Supplementary information

Power flow analysis using quantum and digital annealers: A discrete combinatorial optimization approach

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Supplementary tables

For justifying the partitioned AQPF algorithm applied on the 14-bus test system, the obtained $\vec{\mu}$ and $\vec{\omega}$ with $\epsilon = 1 \times 10^{-3}$ using DA are presented in the following table. The corresponding $\vec{\mu}$ and $\vec{\omega}$ obtained by NR are also included in the table.

Table 1: Comparison of $\vec{\mu}$ and $\vec{\omega}$ obtained by DA using a partitioned AQPF and NR for the 14-bus test system.

	DA		NR	
i	μ_i	ω_i	μ_i	ω_i
0	1.0	0.0	1.0	0.0
1	1.045	-0.106	1.038	-0.118
2	0.986	-0.239	0.978	-0.249
3	1.001	-0.197	0.996	-0.203
4	1.007	-0.154	0.999	-0.176
5	1.051	-0.280	1.029	-0.292
6	1.030	-0.284	1.012	-0.265
7	1.075	-0.257	1.054	-0.276
8	1.010	-0.290	0.989	-0.288
9	1.001	-0.289	0.987	-0.291
10	1.028	-0.296	1.014	-0.293
11	1.063	-0.312	1.028	-0.302
12	1.002	-0.297	1.002	-0.301
13	0.979	-0.302	0.972	-0.306

To justify the expected reduction of problem encoding correctness from the discretization of the PF problem, we compare the results obtained using the Newton-Raphson (NR) method and the Adiabatic Quantum Power Flow (AQPF) algorithm with the QUBO formulation and the Digital Annealer (DA) solver over iterations. The following remarks should be noted:

- Although the number of iterations required by NR is less compared to DA, the clock time consumed per iteration on DA is significantly less by orders of magnitude.
- The accuracy of the results obtained in the final iteration by the DA can be further refined if a smaller threshold is set. This implies that the AQPF algorithm can achieve even higher precision if required.
- One significant advantage of the AQPF algorithm is its ability to mitigate the divergence problem commonly faced by classical solvers. The

Table 2: Performance comparison of Fujitsu's digital annealer V3 (DA) with the Newton-Raphson classical solver (NR) over iterations (it). The AQPF algorithm is implemented using the QUBO formulation for the 4-bus test system. The reference bus $i = 0$ with known values of $\mu_0 = 1$ and $\omega_0 = 0$ is not shown.

	it	μ_1	μ_2	μ_3	ω_1	ω_2	ω_3
NR	1	0.95	0.96	0.94	-0.03	-0.025	-0.04
	2	0.925	0.935	0.915	-0.06	-0.055	-0.07
	3	0.91	0.921	0.901	-0.092	-0.080	-0.104
	4	0.902	0.916	0.890	-0.092	-0.080	-0.104
DA	1	0.96	0.96	0.96	-0.04	-0.04	-0.04
	2	0.94	0.94	0.94	-0.06	-0.06	-0.06
	3	0.94	0.94	0.92	-0.06	-0.06	-0.08
	4	0.92	0.94	0.9	-0.08	-0.08	-0.1
	5	0.91	0.93	0.9	-0.09	-0.08	-0.1
	6	0.91	0.93	0.9	-0.09	-0.08	-0.1
	7	0.91	0.93	0.9	-0.09	-0.08	-0.1
	8	0.91	0.93	0.9	-0.09	-0.08	-0.1
	9	0.909	0.929	0.901	-0.09	-0.079	-0.101
	10	0.91	0.928	0.901	-0.091	-0.078	-0.102
	11	0.909	0.927	0.901	-0.091	-0.079	-0.103
	12	0.91	0.926	0.902	-0.091	-0.079	-0.103
	13	0.91	0.921	0.897	-0.091	-0.079	-0.103
	14	0.91	0.921	0.897	-0.091	-0.079	-0.103
	15	0.91	0.921	0.897	-0.091	-0.079	-0.103
	16	0.91	0.921	0.897	-0.091	-0.079	-0.103
	17	0.91	0.921	0.897	-0.091	-0.079	-0.103
	18	0.91	0.921	0.897	-0.091	-0.079	-0.103
	19	0.91	0.921	0.897	-0.091	-0.079	-0.103
	20	0.91	0.921	0.897	-0.091	-0.079	-0.103
	21	0.909	0.92	0.898	-0.091	-0.079	-0.103
	22	0.908	0.919	0.897	-0.091	-0.079	-0.103
	23	0.904	0.918	0.896	-0.092	-0.079	-0.103
	24	0.904	0.917	0.892	-0.092	-0.079	-0.104

AQPF algorithm demonstrates robustness in scenarios where the power consumption information is incomplete.

