nature portfolio

Peer Review File

Experimental realization of on-chip few-photon control around exceptional points

Corresponding Author: Professor Zhihui Peng

This manuscript has been previously reviewed at another journal. This document only contains reviewer comments, rebuttal and decision letters for versions considered at Nature Communications.

This file contains all reviewer reports in order by version, followed by all author rebuttals in order by version.

Attachments originally included by the reviewers as part of their assessment can be found at the end of this file.

Version 0:

Reviewer comments:

Reviewer #1

(Remarks to the Author)
The question that the authors raise - the interplay between small photon number and non-reciprocal behavior in the vicinity of an exceptional point, including the role of EP noise in the quantum domain - is extremely interesting and relevant. Recent progress on EP realizations in non-photonic open quantum systems makes it even more so, since in those systems, EPs are hard to access.
In this manuscript, by using a highly dissipative cavity coupled to a high-Q cavity, with tunable interaction between them mediated by a superconducting qubit, and two auxiliary qubits, the authors study the scattering matrix for different mean-photon numbers. They characterize the asymmetric transport (isolation) and show that in the weak-coupling limit, it shows clear asymmetries that are absent in the strong-coupling limit. Normalized reflection studies, that go from one, quadratic dip to a quartic one at the EP, to two dips allow identification of the effective, lossy, non-Hermitian Hamiltonian that governs the dynamics of modes in the two coplanar-waveguide cavities. Further observations are presented showing that although the noise is enhanced near the EP, the SNR is reduced for reflection measurements while enhanced for transmission.
The manuscript is fairly well written, with significant details provided in the supplementary material that were helpful with understanding. I find the choice of some of the parameter labels mysterious -- such as "att" fominr the baseline of the exponential function used to characterize mean photon number in the cavity -- but such instances and typos are minimal.
I find the results in the paper very interesting and timely. To my knowledge this is the first example of studies near EP in the low-photon number domain, although multi-photon interferences have been "simulated" using lossy or unitary devices (Klauck et al. Nature Photonics 13, 883 (2019); N. Maraviglia et al, Phys. Rev. Research 4, 013051 (2022)). I therefore think that the manuscript is suitable for Nature Communications.

Reviewer #2

(Remarks to the Author)
Summary of the key results
The article “Experimental realization of on-chip few-photon control around exceptional points” by Pengtao Song et al. presents a couple of experiments realized with a system exhibiting exceptional points. The system is the well-known passive PT-dimer implemented here using a couple of superconducting resonators coupled with a tunable coupler. The authors observe the predicted exceptional point by measuring the complex frequencies of the two modes as a function of the coupling.
The authors then proceed to measure the non-reciprocal transmission through the system caused by the Kerr nonlinearity, which occurs at a few photons level. Finally, they measure the signal-to-noise ratio of a signal transmitted through the system or reflected. They find that the reflected SNR is minimized at the exceptional point while the transmitted SNR is maximized at the same point.

Originality and significance

Exceptional points are a classical phenomenon that has been demonstrated in many systems including electronic oscillators, optical cavities and superconducting resonators. The non-Hermitian nature of the dynamics is restricted to the average of the fields but the extension of this concept to systems in the quantum limit is the subject of ongoing research. For example, one of the main questions of the field is whether or not the increase in sensitivity to the order parameter at the exceptional point also comes with a quantum noise increase which would reduce or cancel the sensing advantage of such systems.

Overall, I found the connection between the three different experiments quite difficult to understand. In the second experiment, the authors measure the non-reciprocal transmission through the two coupled resonators but this phenomenon doesn't have a systematic connection to exceptional points (or quantum mechanics), it is simply a consequence of the nonlinearity introduced to tune this particular system. Similar physics is at play in the hysteretic frequency response of a Duffing oscillator, the classical equivalent of a single non-linear resonator. The large nonlinearity is indeed quite impressive but this is very common in such superconducting circuits, which have been shown to reach extremely high light-matter couplings.

I had even more trouble understanding the third experiment regarding sensing using this system. This experiment purports to help understand the usefulness of EPs for sensing application but the EPs are not used in the experiment. Indeed, EPs have been proposed to sense small variations of the "order parameter" g/κ when close to the EP but here the experiment keeps g/κ constant and simply looks at the transmission through the system.

The system is a completely passive system close to zero temperature and thus any variation of the noise is hard to understand. Indeed, in such system, if the input is a coherent state, then the output should also be a coherent state with the same quadrature variance 1/4 (but with possible less amplitude and a phase shift). Despite this, the authors predict large variations of noise in the zero temperature limit (see below for remarks on the supplementary material).

Extra noise is indeed expected around the EP of a PT-symmetric system because of the gain required to implement such systems but this is not the case here as the system studied here is not PT-symmetric.

Detailed comments

Here are a few specific comments on the figures, the main manuscript and the supplementary information.

Figures

Figure 1: e) and f) do not seem to be Lorentzian shaped as they feature a significant asymmetry. Why is this the case? These two figures would also be ideal to show how the complex frequency is extracted from the spectrum by showing fits for example.

Figure 2: b) The imaginary parts of the frequencies do not seem to match the theoretical prediction at all. They vary almost linearly with the coupling strength g/κ instead of as sqrt(g^2 – κ^2). Why is that?

c), d) and e) Once again the reflection spectra do not seem perfectly lorentzian and showing the fits used for the parameters extraction would be useful.

Figure 3: c) The reflection spectra of this non-linear system might be hysteretic so it would be nice to mention the sweep direction in the caption or the figure.

c) d) The units are unclear to me, how could the transmission be above 0dB?

Figure 4: Clouds of points are difficult to understand because of their overlap. A 2D histogram would be nicer in my opinion.

c) d) e) f): What are the solid lines? They seem quite different from the theoretical curves of the supplementary material.

Main text

Lines 54-63: This prediction of signal and noise enhancement is true for PT-symmetric system, which feature gain. It is not true for the passive system presented in the manuscript. While the mean field evolution can be post-processed (not done here) to simulate a PT-symmetric system, it is not clear to me if the same can be done for the noise.

Lines 65-72: A mention should be made of traditional RF circulators which work regardless of the input power at the cost of some small amount of loss and a bulky magnet. Some active circulators have also been made using Josephson junctions (e.g. Abdo, B., Bronn, N.T., Jinka, O. et al. Active protection of a superconducting qubit with an interferometric Josephson isolator. Nat Commun 10, 3154 (2019)) which also feature operation down to arbitrarily low power and very low losses.

Line 74: Once again, non-reciprocity at the single-photon level is routinely achieved in optics using a magnetic field in a Faraday isolator, without using any nonlinearity.

Line 84: While research is still ongoing, quite a few tunable switches have been demonstrated and the challenge is much more often to increase the power handling, bandwidth and ease of operation and not to reach operation at the single photon level. Here are a few examples among many others

O. Naaman, M. O. Abutaleb, C. Kirby, M. Rennie; On-chip Josephson junction microwave switch. Appl. Phys. Lett. 14 March 2016; 108 (11): 112601

M. Pechal, J.-C. Besse, M. Mondal, M. Oppliger, S. Gasparinetti, and A. Wallraff , Superconducting Switch for Fast On-Chip Routing of Quantum Microwave Fields, Phys. Rev. Applied 6, 024009 – 2016

A. Wagner, L. Ranzani, G. Ribeill, T. A. Ohki; Demonstration of a superconducting nanowire microwave switch. Appl. Phys. Lett. 21 October 2019; 115 (17): 172602

Line 124: The expression of the coupling g does not seem consistent with the one provided in the supplementary information and does not depend on the resonators frequencies. Neglecting the qubit relaxation, the coupling should be (g_a g_b)/Δ, with Δ the detuning between qubit and resonator (see Blais, et al. Phys. Rev. A 75, 032329 (2016)) so I am inclined to think that the expression S.15 is the correct one.

Line 171: How is the power uncertainty computed? It seems a bit pessimistic.

Line 179: Which non linearity is responsible for the non-reciprocity? Presumably the fourth-order (self Kerr) but this is not mentioned in the main text.

Line 224: The EP concept is fundamentally a linear (and classical) property. The nonlinearity introduced here seems orthogonal to the problem of EP sensing. Furthermore, the parameter sensed here is not g/κ for which sensing is predicted to be improved.

Supplementary Information
Line 209: What is ω? How is it chosen?
S.10: What is this parameter Λ? How can it be computed?
S.15: The Kerr coefficient formula is not homogeneous (it has dimension of a frequency squared and not a frequency).
S36: This equation is incorrect. Fundamentally, it does not obey the dissipation-fluctuation theorem. The dissipation is only 2χ (for $g > \kappa$) while the fluctuations are $2\chi + \kappa_{c1}/2$. This dynamical equation should be properly derived from a Hamiltonian and a set of jump operators using the input-output theory but the error here is probably the sqrt(2χ) which should be replaced by sqrt($2\chi - \kappa_{c1}/2$). This unfortunately leads to conclusion that the noise depends on the system parameters instead of being constant as expected for a passive system.

Version 1:

Reviewer comments:

Reviewer #1

(Remarks to the Author)
I have gone over the revised manuscript, a detailed response to the reviewers, and the lightly edited supplementary information. I am particularly impressed by the in-depth and detailed answers the authors have provided to the extensive list of questions from the first report. I therefore recommend the its acceptance.

Reviewer #2

(Remarks to the Author)
I would like to thank the authors for their detailed response and their numerous updates to the manuscript. Unfortunately, I still have some questions and disagreements and I cannot recommend publication as is.

Here is a detailed response to some of the points:

C2.3 I think the author explanation is quite convincing though I still disagree with the non-linearity enhancement near the EP. Non-reciprocity is maximized at small g.

