Supporting Information for

Accurate birth weight prediction from fetal biometry using the Gompertz model

Chandrani Kumari, Gautam Menon, Leelavati Narlikar, Uma Ram, Rahul Siddharthan

Derivation of the Gompertz formula

The Gompertz equation for the growth of a parameter X as a function of time t is:

$$X(t) = Ae^{-e^{-c(t-t_0)}} \tag{1}$$

The formula describes a constrained growing system where a parameter $X(t)$ grows at a rate $r(t)$ which is itself exponentially decreasing with time (due to volume, resource or other constraints):

$$\begin{aligned} \frac{dX}{dt} &= r(t)X, \\ r(t) &= r_0 e^{-ct}. \end{aligned} \tag{2}$$

The solution to this is

$$X(t) = X_0 \exp\left[\frac{r_0}{c}\left(1 - \exp(-ct)\right)\right]$$

which we reparametrize as

$$X(t) = A \exp\left[-\exp\left(-c(t-t_0)\right)\right] \tag{3}$$

identifying $A \equiv X_0 \exp(r_0/c)$ and $t_0 \equiv \frac{1}{c}\log(r_0/c)$. Tjørve and Tjørve [1] observe that this parametrization is more useful than others because of its “easy interpretable parameters”: A is an overall scale factor and the upper asymptote, c is the retardation rate of the growth rate r (eq 2), and t_0 is a time offset (it represents the time at the inflection point of the curve).

References

1. Tjørve KM, Tjørve E. The use of Gompertz models in growth analyses, and new Gompertz-model approach: An addition to the Unified-Richards family. PloS one. 2017;12(6):e0178691.

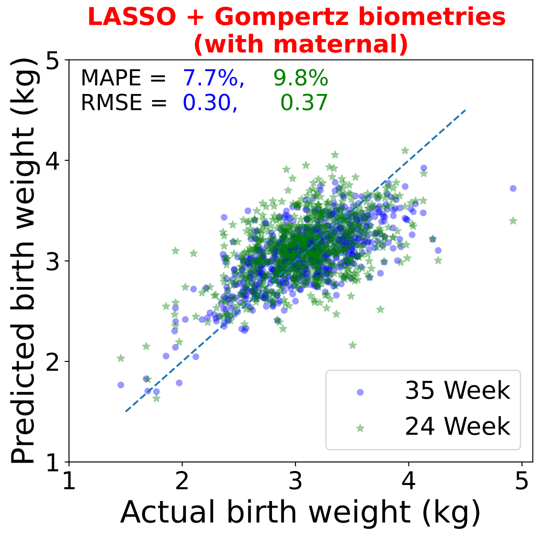

Fig S1. Birth weight prediction by using biometries at delivery predicted by the Gompertz function, A calculated using scans at 24 weeks (green) and 35 weeks (blue) along with maternal parameters mentioned in Table 1, i.e., the first 11 rows. Here linear regression was used with the L1 penalty (also known as the LASSO model) to learn a model over all parameters. Again, leave-one-out was used similar to figure 4 in the main text to predict birth weight. We note a marginal improvement over the model that does not use maternal parameters.

