# **Supplementary information**

# Spectral entropy outperforms MS/MS dot product similarity for small-molecule compound identification

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# **Supplementary Note for**

# Spectral entropy outperforms MS/MS dot product similarity for small molecule compound identification

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## Supplementary Note 1: "Equations for Calculating MS/MS Similarity"

We define m/z as M, and intensity as I.

The test spectrum q is a collection of peaks:  $(M_{q,1}, I_{q,1}), \dots, (M_{q,i}, I_{q,i})$ , the library spectrum r is defined as  $(M_{r,1}, I_{r,1}), \dots, (M_{r,i}, I_{r,i})$ .

 $N_q, N_r$  is the number of fragment ions for spectrum  $q, r; N_m$  is the total number of matching fragment ions.

Thus, we have the average intensity  $\overline{I_q} = \frac{\sum I_{q,i}}{N_q}$ ,  $\overline{I_r} = \frac{\sum I_{r,i}}{N_r}$ .

Here ln is the natural logarithm  $log_e$ .

The spectral similarity is calculated with follow equations:

Unweighted entropy similarity:  $-\frac{2 \times S_{AB} - S_A - S_B}{\ln(4)}$ ,  $S = -\sum_p I_p \ln I_p$ Entropy similarity:  $1 - \frac{2 \times S'_{AB} - S'_A - S'_B}{\ln(4)}$ ,  $S' = -\sum_p I'_p \ln I'_p$ , with  $\begin{cases} I' = I \ (S \ge 3) \\ I' = I^w, w = 0.25 + S * 0.25 \ (S < 3) \end{cases}$  $\frac{\left(\sum I_{q,i} \cdot I_{r,i}\right)^2}{\sum I_{q,i}^2 \cdot \sum I_{r,i}^2}$ Dot-product (cosine) similarity:  $\frac{\left(\sum W_{q,i} W_{r,i}\right)^2}{\sum W_{q,i}^2 \sum W_{r,i}^2}, W_i = M_i^3 I_i^{0.6}$ Weighted dot product similarity:  $\frac{\left(\sum I'_{q,i}, I'_{r,i}\right)^{2}}{\sum I'_{r,i}^{2} \cdot \sum I'_{r,i}^{2}} with I'_{q,i} > 0, I'_{r,i} > 0 and M'_{q,i} = M'_{r,i}$ Reverse dot-product similarity:  $-\frac{N_m^4}{N_a N_r (\Sigma |I_{a,i} - I_{r,i}|)^{0.25}}$ MSforID similarity version 1:  $-\frac{N_m^4 (\sum I_{q,i} + 2\sum I_{r,i})^{1.25}}{(N_a + 2N_r)^2 + \sum |I_{q,i} - I_{r,i}| + \sum |M_{q,i} - M_{r,i}|}$ MSforID similarity:  $1 - \frac{1}{\sqrt{2}} \sqrt{\sum_{i} (I_{q,i} - I_{r,i})^2}$ Euclidean similarity: Manhattan similarity:  $1 - \frac{1}{2} \sum_{i} |I_{q,i} - I_{r,i}|$ Chebyshev similarity:  $1 - \max_{i} (|I_{q,i} - I_{r,i}|)$ Squared euclidean similarity:  $1 - \frac{1}{2}\sum_{i} (I_{q,i} - I_{r,i})^2$  $\sum \sqrt{I_{q,i} \cdot I_{r,i}}$ Fidelity similarity:  $1 - \frac{1}{2} \sqrt{\sum \left(\sqrt{I_{q,i}} - \sqrt{I_{r,i}}\right)^2}$ Matusita similarity: Squared-chord similarity:  $1 - \frac{1}{2} \sum \left( \sqrt{I_{q,i}} - \sqrt{I_{r,i}} \right)^2$ 

