nature portfolio

# Peer Review File

## How synchronized human networks escape local minima

Corresponding Author: Professor Moti Fridman

**This file contains all reviewer reports in order by version, followed by all author rebuttals in order by version.**

Version 0:

Reviewer comments:

Reviewer #1

(Remarks to the Author)
The manuscript by Schniderman et al. reports on a series of experiments that have the goal to investigate the adjustment of rhythms (synchronization) among musicians. The communication between the participants was set to happen according to a unidirectional ring; that is, a given player could only hear one neighbor and be heard by another one. In the experiments, the players should repeat the same musical phrase synchronously. This is a trivial task for musicians under normal circumstances. However, in this study the time delay at which each player listens to one another is tunable. The variation of this time delay revealed different strategies that the group of musicians adopted in order to stay in sync, namely: altering their effective coupling strength, slowing down the tempo, playing the same note indefinitely, and interrupting the musical phrase completely. All these strategies can be interpreted in terms of dynamical states well studied in the context of coupled phase oscillators, such as amplitude and oscillation death.

This is a very interesting study on what seems to be an ideal system to investigate the emergence of synchronization among people: a network of musicians. It is remarkable how a simple model of phase coupled oscillators is able to reproduce the dynamical behavior of a very complicated system. In my opinion, this is one of the best illustrations of the application of the Kuramoto model in describing real-world observations. I say this because the vast majority of the literature on synchronization of empirical systems often deals with setups that are actually experimental implementations of sets of differential equations that are known to exhibit synchronous solutions. Examples of such systems include networks of circuits, electrochemical and mechanical oscillators. The spontaneous synchronization observed in the latter systems often does not come as a surprise, because they are tailored to yield certain types collective phenomena. Here, as in the previous work by some of the authors, the scenario is different: it is a social system, a network of dynamical human interactions, whose experimental outcome is explained by the theory of coupled phase oscillators. Owing to the complexity of the experiments, the agreement with the proposed theory, and the distinct nature of the system (one does not usually encounter studies on networks of violinists), I believe the manuscript is at the level of Nature Communications, and I recommend the publication. Nonetheless, I suggest the authors to address the points below.

The object of the study here is a network of musicians, but this is not mentioned in the title and in the abstract. In the latter, the authors state that "humans have different methods to avoid local minima than other networks". However, it is not clear there what are the circumstances the humans are subjected to and what those "other networks" could be. Thus, I would suggest to be a bit more clear and precise in the abstract regarding those statements.

In the introduction, it is mentioned that "human networks can reach synchronization by finding unique solutions which are more stable compared to other networks due to (...)". What is the meaning of "stability" used here? Is it the size of the basin of the attraction of the synchronous solution? Or some measurement related with the amplitudes of the perturbations and the ability of the synchronous state to withstand them?

What would be "strength" of the coupling between the violinists? In section 2 ("Experiment"), it is mentioned that the connectivity, time-delay and strength can be controlled. While the first two parameters are easily understood in the context of making music, the meaning of "strength" is less clear. Is it related to the volume at which each player listens to one another? I believe this should be clarified.

I would suggest avoiding the term "topological charge" to refer to the parameter $n$, which is related to the order or the commensurability of the periods. "Topological charge" does not seem to have a clear connection with the system studied here. In my opinion, this might be confusing to the reader, especially when it appears in the legend of Fig. 2. I see that the use was inspired by Ref. [30]. If we check that reference, we'll find that the definition of topological charge [Eq. (1)] is related to complex fields of sites on a ring array, which may not have a clear analogy with the network of violinists. Hence, I believe this term might not be easily understandable for an interdisciplinary audience, which probably will be the case of the majority of the readership of this paper.

About Section 3: Why one would interested in calculating the potential V? Wouldn't the order parameter provide a clearer interpretation? For example, in Fig. 1(d) the "in-phase" region is not distinguishable from the "vortex" one. Probably the difference could be more evident by visualizing the order parameter instead of the potential.

Further in section 3, it is mentioned that "the first-order vortex becomes unstable and the next-order vortex become stable". What is the definition of "order" of a vortex? Is it related with the "topological charge"?

Finally, in Figure 5, there are some bursts within the region labeled as "Amplitude death", which is the region where a violinist was supposed to be in a pause. Are these bursts fluctuations?

Minor points: there are typos in references [7] and [33].

Out of curiosity: what was the criteria used to select that specific musical phrase shown in Fig. 1?

Reviewer #2

(Remarks to the Author)
Report on manuscript no. NCOMMS-23-44602

The present manuscript discusses the emergence of synchronization in a population of human agents performing a coordination task (playing the violin). More specifically, given a population of $N$ agents, the evolution of the phase of an oscillator $i$, $\varphi_i$, can be described using a Kuramoto model with a coupling delay $\Delta t$ [Eq. (1)]. Oscillators interact via a directed ring network, where each node is connected exclusively to the next one (i.e., node $i$ is connected to $i+1$, with periodic boundaries such that $i + N$ = $i$). The dynamics possesses two stationary states: one in which all oscillators gets perfectly synchronised and another state (vortex state), instead, in which the phase separation among oscillators' pairs remains stationary. The dynamics can be studied in terms of an effective potential $V$, and the manuscript discusses how the system migrates from a local minimum (i.e., partially synchronized state) to a global one via different mechanisms.

The experimental study of human coordination has drawn the attention of many scientists working in fields like behavioral science, synchronization, and complex systems. The present manuscript provides a nice contribution to this domain; yet I believe it lacks the impact needed to ensure its publication in a venue like Nature Communications. In the following, I provide a more detailed motivation for my assessment, together with some suggestions for the authors to improve the manuscript.

#### MAIN ISSUES ####

-- The results presented in the manuscript are interesting but they are obtained considering a very specific -- and peculiar -- topology (unidimensional directed ring). However, people do not usually interact neither in a ring, nor in a unidirectional way. Hence, my question is: how can we be sure that the phenomenology displayed could be observed also in reality? From an experimental point of view, it should be possible to implement more realistic topologies, as some of the authors of this manuscript have published another similar work in which the topologies adopted were far more realistic.

-- The presentation does not meet the expectations. Pictures are small and difficult to read. The conclusions are merely descriptive and do not frame the results into a broader perspective (strangely, there is no mention to any previous result, as if the results presented in the manuscript are completely unrelated to the body of literature existing on human synchronization). The supplementary materials do not provide enough information to replicate the results (both theoretically and experimentally), and are not well referenced within the main text. The acknowledgement section still contains the guidelines for its redaction, and I have spotted a couple of typos in the text/bibliography. I will provide more details in the minor comments section but, in general, the manuscript needs to improve a lot under this aspect.

#### MINOR COMMENTS ####

-- The conclusions are too shallow and present only a mere summary of the results found. I believe, instead, that the conclusions should place the work into a broader context, compare the results found with those available in the literature, and lay future research avenues. I strongly suggest that the authors put all their efforts in improving this aspect.

-- In Sec. 5 of the main manuscript, the "disconnection" effect induced by the tempo's slow down seems like the byproduct of the agents' evaluation of some cost/benefit ratio. Said in other words, it appears as if when agents realize that their efforts to stay synchronized is "too much", they spontaneously decide to ignore the feedback and continue on their own rhythm. If my intuition is correct, perhaps the authors could consider discussing this phenomenon in terms of the "evolutionary Kuramoto" model proposed in:

https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.118.238301

-- Please, provide an explanation of the term "topological charge" (I have never heard of it).

-- In the first paragraph of the introduction, the phrase "which decision the network is more likely to make" does not make much sense to me, as networks are incapable of taking decisions (the people forming the network do it).

-- Section 2 second paragraph: After the phrase "When $\Delta t$ exceeds ...", please specify that $n$ is an integer number.

-- I have found the definition of the periodic boundary condition a bit hard to understand. Perhaps, something like $\varphi_{i+N} = \varphi_{i}$ with $i = 1,2, \ldots, N$ is more general and easy to get.

-- The content and organization of the supplementary materials is not adequate. More specifically:

1) I warmly recommend to include a section on the experimental setup. As it is right now, it is practically impossible to reproduce the experiment performed. The authors should explain not only how the experiment has been prepared but also how the data have been analyzed.

2) Please amend the format used to mention sections and equation. As it is now, it is very complicated to understand if, for instance, by Eq. (1) one refers to that appearing in the main manuscript or to that of the supplementary materials. A convention like adding an "S" before the equation number (as done in many manuscripts) will improve considerably the readability. The same reasoning applies also to the section numbering. Using a formalism like "Supplementary Note Sx" or "Section Sx" will help to distinguish when some content is available in the main manuscript or in the supplementary materials.

3) In Section 1 "Analytical Models" it seems counter intuitive to me to place first the calculations for a specific case (Sec. 1.1), and then the general ones (Sec. 1.2). To me, the natural order of sections should be the opposite: with the general case first, and the specific one after.

4) I may have missed something but, I have not understood very well how to obtain the effective potential, $V$. Perhaps a more detailed and step-by-step explanation of how to compute $V$ could improve this point. Following the same reasoning, I did not get well which are the algebraic passages to get to Eq. (5).

5) I have found strange the absence of additional results for the oscillation and amplitude deaths regimes. Unless there is some valid reason for that, I suggest to add further results as done for the other two regimes (i.e., "spreading the phase" and "slowing the tempo").

-- The graphics could be improved. In general, I have found that many details are hard to read because the font used is quite small. Specifically:

** Fig. 1: Many details like the axes and tick labels are barely readable. Please, make them bigger.

** Fig. 1(a): The color used to represent the two dataset are rendered with the same tone of gray when displayed in b/w. Besides, the legend is not very visible (please, make it bigger).