Supplementary equations

For the QUBO formulation of the net active power, substitution of equations (7)–(8) into equation (5) yields

$$
\begin{aligned}
p_i = \sum_{j=0}^{n} & \mu_i^0 g_{ij} \mu_j^0 + \mu_i^0 g_{ij} x_{j,0}^{\mu} \Delta\mu_j - \mu_i^0 g_{ij} x_{j,1}^{\mu} \Delta\mu_j \\
+ \; & x_{i,0}^{\mu} \Delta\mu_i g_{ij} \mu_j^0 + x_{i,0}^{\mu} \Delta\mu_i g_{ij} x_{j,0}^{\mu} \Delta\mu_j - x_{i,0}^{\mu} \Delta\mu_i g_{ij} x_{j,1}^{\mu} \Delta\mu_j \\
- \; & x_{i,1}^{\mu} \Delta\mu_i g_{ij} \mu_j^0 - x_{i,1}^{\mu} \Delta\mu_i g_{ij} x_{j,0}^{\mu} \Delta\mu_j + x_{i,1}^{\mu} \Delta\mu_i g_{ij} x_{j,1}^{\mu} \Delta\mu_j \\
+ \; & \omega_i^0 g_{ij} \omega_j^0 + \omega_i^0 g_{ij} x_{j,0}^{\omega} \Delta\omega_j - \omega_i^0 g_{ij} x_{j,1}^{\omega} \Delta\omega_j \\
+ \; & x_{i,0}^{\omega} \Delta\omega_i g_{ij} \omega_j^0 + x_{i,0}^{\omega} \Delta\omega_i g_{ij} x_{j,0}^{\omega} \Delta\omega_j - x_{i,0}^{\omega} \Delta\omega_i g_{ij} x_{j,1}^{\omega} \Delta\omega_j \\
- \; & x_{i,1}^{\omega} \Delta\omega_i g_{ij} \omega_j^0 - x_{i,1}^{\omega} \Delta\omega_i g_{ij} x_{j,0}^{\omega} \Delta\omega_j + x_{i,1}^{\omega} \Delta\omega_i g_{ij} x_{j,1}^{\omega} \Delta\omega_j \\
+ \; & \omega_i^0 b_{ij} \mu_j^0 + \omega_i^0 b_{ij} x_{j,0}^{\mu} \Delta\mu_j - \omega_i^0 b_{ij} x_{j,1}^{\mu} \Delta\mu_j \\
+ \; & x_{i,0}^{\omega} \Delta\omega_i b_{ij} \mu_j^0 + x_{i,0}^{\omega} \Delta\omega_i b_{ij} x_{j,0}^{\mu} \Delta\mu_j - x_{i,0}^{\omega} \Delta\omega_i b_{ij} x_{j,1}^{\mu} \Delta\mu_j \\
- & x_{i,1}^{\omega} \Delta\omega_i b_{ij} \mu_j^0 - x_{i,1}^{\omega} \Delta\omega_i b_{ij} x_{j,0}^{\mu} \Delta\mu_j + x_{i,1}^{\omega} \Delta\omega_i b_{ij} x_{j,1}^{\mu} \Delta\mu_j \\
- \; & \mu_i^0 b_{ij} \omega_j^0 - \mu_i^0 b_{ij} x_{j,0}^{\omega} \Delta\omega_j + \mu_i^0 b_{ij} x_{j,1}^{\omega} \Delta\omega_j \\
- \; & x_{i,0}^{\mu} \Delta\mu_i b_{ij} \omega_j^0 - x_{i,0}^{\mu} \Delta\mu_i b_{ij} x_{j,0}^{\omega} \Delta\omega_j + x_{i,0}^{\mu} \Delta\mu_i b_{ij} x_{j,1}^{\omega} \Delta\omega_j \\
+ \; & x_{i,1}^{\mu} \Delta\mu_i b_{ij} \omega_j^0 + x_{i,1}^{\mu} \Delta\mu_i b_{ij} x_{j,0}^{\omega} \Delta\omega_j - x_{i,1}^{\mu} \Delta\mu_i b_{ij} x_{j,1}^{\omega} \Delta\omega_j .
\end{aligned}
\tag{1}
$$

A similar expression can also be derived for the QUBO formulation of the net reactive power injection by substituting (7)–(8) into equation (6).

