in Fig. 2a should be “participation entropy”.

The manuscript needs to be carefully revised according to the comments above, before I can recommend publication in Nature Communications.

Re: NCOMMS-24-17804-T

Probing spin hydrodynamics on a superconducting quantum simulator
by Yun-Hao Shi, Zheng-Hang Sun, Yong-Yi Wang, *et al.*

Dear Editors,

We hope that this finds you well. Thank you for your message and consideration of our manuscript. In this letter, we provide point-to-point responses and the corresponding revisions. We thank both Reviewers for the careful reading and useful comments on our work. We thank Reviewer 1's recommendation for the publication of our manuscript in Nature Communications, and Reviewer 2's recognition of the novelty of the experimental scheme, which is applied to measure the infinite-temperature correlation functions.

According to the comments raised by both Reviewers, we have spent around two weeks conducting series of additional numerical simulations to further support the conclusions of our manuscript and revised the manuscript with the new data. We are enclosing the new version of our manuscript revised according to the comments and suggestions. The changes are marked in blue in the additional manuscripts. We hope these responses and the revised manuscripts can address the concerns of Reviewers.

We thank you for your time and consideration.

Kind regards,

the authors

Reply to Comments of Reviewer #1

(1-0) Reviewer #1 WROTE THAT:

I would like to thank the authors for their thorough response to my earlier comments. I find that all my comments have been adequately addressed, and believe that the revised manuscript has been significantly improved. I would therefore like to recommend the manuscript for publication in Nature Communications.

OUR REPLY:

We would like to thank the Reviewer for the careful reading of our manuscript and recognition. We also thank Reviewer's recommendation for the publication of our manuscript in Nature Communications.

(1-1) Reviewer #1 WROTE THAT:

Minor note:

Ref. 43 has now been peer-reviewed and published: https://doi.org/10.1038/s41586-024-07325-z

OUR REPLY:

In the revised manuscript, we have updated information of Ref. 43.

Reply to Comments of Reviewer #2

(2-0) Reviewer #2 WROTE THAT:

Yun-Hao Shi et al. perform a quantum simulation of emergent hydrodynamics in a coupled spin ladder of 24 qubits using their superconducting quantum information processor. In this work a novel experimental scheme is applied to measure infinite-temperature correlation functions that is based on typicality arguments. The authors apply this technique to ergodic systems, many-body localized systems, and systems subjected to a tilt. The autocorrelation function of these distinct systems is then analyzed.

OUR REPLY:

We are grateful to the Reviewer for this assessment, and the recognition for the novelty of the experimental scheme. The description explains well the main results of our manuscript.

(2-1) Reviewer #2 WROTE THAT:

Here are a couple of comments and questions:

In order to observe hydrodynamics, the total magnetization needs to be conserved. How does the magnetization evolve over the experimental time scales? In general, there could be leakage to higher occupation states of the resonator or just decay. It would be important to characterize the magnetization as a function of time. Such a plot should be added to the manuscript or supplement.

OUR REPLY:

We would like to thank the Reviewer for this comment. We agree with the Reviewer that for the observation of hydrodynamics, the total magnetization (or equivalently the particle number) should be conserved. As discussed in the Supplementary Note 1, the leakage to higher occupation states can be possibly induced by the finite value of the ratio between the anharmonicity $E_C/2\pi \simeq 200$ MHz and the coupling strength $J/2\pi \simeq 7$ MHz. If $E_C/J \to \infty$, the leakage to higher occupation states can be perfectly suppressed.

In the previous version of the Supplementary Information, we have already shown that the probability of the leakage is around 0.03 (see Fig. S2 of the original version). Here, to provide a more direct evidence for the approximate conservation of the particle number, we numerically simulate the time evolution of the particle number $\langle n(t)\rangle \equiv \langle\psi(t)|\hat{n}|\psi(t)\rangle =$

$\langle\psi(t)|\sum_i \hat{n}_i|\psi(t)\rangle$, with $\hat{n}_i \equiv |0\rangle_i\langle 0| + |1\rangle_i\langle 1|$, up to the experimental time scales $t \simeq 200$ ns. The results are displayed in Fig. R1. We emphasize that only the occupations of the states $|0\rangle$ and $|1\rangle$ are considered in the definition of $\hat{n}_i$, while the finite E_C/J allows the possibility of the leakage to the states with higher occupations, such as the state $|2\rangle$. Consequently, the decay of $\langle n(t)\rangle$ shown in Fig. R1 quantifies the leakage induced by the finite E_C/J. The stable value of $n(t)/2L$ with $t \simeq 200$ ns is about 0.4966, indicating a moderate impact of the finite E_C/J on the leakage.