** Fig. 2(b): Please highlight better the three regimes (e.g., putting a solid thick line) because they are not very distinguishable. Moreover, using white fonts on a light background reduces considerably the contrast. Make sure to specify the end values of the axes (e.g., it is not clear which is the value displayed at the end of the lower axis). The same reasoning applies also to the upper horizontal axis (also in panel a).

** Fig. 3(a) inset: Please, use different symbols to indicate different topological charges, as color alone does not ensure proper discrimination (especially for colorblind people).

** Fig. 5: Please use a colorblind friendly palette to denote the four data series (people) appearing in panels (a) and (b).

-- I have spotted a few typos. Specifically:

** Caption of Fig. 2: It should be $N=8$ instead of $N=6$.

** Section 4 second paragraph: It should be "we can expand" instead of "we can expend".

** Bibliography references no. 7 and 33: There is some missing information, as there are several "?" displayed after the name of the publisher.

** Bibliography reference no. 23: Please add information on the volume of the article.

** Bibliography reference no. 35: The correct spelling is "Kuramoto" and "Crawford" with capital letters.

** Section 1.1 of the Supplementary materials: please replace "insure" with "ensure".

** Section 2 of the Supplementary materials: The first word of the first paragraph should be written with a capital letter.

Reviewer #3

(Remarks to the Author)
I read with interest the paper by Fridman et al. Despite finding the paper well written and clear I have some major concerns about the novelty and the generality of the results presented as they appear to be limited, especially when compared to the previous work by some of the authors [1], on the same journal. In their previous work, it was already shown that vortices could form spontaneously in networks of human beings performing musical phrases at a violin, as an effect of introducing and manipulating a delay in their interconnections. In the present manuscript, three more mechanisms are presented, that are "slowing the tempo", and "oscillation/amplitude death". These are also justified through comparison with the simulation and the analysis of a mathematical model (see below for more specific comments on the model). I am therefore left unconvinced that the contribution in this manuscript is broad enough to justify its publication in Nat Comms while it could be of some interest in more specialised journals on human movement and coordination.

In the following, I list a series of additional concerns.

Major comments
- In the abstract, it is stated that the research "may have implications in politics, economics, pandemic control, decision-making, and predicting the dynamics of networks with artificial intelligence". However, currently this claim is not justified by the research presented.
- The concept of topological charge, although important, is never explained in the manuscript. While a reference is given, I would advise to give more information on this concept to make the paper more self contained.
- Before (2) and in the supplemental material, the authors assume---they say, without loss of generality---that $\Delta \phi_n = \Delta \phi$ (for all n). This should correspond to stating that all nodes in the network have the same phase difference with respect to each other. It is not clear to me how this assumption can preserve generality.
- At the end of Section 5, the authors suggest that an open ring structure is a network topology that makes it easier to synchronize frequencies, with respect to a closed ring. While this is intuitive to some readers, I would suggest the authors explain the reasons why this happens. More generally, it seems to me that the effect of changing the network structure is an important one and should be investigated with respect to the phenomena studied in the manuscript. To me the choice of studying a ring configuration with unidirectional time-delayed coupling is quite a specific one. Other options/configurations should also be explored.
- In Section 1.2 in the supplemental material, before (8), it seems the authors assume that a phase-locked solution (i.e., with equal node frequencies) exists and it is reached. If so, the authors should explicitly state they make this assumption.

Minor comments:
- In (1) and elsewhere, why is the symbol of partial derivative used, rather than that of the total derivative, given that \phi_n is only a function of time?
- After (3), Omega_n is not defined (it is defined only in the supplemental material).
- In the phrase "Therefore, increasing the delay adiabatically", do the authors mean a slow increase that allows the network to get to a steady state after each small increase? In any case, I would suggest clarifying the meaning of "adiabatically" in this context, given that the topic is unrelated to thermodynamics.

[1] Shahal, S., Wurzberg, A., Sibony, I., Duadi, H., Shniderman, E., Weymouth,
D., Davidson, N., Fridman, M.: Synchronization of complex human networks. Nature communications 11(1), 3854 (2020)

Version 1:

Reviewer comments:

Reviewer #1

(Remarks to the Author)
I went through the correspondence as well as the new version of the manuscript, and I believe the authors have convincingly answered the points raised by the referees. I therefore recommend the publication.

Reviewer #3

(Remarks to the Author)
I appreciate the authors' efforts in revising their manuscript. However, I remain of the opinion that the novelty and generality of the work are more suited for a specialised journal, rather than Nature Communications.

In the revised conclusions, the Authors included statements to emphasise the significance and novelty of their findings. Nonetheless, these statements do not convincingly show the broad applicability of the findings. Specifically, it remains unclear why or how exactly the mechanisms observed in maintaining synchronization (a very specific dynamical behavior) in a periodic task can be relevant to "organizational behavior, management, and policy-making" (sic). For instance, what would be the equivalent of a phase in a management or policy making context? The author reference "coordination" as a link, but the coordination observed in these fields typically involves the coordination (not necessarily synchronization) of events or (non-periodic) opinions rather than synchronization of motor/musical behavior. Providing more specific examples, contextualized within the applications mentioned, would have been helpful.
To clarify, I am not asserting that there is no possible connection, but rather that that it appears loose, based on the statements provided in the manuscript.

For the sake of completeness, I list a few additional comments below.

- In the rebuttal, concerning S1.1, the authors state "The fact that all the phase differences are identical is an assumption due to symmetry and we verify it later."
At the end of the revised S1.1 the authors state "a new global minimum appears in \Delta \phi = –\pi/2, namely, the vortex state of synchronization".
This appears to be the point where the authors verify the assumption; however, the verification is simply stated and not demonstrated. Moreover, is it correct that \Delta \phi does not depend on N? (I might be wrong, but I was expecting a first vortex order solution, with phases equally spaced on the circle).

- I attempted to perform the computations leading from (S12) to (S13) but could not obtain the same result as the authors.

- In (S7), it seems to me that the last term should be
"k sin (+ \Delta \phi - omega \Delta t)"
rather than
"k sin (- \Delta \phi - omega \Delta t)"
I believe this is because, using (S6), from (S5) we should get
\phi_N(t - \Delta t) - \phi_{N-1}(t) =
\Delta \phi(t - \Delta t) N + \omega(t - \Delta t) - \Delta \phi(t) (N-1) - \omega t =
\Delta \phi - \omega \Delta t
If I am wrong, I apologize and kindly ask the authors to clarify this point.

After (S15), the authors state "This tempo slowing down ensures that \Delta t < T/N". It is not clear to me how this fact is obtained from (S15).

Version 2:

Reviewer comments:

Reviewer #3

(Remarks to the Author)
The authors have provided several references to justify part of the motivation that I had not found very solid in the previous version of the manuscript, which was my main concern.
I am satisfied with the authors' response. Hence, I can now recommend publication.

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Response letter to the reviewers

We would like to thank the reviewers for their time and effort in reviewing our manuscript. We revised the manuscript according to all the reviewers' comments.

Specifically, due to the comments which were repeated by all reviewers:

1. We replaced the term "topological charge" with "vortex mode" and defined it in the text.
2. We added more results on different network topologies including two-dimensional networks, indicating that unidirectional chains are motifs for many complex networks.
3. We extended the introduction and conclusions and emphasize the motivation and significance of this research.
4. We reordered the Supplemental Materials document and extended it to include: more experimental results, the definition of the effective potential, its importance, numerical calculations of large networks, and detailed analytical derivations of all the equations in the manuscript.

Here are our responses to each comment:

**REVIEWER COMMENTS**

Reviewer #1 (Remarks to the Author):

*"This is a very interesting study on what seems to be an ideal system to investigate the emergence of synchronization among people: a network of musicians. It is remarkable how a simple model of phase coupled oscillators is able to reproduce the dynamical behavior of a very complicated system. In my opinion, this is one of the best illustrations of the application of the Kuramoto model in describing real-world observations. I say this because the vast majority of the literature on synchronization of empirical systems often deals with setups that are actually experimental implementations of sets of differential equations that are known to exhibit synchronous solutions. Examples of such systems include networks of circuits, electrochemical and mechanical oscillators. The spontaneous synchronization observed in the latter systems often does not come as a surprise, because they are tailored to yield certain types collective phenomena. Here, as in the previous work by some of the authors, the scenario is different: it is a social system, a network of dynamical human interactions, whose experimental outcome is explained by the theory of coupled phase oscillators. Owing to the complexity of the experiments, the agreement with the proposed theory, and the distinct nature of the system (one does not usually encounter studies on networks of violinists), I believe the manuscript is at the level of Nature Communications, and I recommend the publication. Nonetheless, I suggest the authors to address the points below."*

1. *"The object of the study here is a network of musicians, but this is not mentioned in the title and in the abstract. In the latter, the authors state that "humans have different methods to avoid local minima than other networks". However, it is not clear there what are the circumstances the humans are subjected to and what those "other networks" could be. Thus, I would suggest to be a bit more clear and precise in the abstract regarding those statements."*

Per comment by the reviewer, we revised the abstract to include:

**“We studied the dynamics of complex networks of violin players and observed that such networks have different methods to avoid local minima than other non-human networks.”**

> *2. “In the introduction, it is mentioned that "human networks can reach synchronization by finding unique solutions which are more stable compared to other networks due to (...)". What is the meaning of "stability" used here? Is it the size of the basin of the attraction of the synchronous solution? Or some measurement related with the amplitudes of the perturbations and the ability of the synchronous state to withstand them?”*

The reviewer is correct, the stability is compared to small perturbations. Therefore, we revised the sentence to:

**“Human networks can reach synchronization by finding unique solutions that are more stable to small perturbations compared to other networks due to the human ability to focus on some inputs while ignoring others.”**