$$
\begin{aligned}
q_i = \sum_{j=0}^{n} \omega_i^0 g_{ij} \mu_j^0 + \omega_i^0 g_{ij} x_{j,0}^{\mu} \Delta\mu_j - \omega_i^0 g_{ij} x_{j,1}^{\mu} \Delta\mu_j \\
+\ x_{i,0}^{\omega} \Delta\omega_i g_{ij} \mu_j^0 + x_{i,0}^{\omega} \Delta\omega_i g_{ij} x_{j,0}^{\mu} \Delta\mu_j - x_{i,0}^{\omega} \Delta\omega_i g_{ij} x_{j,1}^{\mu} \Delta\mu_j \\
-\ x_{i,1}^{\omega} \Delta\omega_i g_{ij} \mu_j^0 - x_{i,1}^{\omega} \Delta\omega_i g_{ij} x_{j,0}^{\mu} \Delta\mu_j + x_{i,1}^{\omega} \Delta\omega_i g_{ij} x_{j,1}^{\mu} \Delta\mu_j \\
-\ \mu_i^0 g_{ij} \omega_j^0 - \mu_i^0 g_{ij} x_{j,0}^{\omega} \Delta\omega_j + \mu_i^0 g_{ij} x_{j,1}^{\omega} \Delta\omega_j \\
-\ x_{i,0}^{\mu} \Delta\mu_i g_{ij} \omega_j^0 - x_{i,0}^{\mu} \Delta\mu_i g_{ij} x_{j,0}^{\omega} \Delta\omega_j + x_{i,0}^{\mu} \Delta\mu_i g_{ij} x_{j,1}^{\omega} \Delta\omega_j \\
+\ x_{i,1}^{\mu} \Delta\mu_i g_{ij} \omega_j^0 + x_{i,1}^{\mu} \Delta\mu_i g_{ij} x_{j,0}^{\omega} \Delta\omega_j - x_{i,1}^{\mu} \Delta\mu_i g_{ij} x_{j,1}^{\omega} \Delta\omega_j \\
-\ \mu_i^0 b_{ij} \mu_j^0 - \mu_i^0 b_{ij} x_{j,0}^{\mu} \Delta\mu_j + \mu_i^0 b_{ij} x_{j,1}^{\mu} \Delta\mu_j \\
-\ x_{i,0}^{\mu} \Delta\mu_i b_{ij} \mu_j^0 - x_{i,0}^{\mu} \Delta\mu_i b_{ij} x_{j,0}^{\mu} \Delta\mu_j + x_{i,0}^{\mu} \Delta\mu_i b_{ij} x_{j,1}^{\mu} \Delta\mu_j \\
+\ x_{i,1}^{\mu} \Delta\mu_i b_{ij} \mu_j^0 + x_{i,1}^{\mu} \Delta\mu_i b_{ij} x_{j,0}^{\mu} \Delta\mu_j - x_{i,1}^{\mu} \Delta\mu_i b_{ij} x_{j,1}^{\mu} \Delta\mu_j \\
-\ \omega_i^0 b_{ij} \omega_j^0 - \omega_i^0 b_{ij} x_{j,0}^{\omega} \Delta\omega_j + \omega_i^0 b_{ij} x_{j,1}^{\omega} \Delta\omega_j \\
-\ x_{i,0}^{\omega} \Delta\omega_i b_{ij} \omega_j^0 - x_{i,0}^{\omega} \Delta\omega_i b_{ij} x_{j,0}^{\omega} \Delta\omega_j + x_{i,0}^{\omega} \Delta\omega_i b_{ij} x_{j,1}^{\omega} \Delta\omega_j \\
+\ x_{i,1}^{\omega} \Delta\omega_i b_{ij} \omega_j^0 + x_{i,1}^{\omega} \Delta\omega_i b_{ij} x_{j,0}^{\omega} \Delta\omega_j - x_{i,1}^{\omega} \Delta\omega_i b_{ij} x_{j,1}^{\omega} \Delta\omega_j.
\end{aligned}
\tag{2}
$$

Supplementary equations (1)–(2) are extended forms of equations (9)–(10) presented in the paper, respectively.

In equation (9), the constant clause is

$$
\sum_{j=0}^{n} \mu_i^0 g_{ij} \mu_j^0 + \omega_i^0 g_{ij} \omega_j^0 + \omega_i^0 b_{ij} \mu_j^0 - \mu_i^0 b_{ij} \omega_j^0, \tag{3}
$$

the linear clause is

$$
\begin{aligned}
\sum_{j=0}^{n} \sum_{k=0}^{1} (-1)^k \mu_i^0 g_{ij} x_{j,k}^{\mu} \Delta\mu_j + (-1)^k x_{i,k}^{\mu} \Delta\mu_i g_{ij} \mu_j^0 \\
+(-1)^k \omega_i^0 g_{ij} x_{j,k}^{\omega} \Delta\omega_j + (-1)^k x_{i,k}^{\omega} \Delta\omega_i g_{ij} \omega_j^0 \\
+(-1)^k \omega_i^0 b_{ij} x_{j,k}^{\mu} \Delta\mu_j + (-1)^k x_{i,k}^{\omega} \Delta\omega_i b_{ij} \mu_j^0 \\
-(-1)^k \mu_i^0 b_{ij} x_{j,k}^{\omega} \Delta\omega_j - (-1)^k x_{i,k}^{\mu} \Delta\mu_i b_{ij} \omega_j^0,
\end{aligned}
\tag{4}
$$

and the quadratic clause is

$$\sum_{j=0}^{n}\sum_{k=0}^{1}\sum_{l=0}^{1}(-1)^{k+l}x_{i,k}^{\mu}\Delta\mu_i g_{ij}x_{j,l}^{\mu}\Delta\mu_j +(-1)^{k+l}x_{i,k}^{\omega}\Delta\omega_i g_{ij}x_{j,l}^{\omega}\Delta\omega_j +(-1)^{k+l}x_{i,k}^{\omega}\Delta\omega_i b_{ij}x_{j,l}^{\mu}\Delta\mu_j -(-1)^{k+l}x_{i,k}^{\mu}\Delta\mu_i b_{ij}x_{j,l}^{\omega}\Delta\omega_j, \tag{5}$$