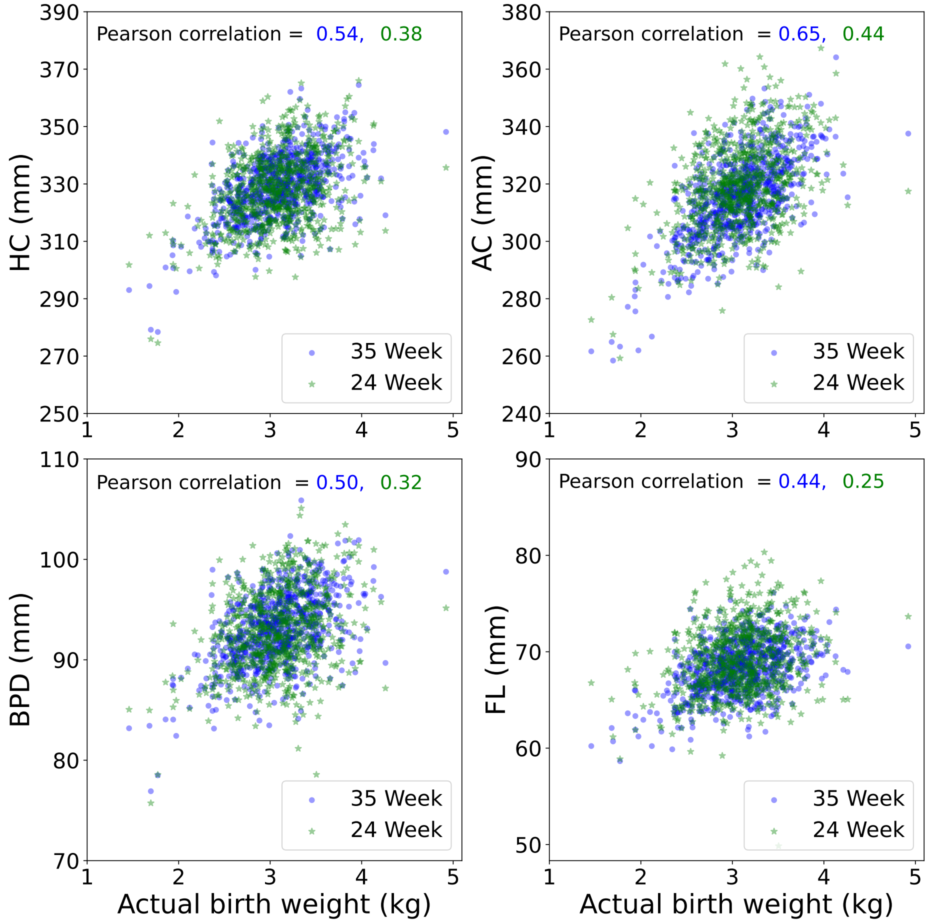

Fig S2. Scatter plot shows the correlation between the actual birth weight and the biometrics at delivery as predicted by the scans at 24 and 35 weeks (green and blue, respectively).

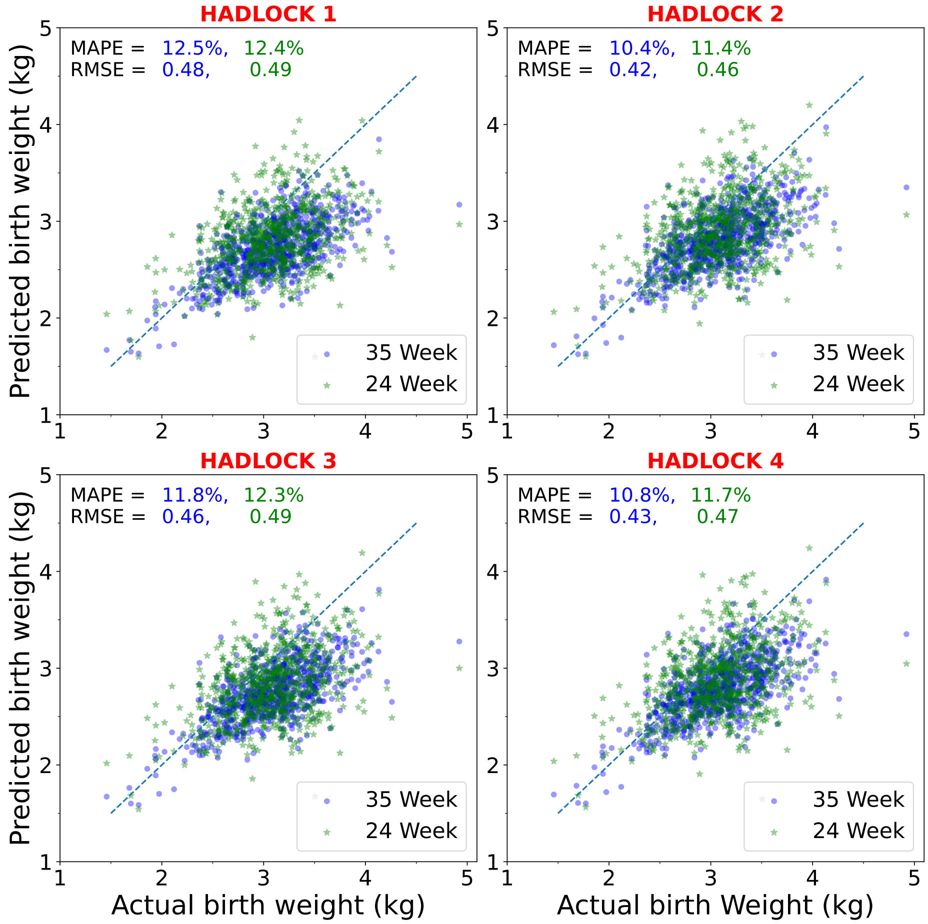

Fig S3. Birth weight prediction at delivery, using four different formulas from Hadlock. Biometries at delivery were predicted using the Gompertz function and A was calculated using scans at 24 weeks (green) and 35 weeks (blue)