> *3. “What would be "strength" of the coupling between the violinists? In section 2 ("Experiment"), it is mentioned that the connectivity, time-delay and strength can be controlled. While the first two parameters are easily understood in the context of making music, the meaning of "strength" is less clear. Is it related to the volume at which each player listens to one another? I believe this should be clarified.”*

Per comment by the reviewer, we clarify the meaning of strength in paragraph 1 in section 2:

**“We define the coupling strength as the volume each player is hearing its neighbors compared to the volume of its own violin.”**

> *4. “I would suggest avoiding the term "topological charge" to refer to the parameter $n$, which is related to the order or the commensurability of the periods. "Topological charge" does not seem to have a clear connection with the system studied here. In my opinion, this might be confusing to the reader, especially when it appears in the legend of Fig. 2. I see that the use was inspired by Ref. [30]. If we check that reference, we'll find that the definition of topological charge [Eq. (1)] is related to complex fields of sites on a ring array, which may not have a clear analogy with the network of violinists. Hence, I believe this term might not be easily understandable for an interdisciplinary audience, which probably will be the case of the majority of the readership of this paper.”*

Per the comment by the reviewer, and according to the same point raised by all the other reviewers, we replaced the term “topological charge” with “vortex order”, and we explain it in paragraph 2 section 2:

**“The $n$ parameter denotes the order of the vortex which is stable [30]. For zero vortex order, $n = 0$, the system is at an in-phase state of synchronization, illustrated at the top of Fig. 1(b). When $n = 1$, we are at the first vortex order, in which adding the delay between the players is equal to $2\pi$, illustrated at the bottom of Fig. 1(b). For higher vortex orders, adding the players' delay equals higher multiples of $2\pi$.”**

We replaced the term in all the figures, figure captions, and in the rest of the text.

> *5. “About Section 3: Why one would interested in calculating the potential V? Wouldn't the order parameter provide a clearer interpretation? For example, in Fig. 1(d) the "in-phase" region is not distinguishable from the "vortex" one. Probably the difference could be more evident by visualizing the order parameter instead of the potential.”*

Effective potential is a powerful tool for analyzing the dynamics of systems and specifically for analyzing the dynamics of coupled networks. This method was used successfully in different coupled systems, including coupled lasers, animals, nonlinear oscillators, and others [19-25]. In particular, the entire landscape of the effective potential, such as that shown in Fig. 1(d) depicts the co-existence of several minima separated by potential barriers that can trap the system in local minima and prevent them from reaching the global minima which is a main theme of our work. To clarify this point, we revised paragraph 3 in section 2:

**“An effective potential is a powerful tool for predicting the dynamics of a coupled system and analyzing its stability [19-25]. Figure 1(c) depicts two representative effective potentials, for $\Delta t = 0$ and $\Delta t = T/N$, respectively, as a function of the phase difference between coupled players. For $\Delta t = 0$, the global minimum potential is at zero phase difference between coupled players, corresponding to the in-phase synchronization state. For $\Delta t = T/N$, the global minimum of the potential is at a phase difference of $-2\pi/N$, corresponding to a vortex state of synchronization, while the in-phase state becomes a local minimum.”.**

> *6. “Further in section 3, it is mentioned that "the first-order vortex becomes unstable and the next-order vortex becomes stable". What is the definition of "order" of a vortex? Is it related with the "topological charge"?”*

Yes, it is related. To prevent confusion and per the comment by the reviewer, and according to the same point raised by all the other reviewers, we replaced the term “topological charge” with “vortex order”, and we explain it in paragraph 2 section 2:

**“The $n$ parameter denotes the order of the vortex which is stable [30]. For zero vortex order, $n = 0$, the system is at an in-phase state of synchronization, illustrated at the top of Fig. 1(b). When $n = 1$, we are at the first vortex order, in which adding the delay between the players is equal to $2\pi$, illustrated at the bottom of Fig. 1(b). For higher vortex orders, adding the players' delay equals higher multiples of $2\pi$.”**

We replaced the term in all the figures, figure captions, and in the rest of the text.

*7. "Finally, in Figure 5, there are some bursts within the region labeled as "Amplitude death", which is the region where a violinist was supposed to be in a pause. Are these bursts fluctuations?"*

Indeed, there is some input in these moments, however, we could not identify the note the player played during this time and we attribute it to noise. Per the reviewer's comment, we added to paragraph 3 in section 5:

**"This player stops playing, as evidenced by its nearly vanishing amplitude, shown in the lower graph in Fig. 5(b). During this time, the player produces some noise, but no notes could be detected."**

*8. "Minor points: there are typos in references [7] and [33]."*

We fixed the typos.

*9. "Out of curiosity: what was the criteria used to select that specific musical phrase shown in Fig. 1?"*

Per the reviewer's comment, we added section 2.3 to the supplemental materials:

**"S2.2 Choosing the musical phrase.**

**When choosing the musical phrase for the violin players, we must consider several points:**

- **Cyclic. It is important for the phrase not to have a clear beginning or end, but rather, have a cyclic characteristic to it.**
- **Same octave. For post-processing with Fourier transform and not confusing with other notes due to over-tones, it is easier when the entire phrase is at the same octave.**
- **Repeating notes. To help the players identify quickly where their neighbor is playing, it is easy to have a minimum number of repeating notes.**
- **4th finger. We need our players to play for a long time and with a minimum number of mistakes, therefore, we prefer finding a musical phrase that does not require using the 4th finger."**

Reviewer #2 (Remarks to the Author):

*"The present manuscript discusses the emergence of synchronization in a population of human agents performing a coordination task (playing the violin). More specifically, given a population of $N$ agents, the evolution of the phase of an oscillator $i$, $\varphi_i$, can*

*be described using a Kuramoto model with a coupling delay $\Delta t$ [Eq. (1)]. Oscillators interact via a directed ring network, where each node is connected exclusively to the next one (i.e., node $i$ is connected to $i+1$, with periodic boundaries such that $i + N$ = $i$). The dynamics possesses two stationary states: one in which all oscillators gets perfectly synchronised and another state (vortex state), instead, in which the phase separation among oscillators' pairs remains stationary. The dynamics can be studied in terms of an effective potential $V$, and the manuscript discusses how the system migrates from a local minimum (i.e., partially synchronized state) to a global one via different mechanisms."*

*"The experimental study of human coordination has drawn the attention of many scientists working in fields like behavioral science, synchronization, and complex systems. The present manuscript provides a nice contribution to this domain; yet I believe it lacks the impact needed to ensure its publication in a venue like Nature Communications. In the following, I provide a more detailed motivation for my assessment, together with some suggestions for the authors to improve the manuscript."*

1. *"The results presented in the manuscript are interesting but they are obtained considering a very specific -- and peculiar -- topology (unidimensional directed ring). However, people do not usually interact neither in a ring, nor in a unidirectional way. Hence, my question is: how can we be sure that the phenomenology displayed could be observed also in reality? From an experimental point of view, it should be possible to implement more realistic topologies, as some of the authors of this manuscript have published another similar work in which the topologies adopted were far more realistic."*

Indeed, in this manuscript, we focus on basic motifs. However, these basic motifs are the motifs that have topological frustration and are the basic building blocks of large networks. The dynamics of these motifs govern the dynamics of large networks. In addition, we perform numerical simulations for large random networks and observe the same dynamics of the motifs. Including the slowing down of the tempo and the spreading of the phase. This is now shown in the Supplemental Materials in section S4. We now cite two papers studying different network motifs in [34, 35] and two papers studying how the dynamics of motifs influence the dynamics of large networks [36, 37]. We also perform experiments with a two-dimension lattice showing the same dynamics of tempo slowing down and phase spreading.

Specifically, we added to paragraph 4 section 1:

**"We are studying here basic network motifs that have global frustration, these motifs are the building blocks of complex networks [34, 35]. In addition, the dynamics of complex networks are dictated by the dynamics of the motifs [36, 37]. We numerically demonstrate the dynamics of the motifs in large random networks in the supplemental materials section S4 and generalize our findings into complex two-dimensional networks in the Supplemental Materials section S3.4."**