Similarly, in equation (10), the constant clause is

$$\sum_{j=0}^{n}\omega_i^0 g_{ij}\mu_j^0 - \mu_i^0 g_{ij}\omega_j^0 - \mu_i^0 b_{ij}\mu_j^0 - \omega_i^0 b_{ij}\omega_j^0, \tag{6}$$

the linear clause is

$$\sum_{j=0}^{n}\sum_{k=0}^{1}(-1)^k\omega_i^0 g_{ij}x_{j,k}^{\mu}\Delta\mu_j + (-1)^k x_{i,k}^{\omega}\Delta\omega_i g_{ij}\mu_j^0 -(-1)^k\mu_i^0 g_{ij}x_{j,k}^{\omega}\Delta\omega_j - (-1)^k x_{i,k}^{\mu}\Delta\mu_i g_{ij}\omega_j^0 -(-1)^k\mu_i^0 b_{ij}x_{j,k}^{\mu}\Delta\mu_j - (-1)^k x_{i,k}^{\mu}\Delta\mu_i b_{ij}\mu_j^0 -(-1)^k\omega_i^0 b_{ij}x_{j,k}^{\omega}\Delta\omega_j - (-1)^k x_{i,k}^{\omega}\Delta\omega_i b_{ij}\omega_j^0, \tag{7}$$

and the quadratic clause is

$$\sum_{j=0}^{n}\sum_{k=0}^{1}\sum_{l=0}^{1}(-1)^{k+l}x_{i,k}^{\omega}\Delta\omega_i g_{ij}x_{j,l}^{\mu}\Delta\mu_j -(-1)^{k+l}x_{i,k}^{\mu}\Delta\mu_i g_{ij}x_{j,l}^{\omega}\Delta\omega_j -(-1)^{k+l}x_{i,k}^{\mu}\Delta\mu_i b_{ij}x_{j,l}^{\mu}\Delta\mu_j -(-1)^{k+l}x_{i,k}^{\omega}\Delta\omega_i b_{ij}x_{j,l}^{\omega}\Delta\omega_j, \tag{8}$$

For the Ising model formulation of the net active power, the substitution of equations (11)–(12) into equation (5) yields