C2.4 Superconducting circuits have been used to study non-Hermitian physics in the past: - There is a series of papers written in the group of Kater Murch where he uses a qubit and (sometimes) post-selection to engineer non-Hermitian Hamiltonians (Chen et al., Phys. Rev. Lett. 127, 140504, 2021; Erdamar et al, arXiv:2309.12393, 2023; Gaikwad et al., Phys. Rev. Research 5, L042024, 2023 and a few more)
- Mikko Mottonen's group has also published one paper where they use tunable losses to explore non Hermitian physics (Teixeira et al., Phys. Rev. Research 5, 033119, 2023)
The nonlinearity coming from the qubit is all but weak since it is the only source of nonlinearity, which occurs at a few photon levels as you point out (though to be fair, the two other ancillary qubits might contribute as well). The non-reciprocity is maximized very far from the EP where g/kappa is very small and I cannot see anything in the data that would suggest an enhancement near the EP.
Section B of the supplementary material attempts to show that the Kerr of the high-Q resonator is amplified near the EP but it also doesn't predict a local maximum at g=kappa but rather proves that the Kerr grows as g^4 in the low coupling regime. I see no reason why the Kerr rate would not continuously increase when g is increased.

C2.6 I do not believe that system complexity or nonlinearity has anything to do with the basic thermodynamics that predict a constant noise. If at zero temperature, with no input signal, the noise rises above the vacuum noise, it would be a violation of the second principle of thermodynamics. This would create infinite energy that could be harvested by a heat engine! Equation S.47 makes such unphysical prediction. Setting the temperature and input amplitude epsilon to zero, the noise can be made arbitrarily high by increasing kappa_c1 or even reducing the loss chi of the resonator.
The calculation is thus obviously wrong. The mistake occurs in S.36: the total relaxation rate of the two supermodes is chi but the noise has an extra term proportional to kappa_c1.

P2.2 The new fits are tremendous improvements over the previous ones! I still have a few more questions. How did you measure the background used for background subtraction? The curves you show in the attached figure do not match the curves in the figure 2 of the paper. In particular, b_0 seems to correspond to subfigure d of the figure 2 but it looks quite different. Is it just the background removal? Could you also show fits in the more difficult strong coupling regime?
I also think that the curves in figure P2.2 should be included in the main article or supplementary material as correct fitting of the measured response is a crucial aspect of the paper.

P2.3 This is a reasonable explanation but I would suggest normalizing the results to avoid confusing readers unaware of this very common experimental difficulty.

P2.4 I find the solid lines in figure 4 to be quite misleading despite the mention in the caption. The choice of a discontinuity of the derivative at g=kappa tries to highlight the exceptional point in a very handed manner. I believe that the only acceptable lines would be a fit to a theoretical model.
P2.5 I strongly disagree with the authors on this point. Apart from the basic thermodynamical argument (see above), none of the references show this fundamental quantum noise rise at zero temperature. For example:

- Kononchuk, R., Cai, J., Ellis, F., Thevamaran, R. & Kottos, T. Exceptional-point-based accelerometers with enhanced signal-to-noise ratio. Nature 607, 697–702 (2022): This experiment is not quantum limited and the noise is thermal and technical
- Wang, H., Lai, Y.-H., Yuan, Z., Suh, M.-G. & Vahala, K. Petermann-factor sensitivity limit near an exceptional point in a Brillouin ring laser gyroscope. Nat Commun 11, 1610 (2020): The noise enhancement is fundamentally due to the laser cavity used to generate the probe signal. In essence, it is the gain of the laser cavity that is the source of the noise (see Grangier, P., Poizat, JP. A simple quantum picture for the Petermann excess noise factor. Eur. Phys. J. D 1, 97–104 (1998). https://doi.org/10.1007/s100530050069 for more details).

At finite temperature, there can be a variation of the noise due to the temperature difference between the drive and resonators.

There is also strong theoretical evidence that there is no SNR improvement near exceptional points when the noise is exclusively quantum in nature Lau, HK., Clerk, A.A. Fundamental limits and non-reciprocal approaches in non-Hermitian quantum sensing. Nat Commun 9, 4320 (2018). https://doi.org/10.1038/s41467-018-06477-7

P2.6 I am confused by the reference Lu et al., PRX Quantum 3, 020305 (2022). This reference discusses isolators becoming transparent at very high frequencies and the implications on thermalization and heating but not really some issue with circulators at low power. Circulators are somewhat lossy (but so is this system) but work at the typical frequencies down to single photons. For example, qubit readout is typically performed with <10 photons and circulators are widely used.

P2.16 As noted earlier, I don't believe that this has been addressed and leads to unphysical conclusions. While the noise is now balanced with the dissipation, there is an extra drive term at a rate kappa_c1. This also means that the homodyne current is not even a stochastic property! There are probably multiple mistakes because even when setting kappa_c1 to zero (to get a correct quantum Langevin equation), equation S47 predicts 0 noise instead of a contribution caused by the vacuum noise. For example, the measurement used is heterodyne and not homodyne which add a factor sqrt(2) to the noise (see "Quantum Measurement and Control" by Wiseman and Milburn) and the noise term in the measurement result was not included in this revision of the supplementary material (though it was in the previous).

Version 2:

Reviewer comments:

Reviewer #2

(Remarks to the Author)
See attached review.docx (needed for the embedded equations)

Version 3:

Reviewer comments:

Reviewer #2

(Remarks to the Author)
I would like to thank the authors once again for their detailed response.

I am afraid that I still disagree with the authors on section IV. of the supplementary material which underpins the 3rd experiment presented in this article in Figure 4.

My arguments are still similar. Here is a point-by-point response.

C4.2.
I unfortunately do not have the time to review close to 8 pages of stochastic calculus calculations but I don't think such detailed and novel calculations are required to analyze what is essentially a single driven cavity given the large existing literature on the subject.
Furthermore, S16 does not qualitatively agree with the predictions shown in Figure 4 (see for example the reflections SNR which is maximized at g = 2kappa whereas the other model predicts a maximum at g = 0) even though averaging the results of the stochastic master equation should yield identical results to the Lindblad master equation.
I suspect that the problem is that the noise is calculated on the Fourier spectrum S and not on the real variables. The Fourier transform of the noise is not the noise of the Fourier transform because the variance is a nonlinear operation which doesn't commute with the Fourier transform.
Overall, I do not the addition of section IV. C is necessary for the reader.

C4.4
The example cited by the authors is quite different. In "Introduction to quantum noise, measurement, and amplification", the case of a "hot" mechanical oscillator parametrically coupled to a measurement cavity is discussed and indeed the noise exhibits a Lorentzian shape caused by the thermal noise of the oscillator.
This is not at all a transient process and is a completely steady state phenomenon.
C4.5

The authors discuss a driven cavity and plot the FFT of the time evolution of one of the quadratures. This FFT represents the signal and not the noise!
Note that a heterodyne measurement should be used and not a homodyne one. A strong continuous homodyne measurement will squeeze the state which does not correspond to the experiment where a phase insensitive amplifier is used and one quadrature later discarded.
The transient gives some spectral width to the signal but does not change the fact that the variance of X is still a time constant. The authors could calculate the variance of X to convince themselves of this fact.
A nice way to see this is to realize that the Hamiltonian is quadratic and the loss linear so the systems remain in a Gaussian state at all times. A drive corresponds to a simple displacement of the state in phase space and does not affect the covariance matrix of the state (see "Gaussian quantum information" by Weedbrook et al.)
Moreover, given the lack of mention of the measurement time, I (and presumably potential readers) have assumed that the problem should be solved in the steady state. The vast majority of qubit measurements are done in the steady state. If transients were such a major factor, they should have been mentioned in the main text and on the figure.

C4.7
The measurement is quantum non-demolition and as such should not have any classical back-action on the system. In practical terms, this is saying that the amplifier can be perfectly isolated from the cavity which is close to true.

Overall, it seems clear that the authors and I fundamentally disagree on this point and I am not sure if more back and forth will ever solve this disagreement.

I think that amputated of the last Figure and the corresponding supplementary material, this article is worth publishing in Nature Communications. In my opinion, leaving this debatable last part of the article muddles the message of an otherwise interesting set of experiments.

Respectfully,

Response to the comments of Reviewer #1

We thank the Reviewer for acknowledging the quality of our work, finding our results and the manuscript suitable for *Nature communications*, and for valuable suggestions regarding the revision of the manuscript. Below we provide our replies to the comments by the Reviewer, including also the changes implemented in the revised manuscript.

Reviewer Comment C1.1 *"The question that the authors raise - the interplay between small photon number and non-reciprocal behavior in the vicinity of an exceptional point, including the role of EP noise in the quantum domain - is extremely interesting and relevant. Recent progress on EP realizations in non-photonic open quantum systems makes it even more so, since in those systems, EPs are hard to access."*

Reply: We thank the reviewer for pointing out the importance of the problems of few-photon non-reciprocal behaviors and the quantum noise in the vicinity of EPs.

Reviewer Comment C1.2 *"In this manuscript, by using a highly dissipative cavity coupled to a high-Q cavity, with tunable interaction between them mediated by a superconducting qubit, and two auxiliary qubits, the authors study the scattering matrix for different mean-photon numbers. They characterize the asymmetric transport (isolation) and show that in the weak-coupling limit, it shows clear asymmetries that are absent in the strong-coupling limit. Normalized reflection studies, that go from one, quadratic dip to a quartic one at the EP, to two dips allow identification of the effective, lossy, non-Hermitian Hamiltonian that governs the dynamics of modes in the two coplanar-waveguide cavities. Further observations are presented showing that although the noise is enhanced near the EP, the SNR is reduced for reflection measurements while enhanced for transmission."*

Reply: We thank the reviewers for summarizing our work and pointing out the significance of our work.

Reviewer Comment C1.3 *"The manuscript is fairly well written, with significant details provided in the supplementary material that were helpful with understanding."*

Reply: We thank the reviewer for his/her careful reading and very positive evaluation.

Reviewer Comment C1.4 *"I find the choice of some of the parameter labels mysterious -- such as "att" fominr the baseline of the exponential function used to characterize mean photon number in the cavity -- but such instances and typos are minimal."*

Reply: We thank the reviewer for pointing out this issue. We have revised the labels in the updated version.