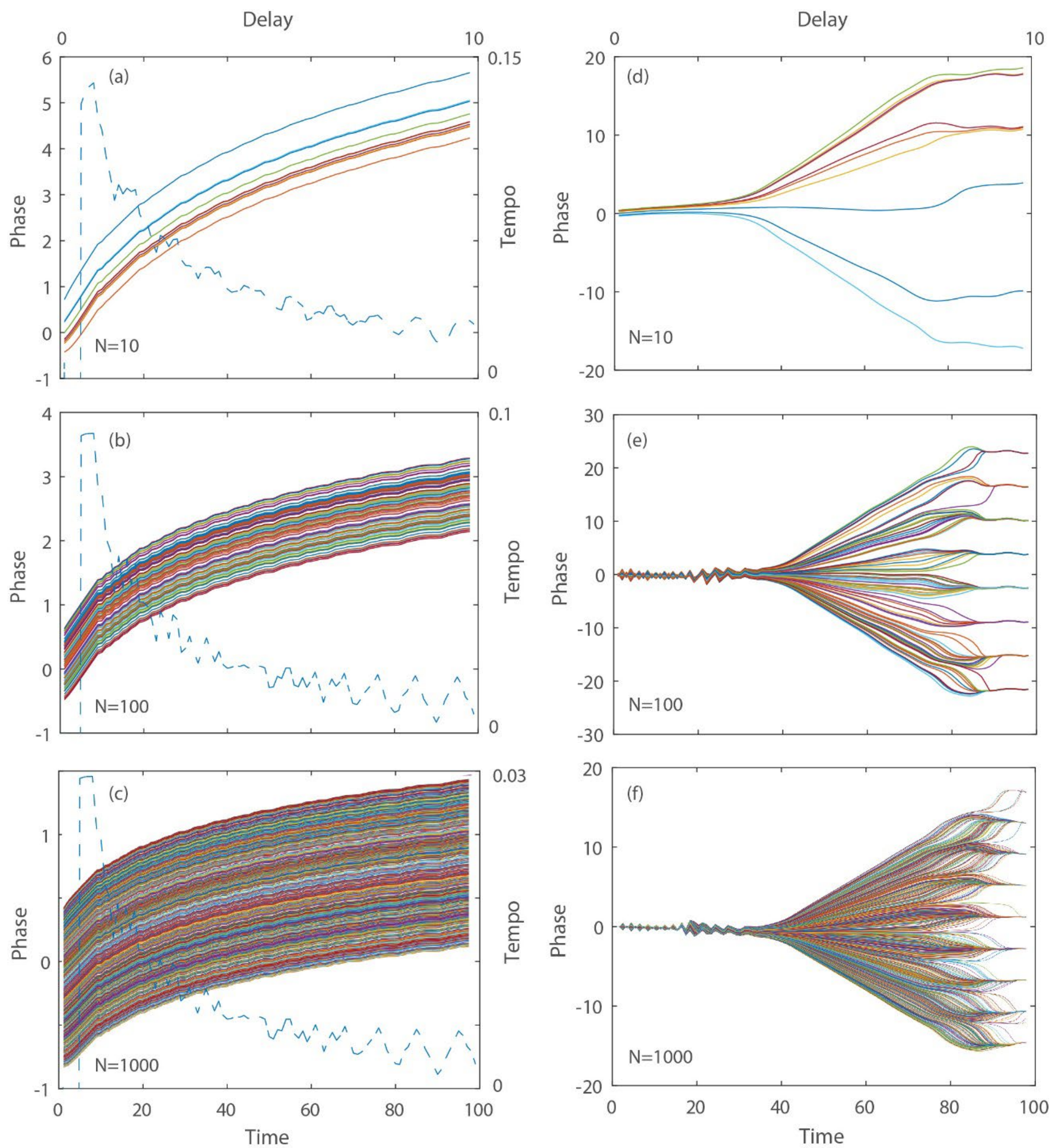
We added section S4 to the supplemental materials:

**"S4 Numerical simulations**

**To study our models in large networks, we performed numerical simulations of random networks with unidirectional and bidirectional coupling. We assume a linear increasing**

**delay between the nodes as a function of time. Two representative results of the different phases of the nodes as a function of time are shown in Fig. S5. We repeated this calculation numerous times for N=10, 100, and 1000 players, and observed the slowing down of the tempo. We show representative results of the phase of the players as a function of time with the average tempo in Fig. S4(a)-(c). Next, we change the coupling strength as a function of time according to $\kappa(t) = \kappa(t = 0)cos^2(\Delta t\, N\pi/T)$ as done in Section 3 in the main text. We repeated the calculations for different networks and observed the spreading of the phases, representative results are shown in Fig. S5(d)-(e). Here, the players are divided into several clusters with a phase shift between them according to the different unidirectional rings in the random networks. These results indicate that the dynamics of the motifs are general dynamics that appear in large complex networks."**

We added a new figure showing representative numerical results:



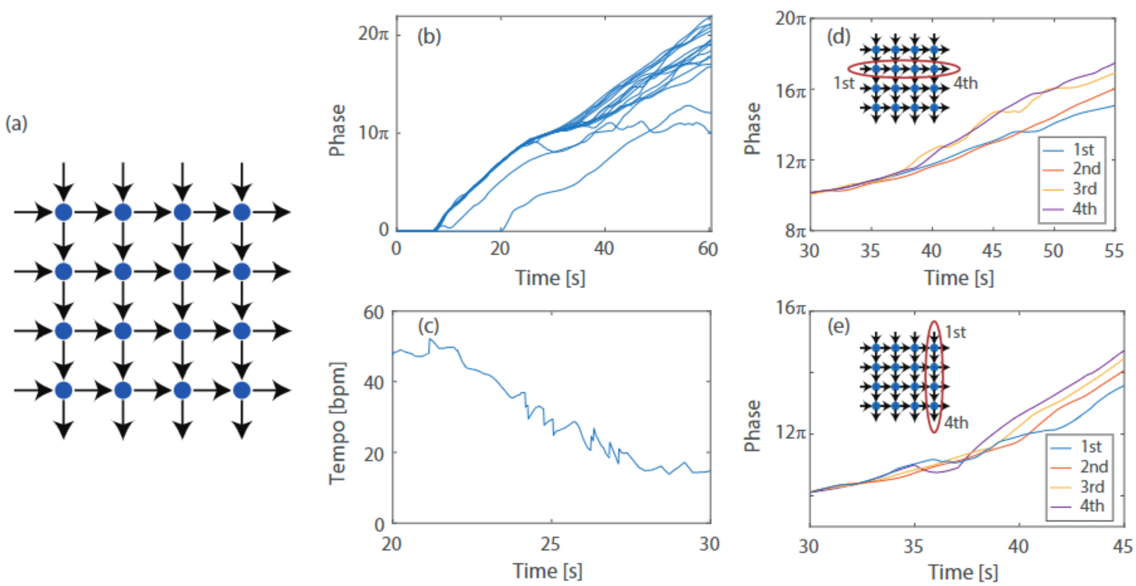

**"Fig. S5. Representative numerical calculations of the phase dynamics in random networks with N=10, 100, and 1000 players. We observe the two main dynamics, namely, the slowing down of the tempo in (a)-(c) and the spreading of the phase in (d)-(e). Solid curves denote the phase of each player as a function of time, and the dashed curves in (a)-(c) denote the average tempo of the players."**

We also added more experimental results in Section S3:

**"S3.4 Experimental results of a two-dimension lattice**

**To generalize our findings into networks with higher complexity, we measured the dynamics of the players when situated in two-dimensional networks with periodic boundary conditions. We start with a square two-dimension lattice shown in Fig. S4(a). We**

**measure the phase as a function of time, shown in Fig. S4(b), and evaluate the average tempo, shown in Fig. S4(c). As evident, we observe the slowing down of the tempo similar to the one shown in the one-dimensional rings. Next, we analyze four players from the lattice that are situated on a ring and observe the same spreading of phase observed in the one-dimensional ring. This is presented with two sets of four players in different orientations, shown in Fig. S4(d) and (e). These results verify that our findings are general to other networks, with unidirectional motifs. We analyze the effective connectivity of the network as a function of time and show it visually in https://youtu.be/h9C0XQgrLPI."**




**"Fig. S4. Coupled violin players in a square lattice with periodic boundary conditions. (a) Schematics of the lattice showing the 16 violin players and their coupled neighbors. (b) The phase of all the players as a function of time as we increase the delay between the players. (c) The average tempo of the players as a function of time, showing the slowing down of the playing tempo due to the delay. (d) and (e) Representative phase as a function of time of four players emphasized by the ellipse. These results show the spreading of the phase similar to the one shown in a one-dimensional ring."**

2. *"Pictures are small and difficult to read."*

Per the comment by the reviewer, we enlarged all the figures.

3. *"The acknowledgment section still contains the guidelines for its redaction, and I have spotted a couple of typos in the text/bibliography."*

We fixed *acknowledgment section and* the typos

4. “*The conclusions are too shallow and present only a mere summary of the results found. I believe, instead, that the conclusions should place the work into a broader context, compare the results found with those available in the literature, and lay future research avenues. I strongly suggest that the authors put all their efforts in improving this aspect.”*

While our research is rooted in the context of human interactions and network dynamics, the insights gleaned from this study may have far-reaching implications across diverse domains, including politics, economics, pandemic control, decision-making, and the development of artificial intelligence systems. By expanding upon these points in the conclusion section, we provide a more comprehensive justification for the potential implications of the research across various fields, addressing the reviewer's comment. Specifically, we added to the conclusions:

**“Our results indicate that human networks are more robust than other networks since they have unique methods for escaping local minima into global ones. The results shed new light on the dynamics of human networks and how a group of humans can reach synchronization while escaping local minima. Our investigation offers insights that extend beyond the immediate domain of network dynamics. While our study focuses on the specific context of human interactions modeled through coupled violin players, the principles and mechanisms uncovered have broader implications across multiple disciplines.**

**In the realm of decision-making theory, our findings highlight the adaptability and resilience of human networks in navigating complex environments. By elucidating the strategies employed by individuals to escape local minima and reach global synchronization states, our research provides valuable insights into the dynamics of group decision-making processes. Understanding these dynamics is crucial in fields such as organizational behavior, management, and policy-making, where the coordination of ideas and actions among individuals is central to achieving collective goals. Moreover, our study has implications for political and economic systems, where the dynamics of human networks play a pivotal role in shaping outcomes. By uncovering how network topology and individual behaviors influence synchronization dynamics, our research offers potential insights into the emergence of leadership, the formation of alliances, and the spread of information within political and economic networks. These insights may inform strategies for enhancing collaboration, fostering innovation, and promoting stability within these systems.**

**Furthermore, our study has implications for artificial intelligence and machine learning, particularly in the development of algorithms and models that mimic or interact with human networks. By elucidating the dynamics of human interactions and the strategies employed to navigate complex network landscapes, our research may inspire new approaches for designing adaptive and resilient artificial systems capable of learning from and interacting with human networks more effectively.**

**An exciting extension of our research is to incorporate real-time analysis of the network and detect the different connections. This will enable us to experiment and study dynamical networks, where the parameters of the networks change according to the state of synchronization between the nodes. With this system, we will study how leaders are**

**formed in a human network and if it is possible to control who will become a leader and who will become a follower."**

*5. "In Sec. 5 of the main manuscript, the "disconnection" effect induced by the tempo's slow down seems like the byproduct of the agents' evaluation of some cost/benefit ratio. Said in other words, it appears as if when agents realize that their efforts to stay synchronized is "too much", they spontaneously decide to ignore the feedback and continue on their own rhythm. If my intuition is correct, perhaps the authors could consider discussing this phenomenon in terms of the "evolutionary Kuramoto" model proposed in: https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.118.238301"*

We thank the reviewer for bringing this paper to our attention. Indeed, it is closely related to our work and we are planning to perform another experiment to study and verify its suggested theoretical models. The paper in PRL focuses on a cost/benefit ratio and shows how different networks evolve with different types of nodes. However, in our case, there is no cost since each player has only one input and their instruction is to synchronize to the input. We are planning on performing another experiment where some players have an external beat in their headphones. This will check the accuracy of the models presented in this paper. Nevertheless, our manuscript emphasizes the importance of our study where we see frustration even when there is a single input to each node. We show a topological frustration that is global to the network and not local frustration as was investigated in the PRL and in our previous paper.

