

1 **Supplimentary information: Thermal imaging**
 2 **through hot emissive windows**

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13 **Supplementary Note 1 : Analytical model for coupled**
 14 **resonator system**

15 We describe the coupled resonator system for AMW using a 2×2 non-
 16 Hermitian Hamiltonian.

$$\hat{H} = \begin{pmatrix} \omega_0 & \kappa \\ \kappa & \omega_0 + i\gamma \end{pmatrix} \quad (\text{S1})$$

17 This Hamiltonian describes a lossless resonator coupled to a lossy resonator
 18 with damping γ . ω_0 is the resonance frequency of the individual resonators.
 19 By varying coupling constant κ , the system can be brought through a phase
 20 transition. The two hybridized modes of this coupled resonator system will

21 have complex frequency values as determined by the two eigenvalues of Eq. S1
22 (see Fig. S1). When the two resonators are strongly coupled (large κ), the
23 two modes have distinct real parts resulting in two emission frequencies. The
24 imaginary part of eigenfrequencies remains the same for both modes. On the
25 contrary, in the weak coupling regime (small κ), the real part of the eigen fre-
26 quencies remains the same. Here, the two complex eigen frequencies associated
27 with the two hybridized modes of the system have distinct imaginary parts,
28 indicating that the losses experienced by the two modes are different.

29 **Supplementary Note 2 : Electric field intensity and** 30 **Poynting vector plots from simulations of CW and** 31 **AMW structures.**

32 Full wave electromagnetic simulations were used to calculate the cross-sectional
33 electric field distribution inside CW resonators at its resonant wavelength of
34 $3.06 \mu\text{m}$ (Fig. S2a). The fields are well confined within the disks as expected
35 for a QBIC mode. The damping from the titanium layer is the source of the
36 absorption loss or thermal emission in both CW and AMW resonators as may
37 be observed in the divergence of the Poynting vector (Fig.S2, c and d).

38 **Supplementary Note 3 : Design of control window (CW)**

39 The two weakly coupled resonators in AMW give rise to two eigen values with
40 the same real part and different imaginary parts. Each eigen mode correspond-
41 ing to its eigen value is mostly confined in the respective resonator. The mode
42 confined to the lossless resonator corresponds to the eigen value with smaller
43 imaginary part and the other corresponds to the eigen value with a larger
44 imaginary part. The spatial localization of the modes with different damping
45 rates (imaginary part of eigen values) leads to a gradient in absorption loss.

46 The CW is chosen to not have such spatial distribution of absorption loss.
47 Hence, only one resonator is array to make the CW. The QBIC and mode
48 localization properties hold good for CW as well as AMW. However, there is
49 only one eigen value with a non-zero imaginary part for the CW. The mode
50 corresponding to it is damped due to its interaction with the thin titanium
51 layer on top of the Si disk. This situation is exactly the same for the top
52 resonator of AMW.

53 Since the AMW and CW are both resonant, support QBIC, and experience
54 the same total damping (from the 10 nm thick Ti layer), their comparison is
55 fair. Also, the CW chosen here has a resonant emissivity comparable to that
56 of common IR windows at elevated temperatures. Fig. S5 shows the emissiv-
57 ity spectra of CW, fused silica, and sapphire at high temperatures. The CW
58 slightly exaggerates the emissivity of fused silica and sapphire windows near
59 their passband edges.

60 The substrate in our proof of concept demonstration has less significance
61 as the Ti topping on the resonator creates the loss asymmetry. We needed a
62 low-loss substrate with minimal emission to minimize the substrate-induced
63 effects and highlight metasurface-induced asymmetry. Materials like ZnSe and
64 ZnS are excellent, widely-used choices for room-temperature applications in
65 the mid-IR and far-IR. However, these materials' stability at higher tempera-
66 tures is poor. Fused silica has high thermal stability and is also transparent at
67 wavelengths shorter than $4.5 \mu\text{m}$ (Fig. S5b) , which is why we used it for our
68 demonstration.

69 A metal layer is not necessary to achieve required functionality provided
70 the substrate contain losses to create the asymmetry. To illustrate this, we
71 show in Fig. S3 that a multilayer structure supporting coupled Fabrey Perot
72 resonances can enhance the contrast by suppressing emission from a lossy

73 dielectric substrate ($n_0 = 3$, $k_0 = 0.05$). The transition from tight coupling to
 74 weak coupling can be observed in the emissivity plot in Fig. S3b suggesting
 75 the concepts discussed in the manuscript (Fig. 2e) hold true here as well.
 76 The weakly coupled system (spacer = 450 nm) suppress the emission from
 77 substrate in ϵ_- (Fig. S3c) and enhances the transmission (Fig. S3d) suggesting
 78 an improvement in χ .