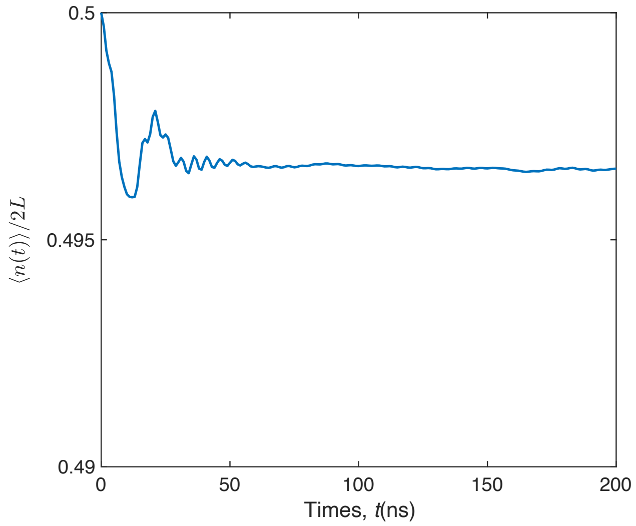

FIG. R1: The dynamics of $\langle n(t)\rangle/2L$ with $U/2\pi \simeq 200$ MHz, $J/2\pi \simeq 7$ MHz for the Bose-Hubbard model, and the $L = 8$ being the length of the ladder.

The Reviewer also suggests that the "decay" can also influence the conservation of the particle number. The "decay" can be induced by the energy relaxation, an effect of decoherence characterized by T_1. To quantify the influence of decoherence, we numerically simulate the dynamics of $n(t)$ by solving the Lindblad master equation

$$\frac{\mathrm{d}\hat{\rho}(t)}{\mathrm{d}t} = i[\hat{H}, \hat{\rho}(t)] + \sum_{j=1}^{N}(\hat{L}_j\hat{\rho}(t)\hat{L}_j^\dagger - \frac{1}{2}\{\hat{L}_j^\dagger\hat{L}_j, \hat{\rho}(t)\}), \tag{R1}$$

where $\hat{\rho}(t) = |\psi(t)\rangle\langle\psi(t)|$ is the density matrix, $\hat{H}$ is the Hamiltonian (1) in the main text, and $\hat{L}_j = \hat{\sigma}_j^-/\sqrt{T_1}$ represents the Lindblad operators for the energy relaxation, with T_1 being the energy lifetime.

For the numerical simulation, we adopt $T_1 = 32.1$ μs, based on the device information as shown in the Table. S1 in the Supplementary Information. Moreover, we directly consider the two-level qubit model, whose Hamiltonian is given by Eq. (1) in the main text, since the influence of the leakage to higher occupations can be neglected. The number of qubits is $N = 16$. We employ the stochastic Schrödinger equation to efficiently solve the Lindblad master equation (R1) [Schaller2014]. The numerical results are plotted in Fig. R2(a). With a evolved time $t = 200$ ns, the value of $\langle n(t)\rangle/2L$ is around 0.497, suggesting that decoherence does not significantly influence the conservation of particle number. We also numerically demonstrate that the decoherence does not strongly affect the dynamics of autocorrelation function $C_{1,1}(t)$ with the evolved time up to 200 ns. The dynamics of $C_{1,1}(t)$, simulated by solving the Lindblad master equation (R1), is more or less the same to the unitary dynamics [see Fig. R2(b)].