We added this to the introduction:

**"In addition, previous studies focus on local frustration where some nodes are in a state of frustration [28, 29], while in real-life networks, many networks have global frustration due to their topology. This type of frustration is not local at a specific position in the network but rather emerges when the entire network is trying to synchronize."**

And to the conclusions:

**"We focus on network motifs with global frustration where the frustration rises from the topology of the network, instead of a local frustration where a node has contradicting inputs."**

We also cite the paper as reference number [29].

*6. "Please, provide an explanation of the term "topological charge".*

Per the comment by the reviewer, and according to the same point raised by all the other reviewers, we replaced the term "topological charge" with "vortex order", and explain it in paragraph 2 section 2:

**"The $n$ parameter denotes the order of the vortex which is stable [30]. For zero vortex order, $n = 0$, the system is at an in-phase state of synchronization, illustrated at the top of Fig. 1(b). When $n = 1$, we are at the first vortex order, in which adding the delay between the**

**players is equal to $2\pi$, illustrated at the bottom of Fig. 1(b). For higher vortex orders, adding the players' delay equals higher multiples of $2\pi$."**

We replaced the term in all the figures, figure captions, and in the rest of the text.

> *7. "In the first paragraph of the introduction, the phrase "which decision the network is more likely to make" does not make much sense to me, as networks are incapable of taking decisions (the people forming the network do it)."*

We revised the sentence to:

**"which decision the humans in the network are more likely to make."**

> *8. "Section 2 second paragraph: After the phrase "When $\Delta t$ exceeds ...", please specify that $n$ is an integer number."*

According to the reviewer's comment, we revised the sentence to:

**"When $\Delta t$ exceeds $nT/N$, where n is an integer number, the in-phase state becomes a local minimum and the global minimum solution is a vortex state, where each player has identical delay compared to its neighbor, satisfying the periodic boundary conditions."**

> *9. "I have found the definition of the periodic boundary condition a bit hard to understand. Perhaps, something like $\varphi_{i+N} = \varphi_{i}$ with $i = 1,2, \ldots, N$ is more general and easy to get."*

Per the comment by the reviewer, we revised the definition of the periodic boundary conditions accordingly.

> *10. "I warmly recommend to include a section on the experimental setup. As it is right now, it is practically impossible to reproduce the experiment performed. The authors should explain not only how the experiment has been prepared but also how the data have been analyzed."*

Per the reviewer's comment, we added section S2 to the supplemental materials describing the experimental setup and the methods for analyzing the results:

## "S2. Experimental details

### S2.1. Experimental setup

**We set sixteen isolated electric violin players to repeatedly play a musical phrase. We collect the output from each violin and control the input to each player via noise-cancellation headphones. The players cannot see or hear each other apart from what is**

**heard in their headphones. All the players start playing the first phrase with the help of an external rhythmical beat, to verify that they all start with the same playing period and phase. The rhythmical beat is stopped after the first phrase and the only instruction to the players is to try to synchronize their rhythm to what they hear in their headphones. We established different network connectivities and introduced delayed coupling between the players while monitoring the phase, playing period, volume, and frequency of each player with a mixing system.**

### S2.2. Choosing the musical phrase

**When choosing the musical phrase for the violin players, we must consider several points:**

- **Cyclic. It is important for the phrase not to have a clear beginning or end, but rather, have a cyclic characteristic to it.**
- **Same octave. For post-processing with Fourier transform and not confusing with other notes due to over-tones, it is easier when the entire phrase is at the same octave.**
- **Repeating notes. To help the players identify quickly where their neighbor is playing, it is easy to have a minimum number of repeating notes.**
- **4th finger. We need our players to play for a long time and with a minimum number of mistakes, therefore, we prefer finding a musical phrase that does not require using the 4th finger.**

### S2.3. Analyzing the results

**We record the output from each player and analyze it off-line. We use a fast Fourier transform algorithm on a moving temporal window to retrieve the spectrogram of the player. If the temporal window is too long we will not have enough temporal resolution and if the temporal window is too short, we will not have enough spectral resolution. We are using a temporal window of 0.15s. Next, we identify the different notes according to their frequency and map the time each player played a specific note. Next, we identified which player is coupled to which, and compensated for the delay between them to check if they are phase-locked or not. All the experimental results and the code to analyze them are available online: 10.6084/m9.figshare.22822103”**

In addition, we uploaded the entire data and the code used to analyze it as open source in 10.6084/m9.figshare.22822103.

> *11. “Please amend the format used to mention sections and equation. As it is now, it is very complicated to understand if, for instance, by Eq. (1) one refers to that appearing in the main manuscript or to that of the supplementary materials. A convention like adding an "S" before the equation number (as done in many manuscripts) will improve considerably the readability. The same reasoning applies also to the section numbering. Using a formalism like "Supplementary Note Sx" or "Section Sx" will help to distinguish when some content is available in the main manuscript or in the supplementary materials.”*

We revised all the supplementary sections and equation numbering accordingly.

*12. "In Section 1 "Analytical Models" it seems counter intuitive to me to place first the calculations for a specific case (Sec. 1.1), and then the general ones (Sec. 1.2). To me, the natural order of sections should be the opposite: with the general case first, and the specific one after."*

According to the reviewer's suggestion, we switched between Sec S1.1 and Sec S1.2. Indeed, it makes more sense this way.

*13. "I may have missed something but, I have not understood very well how to obtain the effective potential, $V$. Perhaps a more detailed and step-by-step explanation of how to compute $V$ could improve this point. Following the same reasoning, I did not get well which are the algebraic passages to get to Eq. (5)."*

Per the comment by the reviewer, we added to the supplemental materials a detailed, step-by-step, derivation of the effective potential in section S1.1, the full derivation leading to Eq. (5) in section S1.2, and we added an analytical derivation for the oscillation death dynamics in S1.3.

*14. "I have found strange the absence of additional results for the oscillation and amplitude deaths regimes. Unless there is some valid reason for that, I suggest to add further results as done for the other two regimes (i.e., "spreading the phase" and "slowing the tempo")."*

Per the reviewer's comment, we added more representative results showing oscillation death and amplitude death. We present oscillation death for N=3, 4, and 5 players, and we show amplitude death for N=3, 5, and 7 players. Per the reviewer's comment, we added to section S.3.2:

**"When the tempo slows too much, the players get stuck in a state of oscillation death, namely, all the players are playing the same note indefinitely, thereby maintaining a degenerate form of synchronization. We present oscillation death for N=3, 4, and 5 coupled violin players, in Fig. S2 (a), (b), and (c), respectively."**

And to section S3.3:

**"The third dynamic we observed was amplitude death. Here, one of the players stops playing breaking the periodic boundary conditions and changing the topology into a unidirectional open-chain network. In such a network, there is no global frustration, and the players find a solution at any value of delay. Once the players reach the vortex solution, the players that stopped playing continue playing at the correct phase. We present amplitude death for N=3, 5, and 7 players in Fig. S3(a), (b), and (c)."**

In addition, we added an analytical derivation demonstrating how the system reach the state of oscillation death in section S1.3.

*"The graphics could be improved. In general, I have found that many details are hard to read because the font used is quite small. Specifically:*
*Fig. 1: Many details like the axes and tick labels are barely readable. Please, make them bigger.*

*Fig. 1(a): The color used to represent the two dataset are rendered with the same tone of gray when displayed in b/w. Besides, the legend is not very visible (please, make it bigger).*
*Fig. 2(b): Please highlight better the three regimes (e.g., putting a solid thick line) because they are not very distinguishable. Moreover, using white fonts on a light background reduces considerably the contrast. Make sure to specify the end values of the axes (e.g., it is not clear which is the value displayed at the end of the lower axis). The same reasoning applies also to the upper horizontal axis (also in panel a).*
*Fig. 3(a) inset: Please, use different symbols to indicate different topological charges, as color alone does not ensure proper discrimination (especially for colorblind people).*
*Fig. 5: Please use a colorblind-friendly palette to denote the four data series (people) appearing in panels (a) and (b).”*

We revised the figures according to all the reviewer’s comments. Specifically, we increased the size of all the fonts to at least 11. We replaced the circles with different shapes to help distinguish between them. We added lines to separate the three regimes. Finally, we checked all our figures in a color-blind simulator to make sure they were readable (https://www.color-blindness.com/coblis-color-blindness-simulator/). We focused on congenital red–green color blindness which is the most common type.

*“I have spotted a few typos. Specifically:*

*** Caption of Fig. 2: It should be $N=8$ instead of $N=6$.*

*** Section 4 second paragraph: It should be "we can expand" instead of "we can expend".*

*** Bibliography references no. 7 and 33: There is some missing information, as there are several "?" displayed after the name of the publisher.*

*** Bibliography reference no. 23: Please add information on the volume of the article.*

*** Bibliography reference no. 35: The correct spelling is "Kuramoto" and "Crawford" with capital letters.*

*** Section 1.1 of the Supplementary materials: please replace "insure" with "ensure".*

*** Section 2 of the Supplementary materials: The first word of the first paragraph should be written with a capital letter.”*

We fixed all the typos.