79 **Supplementary Note 4 : Calculated and measured** 80 **emissivity spectra for CW and AMW at 873 K**

81 The calculated and measured emissivity spectra used to generate the $\Delta\epsilon$
 82 spectra in Fig.3b and 3c are shown in Fig. S4 a and b respectively.

83 **Supplementary Note 5 : Transmitted and emitted power** 84 **for AMW and CW**

85 For finite bandwidth operation, the ratio $\frac{T}{\epsilon}$ translates to the ratio of transmit-
 86 ted power to emitted power by the window. Considering the thermal camera
 87 operates in a 3-3.5 μm band and the windows at 873 K, the power emitted
 88 (at normal angle) with emissivities ϵ_- and ϵ_+ for CW and AMW is calculated
 89 using Eq. S4 and shown in Table 1. We also calculate the integrated trans-
 90 mitted power when a blackbody at the same temperature is placed behind
 91 the windows. For these calculations, we use the experimental emissivity and
 92 transmission data reported in Figs—3C and 3E.

Window	Emitted Power with ϵ_-	Emitted Power with ϵ_+	Transmitted Power
CW	4.81	6.65	2.26
AMW	3.13	4.71	3.27

Table 1 Integrated emitted and transmitted power (at normal angle) by CW and AMW considering a blackbody as the target. Both the window and blackbody is at 873 K and all power is in units $10^8 \text{ W/m}^2\text{ster}$

93 CW, and AMW don't show substantial differences in emission asymmetry.
 94 However, the critical advantage of AMW here is the effective transmission
 95 (44% more than CW) and suppression of emitted power (34% less than CW)
 96 in ϵ_- . This results in the two-times contrast enhancement shown in Fig. 4c of
 97 the main text.

98 **Supplementary Note 6 : Weber's formula for quantifying** 99 **contrast**

100 Weber's formula for quantifying simple contrast for an object placed on a
 101 background at a given wavelength (λ) is given by

$$CR(T_{Obj}) = \frac{|I_{Obj} - I_{BG}|}{I_{BG}} \quad (\text{S2})$$

102 where I_{Obj} and I_{BG} are the values of integrated power emitted per unit area
 103 of the object and background, respectively, that reaches the camera. The inte-
 104 grals for power run in the spectral and spatial bandwidths of the camera. In
 105 the presence of an emissive window between the object and camera, the con-
 106 trast equation should be modified to include the thermal emission (I_W) and
 107 transmission (\mathcal{T}) of the window.

$$CR = \frac{|\mathcal{T}I_{Obj} + I_W - (\mathcal{T}I_{BG} + I_W)|}{\mathcal{T}I_{BG} + I_W} \quad (\text{S3})$$

108 Note that the far-field thermal radiation at temperature T from any surface
 109 of emissivity ϵ is given by:

$$I(T) = \int_0^{\theta_c} \int_{\lambda_2}^{\lambda_1} \epsilon(\lambda, T) \Theta_{BB}(\lambda, T) d\lambda d\theta \quad (\text{S4})$$

110 where $\Theta_{BB}(\lambda, T)$ is the power spectral density of an ideal blackbody at
 111 temperature T , and θ_c is the maximum collection angle of the camera with
 112 an operating bandwidth of (λ_1, λ_2) . In the limit of infinitesimal bandwidth $\Delta\lambda$
 113 around a central wavelength λ , $I(\lambda, T) = \epsilon(\lambda, T)I_{BB}(\lambda, T)\Delta\lambda$. Here, I_{BB} is
 114 the blackbody power spectral density integrated over the camera's acceptance
 115 solid angle. Using this expression for thermal radiation in Eq. S3, the image
 116 contrast at any given wavelength is given by:

$$CR(T_{Obj}, T_W) = \frac{|1 - \frac{\epsilon_{obj}(T_{Obj})}{\epsilon_{BG}(T_{Obj})}|}{1 + \frac{\epsilon(T_W) I_{BB}(T_W)}{\epsilon_{BG}(T_{Obj}) \mathcal{T}(T_W) I_{BB}(T_{Obj})}} \quad (S5)$$

117 where the emissivities of the object, background, and the window (toward
 118 the camera) are ϵ_{obj} , ϵ_{BG} , ϵ , respectively. We always assume that the object
 119 and its background are at the same temperature.