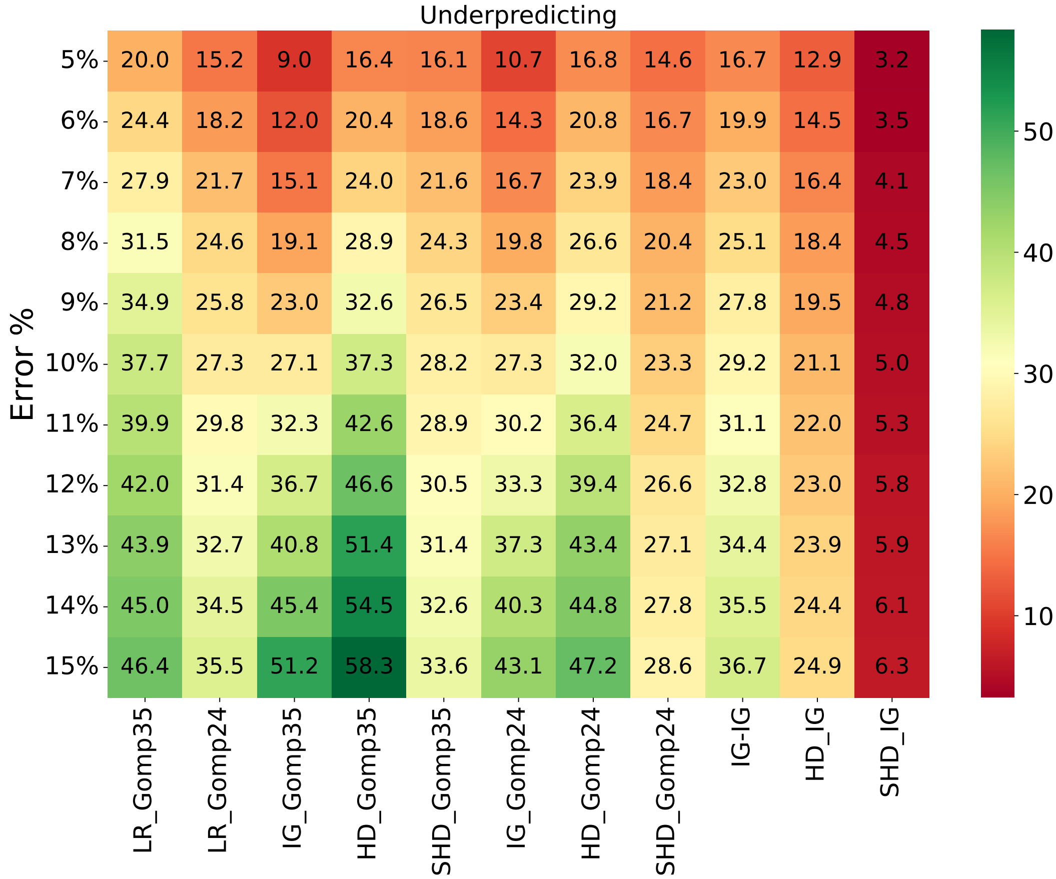

Fig S4. The percentage of fetuses whose birthweight is under-predicted within a given error rate, according to 11 methods.