Reviewer #3 (Remarks to the Author):

*“I read with interest the paper by Fridman et al. Despite finding the paper well written and clear I have some major concerns about the novelty and the generality of the results presented as they appear to be limited, especially when compared to the previous work by some of the authors [1], on the same journal. In their previous work, it was already shown that vortices could form spontaneously in networks of human beings performing musical phrases at a violin, as an effect of introducing and manipulating a delay in their*

*interconnections. In the present manuscript, three more mechanisms are presented, that are "slowing the tempo", and "oscillation/amplitude death". These are also justified through comparison with the simulation and the analysis of a mathematical model (see below for more specific comments on the model). I am therefore left unconvinced that the contribution in this manuscript is broad enough to justify its publication in Nat Comms while it could be of some interest in more specialised journals on human movement and coordination.”*

Per the reviewer’s comment, we further emphasize the difference between this study and previous published. Specifically, we focus here on networks with global frustration. Most of previous studies focus on local frustration where a node has several contradicting inputs while here the frustration is a result of the network topology. This is important for many applications as presented in the revised text:

We added this to the introduction:

**“In addition, previous studies focus on local frustration where some nodes are in a state of frustration [28, 29], while in real-life networks, many networks have global frustration due to their topology. This type of frustration is not local at a specific position in the network but rather emerges when the entire network is trying to synchronize.”**

And to the conclusions:

**“We focus on network motifs with global frustration where the frustration rises from the topology of the network, instead of a local frustration where a node has contradicting inputs.”**

We added the significance and novelty of our finding at the conclusions:

**“Our results indicate that human networks are more robust than other networks since they have unique methods for escaping local minima. The results shed new light on the dynamics of human networks and how a group of humans can reach synchronization while escaping local minima. Our investigation offers insights that extend beyond the immediate domain of network dynamics. While our study focuses on the specific context of human interactions modeled through coupled violin players, the principles and mechanisms uncovered have broader implications across multiple disciplines.**

**In the realm of decision-making theory, our findings highlight the adaptability and resilience of human networks in navigating complex environments. By elucidating the strategies employed by individuals to escape local minima and reach global synchronization states, our research provides valuable insights into the dynamics of group decision-making processes. Understanding these dynamics is crucial in fields such as organizational behavior, management, and policy-making, where the coordination of ideas and actions among individuals is central to achieving collective goals. Moreover, our study has implications for political and economic systems, where the dynamics of human networks play a pivotal role in shaping outcomes. By uncovering how network topology and individual behaviors influence synchronization dynamics, our research offers potential insights into the emergence of leadership, the formation of alliances, and the spread of information within political and economic networks. These insights may inform**

**strategies for enhancing collaboration, fostering innovation, and promoting stability within these systems.**

**Furthermore, our study has implications for artificial intelligence and machine learning, particularly in the development of algorithms and models that mimic or interact with human networks. By elucidating the dynamics of human interactions and the strategies employed to navigate complex network landscapes, our research may inspire new approaches for designing adaptive and resilient artificial systems capable of learning from and interacting with human networks more effectively.**

**The next step we aim to take is to incorporate a real-time analysis of the system and detect the different connections. This will enable us to experiment and study dynamical networks, where the parameters of the networks change according to the state of synchronization between the nodes. With this system, we will study how leaders are formed in a human network and if it is possible to control who will become a leader and who will become a follower.”**

> *2. “In the abstract, it is stated that the research "may have implications in politics, economics, pandemic control, decision-making, and predicting the dynamics of networks with artificial intelligence". However, currently, this claim is not justified by the research presented.”*

While our research is rooted in the context of human interactions and network dynamics, the insights gleaned from this study may have far-reaching implications across diverse domains, including politics, economics, pandemic control, decision-making, and the development of artificial intelligence systems. By expanding upon these points in the conclusion section, we provide a more comprehensive justification for the potential implications of the research across various fields, addressing the reviewer's comment. Specifically, we added to the conclusions:

**“Our results indicate that human networks are more robust than other networks since they have unique methods for escaping local minima into global ones. The results shed new light on the dynamics of human networks and how a group of humans can reach synchronization while escaping local minima. Our investigation offers insights that extend beyond the immediate domain of network dynamics. While our study focuses on the specific context of human interactions modeled through coupled violin players, the principles and mechanisms uncovered have broader implications across multiple disciplines.**

**In the realm of decision-making theory, our findings highlight the adaptability and resilience of human networks in navigating complex environments. By elucidating the strategies employed by individuals to escape local minima and reach global synchronization states, our research provides valuable insights into the dynamics of group decision-making processes. Understanding these dynamics is crucial in fields such as organizational behavior, management, and policy-making, where the coordination of ideas and actions among individuals is central to achieving collective goals. Moreover, our study has implications for political and economic systems, where the dynamics of human networks play a pivotal role in shaping outcomes. By uncovering how network topology**

**and individual behaviors influence synchronization dynamics, our research offers potential insights into the emergence of leadership, the formation of alliances, and the spread of information within political and economic networks. These insights may inform strategies for enhancing collaboration, fostering innovation, and promoting stability within these systems.**

**Furthermore, our study has implications for artificial intelligence and machine learning, particularly in the development of algorithms and models that mimic or interact with human networks. By elucidating the dynamics of human interactions and the strategies employed to navigate complex network landscapes, our research may inspire new approaches for designing adaptive and resilient artificial systems capable of learning from and interacting with human networks more effectively.**

**The next step we aim to take is to incorporate a real-time analysis of the system and detect the different connections. This will enable us to experiment and study dynamical networks, where the parameters of the networks change according to the state of synchronization between the nodes. With this system, we will study how leaders are formed in a human network and if it is possible to control who will become a leader and who will become a follower."**

> *3. "The concept of topological charge, although important, is never explained in the manuscript. While a reference is given, I would advise to give more information on this concept to make the paper more self contained."*

Per the comment by the reviewer, and according to the same point raised by all the other reviewers, we replaced the term "topological charge" with "vortex order", and we explain it in paragraph 2 section 2:

**"The $n$ parameter denotes the order of the vortex which is stable [30]. For zero vortex order, $n = 0$, the system is at an in-phase state of synchronization, illustrated at the top of Fig. 1(b). When $n = 1$, we are at the first vortex order, in which adding the delay between the players is equal to $2\pi$, illustrated at the bottom of Fig. 1(b). For higher vortex orders, adding the players' delay equals higher multiples of $2\pi$."**

We replaced the term in all the figures, figure captions, and in the rest of the text.

> *4. "Before (2) and in the supplemental material, the authors assume---they say, without loss of generality---that $\Delta \phi_n = \Delta \phi$ (for all n). This should correspond to stating that all nodes in the network have the same phase difference with respect to each other. It is not clear to me how this assumption can preserve generality."*

The reviewer is correct, this is a mistake we have. The starting point is arbitrary and can be chosen without loss of generality. The fact that all the phase differences are identical is an assumption due to symmetry and we verify it later. Per the reviewer's comment, we removed this statement but add to the Supplemental Materials:

**“We focus on the end of the ring, without loss of generality since the beginning of the ring is arbitrarily chosen. This is done to consider the periodic boundary conditions of the ring.”**

In addition, it is possible to follow this derivation for arbitrary phase differences, as long as there are at least two adjacent phase differences that have about the same value. We added this to paragraph 3 section 3:

**“The same derivation holds for non-uniform phase differences, as long as two adjacent phase differences are similar.”**

> *5. “At the end of Section 5, the authors suggest that an open ring structure is a network topology that makes it easier to synchronize frequencies, with respect to a closed ring. While this is intuitive to some readers, I would suggest the authors explain the reasons why this happens.”*

Per the reviewer’s comment, we elaborate on the differences between a close and an open ring configuration. We added to paragraph 3 in section 5:

**“When one of the players stops playing, the closed ring switches into an open ring topology where all the other players are free to shift their phases according to the coupling delay. In an open ring, the players are not limited by the periodic boundary conditions, so the network is stable for any value of delay.”**

> *6. “More generally, it seems to me that the effect of changing the network structure is an important one and should be investigated with respect to the phenomena studied in the manuscript. To me, the choice of studying a ring configuration with unidirectional time-delayed coupling is quite a specific one. Other options/configurations should also be explored.”*

To answer this comment, we add new experimental results, demonstrate large networks with numerical simulations, and explain the basics of motifs in complex networks.

Indeed, in this manuscript, we focus on the unidirectional ring. Unidirectional rings are the motifs that have global frustration, namely, frustration resulting from the topology of the system and not local frustration where a single node has several contradicting inputs. These basic motifs are the basic building blocks of large networks, and, the dynamics of these motifs govern the dynamics of large networks. In addition, we perform numerical simulations for large random networks and observe the same dynamics of the basic motifs. These dynamics include the slowing down of the tempo and the spreading of the phase. We added these results to the Supplemental Materials in section S4. We cite two papers studying the basics of different network motifs in [34, 35] and two papers studying how the dynamics of motifs influence the dynamics of large networks [36, 37]. Finally, we performed more experiments and show experimental results of two-dimension lattices showing the same dynamics of tempo slowing down and spreading of the phase.