$$
\begin{aligned}
p_i = \sum_{j=0}^{n} \mu_i^0 g_{ij}\mu_j^0 + \mu_i^0 g_{ij} s_{j,0}^{\mu}\Delta\mu_j + \mu_i^0 g_{ij} s_{j,1}^{\mu}\Delta\mu_j + \mu_i^0 g_{ij} s_{j,2}^{\mu}\Delta\mu_j \\
+\ s_{i,0}^{\mu}\Delta\mu_i g_{ij}\mu_j^0 + s_{i,0}^{\mu}\Delta\mu_i g_{ij} s_{j,0}^{\mu}\Delta\mu_j + s_{i,0}^{\mu}\Delta\mu_i g_{ij} s_{j,1}^{\mu}\Delta\mu_j + s_{i,0}^{\mu}\Delta\mu_i g_{ij} s_{j,2}^{\mu}\Delta\mu_j \\
+\ s_{i,1}^{\mu}\Delta\mu_i g_{ij}\mu_j^0 + s_{i,1}^{\mu}\Delta\mu_i g_{ij} s_{j,0}^{\mu}\Delta\mu_j + s_{i,1}^{\mu}\Delta\mu_i g_{ij} s_{j,1}^{\mu}\Delta\mu_j + s_{i,1}^{\mu}\Delta\mu_i g_{ij} s_{j,2}^{\mu}\Delta\mu_j \\
+\ s_{i,2}^{\mu}\Delta\mu_i g_{ij}\mu_j^0 + s_{i,2}^{\mu}\Delta\mu_i g_{ij} s_{j,0}^{\mu}\Delta\mu_j + s_{i,2}^{\mu}\Delta\mu_i g_{ij} s_{j,1}^{\mu}\Delta\mu_j + s_{i,2}^{\mu}\Delta\mu_i g_{ij} s_{j,2}^{\mu}\Delta\mu_j \\
+\ \omega_i^0 g_{ij}\omega_j^0 + \omega_i^0 g_{ij} s_{j,0}^{\omega}\Delta\omega_j + \omega_i^0 g_{ij} s_{j,1}^{\omega}\Delta\omega_j + \omega_i^0 g_{ij} s_{j,2}^{\omega}\Delta\omega_j \\
+\ s_{i,0}^{\omega}\Delta\omega_i g_{ij}\omega_j^0 + s_{i,0}^{\omega}\Delta\omega_i g_{ij} s_{j,0}^{\omega}\Delta\omega_j + s_{i,0}^{\omega}\Delta\omega_i g_{ij} s_{j,1}^{\omega}\Delta\omega_j + s_{i,0}^{\omega}\Delta\omega_i g_{ij} s_{j,2}^{\omega}\Delta\omega_j \\
+\ s_{i,1}^{\omega}\Delta\omega_i g_{ij}\omega_j^0 + s_{i,1}^{\omega}\Delta\omega_i g_{ij} s_{j,0}^{\omega}\Delta\omega_j + s_{i,1}^{\omega}\Delta\omega_i g_{ij} s_{j,1}^{\omega}\Delta\omega_j + s_{i,1}^{\omega}\Delta\omega_i g_{ij} s_{j,2}^{\omega}\Delta\omega_j \\
+\ s_{i,2}^{\omega}\Delta\omega_i g_{ij}\omega_j^0 + s_{i,2}^{\omega}\Delta\omega_i g_{ij} s_{j,0}^{\omega}\Delta\omega_j + s_{i,2}^{\omega}\Delta\omega_i g_{ij} s_{j,1}^{\omega}\Delta\omega_j + s_{i,2}^{\omega}\Delta\omega_i g_{ij} s_{j,2}^{\omega}\Delta\omega_j \\
+\ \omega_i^0 b_{ij}\mu_j^0 + \omega_i^0 b_{ij} s_{j,0}^{\mu}\Delta\mu_j + \omega_i^0 b_{ij} s_{j,1}^{\mu}\Delta\mu_j + \omega_i^0 b_{ij} s_{j,2}^{\mu}\Delta\mu_j \\
+\ s_{i,0}^{\omega}\Delta\omega_i b_{ij}\mu_j^0 + s_{i,0}^{\omega}\Delta\omega_i b_{ij} s_{j,0}^{\mu}\Delta\mu_j + s_{i,0}^{\omega}\Delta\omega_i b_{ij} s_{j,1}^{\mu}\Delta\mu_j + s_{i,0}^{\omega}\Delta\omega_i b_{ij} s_{j,2}^{\mu}\Delta\mu_j \\
+\ s_{i,1}^{\omega}\Delta\omega_i b_{ij}\mu_j^0 + s_{i,1}^{\omega}\Delta\omega_i b_{ij} s_{j,0}^{\mu}\Delta\mu_j + s_{i,1}^{\omega}\Delta\omega_i b_{ij} s_{j,1}^{\mu}\Delta\mu_j + s_{i,1}^{\omega}\Delta\omega_i b_{ij} s_{j,2}^{\mu}\Delta\mu_j \\
+\ s_{i,2}^{\omega}\Delta\omega_i b_{ij}\mu_j^0 + s_{i,2}^{\omega}\Delta\omega_i b_{ij} s_{j,0}^{\mu}\Delta\mu_j + s_{i,2}^{\omega}\Delta\omega_i b_{ij} s_{j,1}^{\mu}\Delta\mu_j + s_{i,2}^{\omega}\Delta\omega_i b_{ij} s_{j,2}^{\mu}\Delta\mu_j \\
-\ \mu_i^0 b_{ij}\omega_j^0 - \mu_i^0 b_{ij} s_{j,0}^{\omega}\Delta\omega_j - \mu_i^0 b_{ij} s_{j,1}^{\omega}\Delta\omega_j - \mu_i^0 b_{ij} s_{j,2}^{\omega}\Delta\omega_j \\
-\ s_{i,0}^{\mu}\Delta\mu_i b_{ij}\omega_j^0 - s_{i,0}^{\mu}\Delta\mu_i b_{ij} s_{j,0}^{\omega}\Delta\omega_j - s_{i,0}^{\mu}\Delta\mu_i b_{ij} s_{j,1}^{\omega}\Delta\omega_j - s_{i,0}^{\mu}\Delta\mu_i b_{ij} s_{j,2}^{\omega}\Delta\omega_j \\
-\ s_{i,1}^{\mu}\Delta\mu_i b_{ij}\omega_j^0 - s_{i,1}^{\mu}\Delta\mu_i b_{ij} s_{j,0}^{\omega}\Delta\omega_j - s_{i,1}^{\mu}\Delta\mu_i b_{ij} s_{j,1}^{\omega}\Delta\omega_j - s_{i,1}^{\mu}\Delta\mu_i b_{ij} s_{j,2}^{\omega}\Delta\omega_j \\
-\ s_{i,2}^{\mu}\Delta\mu_i b_{ij}\omega_j^0 - s_{i,2}^{\mu}\Delta\mu_i b_{ij} s_{j,0}^{\omega}\Delta\omega_j - s_{i,2}^{\mu}\Delta\mu_i b_{ij} s_{j,1}^{\omega}\Delta\omega_j - s_{i,2}^{\mu}\Delta\mu_i b_{ij} s_{j,2}^{\omega}\Delta\omega_j .
\end{aligned}
\tag{9}
$$

A similar expression can also be derived for the Ising model formulation of the net reactive power injection by substituting (11)–(12) into equation (6).