Reviewer Comment C1.5 *"I find the results in the paper very interesting and timely. To my* *knowledge this is the first example of studies near EP in the low-photon number domain, although multi-photon interferences have been "simulated" using lossy or unitary devices (Klauck et al. Nature Photonics 13, 883 (2019); N. Maraviglia et al, Phys. Rev. Research 4, 013051 (2022)). I therefore think that the manuscript is suitable for Nature Communications."*

Reply: We thank the reviewer for finding that our results in the paper are *"very interesting and timely…"* and for the recommendation of our paper for *Nature Communications*. We have also added those two references listed by the referee in the revised version.

Response to the comments of Reviewer #2

We thank the reviewer for carefully reading the paper and giving valuable suggestions. According to the suggestions, in the updated manuscript, we have updated Figure 2 and Figure 4 in the main text, and Figure S13 about the noise effects and SNRs around the exceptional points in the supplementary materials. Below we provide our replies to the comments by the reviewer including also the changes implemented in the revised manuscript.

Reply for the overall comments

Reviewer Comment C2.1 *"Summary of the key results*
The article "Experimental realization of on-chip few-photon control around exceptional points" by Pengtao Song et al. presents a couple of experiments realized with a system exhibiting exceptional points. The system is the well-known passive PT-dimer implemented here using a couple of superconducting resonators coupled with a tunable coupler. The authors observe the predicted exceptional point by measuring the complex frequencies of the two modes as a function of the coupling.

The authors then proceed to measure the non-reciprocal transmission through the system caused by the Kerr nonlinearity, which occurs at a few photons level. Finally, they measure the signal-to-noise ratio of a signal transmitted through the system or reflected. They find that the reflected SNR is minimized at the exceptional point while the transmitted SNR is maximized at the same point.

Reply: We thank the reviewer for the above comments.

Reviewer Comment C2.2 *"Originality and significance*
Exceptional points are a classical phenomenon that has been demonstrated in many systems including electronic oscillators, optical cavities and superconducting resonators. The non-Hermitian nature of the dynamics is restricted to the average of the fields but the extension of this concept to systems in the quantum limit is the subject of ongoing research. For example, one of the main questions of the field is whether or not the increase in sensitivity to the order parameter at the exceptional point also comes with a quantum noise increase which would reduce or cancel the sensing advantage of such systems."

Reply: We thank the reviewer for pointing out the significance of the subjects considered in our paper.

Reviewer Comment C2.3 **"***Overall, I found the connection between the three different experiments quite difficult to understand. In the second experiment, the authors measure the non-reciprocal transmission through the two coupled resonators but this phenomenon doesn't* *have a systematic connection to exceptional points (or quantum mechanics), it is simply a consequence of the nonlinearity introduced to tune this particular system. Similar physics is at play in the hysteretic frequency response of a Duffing oscillator, the classical equivalent of a single non-linear resonator."*

Reply: We study two phenomena around exceptional points in few-photon regime, i.e., the unidirectional wave transmission and enhanced sensing. For the second experiment, the experimental demonstration of non-reciprocity is a very common method to verify the properties of the non-Hermitian system, especially in the vicinity of the exceptional points (EPs). See, e.g. Peng, B., Ozdemir, S. K., Lei, F. C., Monifi, F., Gianfreda, M., Long, G. L., Fan, S. H., Nori, F., Bender, C. M. & Yang, L. Parity-time-symmetric whispering-gallery microcavities. Nat. Phys. 10, 394-398 (2014); Feng, L. et al. Nonreciprocal Light Propagation in a Silicon Photonic Circuit. Science 333, 729–733 (2011). We agree that the non-reciprocity transmission is due to the nonlinearity. However, the amplification of the nonlinear effects in the vicinity of the EPs is crucial, in order to demonstrate non-reciprocity in such systems. Additionally, whether the EP-enhanced nonlinear effects can still guarantee the non-reciprocity transmission of photon in the few-photon regime is still a problem left open due to the effects of quantum fluctuations and thermal noises.

Reviewer Comment C2.4 *"The large nonlinearity is indeed quite impressive but this is very common in such superconducting circuits, which have been shown to reach extremely high light-matter couplings."*

Reply: Superconducting circuits are a good platform to study quantum optics on a chip, but it has never being used (before our work) to study its non-Hermitian physics. In our system, the non-reciprocity transmission is switchable. The non-reciprocity transmission only occurs in the weak-coupling regime near the EP, rather than in the strong-coupling regime. The nonlinearity in the cavities resulting from the interactions with qubits is weak, and this weak nonlinearity cannot satisfy the requirements for the non-reciprocal transmission. Therefore, we achieve non-reciprocal transmission by enhancing the nonlinearity in the vicinity of the EP.

Reviewer Comment C2.5 *"I had even more trouble understanding the third experiment regarding sensing using this system. This experiment purports to help understand the usefulness of EPs for sensing application but the EPs are not used in the experiment. Indeed, EPs have been proposed to sense small variations of the "order parameter" g/κ when close to the EP but here the experiment keeps g/κ constant and simply looks at the transmission through the system."*

Reply: We have now improved the explanation mentioned above. In our manuscript, g/κ only indicates the normalized coupling strengths between the two cavity-modes and thus distinguishes different regimes (strong- or weak-coupling) of the system rather than an index for sensing the small variations in some other papers in, e.g., Wiersig, J. Enhancing the Sensitivity of Frequency and Energy Splitting Detection by Using Exceptional Points: Application to Microcavity Sensors for Single-Particle Detection. Phys. Rev. Lett. 112, 203901 (2014); Chen, W., Kaya Özdemir, Ş., Zhao, G., Wiersig, J. & Yang, L. Exceptional points enhance sensing in an optical microcavity. Nature 548, 192–196 (2017). For example, if $g < \kappa$, i.e., $g / \kappa < 1$, the system is in the weaking-coupling regime; if $g = k$, i.e., $g / \kappa = 1$, the system is at the EP; if $g > k$, i.e., $g / \kappa > 1$, the system is in the strong-coupling regime.

Reviewer Comment C2.6 *"The system is a completely passive system close to zero temperature and thus any variation of the noise is hard to understand. Indeed, in such system, if the input is a coherent state, then the output should also be a coherent state with the same quadrature variance 1/4 (but with possible less amplitude and a phase shift). Despite this, the authors predict large variations of noise in the zero temperature limit (see below for remarks on the supplementary material).*
Extra noise is indeed expected around the EP of a PT-symmetric system because of the gain required to implement such systems but this is not the case here as the system studied here is not PT-symmetric."

Reply: As the reviewer wrote, it is true that the gain of PT-symmetric systems will introduce extra noise. In our experiments, the passive system is relatively complicated, including nonlinearity. Therefore, it might not be necessary that the coherent input may lead to coherent output and the quadrature variances of the input and output states are the same.

Reply for the detailed points

Reviewer Point P2.1 *"Figure 1: e) and f) do not seem to be Lorentzian shaped as they feature a significant asymmetry. Why is this the case? These two figures would also be ideal to show how the complex frequency is extracted from the spectrum by showing fits for example."*

Reply: We thank the reviewer for pointing this out. Yes, indeed, the reflection spectra shown in Figure 1: e) and f) are not perfect Lorentzian shapes. The imperfect Lorentzian shape of a transmission line resonator is often observed in superconducting quantum circuits, eg., Grabovskij et al., In situ measurement of the permittivity of helium using microwave NbN Resonators. Applied Physics Letters 93, 134102 (2008); Khalil et al., Loss Dependence on Geometry and Applied Power in Superconducting Coplanar Resonators IEEE Trans. Appl. Supercond. 21, 879 (2011); Megrant et al., Planar superconducting resonators with internal quality factors above one million. Applied Physics Letters 100, 113510 (2012).

We think that there are two possible reasons for this to also occur in our work. Firstly, the reflection background of the necessary components of the microwave circuits may come from the impedance mismatch (for example, wire-bond connections), imperfect connections, transmission line geometry, and returning loss of the microwave wave components (cables, filters, circulators, etc.), which leads to the deformations of the Lorentzian shapes of the output spectra. Secondly, in our experiments, the two cavities are tuned to be resonant with two auxiliary qubits (working as cavity frequency shifters) in the dispersive regime. Therefore, the auxiliary qubits will introduce nonlinearity which will be amplified by the EP and affect the shapes of the cavity spectra. We will show some Lorentzian fits in the reply to Point P2.2.

Reviewer Point P2.2 *“Figure 2: b) The imaginary parts of the frequencies do not seem to match the theoretical prediction at all. They vary almost linearly with the coupling strength g/κ instead of as sqrt(g^2 – κ^2). Why is that?*
c), d) and e) Once again the reflection spectra do not seem perfectly Lorentzian and showing the fits used for the parameters extraction would be useful.”

Reply: We thank the reviewer for pointing this out. For the small mismatched imaginary parts, they can be improved by extracting the reflection background and refitting the data. It looks better now (see Figure P2.2).