120 Eq.1 in main text follows from Eq. S5 if $T_{Obj} = T_W$.

121 Since most thermal cameras operate in a spectral bandwidth $(\lambda_1 - \lambda_2)$, we
 122 define an integrated CR (ICR) inside this bandwidth as:

$$ICR(T_{Obj}, T_W) = \frac{|1 - \frac{\int_{\lambda_1}^{\lambda_2} \mathcal{T}(T_W) \epsilon_{obj}(T_{Obj}) I_{BB}(T_{Obj}) d\lambda}{\int_{\lambda_1}^{\lambda_2} \mathcal{T}(T_W) \epsilon_{BG}(T_{Obj}) I_{BB}(T_{Obj}) d\lambda}|}{1 + \frac{\int_{\lambda_1}^{\lambda_2} \epsilon(T_W) I_{BB}(T_W) d\lambda}{\int_{\lambda_1}^{\lambda_2} \mathcal{T}(T_W) \epsilon_{BG}(T_{Obj}) I_{BB}(T_{Obj}) d\lambda d\lambda}} \quad (S6)$$

123 **Supplementary Note 7 : Thermal imaging and contrast** 124 **mapping**

125 The imaging object was fabricated using standard e-beam lithography fol-
 126 lowed by carbon coating on a polished tungsten chip (MTI-corp) 0.5 mm
 127 thick (see Fig. S6a). The object chip was placed inside a vacuum chamber
 128 (MicroOptik-MHCS1200) with a water-cooled ZnSe window on a heating stage.
 129 The metasurface (Fig. S6b) was placed directly on top of this object chip and
 130 the stage was heated to 873 K. A thermal camera (FLIR A6701) that operates

131 in the 3-5 μm wavelength range was used to capture the thermal images of
 132 the object chip through the metasurface. An IR bandpass filter (BP-3.25/BP-
 133 4.75) was placed between the chamber and the camera. The camera exposure
 134 time was fixed at 0.1 ms for all measurements.

135 We use the collected thermal image to map the image contrast. The thermal
 136 imaging camera records the number of thermal photons (counts) collected
 137 by each pixel at location (x,y) . At first, we normalize the entire image with
 138 the highest photon count recorded among image pixels. Then, we use this
 139 normalized photon count at each pixel $(n(x,y))$ to calculate contrast defined
 140 using Weber's formula:

$$CR(x,y) = \frac{|n(x,y) - \langle n_{BG} \rangle|}{\langle n_{BG} \rangle} \quad (\text{S7})$$

141 where $\langle n_{BG} \rangle$ represents the mean photon count from 100 background pixels.

142 Weber contrast maps were generated for CW and AMW when ϵ_+ or ϵ_-
 143 facing the camera (Fig. S9a). Contrast enhancement at each pixel is calculated
 144 by taking the ratio of contrast values for CW(ϵ_-) and AMW(ϵ_-) Fig. S9b.

145 **Supplementary Note 8 : Emissivity of object,** 146 **background, and window at high temperatures**

147 The emissivity of the object (carbon on tungsten), background (tungsten),
 148 and window (AMW (OS)) samples were measured at different temperatures
 149 as detailed under Materials and Methods section. The results are shown in
 150 Fig. S7. These results are employed to generate Fig. 4d in main text using
 151 Eq. S6.

152 **Supplementary Note 9 : Spectral filter transmission**

153 Commercially available spectral filters were employed as BP-3.25 (Thorlabs
 154 FB3250-500) and BP-4.75 (Thorlabs FB4750-500). The transmission spectra
 155 for these filters are plotted in Fig. S8.

156 **Supplementary Note 10: Reflectance spectra of AMW**

157 For a reciprocal system, an asymmetry in emissivity is equivalent to an asymmetry
 158 in reflectance. Asymmetric reflection is observed for AMW as shown in
 159 Fig. S10.

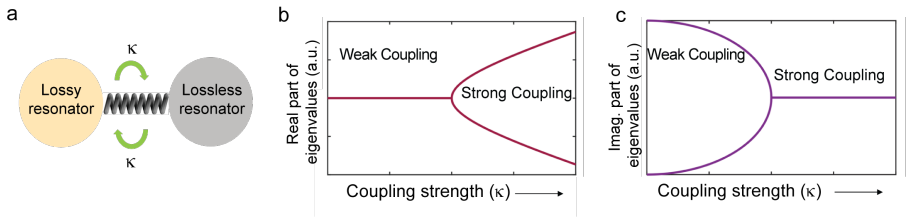


Fig. S1 The system of coupled lossless and lossy resonators: (a) Schematic showing the coupling between two identical resonators with distinct optical losses. (b) Real and (c) imaginary part of the coupled system's eigen frequencies as a function of the coupling strength, κ .