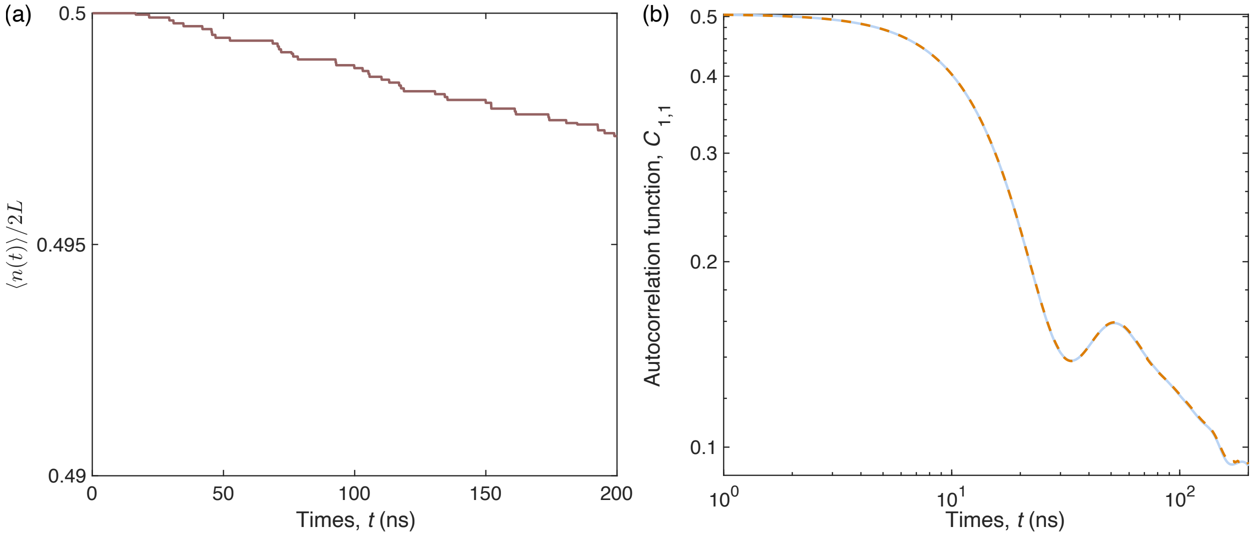

FIG. R2: (a) For the qubit ladder with a length $L = 8$ (the total number of qubits is $N = 16$), the dynamics of $\langle n(t)\rangle/2L$ with the energy relaxation, as an effect of decoherence quantified by $T_1 = 32.1\ \mu$s. (b) Numerical simulation of the autocorrelation function $C_{1,1}(t)$ for the qubit ladder with a length $L = 8$. The solid curve shows the the results obtained from unitary dynamics. The dashed curve represents the results of the dynamics with decoherence governed by Eq. (R1) and $T_1 = 32.1\ \mu$s.

In the revised version of the Supplementary Information, we have added Fig. R1 to the Fig. S2 (as the new Fig. S2**b**), and related discussions to Supplementary Note 1. Moreover, we have added a new section in the revised version of the Supplementary Information, i.e., **Supplementary Note 4: The effect of decoherence**, to discuss the impact of decoherence on the conservation of the particle number and the dynamics of autocorrelation functions. Figure. R2 and related discussions have been presented in this new section.

[Schaller2014] G. Schaller, *Open quantum systems far from equilibrium*, Springer, 2014.

(2-2) Reviewer #2 WROTE THAT:

The authors have slightly rephrased the discussion on page 5 first paragraph and stated that for small t_R, the values of the observables are incompatible with infinite temperature autocorrelation functions. Could it still be that the prepared state is a close to an infinite temperature state (in the sense of ETH), but that time scales are too short to get rid of coherence?

OUR REPLY:

We would like to thank the Reviewer for this comment. Based on Eq. (5) of the manuscript, the measurement of infinite-temperature spin transport requires the Haar-random state (one can see the first section of Method for more details). To prepare the Haar-random state, we perform the evolution $|\psi_R\rangle = \hat{U}_R(t_R)|\psi_0\rangle$ [see Fig. 1**c**]. We agree with the Reviewer that in the sense of ETH, since the initial state has an effective high temperature for the Hamiltonian $\hat{H}_R = \hat{H}_I + \hat{H}_d$, with sufficiently long evolved time t_R, coherence vanished, and the state can approach the Haar-random state.

Here, with a short evolved time $t_R \simeq 15$ ns, the time scale is too short to get rid of coherence, and consequently, the state $|\psi_R\rangle$ with $t_R \simeq 15$ ns is far away from the Haar-random state, which can be further verified by checking the fact that the value of S_{PE} with $t_R \simeq 15$ ns is much smaller than the $S_{\mathrm{PE}}^{\mathrm{T}}$ (see Fig. 2**a** of the main text).

To clarify it based on the comment and question of the reviewer, we have rephrased the second paragraph of the section "**Observation of diffusive transport**" with revised discussions:

"Moreover, in Supplementary Note 4, we show that for a short evolved time $t_R \simeq 15$ ns, the values of the observable defined in Eq. (4) are incompatible with the infinite-temperature autocorrelation functions. Given that the chosen initial state for generating the Haar-random state exhibits a high effective temperature associated with the Hamiltonian $\hat{H}_R$, the state $|\psi^R\rangle$ would asymptotically converge to the Haar-random state with a sufficiently extended

t_R. However, with $t_R \simeq 15$ ns, the time scale is too small to get rid of the coherence, and the value of $S_{\rm PE}$ for the state $|\psi^R\rangle$ is much smaller than the $S^{\rm T}_{\rm PE}$ (see Fig. 2**a**), suggesting that $|\psi^R\rangle$ with $t_R \simeq 15$ ns is far away from the Haar-random state and cannot be employed to measure the infinite-temperature autocorrelation function (2)."