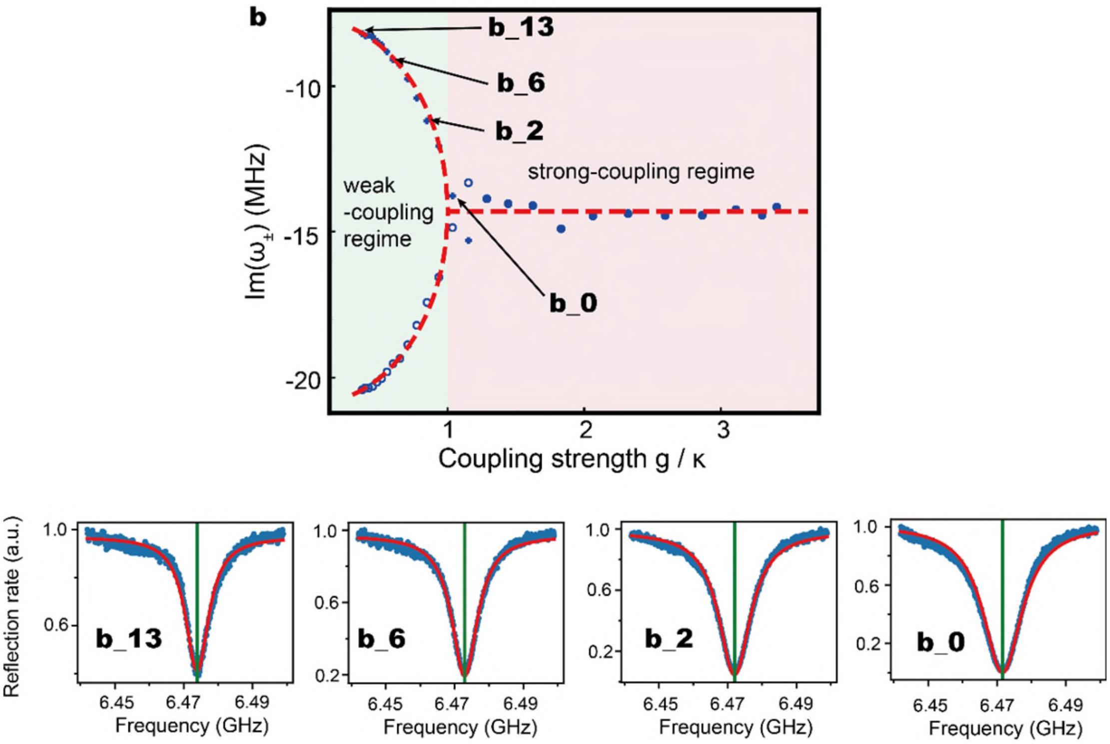

Figure P2.2. Curve fitting for extracting the imaginary part the supermodes. b, the imaginary part of the supermodes (decay rate). The blue circles and crosses represent the experimental data and the dashed curves represent the theoretical curves. **b_13** to **b_0**, The reflection spectra of the left cavity with one-dip Lorentzian fitting, where the blue dots are experimental data, red curves are the Lorentzian fittings and the green curves are the positions of the calculated local minimum points.

Reviewer Point P2.3 *“Figure 3: c) The reflection spectra of this non-linear system might be hysteretic so it would be nice to mention the sweep direction in the caption or the figure.*
c) d) The units are unclear to me, how could the transmission be above 0dB?”

Reply: We thank the reviewer for this suggestion. The sweep direction is from low frequency to high frequency, from forward to backward, and from low power to high power. We have added this information in the caption of Fig. 3.

As shown in the measurement setup in Fig. S7 in the supplementary materials, there are three amplifiers in the measurement circuit (one HEMT at the 4K stage, and two room-temperature amplifiers). The total gain from the three amplifiers is about 100 dB, and that is why the transmission rate can be above 0 dB.

Reviewer Point P2.4 *"Figure 4: Clouds of points are difficult to understand because of their overlap. A 2D histogram would be nicer in my opinion.*
c) d) e) f): What are the solid lines? They seem quite different from the theoretical curves of the supplementary material."

Reply: We thank the reviewer for this suggestion. As suggested, we have added the 2D histograms in the updated Figure 4.

In Figure 4. c,d,e and f, the solid lines are the polynomial fitting curves and we have now added the explanations of the polynomial fitting curves in the caption. We improve the fitting by taking the EPs into account in the revised version. We simulate the noise and SNR with nonlinearities in the two cavities by following the reviewer's suggestions (see the updated supplementary materials) and find that the noise peak will become flat with the increase of the nonlinearities. Therefore, we conclude that the deviation might originate from the nonlinearities of the two cavities.

Reviewer Point P2.5 *"Lines 54-63: This prediction of signal and noise enhancement is true for PT-symmetric system, which feature gain. It is not true for the passive system presented in the manuscript. While the mean field evolution can be post-processed (not done here) to simulate a PT-symmetric system, it is not clear to me if the same can be done for the noise."*

Reply: The noise enhancement in the passive EP system may come from the extra pumping noise or from the eigenbasis collapse (quantum noise) [Kononchuk, R., Cai, J., Ellis, F., Thevamaran, R. & Kottos, T. Exceptional-point-based accelerometers with enhanced signal-to-noise ratio. Nature 607, 697–702 (2022); Wiersig, J. Prospects and fundamental limits in exceptional point-based sensing. Nat Commun 11, 2454 (2020)]. This noise enhancement around exceptional points in a passive system has also been experimentally demonstrated in, e.g., Wang, H., Lai, Y.-H., Yuan, Z., Suh, M.-G. & Vahala, K. Petermann-factor sensitivity limit near an exceptional point in a Brillouin ring laser gyroscope. Nat Commun 11, 1610 (2020) and Zhang, J. et al. A phonon laser operating at an exceptional point. Nature Photonics 12, 479–484 (2018), and the extra noise comes from the mode non-orthogonality, which occurs when the two system eigenvectors (optical modes) begin to coalesce in the vicinity of the exceptional points [early predictions: Berry, M. V. Mode degeneracies and the Petermann excess-noise factor for unstable lasers. Journal of Modern Optics 50, 63–81 (2003); Wenzel, H., Bandelow, U., Wunsche, H.-J. & Rehberg, J. Mechanisms of fast self pulsations in two-section DFB lasers. IEEE Journal of Quantum Electronics 32, 69–78 (1996)]. The 'signal' we mentioned here means mapped sensing signal. The enhanced sensing (Sagnac effect) around exceptional points has also been demonstrated in such a passive system [Lai, Y.-H., Lu, Y.-K., Suh, M.-G., Yuan, Z. & Vahala, K. Observation of the exceptional-point-enhanced Sagnac effect. Nature 576, 65–69 (2019)].

Reviewer Point P2.6 *"Lines 65-72: A mention should be made of traditional RF circulators which work regardless of the input power at the cost of some small amount of loss and a bulky magnet. Some active circulators have also been made using Josephson junctions (e.g. Abdo, B., Bronn, N.T., Jinka, O. et al. Active protection of a superconducting qubit with an interferometric Josephson isolator. Nat Commun 10, 3154 (2019)) which also feature operation down to arbitrarily low power and very low losses. "*

Reply: We thank the reviewer for the above suggestions. It is true that the traditional RF circulators work well in the classical regime. However, it was only recently that people found that the traditional RF circulators *cannot work properly* when the signal is around the single microwave photon level [for example, Lu et al., PRX Quantum 3, 020305 (2022)]. The reference mentioned by the referee already cited as Ref.[39] in our old version of main text.

Reviewer Point P2.7 *"Line 74: Once again, non-reciprocity at the single-photon level is routinely achieved in optics using a magnetic field in a Faraday isolator, without using any nonlinearity."*

Reply: The Faraday effect can induce non-reciprocal effects, but in our non-Hermitian system, non-linearity must be present to produce non-reciprocity transmission. Thus, we revise our description from "Demonstrating nonreciprocity in the few-photon regime requires strong nonlinearity, which is quite difficult to achieve in optical systems." to "Demonstrating nonreciprocity in non-Hermitian systems in the few-photon regime requires strong nonlinearity, which is quite difficult to achieve in optical systems."

Reviewer Point P2.8 *"Line 84: While research is still ongoing, quite a few tunable switches have been demonstrated and the challenge is much more often to increase the power handling, bandwidth and ease of operation and not to reach operation at the single photon level. Here are a few examples among many others*

O. Naaman, M. O. Abutaleb, C. Kirby, M. Rennie; On-chip Josephson junction microwave switch. Appl. Phys. Lett. 14 March 2016; 108 (11): 112601

M. Pechal, J.-C. Besse, M. Mondal, M. Oppliger, S. Gasparinetti, and A. Wallraff , Superconducting Switch for Fast On-Chip Routing of Quantum Microwave Fields, Phys. Rev. Applied 6, 024009 – 2016

A. Wagner, L. Ranzani, G. Ribeill, T. A. Ohki; Demonstration of a superconducting nanowire microwave switch. Appl. Phys. Lett. 21 October 2019; 115 (17): 172602"

Reply: Increasing power handling may indeed represent an important research direction, but moving in the direction of low power, especially at single-photon level, is also important and plays a crucial role in quantum networks and quantum sensing. Even the second reference (M. Pechal, J.-C. Besse, M. Mondal, M. Oppliger, S. Gasparinetti, and A. Wallraff, Superconducting Switch for Fast On-Chip Routing of Quantum Microwave Fields, Phys. Rev. Applied 6, 024009 – 2016) listed by the reviewer demonstrated the single microwave photon switch with a type of single microwave photon source.

We revised our description from "However, the control of the transmission of single microwave photon on-chip, which is crucial for quantum networks, is a problem yet to be solved." to "The control of the transmission of single microwave photon on-chip is still an ongoing research, which is one of crucial problems for quantum networks." We have also added those three references listed by the referee in the revised version.

Reviewer Point P2.9 *"Line 124: The expression of the coupling g does not seem consistent with the one provided in the supplementary information and does not depend on the resonators frequencies. Neglecting the qubit relaxation, the coupling should be (g_a g_b)/Δ, with Δ the detuning between qubit and resonator (see Blais, et al. Phys. Rev. A 75, 032329 (2016)) so I am inclined to think that the expression S.15 is the correct one."*

Reply: We thank the reviewer for pointing out this typo. As pointed out by the reviewer, the expression in Eq. S.15 in the supplementary materials is the correct one and we have corrected the typos in the updated version of the main text and supplementary materials.

Reviewer Point P2.10 *"Line 171: How is the power uncertainty computed? It seems a bit pessimistic."*

Reply: Please find the details about power uncertainty calibration in the part of 1.C 'Calibration of the photon numbers inside the cavities' in supplementary materials. The power uncertainty is computed by , where is the noise temperature of the preamplifier, is the calibrated effective noise temperature of output channel from noise spectral density of the preamplifier [see Astafiev, O. et al. Resonance Fluorescence of a Single Artificial Atom. Science 327, 840–843 (2010); also, Peng et al., Correlated Emission Lasing in Harmonic Oscillators Coupled via a Single Three-Level Artificial Atom. Phys. Rev. Lett. 115, 223603(2015)]. In our experiments, is about 3.6 K, is about 7.5 K, so the power uncertainty is about 3 dB. We suppose that the factor of 3 dB is a good enough accuracy because it does not affect the fact that the system works in a few-photon regime.

