

Increasing wind force (La.)

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 27 23 **Supplementary Fig. 1 Turbulence regimes in different parameter slices in summer.** The regimes (GSP: geostrophic shear production turbulence; LSP: Langmuir shear production turbulence; VBP: vertical buoyancy production turbulence; AGSP: ageostrophic shear production turbulence) denoted by different color patches are defined by the dominant production terms in 27 the TKE budget in different parameter slices. The white contours enclose 30%, 60%, and 90% of
28 the locations with the corresponding values. A regime is considered dominant when its dissipation
29 contribution exceeds the locations with the corresponding values. A regime is considered dominant when its dissipation 29 contribution exceeds 75% of the total dissipation, otherwise, it is a two-turbulence-mixed regime
30 when two TKE sources both contribute more than 25% while all others contribute less than 25%
31 and lastly, it is a mi when two TKE sources both contribute more than 25% while all others contribute less than 25%, 31 and lastly, it is a mixed regime if more than three kinds of turbulence contribute more than 25% (2.32) (Li et al., 2019). Compared with **Fig. 2**, the magnitude of GSP is much weakened in summer. 32 (Li et al., 2019). Compared with **Fig. 2**, the magnitude of GSP is much weakened in summer.

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40 35 **Supplementary Fig. 2. Global distributions of the dissipation rates. a**-**d**, Langmuir shear 36 production turbulence (LSP). **e**-**h**, geostrophic shear production turbulence (GSP). **i**-**l,** vertical 37 buoyancy production turbulence (VBP). **m**-**p**, ageostrophic shear production turbulence (AGSP). The dissipation rates are derived from the TKE model based on the LLC4320 data. The columns from left to right show the means in winter, the medians in winter, the means in summer and the 40 medians in summer, respectively. Both means and medians suggest the importance of GSP
41 turbulence over the globe. turbulence over the globe.

44 **Supplementary Fig. 3 The relative contribution percentages of geostrophic shear**

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 48 45 **production turbulence (GSP) to the dissipation rate in the surface boundary layer. a, d,** The results based on uncorrected buoyancy gradients. **b**, **e**, The results based on corrected buoyancy gradients. **c**, **f**, The results based on no-slope corrected buoyancy gradients (left: winter; right:

48 summer). The relative contribution of GSP explicitly shows where GSP dominates the boundary
49 layer turbulence, and also suggests a robust role of GSP turbulence due to the buoyancy gradier
50 correction. layer turbulence, and also suggests a robust role of GSP turbulence due to the buoyancy gradient

- correction.
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53 **Supplementary Fig. 4 Global distributions of the two most likely dominant sources at each location.** The results are based on the uncorrected buoyancy gradients. **a**, The first most likely for dominant sources (GSP: geostrophic shear production turbulence; LSP: Langmuir shear 55 dominant sources (GSP: geostrophic shear production turbulence; LSP: Langmuir shear
56 production turbulence; VBP: vertical buoyancy production turbulence; AGSP: ageostroph 56 production turbulence; VBP: vertical buoyancy production turbulence; AGSP: ageostrophic shear
57 production turbulence) in winter. **b**, The second most likely dominant sources in winter. **c**, The 57 production turbulence) in winter. **b**, The second most likely dominant sources in winter. **c**, The first most likely dominant sources in summer. **d**, The second most likely dominant sources in summer. Their relative con first most likely dominant sources in summer. **d**, The second most likely dominant sources in 59 summer. Their relative contribution percentages to the total mean dissipation (%) are shown in **e-**
60 **h**. Despite that GSP turbulence is weakened, it is still a significant contributor at low latitudes in 60 **h**. Despite that GSP turbulence is weakened, it is still a significant contributor at low latitudes in winter, and the second largest contributor at high latitudes in both seasons. S 62

 location. The results are based on the no-slope corrected buoyancy gradients. **a,** The first most 67 likely dominant sources (GSP: geostrophic shear production turbulence; LSP: Langmuir shear
68 production turbulence; VBP: vertical buoyancy production turbulence; AGSP: ageostrophic she 68 production turbulence; VBP: vertical buoyancy production turbulence; AGSP: ageostrophic shear production turbulence) in winter. **c**. The production turbulence) in winter. **b,** The second most likely dominant sources in winter. **c,** The 70 first most likely dominant sources in summer. **d**, The second most likely dominant sources in
71 summer. Their relative contribution percentages to the total mean dissipation (%) are shown i 71 summer. Their relative contribution percentages to the total mean dissipation (%) are shown in **e-**
72 **h.** As GSP turbulence is strengthened, it becomes the most prevalent contributor of the first-

