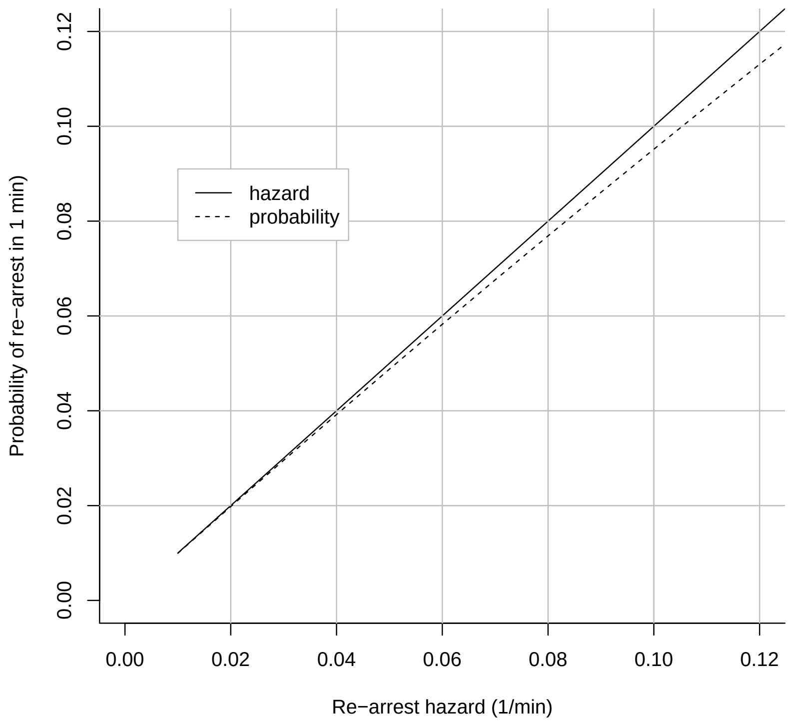

As illustrated in the figure above, we describe the approximation to the short-term probability of a specific event. Let T be the time until event and $C = j, j = 1,2,...,k$, be the type of event (here, the time is time until recurrence of cardiac arrest, and the type of event is the clinical arrest state the patient returns to). Let h_j be the corresponding specific hazard rate for this event in a multi-state model. The event j specific hazard rate (often called cause-specific hazard rate) function is given as

$$h_j(t) = \lim_{\Delta t \to 0} \frac{P(t < T < t + \Delta t \cap C = j | T > t)}{\Delta t}$$

or approximately

$$h_j(t)\Delta t \approx P(t < T < t + \Delta t \cap C = j | T > t)$$

If the time scale is such that one time unit is ”small” , then we further get:

$$h_j(t) \approx P(t < T < t + 1 \cap C = j | T > t)$$

So roughly, the hazard for event $C = j$ at time t is approximately equal to the probability that event j happens the next time unit, given that no event has happened up to time t. This is an acceptable approximation if all competing events have small probabilities of happening during one time unit.

Assuming constant hazards

If one, as in a time-homogeneous Markov process, can assume constant hazard rates λ_i for all the competing risks i, we can illustrate this further. Let T_i be the time until event i, and let $T = \min(T_1, T_2, ..., T_k)$. Due to the constant hazard assumption T_i will have an exponential distribution with rate λ. Then:

$$
\begin{aligned}
P(t < T_j < t+\Delta t | T > t) &= \frac{P(t < T_j < t+\Delta t \cap T_i > T_j \; \forall\, i \neq j)}{P(T > t)} \\
&\overset{\text{indep.}}{\approx} \frac{[F_j(t+\Delta t) - F_j(t)] \cdot \prod_{i \neq j} [1 - F_i(t+\Delta t)]}{1 - F(t)} \\
&= \frac{[(1 - e^{-\lambda_j (t+\Delta t)}) - (1 - e^{-\lambda_j t})] \cdot \prod_{i \neq j} e^{-\lambda_i (t+\Delta t)}}{e^{-\sum_i \lambda_i t}} \\
&= \frac{\left[e^{-\lambda_j t} - e^{-\lambda_j (t+\Delta t)}\right] \cdot e^{-\sum_{i \neq j} \lambda_i t - \sum_{i \neq j} \lambda_i \Delta t}}{e^{-\sum_i \lambda_i t}} \\
&= (1 - e^{-\lambda_j \Delta t}) \cdot e^{-\sum_{i \neq j} \lambda_i \Delta t} \\
&\overset{\text{Taylor expansion}}{\approx} \lambda_j \Delta t \cdot (1 - \sum_{i \neq j} \lambda_i \Delta t) + \mathcal{O}((\lambda_j \Delta t)^2) + \mathcal{O}((\sum_{i \neq j} \lambda_i \Delta t)^2) \\
&\approx \lambda_j \Delta t
\end{aligned}
$$

The Taylor expansion (using only the first term) becomes more precise the smaller the $\lambda \Delta t$ -terms are. If $\lambda_j \Delta t < 0.1$ and $\sum_{i \neq j} \lambda_i \Delta t < 0.1$, the error term will be of the order of 0.01, and so on (error $\sim$ term^2). E.g., for combinations of λ's and Δt such that $\lambda_j \Delta t < 0.1$ and $\sum_{i \neq j} \lambda_i \Delta t < 0.1$ the approximation

$$
h_j(t)\Delta t \approx P(t < T_j < t+\Delta t | T > t)
$$

is OK. So, if the time scale is chosen such that $\lambda_j \Delta t < 0.1$ and $\sum_{i \neq j} \lambda_i \Delta t$ are both small (e.g. < 0.1) even when Δt=1, we get

$$
\lambda_j = h_j(t) \approx P(t < T_j < t+1 | T > t)
$$