Reviewer Point P2.11 *"Line 179: Which non linearity is responsible for the non-reciprocity? Presumably the fourth-order (self Kerr) but this is not mentioned in the main text."*

Reply: We thank the reviewer for pointing this out. The four-order nonlinearity ($a^\dagger a a^\dagger a + b^\dagger b b^\dagger b$) as described in equation S.14 in the supplementary materials, which is amplified in the vicinity of the exceptional point, is responsible for the non-reciprocal effects. We have added this information in the updated version.

Reviewer Point P2.12 *"Line 224: The EP concept is fundamentally a linear (and classical) property. The nonlinearity introduced here seems orthogonal to the problem of EP sensing. Furthermore, the parameter sensed here is not g/κ for which sensing is predicted to be improved."*

Reply: We thank the reviewer for pointing this out. We agree that the EP concept is a linear property, however the nonlinearity in the system will also affect the behaviors of the system in the vicinity of EP. Therefore, we simulate how the nonlinearity affect the signal and noise in our system and the results are presented in the updated version of supplementary materials. As shown in Fig. S13, we simulate the system in three cases, including no nonlinearity (Kerr strength), weak nonlinearity ( ), and stronger nonlinearity ( ), respectively. With increasing the nonlinearity, the SNR and noise of the system are indeed enhanced in the vicinity of the EP by the transmission measurement, which indicates that the nonlinearity does affect the EP sensing.

Our experiments are designed for resolving two coherent states, so the signal discrimination and the noise effects are defined as the separation and the variances of the two output coherent states, which is different to the linear EP-enhancement sensing experiments (see, e.g., our experiment in Chen, W., Kaya Özdemir, Ş., Zhao, G., Wiersig, J. & Yang, L. Exceptional points enhance sensing in an optical microcavity. Nature 548, 192–196 (2017) which uses EP system for particle sensing). As our reply to "**Reviewer Comment C2.5**", in our experiments, g/κ indicates the normalized coupling strengths between the two cavity-modes and thus distinguishes different (strong- or weak-coupling) regimes of the system.

Reviewer Point P2.13 *"Line 209: What is ω? How is it chosen?"*

Reply: We thank the reviewer for the above comment. ω is the frequency of the input driving field. To avoid misunderstandings, we change ω to in the updated version of the main text and the supplementary materials.

Reviewer Point P2.14 *"S.10: What is this parameter Λ? How can it be computed?"*

Reply: We thank the reviewer for pointing this out. We feel very sorry for forgetting to give the specific expression of Λ, which should be . Here are the specific calculation details. From Eqs. S6-S8, we can obtain the dynamical equation of as

using the relation . Then we defined and we obtain Eq. S10 as

To avoid misunderstandings, we delete the symbol Λ and use directly in Eq. S10.

Reviewer Point P2.15 *"S.15: The Kerr coefficient formula is not homogeneous (it has dimension of a frequency squared and not a frequency)."*

Reply: We thank the reviewer for pointing this out. Sorry for this typo. From Eq. S12 and Eq. S13, the Kerr coefficient formula should be . We have corrected it in the updated version of the supplementary materials.

Reviewer Point P2.16 *"S36: This equation is incorrect. Fundamentally, it does not obey the dissipation-fluctuation theorem. The dissipation is only 2χ (for g > κ) while the fluctuations are 2χ + κ_{c1}/2. This dynamical equation should be properly derived from a Hamiltonian and a set of jump operators using the input-output theory but the error here is probably the sqrt(2χ) which should be replaced by sqrt(2χ - κ_{c1}/2). This unfortunately leads to conclusion that the noise depends on the system parameters instead of being constant as expected for a passive system."*

Reply: We thank the reviewer for pointing this out. We have revised the equation to obey the dissipation-fluctuation theorem in the updated version of the supplementary materials. We deeply apologize for the typos, all fixed now.

LIST OF CHANGES:

Below we give a brief summary of changes made in the manuscript to address the comments of the Reviewers. The changes are labeled with red color in the revised manuscript and in our responses to the Reviewers' comments.

Main text:

1. We added some references (Ref. [25], [28],[29],[46-48]) in the revised main text according to suggestions from the two referees.[***Reviewer # 1, Point C1.5; Reviewer # 2,*** **P2.8**] (*Line 49, 51 and 78 of the revised manuscript*)
2. We revised our expressions as suggested from “However, the control of the transmission of single microwave photon on-chip, which is crucial for quantum networks, is a problem yet to be solved.” to “The control of the transmission of single microwave photon on-chip is still an ongoing research, which is one of crucial problems for quantum networks.” ***[Reviewer # 2, Point P2.8]*** *(Line 83-35 of the revised manuscript)*
3. We improved Figure 2 for the imaginary part as suggested. ***[Reviewer # 2, Point P2.2]*** *(Figure 2 of the revised manuscript)*
4. We added the sweep direction in the caption of Figure 3. ***[Reviewer # 2, Point P2.3]*** *(Line 203-205 of the revised manuscript)*
5. We updated Figure 4 and the corresponding expressions in the main text as suggested. ***[Reviewer # 2, Point P2.4]*** *(Figure 4 of the revised manuscript, Line 235-274 of the revised manuscript)*

Supplementary materials:

6. We changed the symbol from *att* to in the supplement. ***[Reviewer # 1, Point P1.2]*** *(Line 116-138 of the revised supplementary materials)*

7. We changed the symbol for the frequency of the driving field from ω to . ***[Reviewer # 2, Point P2.13]*** (*Line 209-213 and Line 248 of the revised supplementary materials*)

8. We replaced as . ***[Reviewer # 2, Point P2.14]*** (*Line 229-233 of the revised supplementary materials*).

9. We revised the expression of in Eq. (S.15). ***[Reviewer # 2, Point P2.15]*** (*Line 246 of the revised supplementary materials*).

10. We revised Eq. (S.36) and Eq. (S.50) and the corresponding explanations for these two equations. ***[Reviewer # 2, Point P2.16]*** (*Line 490 - 532 of the revised supplementary materials*).

11. We updated Figure S13 with some new simulations and the corresponding explanations as suggested. ***[Reviewer # 2, Point P2.12 and Point P2.16]*** (*Line 554-561 of the revised supplementary materials*).

12. There are some typos are corrected.

Responses to the comments of Reviewer 2

We thank the reviewer for the concerns and valuable suggestions. According to the suggestions, in the updated manuscript, we have updated Figure 3, Figure 4 in the main text, and Figure S10, Figure 15, and relevant equations in the supplementary materials. Below we provide our replies to the comments raised by the reviewer including also the changes implemented in the revised manuscript.

Reviewer comment C3.0 "*I would like to thank the authors for their detailed response and their numerous updates to the manuscript. Unfortunately, I still have some questions and disagreements and I cannot recommend publication as is.*

Here is a detailed response to some of the points:"

Reply: We thank the reviewer for his/her time to carefully read our paper and give valuable suggestions.

Reviewer comment C3.1 "*C2.3 I think the author explanation is quite convincing though I still disagree with the non-linearity enhancement near the EP. Non-reciprocity is maximized at small g.*"

Reply: We thank the reviewer for thinking that our explanation is convincing. Indeed, the non-reciprocal effects are maximized not at EP, but in the weak-coupling regime (similar experimental results can be found in other papers, see, e.g., Peng, Bo, et al. Parity–time-symmetric whispering-gallery microcavities. Nature Physics 10.5 (2014): 394-398.). Despite the nonlinear effects, the non-reciprocity phenomenon is also related to the field distribution in EP system. In the weak-coupling regime, the field with strong nonlinear effects is mainly localized in the high-Q cavity and the distribution of the field is very low in the low-Q cavity, which leads to the non-reciprocal transmission of photon. In the strong-coupling regime and in the vicinity of EP, the field distributes almost equally in the two cavities, thus we cannot observe the non-reciprocal transmission of photon. Even though the non-linearity is enhanced near EP (see equation S.34 in supplementary), but the field is not highly localized, thus non-reciprocal effects are not maximized near EP.

Reviewer comment C3.2 "*C2.4 Superconducting circuits have been used to study non-Hermitian physics in the past: - There is a series of papers written in the group of Kater Murch where he uses a qubit and (sometimes) post-selection to engineer non-Hermitian Hamiltonians (Chen et al., Phys. Rev. Lett. 127, 140504, 2021; Erdamar et al, arXiv:2309.12393, 2023; Gaikwad et al., Phys. Rev. Research 5, L042024, 2023 and a few more)*
- Mikko Mottonen's group has also published one paper where they use tunable losses to explore non Hermitian physics (Teixeira et al., Phys. Rev. Research 5, 033119, 2023)"

Reply: We thank the reviewer for pointing out these works in superconducting circuits. We have cited these papers in the updated version of manuscript. Different from the previous papers, the main contribution of this paper is to show the non-reciprocity effects, the enhancement of noise, the SNR of quantum measurements in the few-photon regime in non-Hermitian systems which has never been systematically discussed in the previous work.

Reviewer comment C3.3 *"The nonlinearity coming from the qubit is all but weak since it is the only source of nonlinearity, which occurs at a few photon levels as you point out (though to be fair, the two other ancillary qubits might contribute as well). The non-reciprocity is maximized very far from the EP where g/kappa is very small and I cannot see anything in the data that would suggest an enhancement near the EP."*

Reply: We thank the reviewer for the above comments. As we replied in comment C 3.1, the maximal non-reciprocity occurs when the field is highly localized in the weak-coupling regime.

Reviewer comment C3.4 *"Section B of the supplementary material attempts to show that the Kerr of the high-Q resonator is amplified near the EP but it also doesn't predict a local maximum at g=kappa but rather proves that the Kerr grows as g^4 in the low coupling regime. I see no reason why the Kerr rate would not continuously increase when g is increased."*

Reply: The equation S.34 in the updated supplement shows that

$$\Box_{K} \quad \Box$$

The parameter *g* also appears in the denominator of the Kerr rate. When the system approaches the EP, the denominator of the Kerr rate decreases very rapidly, and thus the whole Kerr rate increases rapidly when the system approaches the EP.