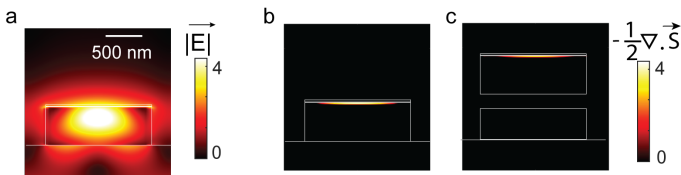


Fig. S2 (a) The cross-sectional electric field profile at the resonance (3.06 μm) of CW resonators showing confined mode. (b,c) The Poynting vector plots for (b) CW and (c) AMW structures at resonance showing the source of emission is the 10-nm thin Ti layer

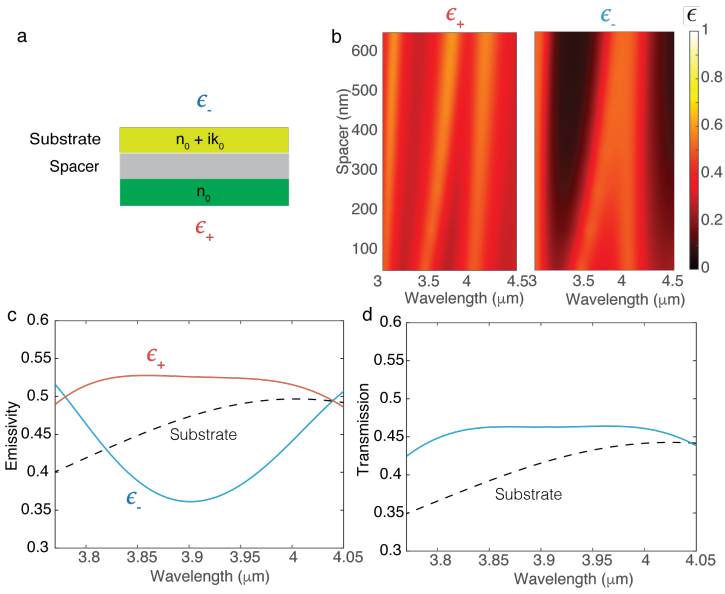


Fig. S3 (a) Multilayered structure that supports coupled Fabry Perot resonances ($n_0 = 3, k_0 = 0.05, t_{\text{layer}} = 2 \mu\text{m}$). Top resonator has increased dielectric losses breaking the parity symmetry of the structure. (b) Simulated emissivity spectra from the structure for two surfaces (ϵ_+ and ϵ_-) as function of spacer thickness. (c) and (d) show the emissivity and transmission spectra when spacer thickness = 450 nm. Dashed lines represent the emissivity and transmission from the substrate layer.

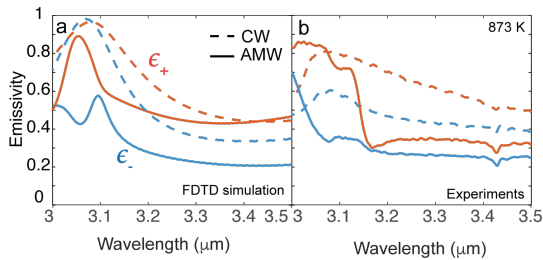


Fig. S4 (a) The calculated and (b) measured emissivity spectra for CW and AMW towards substrate side (ϵ_+) and opposite side (ϵ_-).

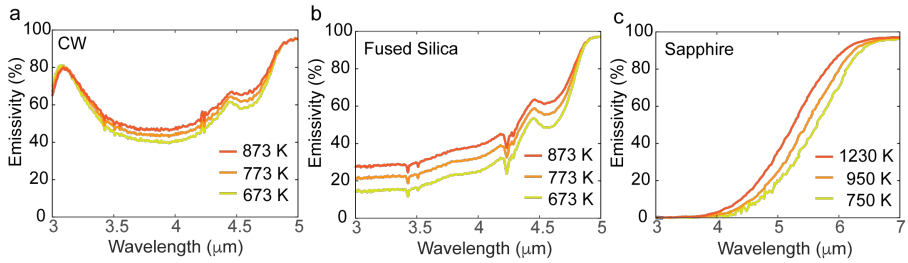


Fig. S5 Emissivity curves of (a) control window (0.5 mm thick), (b) fused silica (0.5 mm thick), and (c) sapphire (1 mm thick) at elevated temperatures. The emissivities of CW and fused silica were measured and emissivity data of sapphire was taken from Ref. 27.