Specifically, we added to paragraph 4 section 1:

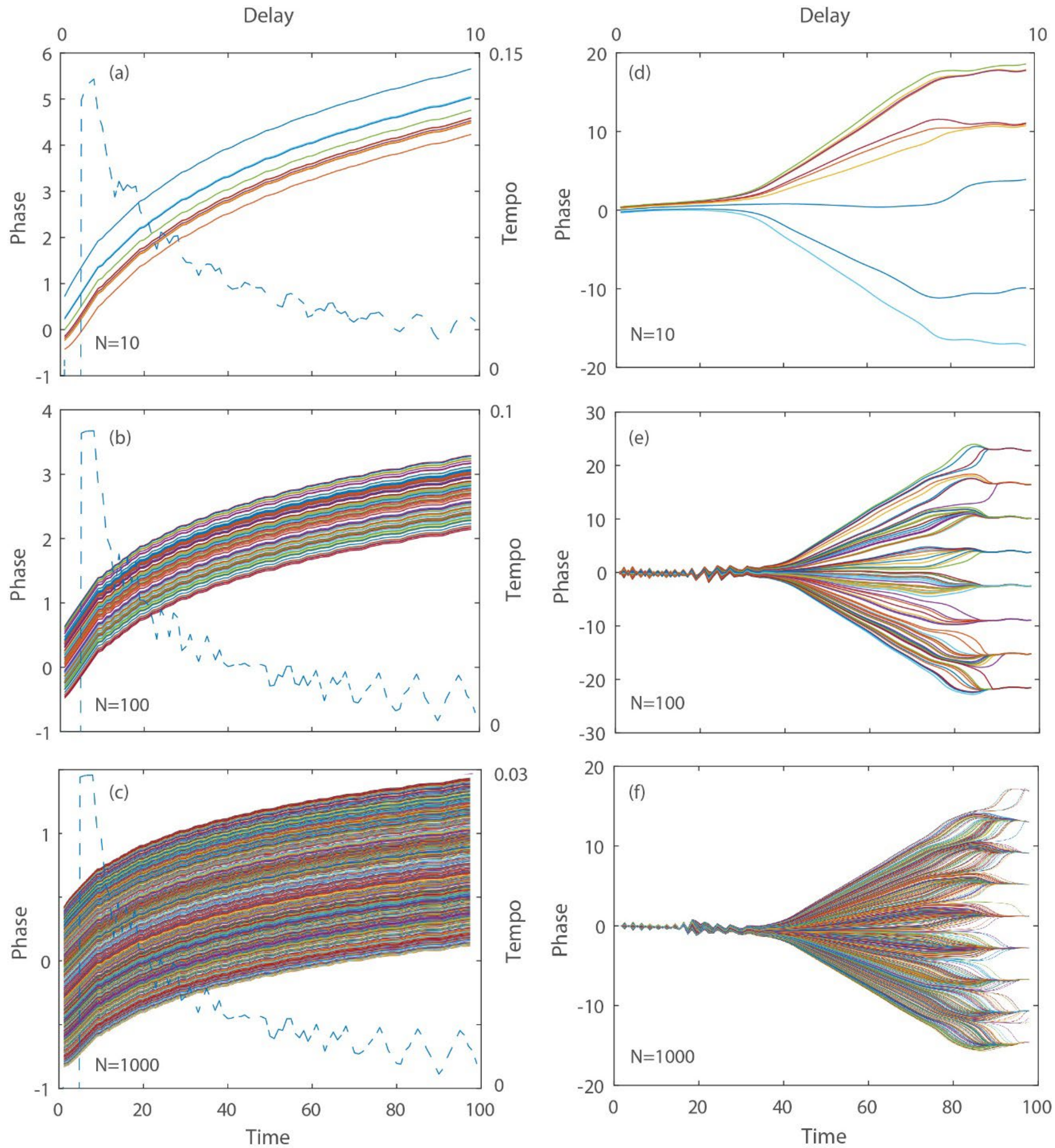
**"We are studying here basic network motifs that have global frustration, these motifs are the building blocks of complex networks [34, 35]. In addition, the dynamics of complex networks are dictated by the dynamics of the motifs [36, 37]. We numerically demonstrate the dynamics of the motifs in large random networks in the supplemental materials section S4 and generalize our findings into complex two-dimensional networks in the supplemental materials section S3.4."**

We also added section S4 to the supplemental materials:

**"S4 Numerical simulations**

**To study our models in large networks, we performed numerical simulations of random networks with unidirectional and bidirectional coupling. We assume a linear increasing delay between the nodes as a function of time. Two representative results of the different phases of the nodes as a function of time are shown in Fig. S5. We repeated this calculation numerous times for N=10, 100, and 1000 players, and observed the slowing down of the tempo. We show representative results of the phase of the players as a function of time with the average tempo in Fig. S5(a)-(c). Next, we change the coupling strength as a function of time according to $\kappa(t) = \kappa(t = 0)cos^2(\Delta t\, N\pi/T)$ as done in Section 3 in the main text. We repeated the calculations for different networks and observed the spreading of the phases, representative results are shown in Fig. S5(d)-(e). Here, the players are divided into several clusters with a phase shift between them according to the different unidirectional rings in the random networks. These results indicate that the dynamics of the motifs are general dynamics that appear in large complex networks."**

We added new figure showing representative numerical results:



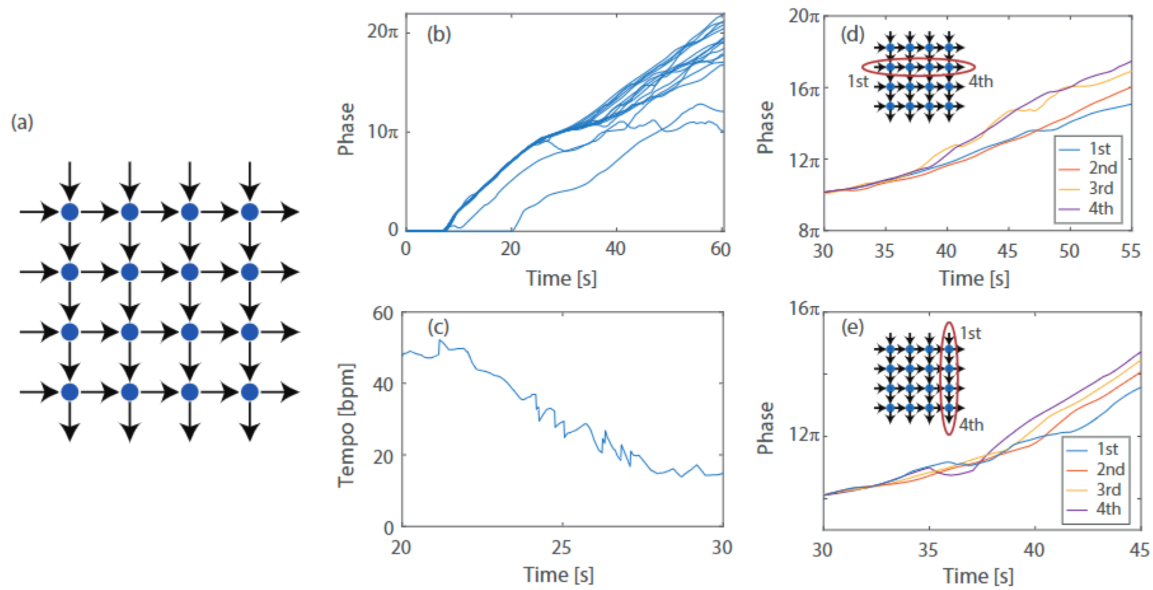

**“Fig. S5. Representative numerical calculations of the phase dynamics in random networks with N=10, 100, and 1000 players. We observe the two main dynamics, namely, the slowing down of the tempo in (a)-(c) and the spreading of the phase in (d)-(e). Solid curves denote the phase of each player as a function of time, and the dashed curves in (a)-(c) denote the average tempo of the players.”**

We also added more experimental results in Section S5:

**“S3.4 Experimental results of a two-dimension lattice**

**To generalize our findings into networks with higher complexity, we measured the dynamics of the players when situated in two-dimensional networks with periodic boundary conditions. We start with a square two-dimension lattice shown in Fig. 4(a). We**

**measure the phase as a function of time, shown in Fig. S4(b) and evaluate the average tempo, shown in Fig. S4(c). As evident, we observe the slowing down of the tempo similar to the one shown in the one-dimensional rings. Next, we analyze four players from the lattice that are situated on a ring and observe the same spreading of phase observed in the one-dimensional ring. This is presented with two sets of four players in different orientations, shown in Fig. S4(d) and (e). These results verify that our findings are general to other networks, with unidirectional motifs. We analyze the effective connectivity of the network as a function of time and show it visually in https://youtu.be/h9C0XQgrLPI."**




**"Fig. S4. Coupled violin players in a square lattice with periodic boundary conditions. (a) Schematics of the lattice showing the 16 violin players and their coupled neighbors. (b) The phase of all the players as a function of time as we increase the delay between the players. (c) The average tempo of the players as a function of time, showing the slowing down of the playing tempo due to the delay. (d) and (e) Representative phase as a function of time of four players emphasized by the ellipse. These results show the spreading of the phase similar to the one shown in a one-dimensional ring."**

> 7. *"In Section 1.2 in the supplemental material, before (8), it seems the authors assume that a phase-locked solution (i.e., with equal node frequencies) exists and it is reached. If so, the authors should explicitly state they make this assumption."*

Per the comment by the reviewer, we added to the supplemental materials a detailed, step-by-step, derivation of the effective potential in section S1.1 and the derivation leading to Eq. (5) in section S1.2. Specifically, we added:

**"When the system is close to phase locking, the phases of all the oscillators change linearly in time with about the same tempo. We assume that all the oscillators have the**

**same frequency and are equally separated in the phase-space, so $\Delta\varphi_n = \Delta\varphi$, for all $n < N$, and $\Delta\varphi_N = -(N-1)\Delta\varphi$."**

> *8. "In (1) and elsewhere, why is the symbol of partial derivative used, rather than that of the total derivative, given that \phi_n is only a function of time?"*

The reviewer is correct, in our case there is no difference between a full derivative and a partial derivative. However, when considering a large network, and approximating to continuum, then these equations are also a function of space. Therefore, it is convenient to use partial derivatives.