Reviewer comment C3.5 *"C2.6 I do not believe that system complexity or nonlinearity has anything to do with the basic thermodynamics that predict a constant noise. If at zero temperature, with no input signal, the noise rises above the vacuum noise, it would be a violation of the second principle of thermodynamics. This would create infinite energy that could be harvested by a heat engine!*
Equation S.47 makes such unphysical prediction. Setting the temperature and input amplitude epsilon to zero, the noise can be made arbitrarily high by increasing kappa_c1 or even reducing the loss chi of the resonator.
The calculation is thus obviously wrong. The mistake occurs in S.36: the total relaxation rate of the two supermodes is chi but the noise has an extra term proportional to kappa_c1."

Reply: We thank the reviewer for the concerns. As has been pointed out by the reviewer, our experimental system contains various noise-source channels. The output noise of each channel cannot exceed the vacuum-noise-induced output noise when the temperature tends to zero.

According to the reviewer's comment, we have double-checked our calculations, and find that we ignored the inherent noise induced by the measurement channel in the old version. In the updated version, we have revised the equation previously labeled as Eq. S47 in the old supplement, which is now Eq. S53 in the updated supplement. The new equation shows that

(*)

The first term of this Equation (*) represents the inherent noise in the measurement channel, and the second term is the output noise from the system.

Here χ is defined as $\chi = (\kappa_a + \kappa_b)/4$. $\kappa_a = \kappa_{a0} + \kappa_{c1}$. $\kappa_b = \kappa_{b0} + \kappa_{c2}$. and (see line 222, 525, 530 in the updated supplementary material). Note that χ contains κ_{c1}, so the second term in Eq. (*), i.e., the output noise will not increase infinitely with increasing κ_{c1}.

When , equation (*) can be simplified as

(**)

Note that we normalize the noise strength by the vacuum noise. Therefore, will not become infinite, and cannot exceed the vacuum-noise-induced output noise when .

More interestingly, it can be seen from Eq. (**) that when . In this case, the system is decoupled from the measurement channel, and only the inherent vacuum noise in the measurement channel can be "seen" in the output signal, which matches well with the last concern of the reviewer.

Reviewer comment C3.6 "*P2.2 The new fits are tremendous improvements over the previous ones! I still have a few more questions. How did you measure the background used for background subtraction? The curves you show in the attached figure do not match the curves in the figure 2 of the paper. In particular, b_0 seems to correspond to subfigure d of the figure 2 but it looks quite different. Is it just the background removal? Could you also show fits in the more difficult strong coupling regime?*
I also think that the curves in figure P2.2 should be included in the main article or supplementary material as correct fitting of the measured response is a crucial aspect of the paper."

Reply: We thank the reviewer for the questions and suggestions. The fitting curves we showed in the response of the last round correspond to the left cavity, while the subfigures of figure 2 correspond to the right cavity. We extract the real part of the complex frequency by finding the extreme points of the polynomial fit, and extract the imaginary part by a one-dip or two-dip Lorentzian fit (see more details in section II.E of the updated supplement). For the real part extraction, we utilize the reflection spectra from the right cavity. For the imaginary part extraction, we utilize the reflection spectra from the right cavity in the strong-coupling regime and from the left cavity in the weak-coupling regime. The reason that the reflection spectra from the right cavity do not fit very well with those from the left one may come from the small differences of the experimental parameters and extra noises introduced by the measurement circuits, which cannot be completely eliminated.

We thank the suggestion raised by the reviewer for adding fitting curves in the strong-coupling regime, and we have added the corresponding figures in fig. S10 of the updated supplement.

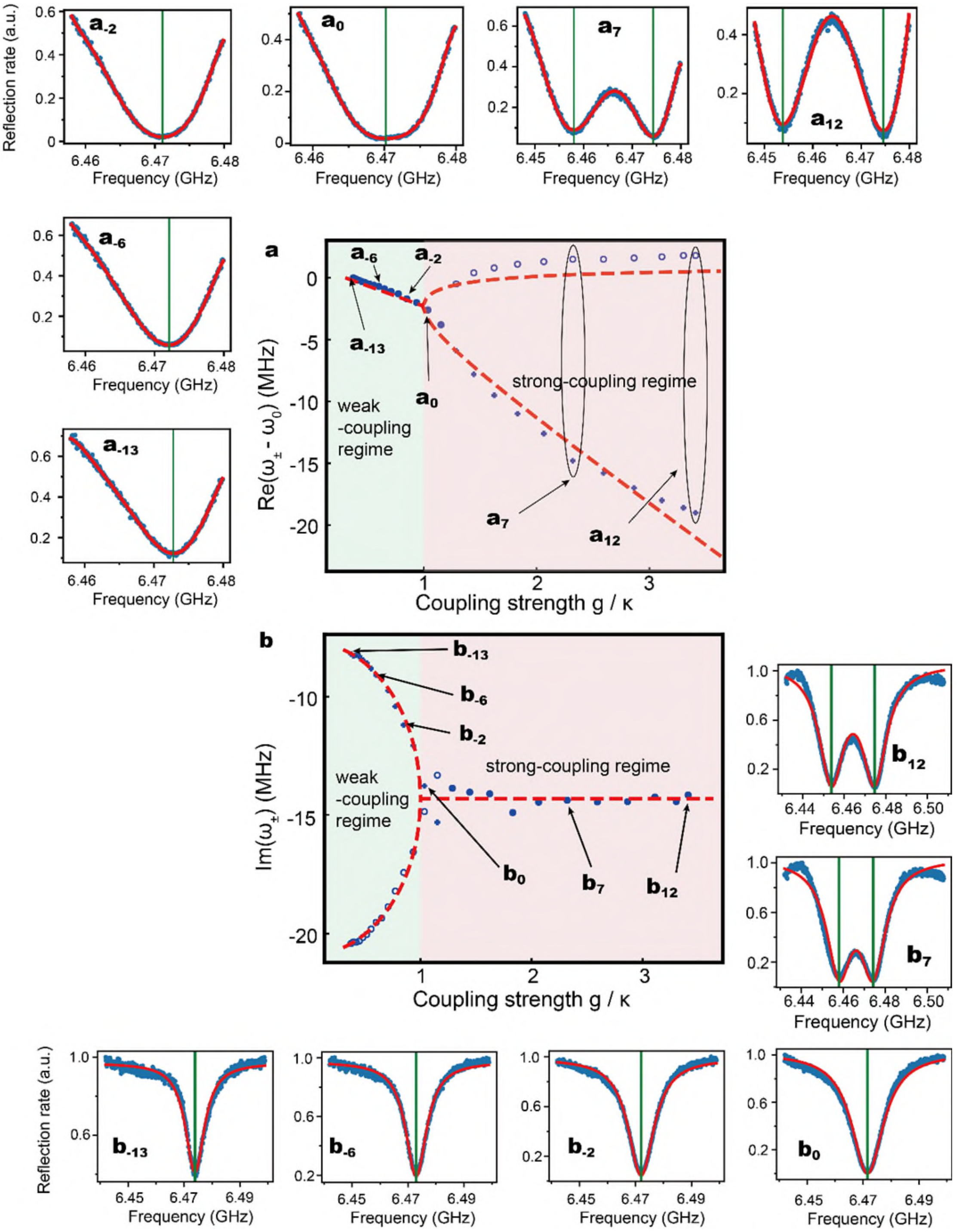

Figure S10. Curve fitting details for extracting the complex frequencies of the supermodes. **a**$_{-13}$ - **a**$_{12}$, Polynomial fittings of the right cavity spectra to extract the real parts of the supermodes. The real parts are gotten from the one or two local minimum points of every polynomial fitting. **b**$_{-13}$ - **b**$_{12}$, The one-dip Lorentzian fittings of the left cavity spectra (in the weak-coupling regime) and the two-dip Lorentzian fittings of the right cavity spectra (in the strong-coupling regime).

Reviewer comment C3.7 “*P2.3 This is a reasonable explanation but I would suggest normalizing the results to avoid confusing readers unaware of this very common experimental difficulty.*”

Reply: We thank the reviewer for this suggestion. We have normalized the results in the updated manuscript.

Reviewer comment C3.8 “*P2.4 I find the solid lines in figure 4 to be quite misleading despite the mention in the caption. The choice of a discontinuity of the derivative at g=kappa tries to highlight the exceptional point in a very handed manner. I believe that the only acceptable lines would be a fit to a theoretical model.*”

Reply: We thank the reviewer for this suggestion. We have fitted the experimental data with the theoretical model in the updated manuscript and the fitting details are shown in Fig. S15 in the updated supplement.

Reviewer comment C3.9 “*P2.5 I strongly disagree with the authors on this point. Apart from the basic thermodynamical argument (see above), none of the references show this fundamental quantum noise rise at zero temperature. For example:*
- Kononchuk, R., Cai, J., Ellis, F., Thevamaran, R. & Kottos, T. Exceptional-point-based accelerometers with enhanced signal-to-noise ratio. Nature 607, 697–702 (2022): This experiment is not quantum limited and the noise is thermal and technical
- Wang, H., Lai, Y.-H., Yuan, Z., Suh, M.-G. & Vahala, K. Petermann-factor sensitivity limit near an exceptional point in a Brillouin ring laser gyroscope. Nat Commun 11, 1610 (2020): The noise enhancement is fundamentally due to the laser cavity used to generate the probe signal. In essence, it is the gain of the laser cavity that is the source of the noise (see Grangier, P., Poizat, JP. A simple quantum picture for the Petermann excess noise factor. Eur. Phys. J. D 1, 97–104 (1998). https://doi.org/10.1007/s100530050069 for more details).
At finite temperature, there can be a variation of the noise due to the temperature difference between the drive and resonators.

There is also strong theoretical evidence that there is no SNR improvement near exceptional points when the noise is exclusively quantum in nature Lau, HK., Clerk, A.A. Fundamental limits and non-reciprocal approaches in non-Hermitian quantum sensing. Nat Commun 9, 4320 (2018). https://doi.org/10.1038/s41467-018-06477-7”

Reply: We thank the reviewer for the concerns. For the thermodynamical argument, as we replied in C3.5, the equation S.53 in the updated supplement shows that

(*)

As we can see in the above equation, is always no more than the vacuum-noise-induced output noise, when we set the external thermal photon $\bar{n}_{th} = 0$.