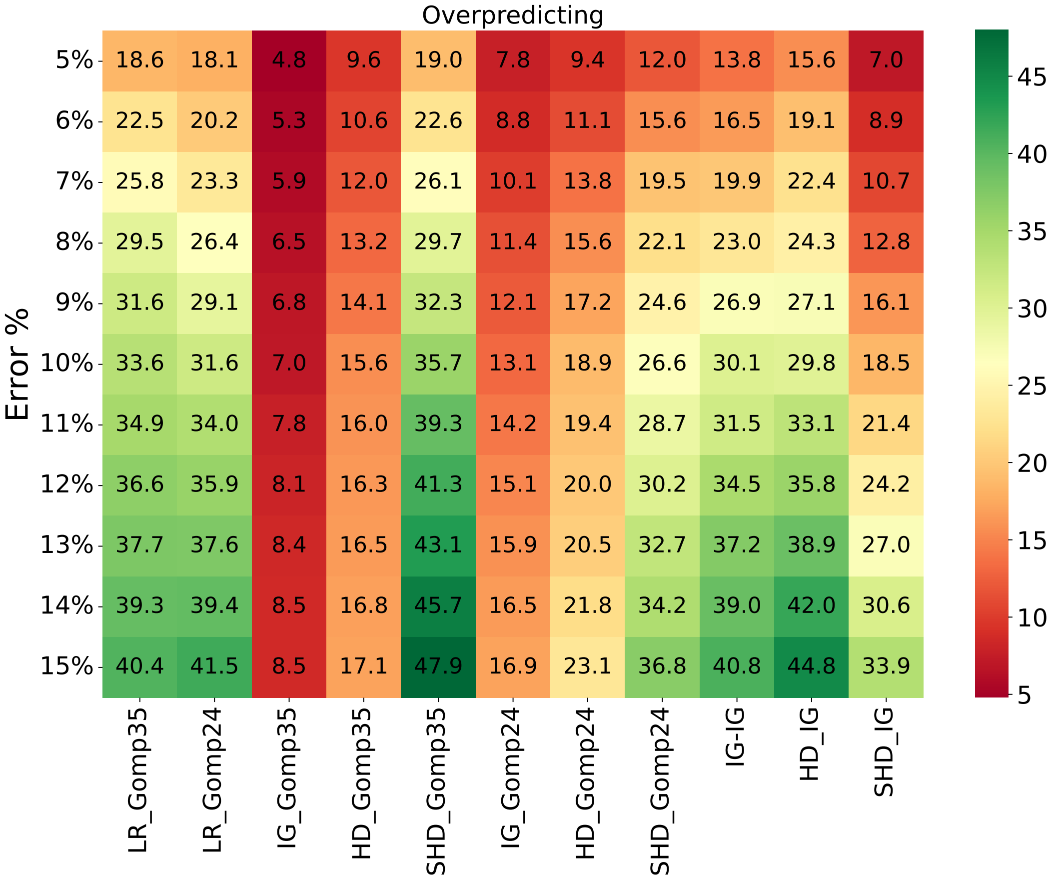

Fig S5. The percentage of fetuses whose birthweight is over-predicted within a given error rate, according to 11 methods.

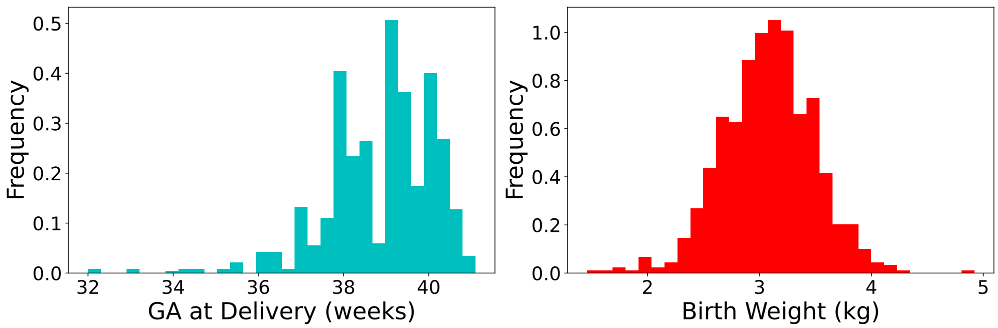

Fig S6. Distribution of GA at delivery and birth weight considered for the study

Table S1. Four different HADLOCK regression formula for birth weight estimation

	HADLOCK regression formula
HADLOCK 1	$\log_{10}(\mathrm{EFW}) = 1.304 + 0.05281 \times AC + 0.1938 \times FL - 0.004 \times AC \times FL$
HADLOCK 2	$\log_{10}(\mathrm{EFW}) = 1.335 - 0.0034 \times AC \times FL + 0.0316 \times BPD + 0.0457 \times AC + 0.1623 \times FL$
HADLOCK 3	$\log_{10}(\mathrm{EFW}) = 1.326 - 0.00326 \times AC \times FL + 0.0107 \times HC + 0.0438 \times AC + 0.158 \times FL$
HADLOCK 4	$\log_{10}(\mathrm{EFW}) = 1.3596 - 0.00386 \times AC \times FL + 0.0064 \times HC + 0.00061 \times BPD \times AC + 0.0424 \times AC + 0.174 \times FL$