Bhattacharya 1 similarity: 
$$1 - \frac{1}{(\cos^{-1}0)^{2}} (\cos^{-1}(\sum \sqrt{l_{q,l}} \cdot l_{r,l}))^{2}$$
Bhattacharya 2 similarity: 
$$\frac{1}{1 - \ln(\sum \sqrt{l_{q,l}} \cdot l_{r,l})}$$
Harmonic mean similarity: 
$$2 \cdot \sum \left(\frac{l_{q,l} t_{r,l}}{l_{q,l} + l_{r,l}}\right)$$
Probabilistic symmetric  $\chi^{2}$  similarity: 
$$1 - 2 \cdot \sum \frac{(l_{q,l} - l_{r,l})^{2}}{l_{q,l} \cdot t_{r,l}}$$
Ruzicka similarity: 
$$1 - \frac{\sum (l_{q,l} - l_{r,l})}{\sum \max(l_{q,l} + l_{r,l})}$$
Roberts similarity: 
$$\sum \frac{(l_{q,l} + l_{r,l})}{(m (l_{q,l} + l_{r,l}))}$$
Roberts similarity: 
$$\sum \frac{\sum (\frac{m(l_{q,l} + l_{r,l})}{m (m (l_{q,l} + l_{r,l}))}$$
Roberts similarity: 
$$\sum \frac{\sum (l_{q,l} + l_{r,l})}{\min (\Sigma l_{q,l} \times L_{r,l})}$$
Motyka similarity: 
$$2 \cdot \frac{\sum \min(l_{q,l} + l_{r,l})}{\sum (l_{q,q} + l_{r,l})}$$
Ramoni-Urbani-Buser similarity: 
$$\frac{1}{m \times \sum \frac{l_{q,q} - l_{r,l}}{2 l_{q,q} - l_{r,l}}}$$
Mean character similarity: 
$$1 - \frac{1}{m \sqrt{N_{q}}} \sum (l_{q,l} - l_{r,l}]$$
Mean character similarity: 
$$1 - \frac{1}{m \sqrt{N_{q}}} \sum (l_{q,l} - l_{r,l}]$$
Penrose shape similarity: 
$$1 - \frac{1}{\sqrt{N_{q}}} \sum (\sum ((l_{q,l} - l_{r,l}) - (l_{r,l} - l_{r,l}))^{2}$$
Clark similarity: 
$$\frac{1}{m \sqrt{N_{q}}} \sum (l_{q,l} - l_{r,l})^{2}$$
Hellinger similarity: 
$$\frac{1}{m \sqrt{N_{q}}} \sum (l_{q,l} - l_{r,l})^{2}$$
Whittaker index of association similarity: 
$$\frac{1}{m \sqrt{N_{q}}} \sum (l_{q,l} - l_{r,l})^{2}$$
Symmetric  $\chi^{2}$  similarity: 
$$1 - \sqrt{2 \cdot \sum \frac{l_{q} + l_{r,l}}{N_{q} (l_{q} - l_{r,l} - l_{r,l})^{2}}}$$

 $\frac{1}{2} \left( 1 + \frac{\sum \left[ (I_{q,i} - \overline{I_q})(I_{r,i} - \overline{I_r}) \right]}{\sqrt{\sum (I_{q,i} - \overline{I_q})^2 \sum (I_{r,i} - \overline{I_r})^2}} \right)$ Pearson/Spearman similarity:  $\frac{1}{1 + \sqrt{\frac{1}{Nq} \sum \left(\frac{I_{q,i} - I_{r,i}}{I_{q,i} + I_{r,i}}\right)^2}}$ Improved similarity:  $\frac{1}{1+\frac{\Sigma(\left|I_{q,i}-I_{r,i}\right|)}{\Sigma I_{q,i}}}$ Absolute Value similarity:  $\frac{\sum I_{q,i} \cdot I_{r,i}}{\sqrt{\sum I_{q,i}^2 \cdot \sum I_{r,i}^2}}$ Spectral contrast angle similarity:  $1 - \sum \frac{|I_{q,i} - I_{r,i}|}{\max(I_{q,i}, I_{r,i})}$ Wave Hedges similarity:  $1 - \frac{\sum (I_{q,i} - I_{r,i})^2}{\sum I_{q,i}^2 + \sum I_{r,i}^2 - \sum I_{q,i'} I_{r,i}}$ Jaccard similarity: Dice similarity:  $1 - \frac{\Sigma (I_{q,i} - I_{r,i})^2}{\Sigma I_{q,i}^2 + \Sigma I_{r,i}^2}$ Inner product similarity:  $\sum I_{q,i} \cdot I_{r,i}$  $\frac{1}{1+2\sum_{(I_{q,i}-I_{r,i})^{2}}^{(I_{q,i}-I_{r,i})^{2}}}$ **Divergence similarity:** Avg (L1, L $^{\infty}$ ) similarity:  $1 - \frac{1}{3} (\sum |I_{q,i} - I_{r,i}| + \max |I_{q,i} - I_{r,i}|)$ Vicis-Symmetric  $\chi 2$  3 similarity:  $1 - \frac{1}{2} \sum \frac{(I_{q,i} - I_{r,i})^2}{\max(I_{q,i},I_{r,i})}$ 

## Supplementary Note 2: "Equations for entropy similarity"

We define m/z as M, and intensity as I. A spectrum A is a collection of peaks:  $(M_{A,1}, I_{A,1}), \dots, (M_{A,i}, I_{A,i})$ .