The mentioned paper, i.e., *Nat. Commun.* 9, 4320 (2018), is mainly considered in the linear regime. Whether the SNR may be improved or not near EP in linear system is still arguable (see, e.g., *Phys. Rev. Lett.* 123, 180501). Despite this argument, there are nonlinear elements in our system, and the investigation of whether the SNR is enhanced near EP in such nonlinear systems still requires further considerations. Our study indicates that the SNR of the non-Hermitian sensing system is enhanced in the transmission channel but suppressed in the reflection channel in the vicinity of the EPs in the case of two coherent states injection.

Reviewer comment C3.10 "*P2.6 I am confused by the reference Lu et al., PRX Quantum 3, 020305 (2022). This reference discusses isolators becoming transparent at very high frequencies and the implications on thermalization and heating but not really some issue with circulators at low power. Circulators are somewhat lossy (but so is this system) but work at the typical frequencies down to single photons. For example, qubit readout is typically performed with <10 photons and circulators are widely used.*"

Reply: We thank the reviewer for the comment. We agree with the reviewer that the traditional RF circulators can work with qubit readout in the low-power regime. These existing RF isolators/circulators are very large and difficult to integrate on-chip. Our device is very small in size and provides a new method for on-chip integrated isolators. Some existing isolators expected to be integrated on-chip are active devices, which require additional pumping tones, thereby may introduce extra noise. Our device provides a possible way to implement an on-chip integrated isolator without the need for additional pumping tones, simplifying circuit design without introducing extra noise.

Reviewer comment C3.11 "*P2.16 As noted earlier, I don't believe that this has been addressed and leads to unphysical conclusions. While the noise is now balanced with the dissipation, there is an extra drive term at a rate kappa_c1. This also means that the homodyne current is not even a stochastic property! There are probably multiple mistakes because even when setting kappa_c1 to zero (to get a correct quantum Langevin equation), equation S47 predicts 0 noise instead of a contribution caused by the vacuum noise. For example, the measurement used is heterodyne and not homodyne which add a factor sqrt(2) to the noise (see "Quantum Measurement and Control" by Wiseman and Milburn) and the noise term in the measurement result was not included in this revision of the supplementary material (though it was in the previous).*"

Reply: We thank the reviewer for pointing out this issue. In our experiment, we extract information via the detection channel with damping rate κ_{c1}. Thus, κ_{c1} can be understood as the detection efficiency of the measurement channel, and remains a constant value determined by the chip design and fabrication. When κ_{c1} is close to 0, it means that we can hardly extract information from the system. In the previous version of our analysis, we include all the noise into , thus the noise caused by the measurement channel has been included in and then cannot been clearly seen. We now give a more specific discussion in which the noise is divided into two parts—input noise from the measurement channel and the effect noise from other channels (see update supplement line 527-536), and we also include the inherent noise in the measurement channel which has been ignored in the old version of our paper (see also our reply to comment C3.5). Then the reflection noise in Eq. (S47) in the old supplement will be updated as

When , is the inherent noise in the measurement channel.

Finally, we want to thank the reviewer again for her/his specific and valuable comments. In responding to these comments, we have gained a deeper understanding of many issues that may still be under debate in various aspects, such as the distribution of noise in EP systems and how EP systems affect the input noise.

Responses to the comments of Reviewer 2

Reviewer comment C4.0 *"I would like to thank the authors for their detailed response and their numerous updates to the manuscript.*
In particular, I appreciate the addition of figure S10 and the associated section dedicated to curve fitting which will help readers to better understand or even reproduce these results. I would also like to thank the authors for the improvements to Figure 4, which now directly compares theory to experiment."

Reply: We thank the reviewer for the valuable suggestions, which have helped us improve the quality of our manuscript.

Reviewer comment C4.1 *"Unfortunately, I still disagree with the theoretical curves of Figure 4 (as does the experimental data to some extent). The calculation of the theoretical curves is performed in section IV. B. of the supplementary material."*

Reply: We thank the reviewer for carefully examining the theory presented in Figure 4. We believe the stochastic master equation model proposed by the reviewer is an excellent suggestion. We have done some theoretical analysis for Figure 4 with stochastic master equation (SME), which is shown in section IV. C. of the supplementary material. We will address the reviewer's concerns in our subsequent replies.

Reviewer comment C4.2 *"My last remaining issue lies with the definition of the homodyne current. The homodyne current is a stochastic quantity that presents some difficulties for calculations. In particular, when dealing with stochastic quantities, one should note that there are two types of averages: average over the quantum state used to compute average values of operators (usually using the <.> notation) as well as a stochastic average used to average over different realization of the experiment i.e. shots."*

Reply: We thank the reviewer for the above comments. As the reviewer mentioned, there are types of averages, one is the average over the input quantum noise, another one is the average over the Wiener noise induced by the post-selection process. In the updated supplementary material, we have considered both of the two noise channels as the reviewer pointed out, and have written down the stochastic master equations (SMEs) after considering the post-selection process. By introducing truncation approximations, the systematic dynamic equations of the first-order and the second-order moments are obtained. The theoretical results thus obtained (Figure S16) is qualitatively consistent with the experimental results.

Reviewer comment C4.3 *"According to Wiseman and Milburn in Measurement and Control, the homodyne current is defined as* $I(t) = \langle a + a^{\dagger}\rangle(t) + \xi(t)$ *with* $\xi(t) = \frac{dW}{dt}$ *and* $W(t)$ *a Wiener process i.e. white noise (note that they use a different convention for x without the* *sqrt(2)). This is quite similar to the definition used by the authors except that the field appears as an average value in the proper definition and not as an operator: that is because the homodyne current is a (stochastic) scalar and not an operator."*

Reply: We thank the reviewer for pointing out the way to write down the homodyne current with Wiener process. As we replied above, we have considered the two types of the averages as the reviewer mentioned and have updated the results by solving the stochastic master equations in section IV. C. of the supplementary material.

Reviewer comment C4.4 *"This small change does not affect equation S.51 (though now the average must be interpreted as a statistical average and not a "quantum" average) but it completely changes S.52.*
Indeed, using $\bar{x}$ to represent the statistical average of the stochastic quantity x, we now have

$$\overline{m^2(t)} = \langle a + a^\dagger \rangle(0)^2\tau^2 + 2\,\overline{\langle a + a^\dagger \rangle(0)\tau W(\tau)} + \overline{W(\tau)^2} = \overline{m(t)}^2 + \tau(1 + 2n_{th}) *$$

Thus, S.53 simply becomes

$$\bar{n} = 2\bar{n}_{th} + 1$$

It should also be noted that given the fact that the detection scheme is phase insensitive, the heterodyne current should be used instead which comes with an additional factor 2 on the vacuum noise but this doesn't change any of the conclusions."

Reply: We thank the reviewer for the above comments. From the given equation $\overline{m^2(t)} = \langle a + a^\dagger \rangle(0)^2\tau^2 + 2\,\overline{\langle a + a^\dagger \rangle(0)\tau W(\tau)} + \overline{W(\tau)^2} = \overline{m(t)}^2 + \tau(1 + 2n_{th})$, we suppose that the reviewer introduced a steady-state approximation, i.e., $\langle a + a^\dagger \rangle(t) \approx \langle a + a^\dagger \rangle(\infty) \approx \langle a + a^\dagger \rangle(0)$. We define $S(\omega)$ as the frequency spectrum density of $m(t)$. In the case of this approximation, we can get that $S(\omega) \approx S(0)$, which means that the output noise spectrum of the system is always a white noise spectrum. We know that traditionally the output noise spectrum of the cavity modes has a Lorentzian shape (see, e.g., Eq. (3.50) in "Introduction to quantum noise, measurement, and amplification," in *Rev. Mod. Phys.* **82**, 1155 (2010)). The output spectrum of the cavity mode has a Lorentzian shape (not a white noise spectrum) due to the transient process, i.e., the process from the initial state to the steady state.

Reviewer comment C4.5 "*The current equation S.53 while revised still predicts that an undriven system thermalized at a constant temperature of 0 has a Lorentzian noise profile instead of a white (quantum) noise!*

This can be used to break thermodynamics by coupling this supermode to another cavity and capturing a steady net energy flux of

$$\frac{\kappa_{c1}}{2}\left(\frac{\chi}{\chi^2 + (\Delta_0 - \beta)^2}\right) > 0$$

to heat up that secondary above 0K without any work!"

Reply: As we replied in comment C4.4, if we consider the transient process, the noise spectrum emitted by a cavity will be in a Lorentzian shape. Let us see as an example of a single cavity case.

The Hamiltonian of single mode cavity under the rotating frame can be written as

with .

The corresponding stochastic master equation can thus be written as

Let us define thus we have

(1)

(2)

(3)

(4)

(5)

From Eqs. (1-5), we can obtain the numerical results as shown in Figure C1. By considering the stochastic noise in the transient process, as shown in Figure C1a, we then transfer the time domain data to frequency domain with average as shown in Figure C1b, and the corresponding variance is shown in Figure C1c. Both Figure C1b and C1c fit very well with a Lorentzian shape.