- 72 **h.** As GSP turbulence is strengthened, it becomes the most prevalent contributor of the first-
73 dominant sources in winter. dominant sources in winter.
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 $\frac{76}{77}$ 77 **Supplementary Fig. 6 Probability density function differences of the first-order structure** 78 **functions of sea surface temperature (SST) in different regions. a, b,** the Kuroshio Extension (KE; 32~38 °N, 150~156 °E). **c, d,** the Northern Subtropical Pacific (NSP; 15~21 °N, 180~186 °E).
80 **e, f,** the Southern Subtropical Pacific (SSP; 20~26 °S, 120~126 °W). **g, h**, the Gulf Stream (GS; **e, f,** the Southern Subtropical Pacific (SSP; 20~26 °S, 120~126 °W). **g, h,** the Gulf Stream (GS; 81 28~34 °N, 60~66 °W). **i, j,** the Antarctic Circumpolar Current (ACC; 50~56 °S, 115~121 °E) (left: 28~34 °N, 60~66 °W). **i, j**, the Antarctic Circumpolar Current (ACC; 50~56 °S, 115~121 °E) (left:
82 winter; right: summer). The dashed lines denote the minimum wavelengths that the effective
83 resolution resolves (i.e., winter; right: summer). The dashed lines denote the minimum wavelengths that the effective 83 resolution resolves (i.e., two times the effect resolution 7∆x). The positive bias in probability at 84 small SST jump magnitude and negative bias in probability at large SST jump magnitude imply
85 that at small spatial scale LLC4320 underpredicts large SST jumps compared to the real ocean 85 that at small spatial scale LLC4320 underpredicts large SST jumps compared to the real ocean,
86 which needs to be corrected. which needs to be corrected.

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91 **Supplementary Fig. 7 Comparison of the ocean surface boundary layer (OSBL) and sea
92 surface mixed layer thicknesses in different seasons (m). a, b, c,** the thicknesses in Febru
93 2012. d, e, f, the thicknesses in A

surface mixed layer thicknesses in different seasons (m). a, b, c, the thicknesses in February

2012. **d, e, f,** the thicknesses in August 2012. The upper, middle, and lower panels show the mixed layer thickness from Argo, LLC4320, and the OSBL thickness from LLC4320, respective 15 The mixed layer depth is defined as

mixed layer thickness from Argo, LLC4320, and the OSBL thickness from LLC4320, respectively.

The mixed layer depth is defined as the depth where a temperature variance of 0.2 \degree C occurs 96 compared to the 10-m depth temperature. The similarity of the global pattern demonstrates the

96 compared to the 10-m depth temperature. The similarity of the global pattern demonstrates the 97 capability of LLC4320 to reproduce the ocean surface layer.

capability of LLC4320 to reproduce the ocean surface layer.

 $^{99}_{100}$ 100 **Supplementary Fig. 8 Dissipation rates at OSMOSIS and its comparison to LLC4320. a**, 101 Time series of the dissipation rates at the boundary layer mid-depth of the OSMOSIS site over
102 the winter time (January 2013–April 2013). **b**, probability density functions of the dissipation rat 102 the winter time (January 2013–April 2013). **b**, probability density functions of the dissipation rates.
103 The gray dots and lines denote the observed values, while the blue and red ones denote the 103 The gray dots and lines denote the observed values, while the blue and red ones denote the calculated values (the blues are the summation of dissipation from Langmuir shear production 104 calculated values (the blues are the summation of dissipation from Langmuir shear production
105 turbulence (LSP), vertical buoyancy production turbulence (VBP) and ageostrophic shear 105 turbulence (LSP), vertical buoyancy production turbulence (VBP) and ageostrophic shear
106 production turbulence (AGSP), while the reds are from geostrophic shear production turbu 106 production turbulence (AGSP), while the reds are from geostrophic shear production turbulence
107 (GSP), LSP, VBP and AGSP). A comparison of the non-dimensional dissipation magnitudes 107 (GSP), LSP, VBP and AGSP). A comparison of the non-dimensional dissipation magnitudes 108 between observations (solid lines) and LLC4320 (dotted and dotted-dash lines; the same winter 109 time but in 2012) is shown in **c** (dash blue: observed LSP+VBP+AGSP; solid blue: observed 109 time but in 2012) is shown in **c** (dash blue: observed LSP+VBP+AGSP; solid blue: observed 110 LSP+ corrected GSP+VBP+AGSP; dash red: simulated LSP+GSP+VBP+AGSP; dotted line:
111 simulated LSP+ uncorrected GSP+VBP+AGSP; dotted dash line: simulated LSP+ corrected 111 simulated LSP+ uncorrected GSP+VBP+AGSP; dotted dash line: simulated LSP+ corrected 112 GSP+VBP+AGSP). GSP+VBP+AGSP).

116 **Supplementary Fig. 9 Probability density functions (PDFs) of the nondimensional** 117 **dissipation rates of turbulence sources from different simulations. The four sources are** 118 geostrophic shear production turbulence (GSP; orange), Langmuir shear production turbulence
119 (LSP; dark blue;), vertical buoyancy production turbulence (VBP; light blue), and ageostrophic 119 (LSP; dark blue;), vertical buoyancy production turbulence (VBP; light blue), and ageostrophic
120 shear production turbulence (AGSP dark red)The dash and solid lines show the results from 120 shear production turbulence (AGSP dark red)The dash and solid lines show the results from
121 LLC4320 and eNATL60, respectively. The similarity of the PDF distributions demonstrates the LLC4320 and eNATL60, respectively. The similarity of the PDF distributions demonstrates the robust role of GSP turbulence. $\frac{122}{123}$

125 **Supplementary Fig. 10 The frontal arrest scale.** The estimated frontal arrested scale under the 126 turbulent thermal-wind balance at the OSMOSIS site based on the method of Bodner et al. $\begin{array}{c} 125 \\ 126 \\ 127 \\ 128 \end{array}$

127 (2023). The frontal scale cannot be resolved by the OSMOSIS mooring array.