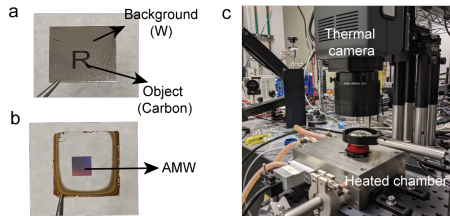


Fig. S6 (a) The object (Letter 'R') made of carbon on a 1 cm x 1 cm tungsten chip (b) Fabricated AMW on a fused silica substrate (c) An image of the thermal imaging set-up showing the thermal camera and heating chamber.

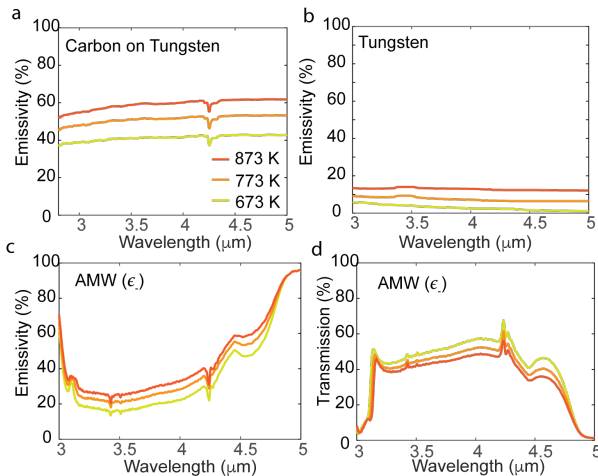


Fig. S7 Measured emissivity of (a) Carbon film on tungsten (b) bare tungsten substrate and (c) AMW toward the open side (ϵ_{-}). (d) Measured transmission spectra of AMW window. Different colors indicate the temperature at which data were taken.

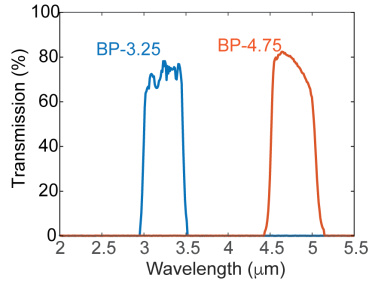


Fig. S8 The transmission spectra of BP-3.25 (Thorlabs FB3250-500) and BP-4.75 (Thorlabs FB4750-500)

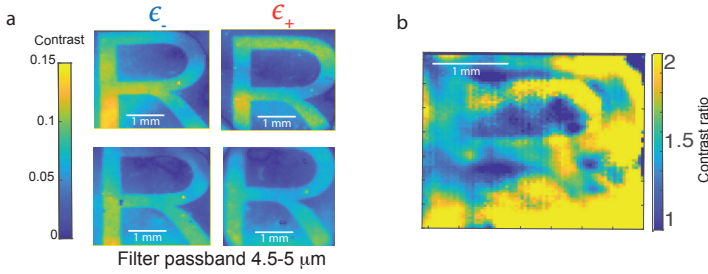


Fig. S9 (a) Weber contrast maps generated for CW and AMW when ϵ_+ or ϵ_- facing the camera. The images were acquired with a BP-4.75 (4.5–5 μm) bandpass filter in front of the camera. (b) Ratio of weber contrast from AMW (ϵ_-) to that from CW (ϵ_-). Weber contrast map from Fig. 4b was used to generate the ratio at each pixel.

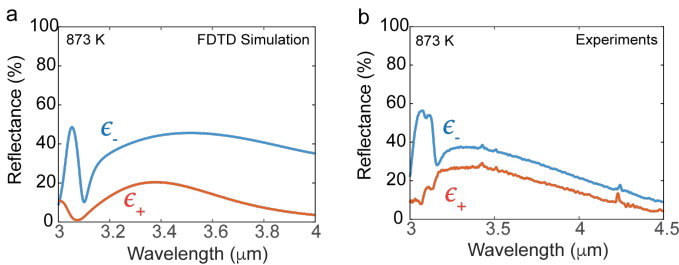


Fig. S10 Calculated (a) and measured (b) reflectance spectra of AMW in ϵ_- and ϵ_+ directions showing strong asymmetry in reflection.