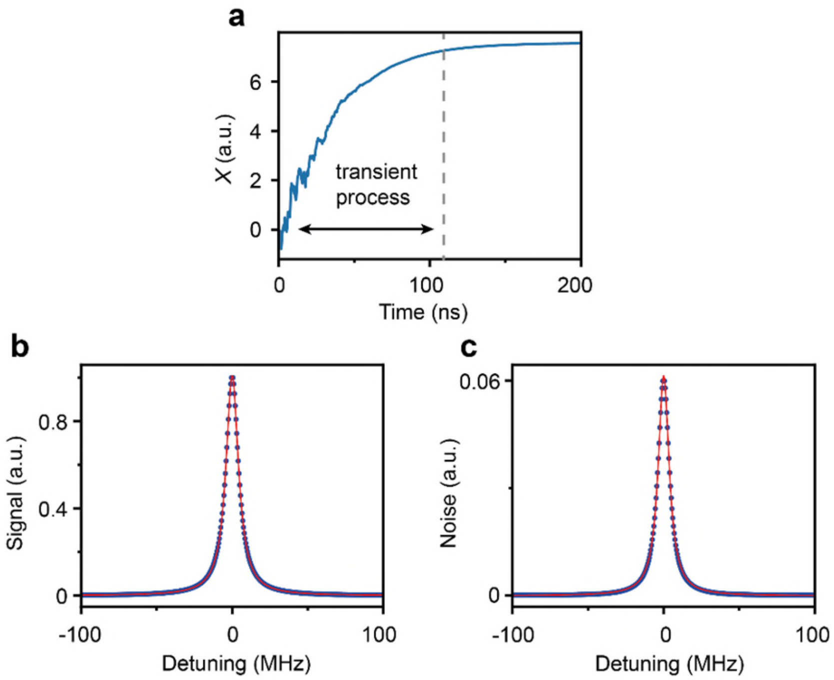

Figure C1. Dynamics of a single cavity mode with stochastic master equation. a, Time-domain evolution of the X component for a single cavity mode with a decay rate of $2\pi \times 10$ MHz. The stochastic noise will be induced during the transient process. The time-domain evolution *X(t)* from **a** is transformed into frequency-domain spectrum $S(\omega)$ using a Fast Fourier Transform (FFT), repeated 500 times. The square of the mean value of $|S(\omega)|$ from these repetitions is depicted in **b** as signal (blue dots), while the variance of $|S(\omega)|$ across the 500 datasets is shown in **c** as noise (blue dots). The blue dots in **b** and **c** are fitted with Lorentzian functions (red solid lines), suggesting that both the signal and noise emitted by the cavity can be modeled by a Lorentzian function.

Reviewer comment C4.6 *"I do not know why the authors observe a noise that varies with the coupling g but the variation is quite small (about 10-20%) and could possibly be explained by some technical noise. This seems reinforced by the fact that the measured noise depends on the drive amplitude even though the theory doesn't predict it (neither mine nor the authors')."*

Reply: We thank the reviewer for the careful checking. As we replied above and also the results in section IV. C. of the supplementary material, if the transient process is considered, the noise spectrum should not be constant. As *g* changes, the frequency and decay rate of the supermode will vary, which will consequently change the noise spectrum resulting from the transient process. For the issue of the noise depending on the drive amplitude, we believe that as the drive amplitude increases, the chip will be heated up, leading to an increase in the thermal photon number n_{th}.

Reviewer comment C4.7 "I do not understand the authors' argument that the noise is bounded to 2. The noise from the waveguide and the cavity are at the same temperature and do not (even partially) add up in average power and thus should be constant.”

Reply: We thank the reviewer for the comment. The noises we consider come from two noise channels, one is from the input noise and another is from the backaction noise, and the backaction noise varies with the coupling strength *g* due to the transient process.

Reviewer comment C4.8 *“I haven’t carefully examined the transmission case but I think that similar remarks do apply.”*

Reply: We thank the reviewer for the concern. Based on our discussion above, the transmission spectrum should be understood in the same manner.

Responses to the comments of Reviewer 2

Reviewer comment C5.0 *"I would like to thank the authors once again for their detailed response.*
I am afraid that I still disagree with the authors on section IV. of the supplementary material which underpins the 3rd experiment presented in this article in Figure 4.
My arguments are still similar. Here is a point-by-point response."

Reply: We sincerely thank the reviewer for the insightful comments and helpful suggestions in each review, which have significantly enhanced our understanding of the controversial issue and improved the quality of our manuscript.

Reviewer comment C5.1 *"C4.2.I unfortunately do not have the time to review close to 8 pages of stochastic calculus calculations but I don't think such detailed and novel calculations are required to analyze what is essentially a single driven cavity given the large existing literature on the subject. Furthermore, S16 does not qualitatively agree with the predictions shown in Figure 4 (see for example the reflections SNR which is maximized at g = 2kappa whereas the other model predicts a maximum at g = 0) even though averaging the results of the stochastic master equation should yield identical results to the Lindblad master equation.*
I suspect that the problem is that the noise is calculated on the Fourier spectrum S and not on the real variables. The Fourier transform of the noise is not the noise of the Fourier transform because the variance is a nonlinear operation which doesn't commute with the Fourier transform.
Overall, I do not the addition of section IV. C is necessary for the reader."

Reply: We thank the reviewer for the roughly checking the stochastic calculus calculations and the corresponding results. We agree with the reviewer that the section IV. C in the supplementary materials is not necessary for reader, so we have removed that part in the revised supplementary materials.

Reviewer comment C5.2 *"C4.4*
The example cited by the authors is quite different. In "Introduction to quantum noise, measurement, and amplification", the case of a "hot" mechanical oscillator parametrically coupled to a measurement cavity is discussed and indeed the noise exhibits a Lorentzian shape caused by the thermal noise of the oscillator.
This is not at all a transient process and is a completely steady state phenomenon."

Reply: We thank the reviewer for pointing out this difference. We partially agree with the reviewer's point and the corresponding controversial parts have been deleted.

Reviewer comment C5.3 *"C4.5*
The authors discuss a driven cavity and plot the FFT of the time evolution of one of the quadratures. This FFT represents the signal and not the noise!

Note that a heterodyne measurement should be used and not a homodyne one. A strong continuous homodyne measurement will squeeze the state which does not correspond to the experiment where a phase insensitive amplifier is used and one quadrature later discarded.
The transient gives some spectral width to the signal but does not change the fact that the variance of X is still a time constant. The authors could calculate the variance of X to convince themselves of this fact.
A nice way to see this is to realize that the Hamiltonian is quadratic and the loss linear so the systems remain in a Gaussian state at all times. A drive corresponds to a simple displacement of the state in phase space and does not affect the covariance matrix of the state (see "Gaussian quantum information" by Weedbrook et al.)
Moreover, given the lack of mention of the measurement time, I (and presumably potential readers) have assumed that the problem should be solved in the steady state. The vast majority of qubit measurements are done in the steady state. If transients were such a major factor, they should have been mentioned in the main text and on the figure."

Reply: We thank the reviewer for the above comments. We agree with the reviewer that the calculation of the noise after FFT might not be rigorous. We have removed the corresponding controversial parts.

Reviewer comment C5.4 *"C4.7*
The measurement is quantum non-demolition and as such should not have any classical back-action on the system. In practical terms, this is saying that the amplifier can be perfectly isolated from the cavity which is close to true."

Reply: We thank the reviewer for the above comments. We agree with the reviewer that there will be no classical back-action if the measurement is quantum non-demolition.

Reviewer comment C5.5 *"Overall, it seems clear that the authors and I fundamentally disagree on this point and I am not sure if more back and forth will ever solve this disagreement."*

Reply: We thank the reviewer for the careful checking of our theory and the valuable comments. We have removed all the parts that may cause misunderstanding.

Reviewer comment C5.6 *"I think that amputated of the last Figure and the corresponding supplementary material, this article is worth publishing in Nature Communications. In my opinion, leaving this debatable last part of the article muddles the message of an otherwise interesting set of experiments."*

Reply: We thank the reviewer for the recommendation for publication. We have removed all the relevant figures and text in accordance with the reviewers' requirements. We believe that the current revised manuscript with removing Figure 4 and corresponding text is worth publishing in *Nature communications*.

I would like to thank the authors for their detailed response and their numerous updates to the manuscript.

In particular, I appreciate the addition of figure S10 and the associated section dedicated to curve fitting which will help readers to better understand or even reproduce these results. I would also like to thank the authors for the improvements to Figure 4, which now directly compares theory to experiment.

Unfortunately, I still disagree with the theoretical curves of Figure 4 (as does the experimental data to some extent). The calculation of the theoretical curves is performed in section IV. B. of the supplementary material.

My last remaining issue lies with the definition of the homodyne current. The homodyne current is a stochastic quantity that presents some difficulties for calculations. In particular, when dealing with stochastic quantities, one should note that there are two types of averages: average over the quantum state used to compute average values of operators (usually using the <.> notation) as well as a stochastic average used to average over different realization of the experiment i.e. shots.

According to Wiseman and Milburn in *Measurement and Control,* the homodyne current is defined as $I(t) = \langle a + a^\dagger \rangle(t) + \xi(t)$ with $\xi(t) = \frac{dW}{dt}$ and $W(t)$ a Wiener process i.e. white noise (note that they use a different convention for x without the sqrt(2)). This is quite similar to the definition used by the authors except that the field appears as an average value in the proper definition and not as an operator: that is because the homodyne current is a (stochastic) scalar and not an operator.

This small change does not affect equation S.51 (though now the average must be interpreted as a statistical average and not a "quantum" average) but it completely changes S.52.

Indeed, using $\bar{x}$ to represent the statistical average of the stochastic quantity x, we now have

$$\overline{m^2(t)} = \langle a + a^\dagger \rangle(0)^2 \tau^2 + 2\,\overline{\langle a + a^\dagger \rangle(0) \tau W(\tau)} + \overline{W(\tau)^2} = \overline{m(t)}^2 + \tau(1 + 2n_{th})$$

Thus, S.53 simply becomes

$$\bar{n} = 2\bar{n}_{th} + 1$$

It should also be noted that given the fact that the detection scheme is phase insensitive, the heterodyne current should be used instead which comes with an additional factor 2 on the vacuum noise but this doesn't change any of the conclusions.

The current equation S.53 while revised still predicts that an undriven system thermalized at a constant temperature of 0 has a Lorentzian noise profile instead of a white (quantum) noise! This can be used to break thermodynamics by coupling this supermode to another cavity and capturing a steady net energy flux of

$$\frac{\kappa_{c1}}{2}\left(\frac{\chi}{\chi^2 + (\Delta_0 - \beta)^2}\right) > 0$$

to heat up that secondary above 0K without any work!

I do not know why the authors observe a noise that varies with the coupling g but the variation is quite small (about 10-20%) and could possibly be explained by some technical noise. This seems reinforced by the fact that the measured noise depends on the drive amplitude even though the theory doesn't predict it (neither mine nor the authors').

I do not understand the authors' argument that the noise is bounded to 2. The noise from the waveguide and the cavity are at the same temperature and do not (even partially) add up in average power and thus should be constant.

I haven't carefully examined the transmission case but I think that similar remarks do apply.