$$
\begin{aligned}
q_i = \sum_{j=0}^{n} & \omega_i^0 g_{ij}\mu_j^0 + \omega_i^0 g_{ij} s_{j,0}^{\mu}\Delta\mu_j + \omega_i^0 g_{ij} s_{j,1}^{\mu}\Delta\mu_j + \omega_i^0 g_{ij} s_{j,2}^{\mu}\Delta\mu_j \\
+ \; & s_{i,0}^{\omega}\Delta\omega_i g_{ij}\mu_j^0 + s_{i,0}^{\omega}\Delta\omega_i g_{ij} s_{j,0}^{\mu}\Delta\mu_j + s_{i,0}^{\omega}\Delta\omega_i g_{ij} s_{j,1}^{\mu}\Delta\mu_j + s_{i,0}^{\omega}\Delta\omega_i g_{ij} s_{j,2}^{\mu}\Delta\mu_j \\
+ \; & s_{i,1}^{\omega}\Delta\omega_i g_{ij}\mu_j^0 + s_{i,1}^{\omega}\Delta\omega_i g_{ij} s_{j,0}^{\mu}\Delta\mu_j + s_{i,1}^{\omega}\Delta\omega_i g_{ij} s_{j,1}^{\mu}\Delta\mu_j + s_{i,1}^{\omega}\Delta\omega_i g_{ij} s_{j,2}^{\mu}\Delta\mu_j \\
+ \; & s_{i,2}^{\omega}\Delta\omega_i g_{ij}\mu_j^0 + s_{i,2}^{\omega}\Delta\omega_i g_{ij} s_{j,0}^{\mu}\Delta\mu_j + s_{i,2}^{\omega}\Delta\omega_i g_{ij} s_{j,1}^{\mu}\Delta\mu_j + s_{i,2}^{\omega}\Delta\omega_i g_{ij} s_{j,2}^{\mu}\Delta\mu_j \\
- \; & \mu_i^0 g_{ij}\omega_j^0 - \mu_i^0 g_{ij} s_{j,0}^{\omega}\Delta\omega_j - \mu_i^0 g_{ij} s_{j,1}^{\omega}\Delta\omega_j - \mu_i^0 g_{ij} s_{j,2}^{\omega}\Delta\omega_j \\
- \; & s_{i,0}^{\mu}\Delta\mu_i g_{ij}\omega_j^0 - s_{i,0}^{\mu}\Delta\mu_i g_{ij} s_{j,0}^{\omega}\Delta\omega_j - s_{i,0}^{\mu}\Delta\mu_i g_{ij} s_{j,1}^{\omega}\Delta\omega_j - s_{i,0}^{\mu}\Delta\mu_i g_{ij} s_{j,2}^{\omega}\Delta\omega_j \\
- \; & s_{i,1}^{\mu}\Delta\mu_i g_{ij}\omega_j^0 - s_{i,1}^{\mu}\Delta\mu_i g_{ij} s_{j,0}^{\omega}\Delta\omega_j - s_{i,1}^{\mu}\Delta\mu_i g_{ij} s_{j,1}^{\omega}\Delta\omega_j - s_{i,1}^{\mu}\Delta\mu_i g_{ij} s_{j,2}^{\omega}\Delta\omega_j \\
- \; & s_{i,2}^{\mu}\Delta\mu_i g_{ij}\omega_j^0 - s_{i,2}^{\mu}\Delta\mu_i g_{ij} s_{j,0}^{\omega}\Delta\omega_j - s_{i,2}^{\mu}\Delta\mu_i g_{ij} s_{j,1}^{\omega}\Delta\omega_j - s_{i,2}^{\mu}\Delta\mu_i g_{ij} s_{j,2}^{\omega}\Delta\omega_j \\
- \; & \mu_i^0 b_{ij}\mu_j^0 - \mu_i^0 b_{ij} s_{j,0}^{\mu}\Delta\mu_j - \mu_i^0 b_{ij} s_{j,1}^{\mu}\Delta\mu_j - \mu_i^0 b_{ij} s_{j,2}^{\mu}\Delta\mu_j \\
- \; & s_{i,0}^{\mu}\Delta\mu_i b_{ij}\mu_j^0 - s_{i,0}^{\mu}\Delta\mu_i b_{ij} s_{j,0}^{\mu}\Delta\mu_j - s_{i,0}^{\mu}\Delta\mu_i b_{ij} s_{j,1}^{\mu}\Delta\mu_j - s_{i,0}^{\mu}\Delta\mu_i b_{ij} s_{j,2}^{\mu}\Delta\mu_j \\
- \; & s_{i,1}^{\mu}\Delta\mu_i b_{ij}\mu_j^0 - s_{i,1}^{\mu}\Delta\mu_i b_{ij} s_{j,0}^{\mu}\Delta\mu_j - s_{i,1}^{\mu}\Delta\mu_i b_{ij} s_{j,1}^{\mu}\Delta\mu_j - s_{i,1}^{\mu}\Delta\mu_i b_{ij} s_{j,2}^{\mu}\Delta\mu_j \\
- \; & s_{i,2}^{\mu}\Delta\mu_i b_{ij}\mu_j^0 - s_{i,2}^{\mu}\Delta\mu_i b_{ij} s_{j,0}^{\mu}\Delta\mu_j - s_{i,2}^{\mu}\Delta\mu_i b_{ij} s_{j,1}^{\mu}\Delta\mu_j - s_{i,2}^{\mu}\Delta\mu_i b_{ij} s_{j,2}^{\mu}\Delta\mu_j \\
- \; & \omega_i^0 b_{ij}\omega_j^0 - \omega_i^0 b_{ij} s_{j,0}^{\omega}\Delta\omega_j - \omega_i^0 b_{ij} s_{j,1}^{\omega}\Delta\omega_j - \omega_i^0 b_{ij} s_{j,2}^{\omega}\Delta\omega_j \\
- \; & s_{i,0}^{\omega}\Delta\omega_i b_{ij}\omega_j^0 - s_{i,0}^{\omega}\Delta\omega_i b_{ij} s_{j,0}^{\omega}\Delta\omega_j - s_{i,0}^{\omega}\Delta\omega_i b_{ij} s_{j,1}^{\omega}\Delta\omega_j - s_{i,0}^{\omega}\Delta\omega_i b_{ij} s_{j,2}^{\omega}\Delta\omega_j \\
- \; & s_{i,1}^{\omega}\Delta\omega_i b_{ij}\omega_j^0 - s_{i,1}^{\omega}\Delta\omega_i b_{ij} s_{j,0}^{\omega}\Delta\omega_j - s_{i,1}^{\omega}\Delta\omega_i b_{ij} s_{j,1}^{\omega}\Delta\omega_j - s_{i,1}^{\omega}\Delta\omega_i b_{ij} s_{j,2}^{\omega}\Delta\omega_j \\
- \; & s_{i,2}^{\omega}\Delta\omega_i b_{ij}\omega_j^0 - s_{i,2}^{\omega}\Delta\omega_i b_{ij} s_{j,0}^{\omega}\Delta\omega_j - s_{i,2}^{\omega}\Delta\omega_i b_{ij} s_{j,1}^{\omega}\Delta\omega_j - s_{i,2}^{\omega}\Delta\omega_i b_{ij} s_{j,2}^{\omega}\Delta\omega_j.
\end{aligned}
\tag{10}
$$