For calculating the entropy similarity between a spectrum *A* and a spectrum *B* defined as  $(M_{B,1}, I_{B,1}), \dots, (M_{B,i}, I_{B,i})$ , we generate a combined spectrum *AB* with:

$$I_{AB,i} = \frac{1}{2}I_{A,i} + \frac{1}{2}I_{B,i}$$

By the definition of spectral entropy, we have:

$$S = -\sum_{i} I_i \ln I_i$$

Hence, the entropy distance is calculated by:

$$2 \times S_{AB} - S_A - S_B$$
  
=  $-\left(\sum_{i} (2 \times I_{AB,i} \ln I_{AB,i}) - \sum_{i} I_{A,i} \ln I_{A,i} - \sum_{i} I_{B,i} \ln I_{B,i}\right)$   
=  $-\sum_{i} ((I_{A,i} + I_{B,i}) \ln I_{AB,i} - I_{A,i} \ln I_{A,i} - I_{B,i} \ln I_{B,i})$   
=  $\sum_{i} (I_{A,i} \ln \frac{I_{A,i}}{I_{AB,i}} + I_{B,i} \ln \frac{I_{B,i}}{I_{AB,i}})$   
=  $\sum_{i} (I_{A,i} \ln \frac{2 \times I_{A,i}}{I_{A,i} + I_{B,i}} + I_{B,i} \ln \frac{2 \times I_{B,i}}{I_{A,i} + I_{B,i}})$ 

If spectra A and B are identical, we have  $I_{A,i} = I_{B,i}$ , and the entropy distance becomes zero:

$$2 \times S_{AB} - S_A - S_B = \sum_{i} \left( I_{A,i} \ln \frac{2 \times I_{A,i}}{I_{A,i} + I_{A,i}} + I_{A,i} \ln \frac{2 \times I_{A,i}}{I_{A,i} + I_{A,i}} \right) = 0$$

If there are no common fragment ions found between spectra *A* and *B*, the entropy distance becomes maximal:

$$2 \times S_{AB} - S_A - S_B = \sum_{i} \left( I_{A,i} \ln \frac{2 \times I_{A,i}}{I_{A,i} + I_{B,i}} + I_{B,i} \ln \frac{2 \times I_{B,i}}{I_{A,i} + I_{B,i}} \right)$$
$$= \sum_{i} \left( I_{A,i} \ln \frac{2 \times I_{A,i}}{I_{A,i}} \right) + \sum_{j} \left( I_{B,j} \ln \frac{2 \times I_{B,j}}{I_{B,j}} \right)$$
$$= \ln 2 \times \sum_{i} I_{A,i} + \ln 2 \times \sum_{j} I_{B,j} = \ln 2 + \ln 2 = \ln 4$$

Therefore, the entropy distance ranges from 0 to  $\ln 4$ .

We then defined the unweighted spectral entropy similarity by normalizing the entropy distance to [0,1].

We obtain the unweighted spectral entropy similarity:

$$1 - \frac{2 \times S_{AB}' - S_A' - S_B'}{\ln(4)}$$



## Supplementary Figure 1.

Distribution of spectral entropy values of all spectra in MassBank.us, NIST20, and the GNPS database after removing noise peaks defined as less than 1% base peak intensity.



## Supplementary Figure 2.

The distribution of candidate hits for NIST20 similarity algorithm benchmarking tests.

- (a) Number of candidate spectra per test spectrum.
- (b) Number of candidate molecules per test spectrum.



#### Supplementary Figure 3.

The distribution of candidate hits for Massbank.us similarity algorithm benchmarking tests.

- (a) Number of candidate spectra per test spectrum.
- (b) Number of candidate molecules per test spectrum.



Receiver-operator characteristic curves for all algorithms when searching NIST20 MS/MS spectra separated by type of mass spectrometer. The entropy similarity, unweighted entropy similarity and dot product similarity are highlighted. All other methods are shown in grey.



Examples of MS/MS similarities of isomeric molecules.

(a) Comparison of structures and MS/MS spectra for vanillic acid (top) and 3-Hydroxy-4-

methoxybenzoic acid (bottom) as example of one-bond different isomers.

(b) Comparison of structures and MS/MS spectra for Flavanomarein (top) and Marein (bottom) as example of multiple-bond different isomers



Supplementary Figure 6. The distribution of bond difference in misidentified molecular pairs.