(2-3) Reviewer #2 WROTE THAT:

Any finite-time, finite-size system may exhibit intermediate regimes, in which the decay of correlation functions may look like a power law of some sort. In the case of the system with a tilt, the observed power laws might be crossover effects; see for example the cold atom experiment: Phys. Rev. X 10, 011042 (2020), where a tilted Fermi-Hubbard model (2D) is analyzed (I think this work should be cited). There, the authors find a crossover from diffusive ($z = 2$) to subdiffusive ($z = 4$) dynamics. As discussed in detail there, this is a finite size effect. For a thermodynamically large system, any tilt would do to observe $z = 4$. Assuming that the tilted two-leg ladder is ergodic, similarly to the 2D Hubbard model, one would expect that at late times and for large systems $z = 4$ is attained. I believe that the manuscript would benefit from including a discussion of that type.

OUR REPLY:

We would like to thank the Reviewer for this comment. In the **Discussion** section of the revised manuscript, we have added following sentences:

"Theoretically, it has been suggested that for a thermodynamically large system, non-zero tilted potentials, i.e., $\Delta > 0$, will lead to a subdiffusive transport with $\alpha \simeq 1/4$ [Feldmeier2020,Nandy2024]. In finite-size systems, both results as shown in Fig. 4 and the cold atom experiments on the tilted Fermi-Hubbard model [Guardado-Sanchez2020] demonstrate a crossover from the diffusive regime to the subdiffusive one. Investigating how this crossover scales with an increasing system size is a further experimental task, which requires for quantum simulators with a larger number of qubits."

Moreover, in the revised manuscript, we have added two new references [Guardado-Sanchez2020,Nandy2024].

[Feldmeier2020] J. Feldmeier, P. Sala, G. De Tomasi, F. Pollmann, and M. Knap, "Anomalous Diffusion in Dipole- and Higher-Moment-Conserving Systems", Phys. Rev. Lett. 125,

245303 (2020).

[Nandy2024] S. Nandy, J. Herbrych, Z. Lenarčič, A. Głódkowski, P. Prelovšek, and M. Mierzejewski, "Emergent dipole moment conservation and subdiffusion in tilted chains", Phys. Rev. B 109, 115120 (2024).

[Guardado-Sanchez2020] E. Guardado-Sanchez, A. Morningstar, B. M. Spar, P. T. Brown, D. A. Huse, and W. S. Bakr, "Subdiffusion and Heat Transport in a Tilted Two-Dimensional Fermi-Hubbard System", Phys. Rev. X 10, 011042 (2020).

(2-4) Reviewer #2 WROTE THAT:

Minor comments: (i) Usually in the recent literature z is the exponent relating space time in such a way that $z = 2$ is diffusion and $z = 1$ is ballistic. This corresponds to $1/z$ as used in this manuscript. To avoid confusion, I would suggest to go with the conventional definition of z or replace it by some other label. (ii) y-label in Fig. 2a should be "participation entropy".

OUR REPLY:

We would like to thank the reviewer for the comments. In the revised manuscript, we have replaced the z by the another label α to avoid confusion. Note that the labels α and β appear in the original manuscript for the definition of Eq. (4), and we have also replaced them by other labels (α and β are replaced by μ and nu respectively). We also change the y-label of the Fig. 2a as "participation entropy".

Summary of changes in the main text

All our changes are marked in blue in the additional manuscript of the main text.

1. The notation of the transport exponent z is replaced by α.
2. The notations α and β in the original version of the manuscript are replace by μ and ν.
3. The second paragraph of the section "**Observation of diffusive transport**" is revised.
4. Additional discussions are added in the second paragraph of the section "**Discussion**".
5. Two references, i.e., Ref. [62,63] are added.

Summary of changes in the supplementary information

All our changes are marked in blue in the additional manuscript of the supplementary information.

1. Additional discussions in the second paragraph of the Supplementary Note 1 are added.

2. Fig. S2 is revised.

3. A new figure, Fig. S7, is added.

4. A new section, Supplementary Note 4, is added to discuss the influence of decoherence.

Reviewers' Comments:

Reviewer #2:

Remarks to the Author:

The authors have addressed most of my comments (except for showing experimental data on the particle number conservation, which I think would have been good to have). Anyway, overall the manuscript is now ready for being accepted according to my opinion.