Supplementary equations (9)–(10) are extended forms of equations (13)–(14) presented in the paper, respectively.

In equation (13), the constant clause is

$$
\sum_{j=0}^{n} \mu_i^0 g_{ij}\mu_j^0 + \omega_i^0 g_{ij}\omega_j^0 + \omega_i^0 b_{ij}\mu_j^0 - \mu_i^0 b_{ij}\omega_j^0, \tag{11}
$$

the linear clause is

$$\sum_{j=0}^{n}\sum_{k=0}^{2}(k+1)\mu_i^0 g_{ij}s_{j,k}^{\mu}\Delta\mu_j + (k+1)s_{i,k}^{\mu}\Delta\mu_i g_{ij}\mu_j^0$$
$$+(k+1)\omega_i^0 g_{ij}s_{j,k}^{\omega}\Delta\omega_j + (k+1)s_{i,k}^{\omega}\Delta\omega_i g_{ij}\omega_j^0$$
$$+(k+1)\omega_i^0 b_{ij}s_{j,k}^{\mu}\Delta\mu_j + (k+1)s_{i,k}^{\omega}\Delta\omega_i b_{ij}\mu_j^0$$
$$-(k+1)\mu_i^0 b_{ij}s_{j,k}^{\omega}\Delta\omega_j - (k+1)s_{i,k}^{\mu}\Delta\mu_i b_{ij}\omega_j^0, \tag{12}$$

and the quadratic clause is

$$\sum_{j=0}^{n}\sum_{k=0}^{2}\sum_{l=0}^{2}(k+1)(l+1)s_{i,k}^{\mu}\Delta\mu_i g_{ij}s_{j,l}^{\mu}\Delta\mu_j$$
$$+(k+1)(l+1)s_{i,k}^{\omega}\Delta\omega_i g_{ij}s_{j,l}^{\omega}\Delta\omega_j$$
$$+(k+1)(l+1)s_{i,k}^{\omega}\Delta\omega_i b_{ij}s_{j,l}^{\mu}\Delta\mu_j$$
$$-(k+1)(l+1)s_{i,k}^{\mu}\Delta\mu_i b_{ij}s_{j,l}^{\omega}\Delta\omega_j. \tag{13}$$

