



Figure S1: IAC model for $N = 2$ enhancers as a product graph of product graphs. **(a)** Graphs describing the activation and communication status of enhancer 1. **(b)** Graphs describing the activation and communication status of enhancer 2. **(c)** The graph H_1 whose underlying regulatory graph is $K_{a,1} \otimes K_{c,1}$. **(d)** The graph H_2 whose underlying regulatory graph is $K_{a,2} \otimes K_{c,2}$. **(e)** The graph $H_1 \otimes H_2$ satisfying the default model Assumptions 1-4 with components H_1 and H_2 . The regulatory graph of $H_1 \otimes H_2$ is given by $(K_{a,1} \otimes K_{c,1}) \otimes (K_{a,2} \otimes K_{c,2})$. Each vertex of this graph corresponds to the activation and communication statuses of both enhancers. For the vertices along the top of the graph, the product-graph binary notation is also provided using the coordinate system $((a_1, c_1), (a_2, c_2))$. Reverse edges and labels of most edges are omitted for clarity. Production states are highlighted in purple with corresponding production rates also in purple font.