Supplementary Fig. 11 Spectral slopes of the horizontal buoyancy gradient in different 132 regions. a, b, the Kuroshio Extension (KE; 32~38°N, 150~156°E). c, d, the Northern Subtrop **regions. a, b,** the Kuroshio Extension (KE; 32~38°N, 150~156°E). **c**, **d**, the Northern Subtropical Pacific (NSP; 15~21 °N, 180~186 °E). **e**, **f**, the Southern Subtropical Pacific (SSP; 20~26 °S, Pacific (NSP; 15~21 °N, 180~186 °E). **e**, **f**, the Southern Subtropical Pacific (SSP; 20~26 °S, 134 120~126 °W). **g**, **h**, the Gulf Stream (GS; 28~34 °N, 60~66 °W). **i**, **j**, the Antarctic Circumpola 134 120~126 °W). **g**, **h**, the Gulf Stream (GS; 28~34 °N, 60~66 °W). **i**, **j**, the Antarctic Circumpolar Current (ACC; 50~56 °S, 115~121 °E) (left: winter; right: summer). The blue and pinks lines 135 Current (ACC; 50~56 °S, 115~121 °E) (left: winter; right: summer). The blue and pinks lines 136 denote the power spectral densities of the horizontal buoyancy gradient in zonal and in 136 denote the power spectral densities of the horizontal buoyancy gradient in zonal and in
137 meridional, respectively. The dashed gray lines denote the corresponding linearly fitted 137 meridional, respectively. The dashed gray lines denote the corresponding linearly fitted slopes of 138 the inertial range. The vertical dash lines denote the maximum wavenumber that the effective 138 the inertial range. The vertical dash lines denote the maximum wavenumber that the effective 139 resolution resolves (i.e., two times the effective resolution $7\Delta x$). The spectral slopes from 139 resolution resolves (i.e., two times the effective resolution 7∆x). The spectral slopes from 140 LLC4320 generally have slightly negative slopes, rather than zero slopes. LLC4320 generally have slightly negative slopes, rather than zero slopes.

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147 amplification factor of the zonal buoyancy gradient. **c**, **d**, the amplification factor of the meridional buoyancy gradient (left: winter; right: summer). This amplification is because the spatial resolution

148 buoyancy gradient (left: winter; right: summer). This amplification is because the spatial resolution 149 of LLC4320 cannot resolve the arrested fronts under the turbulent thermal-wind balance. of LLC4320 cannot resolve the arrested fronts under the turbulent thermal-wind balance.

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 $\begin{array}{c} 152 \\ 153 \\ 154 \\ 155 \\ 156 \end{array}$ 153 **Supplementary Fig. 13 Global distributions of the frontal scale** *Lf* **(km). a,** *Lf* in winter. **b,** *Lf* in summer. **c**, the zonal median *L_f* (winter in pink and summer in blue). The solid and dashed lines in c denote the values derived from the LCC4320 and GOTM results and the shaded intervals 156 denote the corresponding bounds of the 10th and 90th percentile *Lf* values of the LLC4320
157 zonally. The frontal arrested scale is latitude-dependent and insensitive to the turbulence 157 zonally. The frontal arrested scale is latitude-dependent and insensitive to the turbulence closures.

160 **Supplementary Table 1 Percentages of locations and contributions.** Percentages of locations globally where each energy source is either the first or second largest contribution and their global averages ε_{ava} and 161 first or second largest contribution and their global averages ε_{avg} and 10th and 90th percentiles, ε_{10th} , ε_{90th} of the dissipation rate (×10⁻⁸ W kg⁻¹), by 162 season. The differences in results due to different levels of correction for limited horizontal model resolution are shown. GSP: geostrophic shear 163 production turbulence; LSP: Langmuir shear production turbulence; VBP: vertical buoyancy production turbulence; AGSP: ageostrophic shear
164 production turbulence. production turbulence.

Supplementary References

- 1. Li Q, Reichl BG, Fox-Kemper B, Adcroft AJ, Belcher SE, Danabasoglu G*, et al.* Comparing ocean surface boundary vertical mixing schemes including Langmuir turbulence. *Journal of Advances in Modeling Earth Systems* 2019, **11**(11)**:** 3545-3592
- 2. Bodner AS, Fox-Kemper B, Johnson L, Van Roekel LP, McWilliams JC, Sullivan PP*, et al.* 173 Modifying the Mixed Layer Eddy Parameterization to Include Frontogenesis Arrest by
174 Boundary Layer Turbulence. Journal of Physical Oceanography 2022, 53(1), 323-339 Boundary Layer Turbulence. *Journal of Physical Oceanography* 2022, **53**(1), 323-339