Similarly, in equation (14), the constant clause is

$$\sum_{j=0}^{n}\omega_i^0 g_{ij}\mu_j^0 - \mu_i^0 g_{ij}\omega_j^0 - \mu_i^0 b_{ij}\mu_j^0 - \omega_i^0 b_{ij}\omega_j^0, \tag{14}$$

the linear clause is

$$\sum_{j=0}^{n}\sum_{k=0}^{2}(k+1)\omega_i^0 g_{ij}s_{j,k}^{\mu}\Delta\mu_j + (k+1)s_{i,k}^{\omega}\Delta\omega_i g_{ij}\mu_j^0$$
$$-(k+1)\mu_i^0 g_{ij}s_{j,k}^{\omega}\Delta\omega_j - (k+1)s_{i,k}^{\mu}\Delta\mu_i g_{ij}\omega_j^0$$
$$-(k+1)\mu_i^0 b_{ij}s_{j,k}^{\mu}\Delta\mu_j - (k+1)s_{i,k}^{\mu}\Delta\mu_i b_{ij}\mu_j^0$$
$$-(k+1)\omega_i^0 b_{ij}s_{j,k}^{\omega}\Delta\omega_j - (k+1)s_{i,k}^{\omega}\Delta\omega_i b_{ij}\omega_j^0, \tag{15}$$

and the quadratic clause is

$$\sum_{j=0}^{n}\sum_{k=0}^{2}\sum_{l=0}^{2}(k+1)(l+1)s_{i,k}^{\omega}\Delta\omega_i g_{ij}s_{j,l}^{\mu}\Delta\mu_j$$
$$-(k+1)(l+1)s_{i,k}^{\mu}\Delta\mu_i g_{ij}s_{j,l}^{\omega}\Delta\omega_j$$
$$-(k+1)(l+1)s_{i,k}^{\mu}\Delta\mu_i b_{ij}s_{j,l}^{\mu}\Delta\mu_j$$
$$-(k+1)(l+1)s_{i,k}^{\omega}\Delta\omega_i b_{ij}s_{j,l}^{\omega}\Delta\omega_j. \tag{16}$$

Supplementary figures

To further clarify the performance of different solvers using the QUBO and Ising model formulations, we present Figure 1 from the main manuscript with a logarithmic scale on the x-axis. This adjustment emphasizes the solver's behavior and progress during the initial iterations.

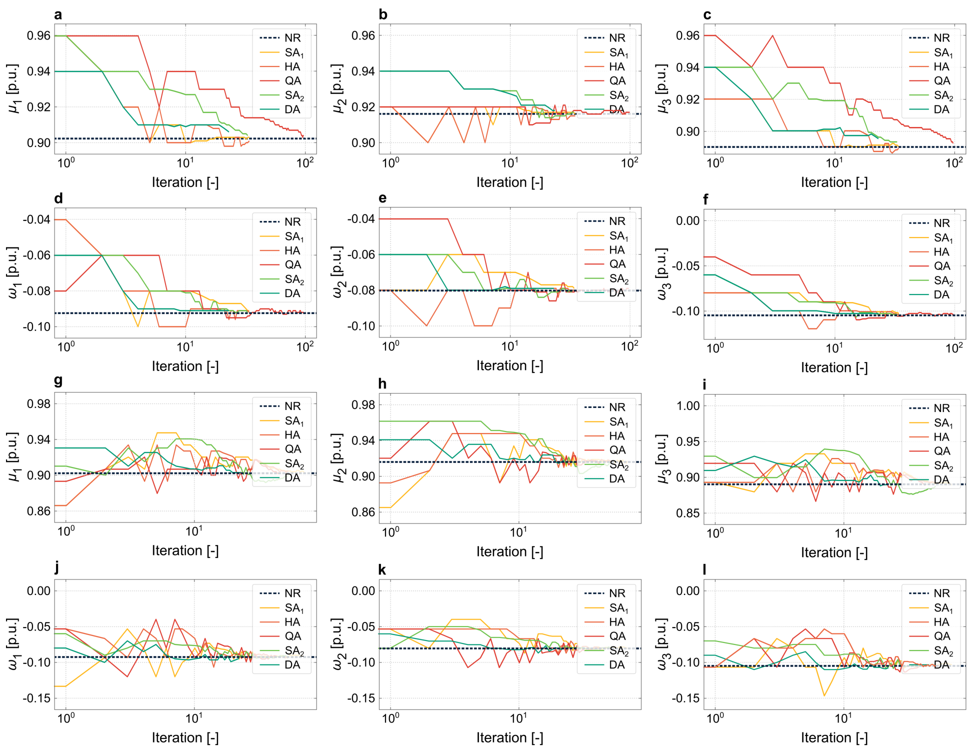

Figure 1: Logarithmic representation of $\vec{\mu} = [\mu_1, \mu_2, \mu_3]$ and $\vec{\omega} = [\omega_1, \omega_2, \omega_3]$ obtained by the combinatorial power flow algorithm. Experiments are performed using D-Wave's classical simulated annealer Neal (SA_1), D-Wave's quantum-classical hybrid annealer (HA), D-Wave's Advantage™ system (QA), Fujitsu's classical simulated annealer (SA_2), and Fujitsu's digital annealer V3 (DA) using both (a-f) QUBO and (g-l) Ising model formulations for the 4-bus test system. The reference bus $i = 0$ is not shown. The graphs include μ_i and ω_i obtained from the Newton-Raphson classical solver (NR).