> *9. "After (3), Omega_n is not defined (it is defined only in the supplemental material)."*

Per the comment by the reviewer, we added after Eq.(3)

**"where $\Omega_n = \omega_{n+1} - \omega_n$, and $\omega = \sum \omega_n / N$ "**

> *10. "In the phrase "Therefore, increasing the delay adiabatically", do the authors mean a slow increase that allows the network to get to a steady state after each small increase? In any case, I would suggest clarifying the meaning of "adiabatically" in this context, given that the topic is unrelated to thermodynamics."*

The reviewer is correct. Per his comment, we revised the sentence to

**"Therefore, increasing the delay adiabatically (slowly), so the network remains in the in-phase state of synchronization, transfers the system to a local minimum."**

# Response letter

We thank the reviewers for their time and effort in reviewing our manuscript titled: “How synchronized human networks escape local minima”. We address all the comments by the third reviewer and hope you will find the revised manuscript suitable for publication as recommended by the first reviewer. Here is the main comment by the third reviewer together with our response. The other minor comments and response follows:

## Reviewer #3:

The main point raised by the reviewer:

> *“In the revised conclusions, the Authors included statements to emphasize the significance and novelty of their findings. Nonetheless, these statements do not convincingly show the broad applicability of the findings. Specifically, it remains unclear why or how exactly the mechanisms observed in maintaining synchronization (a very specific dynamical behavior) in a periodic task can be relevant to "organizational behavior, management, and policy-making" (sic). For instance, what would be the equivalent of a phase in a management or policy-making context? The author reference "coordination" as a link, but the coordination observed in these fields typically involves the coordination (not necessarily synchronization) of events or (non-periodic) opinions rather than synchronization of motor/musical behavior. Providing more specific examples, contextualized within the applications mentioned, would have been helpful. To clarify, I am not asserting that there is no possible connection, but rather that that it appears loose, based on the statements provided in the manuscript.”*

We appreciate the reviewer's feedback and the opportunity to clarify and strengthen our manuscript. We address the critique from four different directions:

1. **Emphasizing the Importance of Synchronization of Periodic Behavior:**

Synchronization of periodic and rhythmic behaviors is crucial for understanding human dynamics and is well-documented in various fields, including organizational behavior and policy-making. For instance:

- **A.W. Wiseman et al. (2018)** discuss the rhythmic application of evidence-based policy in national educational systems, highlighting the importance of timing and coordination in policy implementation.

- **G. Koehler (2003)** presents a theoretical foundation for adaptive policy-making, emphasizing the role of time and complex systems in public policy.
- **A.G. Morçöl (2013)** explores complexity theory in public policy, which often involves coordinated actions and timing.
- **P. Hawe et al. (2009)** theorize interventions as events in systems, where rhythmic and coordinated interventions are crucial for effective outcomes.

These references underscore that rhythmic behaviors and synchronization are not only relevant but also critical in fields like organizational behavior, management, and policy-making.

2. **Using Periodicity and Rhythmics to Study Mechanisms of Escaping Local Minima:**

The periodicity in our experiment serves as a quantitative and well-controlled tool to induce a local minimum state, allowing us to investigate how the network finds the global minimum accurately. This concept of escaping local minima is significant in various systems, including:

- **Biological Systems:** Insects like ants and fireflies exhibit mechanisms to escape local minima and find optimal states. **[O. Feinerman *Nat. Phys.* (2018)]**
- **Physical Systems:** Lasers often undergo state transitions to find stable operating points. **[V. Pal *PRL* (2017)]**
- **Deep Learning Networks:** These systems frequently move from local to global minima to optimize performance. **[K. Kawaguchi *Adv. Neur. Info.* (2016)]**

In human networks and communities, adapting and finding new stable states amid changing conditions (e.g., conflicts, climate changes, disasters) is crucial. Our research models these transitions, using rhythmic behavior to prepare the network at a local minimum and studying the mechanisms the network uses to return to a stable global state.

3. **Utility of Simple Cases for Studying Complex Systems:**

Simple systems can illuminate complex dynamics that might be otherwise obscured. For example:

- **Traffic Studies:** Research on cars on circular roads (e.g., Sugiyama et al. 2008) has led to breakthroughs in understanding traffic dynamics, despite the simplicity of the model. These studies involve the rhythmic behaviors of drivers, reinforcing the relevance of rhythmic behavior in human interaction.

Our research similarly uses a simplified and well-controlled system to accurately study human dynamics, providing clean insights that can be applied to more complex scenarios.

4. **Direct mapping of Synchronization of Periodic Behavior onto none-periodic coordination**

The Synchronization of periodic motion, generally described by the celebrated Kuramoto model, has a direct mathematical mapping onto the coordinated directional alignment of aperiodic systems. For example:

- Physical systems: research on coupled lasers [e.g. Nixon et. al. (2013)] established an exact mapping between laser phase synchronization and minimizing the energy of a spin Hamiltonian by aligning the direction of the spins. Such spin Hamiltonians are themselves mapped to a variety of coordinated decision-making such as optimal cooperative transport in groups of ants [Feinerman, et. al. 2018; Gelblum, et. al. (2015)]

These mappings imply that our research on the synchronization of periodic behavior can be applied to a variety of aperiodic coordinated behaviors in humans, animals, and physical systems.

By addressing these points, we aim to clarify the broader applicability of our findings and demonstrate the relevance of synchronization and rhythmic behavior in various fields. We have revised the manuscript to include these examples and references to strengthen our claims.

## Changes in the manuscript according to the reviewer's comment:

Per the comment by the reviewer, we added references for organizational behavior, management, and policy-making [34 – 37], for the significance of escaping local minima [25 – 27], and for examples of how to map our periodic results into aperiodic systems [50, 51].

We revised the beginning of the abstract to:

**"Finding the global minimum in complex networks while avoiding local minima is challenging in many types of networks. In human networks and communities, adapting and finding new stable states amid changing conditions due to conflicts, climate changes, or disasters, is crucial. We studied the dynamics of complex networks of violin players and observed that such human networks have different methods to avoid local minima than other non-human networks."**

We added to the introduction paragraph 2:

**"The concept of escaping local minima is significant in various systems including biological systems [25], physical systems [26], and deep neural network learning [27, 28]. It also has implications for increased stability in other types of networks [29], and optimization problems in spin-glass dynamics [30]."**

And to paragraph 4:

**"We study the rhythmic behavior of humans since it can reveal aspects of human network dynamics that are usually hard to identify [33]. Human synchronization in general and specifically rhythmic behaviors are critical in fields like organizational behavior, management, and policy-making [34 – 37]."**

Finally, we added to the conclusions:

**"The Synchronization of periodic motion, generally described by the celebrated Kuramoto model, has a direct mathematical mapping onto the coordinated directional alignment of aperiodic systems [27]. These mappings imply that our research on the synchronization of periodic behavior can be applied to a variety of aperiodic coordinated behaviors in humans, animals, and physical systems [51, 52]."**

## Here are more comments by the reviewer and our answers:

1. *"In the rebuttal, concerning S1.1, the authors state "The fact that all the phase differences are identical is an assumption due to symmetry and we verify it later." At the end of the revised S1.1 the authors state "a new global minimum appears in \Delta \phi = −\pi/2, namely, the vortex state of synchronization". This appears to be the point where the authors verify the assumption; however, the verification is simply stated and not demonstrated."*

This assumption is justified by the experimental results. Per the comment by the reviewer, we refer to the measured results which verify our assumption. Specifically, we added to the paragraph:

**"This assumption is verified by the experimental results shown in Figs. 3, 4, and 5, in the manuscript, where there is an equal spacing between the phases, apart from the first and the last player."**

2. *“Moreover, is it correct that \Delta \phi does not depend on N? (I might be wrong, but I was expecting a first vortex order solution, with phases equally spaced on the circle).”*

Indeed, the reviewer is correct and in the first vortex the phases are equally spaced. We had a typo in the inline equation in the paragraph. Per the comment by the reviewer, we fixed the typo to:

**“However, when the delay increases beyond $\omega\,\Delta t > \pi/2N$, the $\Delta\varphi = 0$ becomes a local minimum and a new global minimum appears in $\Delta\varphi = 2\pi/N$, namely, the vortex state of synchronization.”**

3. *“I attempted to perform the computations leading from (S12) to (S13) but could not obtain the same result as the authors.*

*- In (S7), it seems to me that the last term should be*
*"k sin (+ \Delta \phi - omega \Delta t)"*
*rather than*
*"k sin (- \Delta \phi - omega \Delta t)"*
*I believe this is because, using (S6), from (S5) we should get*
*\phi_N(t - \Delta t) - \phi_{N-1}(t) =*
*\Delta \phi(t - \Delta t) N + \omega(t - \Delta t) - \Delta \phi(t) (N-1) - \omega t =*
*\Delta \phi - \omega \Delta t*
*If I am wrong, I apologize and kindly ask the authors to clarify this point.”*

The reviewer is correct, we are sorry for the typo and fixed it. We also checked the entire derivation in the paper to verify that there were no other typos and revised accordingly the manuscript.

4. “After (S15), the authors state "This tempo slowing down ensures that \Delta t < T/N". It is not clear to me how this fact is obtained from (S15).”

Per the comment by the reviewer, we explain this condition after S15, as:

**“Thus, the tempo of the coupled oscillators slows down as long as the players stay phase-locked. Assuming that $\kappa\,\Delta t > 1$, we obtain that $T = 2\pi\kappa\,\Delta t/\omega$ . Therefore, this tempo slowing down ensures that $\Delta t < T/N$, indicating that our assumptions are valid.”**

On behalf of all the authors

Moti Fridman